1. Consider the structures shown in Fig. RI.1, with input transforms and filter responses as indicated. Sketch the quantities $Y_0(e^{j\omega})$ and $Y_1(e^{j\omega})$. (Decimator-expander)

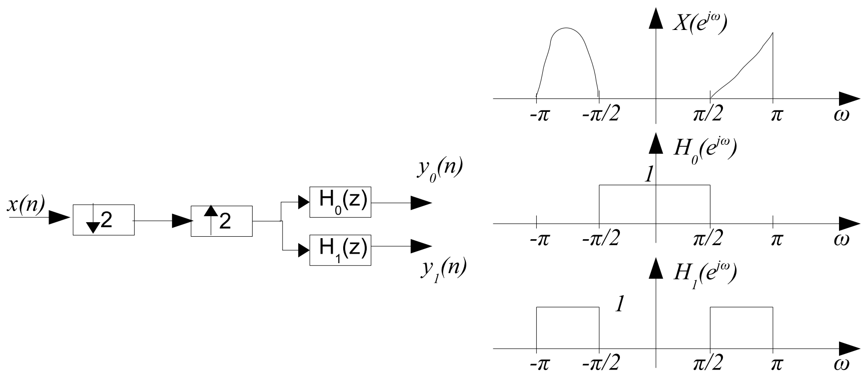


Figure RI.1:

2. For each case shown in the Fig. RI.2, prove or disprove whether the left system is equivalent to the right system? Assume M, L, K are all integers larger than 1. (Decimator-expander cascade)

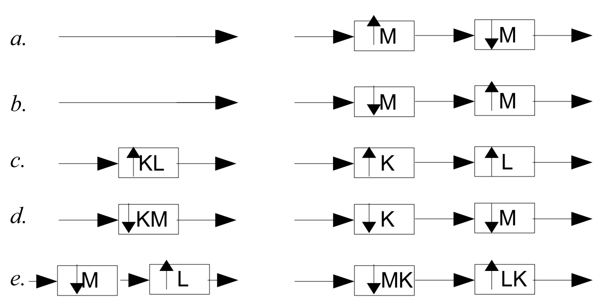


Figure RI.2:

3. Simplify the systems in Fig. RI.3. (Decimator-expander cascade)

4. In this problem, the term 'polyphase components' stands for the Type 1 components with $M = 2$. (polyphase component)

(a) Let $H(z)$ represent an FIR filter of length 10 with impulse response coefficients $h(n) = (1/2)^n$ for $0 \leq n \leq 9$ and zero otherwise. Find the polyphase components $E_0(z)$ and $E_1(z)$.

(b) Let $H(z)$ be IIR with $h(n) = (1/2)^n u(n) + (1/3)^n u(n-3)$. Find the polyphase components $E_0(z)$ and $E_1(z)$. Give simplified, closed form expressions. (Hint: $\frac{1}{1+x} = \frac{1-x}{1-x^2}$)

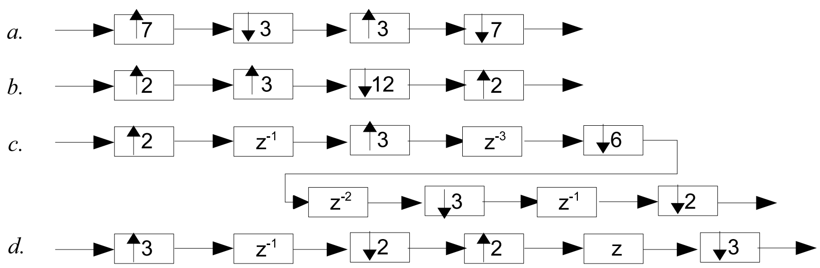
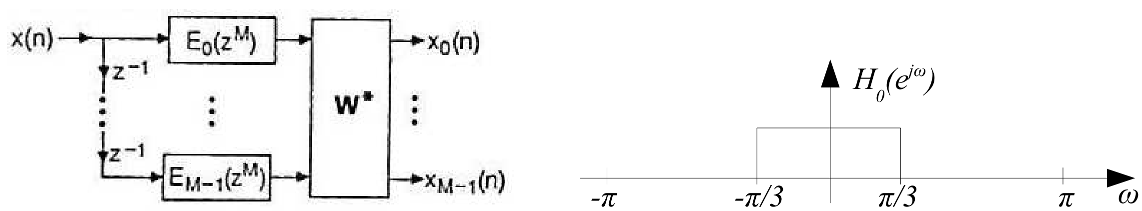


Figure RI.3:

(c) Let $H(z) = 1/(1 - 2R\cos\theta z^{-1} + R^2 z^{-2})$, with $R > 0$ and θ real. This is a system with a pair of complex conjugate poles at $Re^{\pm j\theta}$. Find the polyphase components $E_0(z)$ and $E_1(z)$.

