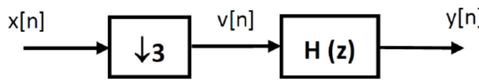
ENEE 630 Review[1]

1. Consider the following processing where $x[n]$ is a zero-mean wide sense stationary process. Is $v[n]$ wide sense stationary? Is $y[n]$ wide sense stationary? Explain why or why not. If yes to both, determine the relation between autocorrelation functions of $x[n]$ and $y[n]$.



2. An adventure on cross-correlation and cross-spectrum.

(a) Let $s[n]$ be a process generated by a stable LTI filter with impulse response $h[n]$ to filter a white noise process $v[n]$. For the $v[n]$ process, we have $E[v[n]] = \mu \neq 0$, and $Var[v[n]] = \sigma^2$. Derive the cross-correlation function $r_{sv}(k)$ between $s[n]$ and $v[n]$ in terms of $h[n]$, μ, and σ.

(b) Let $x[n]$ and $y[n]$ be two zero-mean, jointly wide sense stationary random processes with power spectra $P_x(\omega)$ and $P_y(\omega)$, respectively. The cross power spectral density $P_{xy}(\omega)$ of the two processes $x[n]$ and $y[n]$ is defined as the DTFT of their cross-correlation function $r_{xy}(k)$, i.e.,

$$P_{xy}(\omega) = \sum_{k=-\infty}^{\infty} r_{xy}(k) e^{-j\omega k}.$$

A normalized cross spectrum $G_{xy}(\omega)$ is given by

$$G_{xy}(\omega) = \frac{P_{xy}(\omega)}{\sqrt{P_x(\omega) P_y(\omega)}}.$$

Show that $|G_{xy}(\omega)|^2$ is invariant under linear transform. That is, if $x_1[n] = h_1[n] * x[n]$ and $y_2[n] = h_2[n] * y[n]$, then $|G_{xy}(\omega)|^2 = |G_{x_1 y_2}(\omega)|^2$.

3. Noise cancellation

Let $g[n]$ be real-valued white noise with zero mean and variance σ_g^2, and $v_1[n]$ be a real-valued zero-mean process generated by the difference equation

$$v_1[n] = a v_1[n-1] + g[n], \qquad -1 < a < 1.$$

Let $x[n] = d[n] + v_1[n]$, where $d[n]$ is real valued, zero mean, wide sense stationary, and uncorrelated with $g[n]$.

(a) Assume σ_g^2, a, and the autocorrelation function $r_d(\cdot)$ of $d[n]$ are all known. Design a p^{th} order causal FIR filter to be applied to $x[n]$, such that the filter provides a minimum mean square error (MMSE) estimate for $d[n]$.

[1] ver. 201212

(b) Consider now another real-valued zero-mean process

$$v_2[n] = bv_2[n-1] + g[n], \qquad -1 < b < 1.$$

Suppose that we observe N samples from the $\{x[n]\}$ process as well as N synchronized samples from the $\{v_2[n]\}$ process to study the statistical properties of the two random processes. Other than these, we do not know explicitly σ_g^2, a, b, and the second order statistics of $d[n]$ and $v_1[n]$. Design a practical MMSE estimate for $d[n]$ using a linear combination of $x[n]$, $v_2[n]$, $v_2[n-1]$, ... $v_2[n-p]$, where $n \gg p$, $N \gg p$.

Hint: see if estimating $v_1[n]$ may help.