

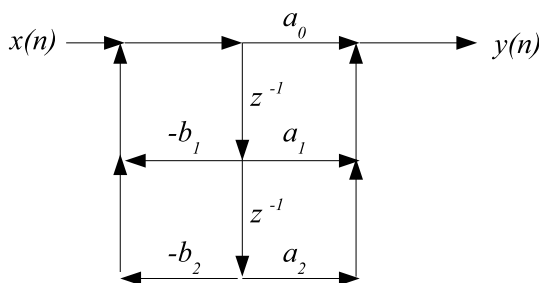
Material covered: Transforms and properties of the transform function of a discrete-time LTI system.

1. Define $H(z) = \sum_{n=-\infty}^{+\infty} h(n)z^{-n}$.
 - (1). If $h(n) = a^n u(n)$, what is $H(z)$?
 - (2). If $h(n) = -a^n u(-n - 1)$, what is $H(z)$?
 - (3). Given $H(z) = \frac{3}{1-0.5z^{-1}} + \frac{1}{1+0.75z^{-1}} + \frac{1}{1-2z^{-1}}$, what are the possible $h(n)$? If $h(n)$ is the impulse response of an LTI system, discuss whether the system is causal or stable.

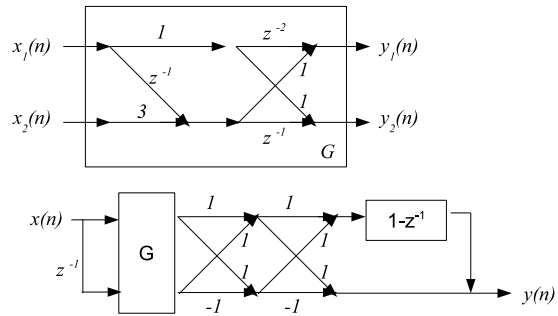
2. A causal system can be represented by the block diagram in Fig. R1.2.
 - (1). What is the system transfer function $H(z)$?
 - (2). Given $a_0 = 1, a_1 = -0.1, a_2 = -0.9$, and $b_1 = 0.9, b_2 = 0.81$, is the system stable? minimum phase? Sketch its frequency response $|H(e^{j\omega})|$.
 - (3). Suppose b_1 and b_2 are real numbers, prove that $b_2 < 1, b_1 + b_2 > -1$, and $b_1 - b_2 < 1$ is the condition that the system is stable. (Hint: discuss the cases when the two roots are real and when the roots are complex conjugate.)

3. If $X(z)$ and $Y(z)$ are z-transform of $x(n)$ and $y(n)$, respectively. Prove the following:
 - (1). The z-transform of $x^*(n)$ is $X^*(z^*)$ (Conjugate).
 - (2). The z-transform of $x(-n)$ is $X(1/z)$ (Time reversal).
 - (3). If $R_{xy}(k) = \sum_{n=-\infty}^{+\infty} x(n)y^*(n-k)$, its z-transform $S_{xy}(z) = X(z)Y^*(\frac{1}{z^*})$ (Cross correlation).

4. See the block diagram shown in Fig. R1.4.
 - (1). For the two-input two-output sub-block G , what are the impulse responses $h_{ij}(n)$ (i.e., from input x_j to output $y_i, i, j = 1, 2$)?
 - (2). For the whole system, what is the impulse response?



(a) Figure.R1.2



(b) Figure.R1.4