5. A uniform DFT analysis bank (Type 1) is shown in Fig. RI.5, where $\boldsymbol{W}^*$ is the $M \times M$ IDFT matrix, i.e., the (m, n)-th entry is W_M^{-mn} with indices m, n starting from 0. The transfer function from input port $x(n)$ to output port $x_k(n)$ is denoted by $H_k(z)$. Answer the following questions. (Uniform DFT filter banks)



(a) (b)

Figure RI.5:

(a). Prove $H_k(z) = H_0(zW_M^k)$ for $0 \le k \le M-1$. Given $H_0(e^{j\omega})$ in Fig. RI.5, sketch $H_1(e^{j\omega})$ when $M = 2$, and $H_1(e^{j\omega})$, $H_2(e^{j\omega})$ when $M = 3$.

(b). $M = 4$. Assume $E_0(z) = 1 + z^{-1}$, $E_1(z) = 1 + 2z^{-1}$, $E_2(z) = 2 + z^{-1}$, and $E_3(z) = 0.5 + z^{-1}$. Find numerical values of these filter coefficients for $H_k(z)$, $0 \le k \le 3$.

(c). $M = 2$. Let $H_0(z) = 1 + 2z^{-1} + 4z^{-2} + 2z^{-3} + z^{-4}$, and let $H_1(z) = H_0(-z)$. Draw an implementation for the pair $[H_0(z), H_1(z)]$ in the form of a uniform DFT analysis bank, explicitly showing the polyphase components, the 2×2 IDFT box, and other relevant details.

6. Find the efficient structure for fractional decimation in Fig. RI.6, with $H(z) = \sum_{n=0}^{10} nz^{-n}$. Please first apply the Type 1 polyphase decomposition. Illustrate your implementation. (Efficient structure)

7. Prove or disprove: the systems in Fig. RI.7 are linear time-invariant (LTI). (LTI system)

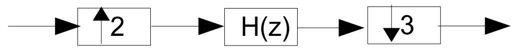


Figure RI.6:

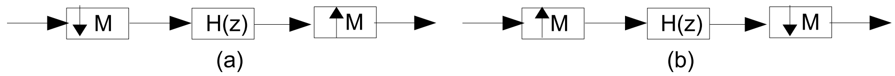


Figure RI.7:

8. Prove that the block diagram shown in Fig. RI.8 is a perfect reconstruction system. Based on this, answer the following questions (No knowledge of QMF bank is needed though. We will briefly revisit this problem in next recitation): (Perfect reconstruction)

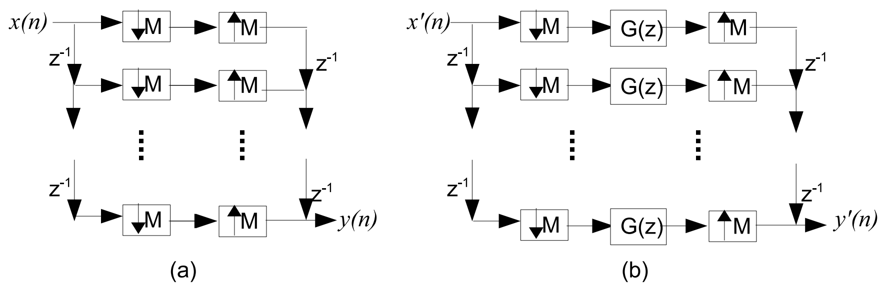


Figure RI.8:

(a) What is the transfer function $H'(z)$ for the block diagram in Fig. R4.2(b)?

(b) Suppose that we are to design a system with the desired system function $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 5z^{-4} + 6z^{-5} + 7z^{-6}$ in a very high-speed environment. If the speed of the devices used to implement the system is three times slower than the required operating speed, please devise a scheme that can implement $H(z)$ under the constraint of the device. (Hint: implement $z^{-2}H(z)$)

9. Consider the QMF bank in Fig. RI.9, where $H_0(z) = 1 + 2z^{-1} + 3z^{-2} + z^{-3}$, and $H_1(z) = H_0(-z)$. (QMF bank, Alias free system)

(a) Design an FIR synthesis bank $F_0(z), F_1(z)$ such that the system is alias-free.

(b) Design an IIR synthesis bank $F_0(z), F_1(z)$ such that the system is perfect reconstruction. Is this bank stable? (By default, we assume it is causal)

(c) Design a stable IIR synthesis bank $F_0(z), F_1(z)$ such that the system is alias-free and amplitude-distortion-free. (Hint: $\frac{a^*+z^{-1}}{1+az^{-1}}$ is an all-pass filter.)

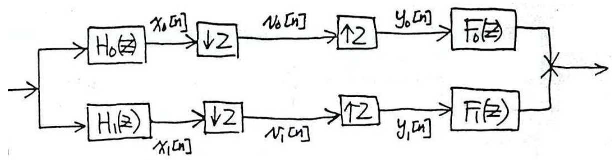


Figure RI.9:

10. Consider a 4-channel filter bank which is alias-free. If we know part of the matrix $\boldsymbol{P}(z)$ as

$$\boldsymbol{P}(z) = \begin{bmatrix} -- & 3z^{-1} & -- & -- \\ -- & 1 & -- & -- \\ -- & 2z^{-2} & -- & -- \\ -- & z^{-3} & -- & -- \end{bmatrix},$$

write down the entire matrix $\boldsymbol{P}(z)$. What is the distortion function $T(z)$? (General Conditions on alias free)

11. Problem 8 revisited. First find the transfer function for Fig. RI.11(a) and (b), Then, given the desired $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 5z^{-4} + 6z^{-5} + 7z^{-6}$, how to implement it on a hardware platform which is 3 times slower than needed? (Hint: using pseudo-circulant representation) (Pseudo-circulant representation)

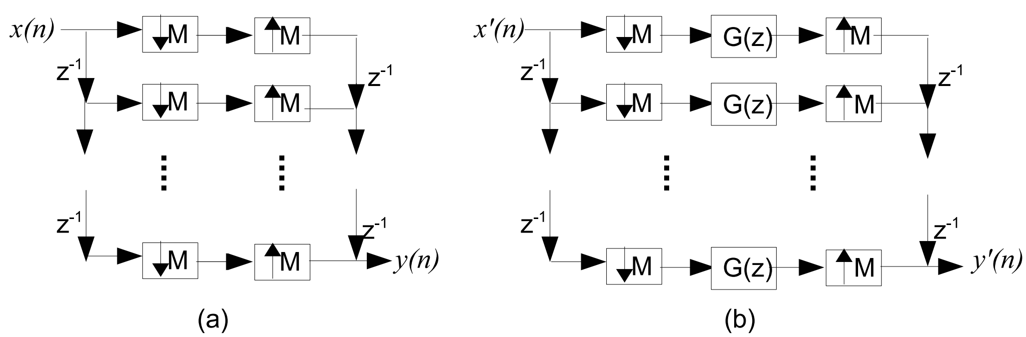
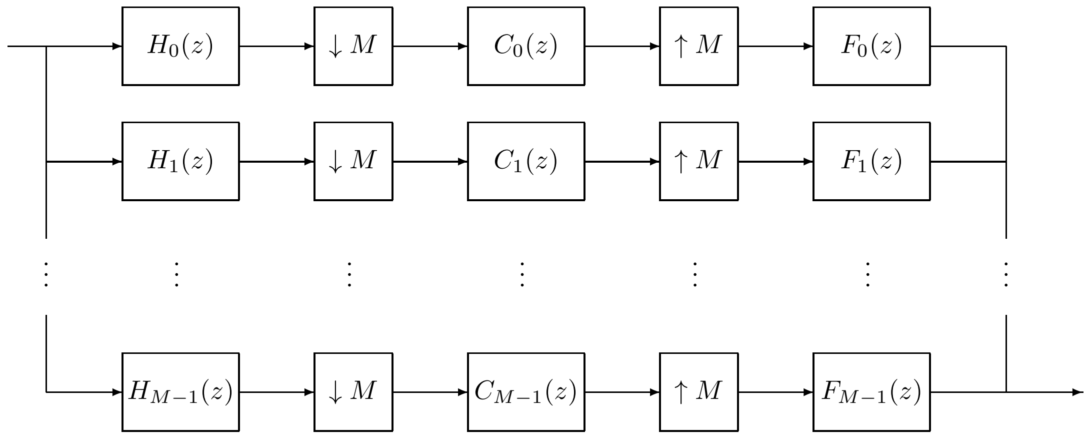


Figure RI.11:

12. Consider the following M-channel multirate system.

Assume throughout that the functions $F_k(z)$, $H_k(z)$, and $C_k(z)$ are rational and stable. (Amplitude distortion free, Perfect reconstruction in multirate system)

(a) Suppose $H_k(z)$ and $F_k(z)$ are such that the system is alias-free in absence of channel distortion $C_k(z)$, i.e. when $C_k(z)$ is unity for all k. Now with $C_k(z)$ present, find a modified set of synthesis filters $F_k(z)$ to retain alias-free property.



(b) Assume $C_k(z)$ has no zeros on the unit circle and has the form of

$$C_k(z) = \frac{A_k(z)B_k(z)}{D_k(z)}$$

where $A_k(z)$ has all the zeros inside the unit circle and $B_k(z)$ has them outside. Repeat part (a) by replacing "alias-free" condition with "alias-free and amplitude distortion free" condition.

(c) Now assume that $C_k(z)$ has no zeros on or outside the unit circle, repeat part (a) by replacing "alias-free" with "perfect reconstruction".