ENEE 630 Homework 8[1]

Material Covered: Lattice Predictor, Reflection Coefficient, Non-parametric Spectrum Estimation and ACF estimator

Problem 1 In a *normalized* lattice predictor, the forward and backward prediction errors at the various stage of the predictor are all normalized to have *unit variance*. Such an operation makes it possible to utilize the full dynamic range of multipliers used in the predictor, the normalized forward and backward prediction errors are defined as follows, respectively:

$$\begin{aligned} \overline{f}_m(n) &= \frac{f_m(n)}{P_m^{1/2}} \\ \overline{b}_m(n) &= \frac{b_m(n)}{P_m^{1/2}} \end{aligned}$$

where P_m is the average power (or variance) of the forward prediction error $f_m(n)$ or that of the backward prediction error $b_m(n)$. Show that the structure of stage m of the normalized lattice predictor is as shown in Figure 1 for real-valued data.

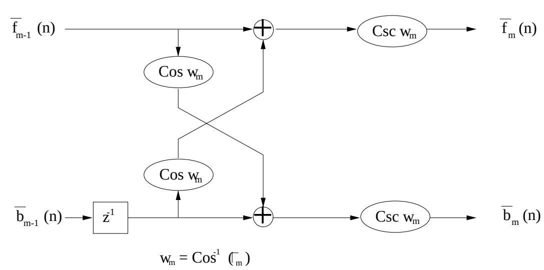

Figure 1: P-1

Problem 2 Consider the problem of optimizing stage m of the lattice predictor. The cost function to be used in the optimization is described by

$$J_m(\Gamma_m) = \alpha E[|f_m(n)|^2] + (1-\alpha)E[|b_m(n)|^2]$$

where α is a constant that lies between zero and one; $f_m(n)$ and $b_m(n)$ denote the forward and backward prediction errors at the output of stage m, respectively.

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(a) Show that the optimum value of the reflection coefficient Γ_m for which J_m is at minimum equals

$$\Gamma_{m,o}(\alpha) = -\frac{E[b_{m-1}(n-1)f^*_{m-1}(n)]}{(1-\alpha)E[|f_{m-1}(n)|^2] + \alpha E[|b_{m-1}(n-1)|^2]}$$

(b) Evaluate $\Gamma_{m,o}(\alpha)$ for each of the following three special conditions:

(1) $\alpha = 1$

(2) $\alpha = 0$

(3) $\alpha = \frac{1}{2}$

Notes: When the parameter α=1, the cost function reduces to

$$J_m(\Gamma_m) = E[|f_m(n)|^2]$$

We refer to this criterion as the *forward method*.

When the parameter α=0, the cost function reduces to

$$J_m(\Gamma_m) = E[|b_m(n)|^2]$$

We refer to this criterion as the *backward method*.

When the parameter $\alpha = \frac{1}{2}$, the formula for $\Gamma_{m,o}(\alpha)$ reduces to what is known as the *Burg formula*.

Problem 3 In this problem we show that the periodogram is an inconsistent estimator by examining the estimator at $f = 0$, or

$$\hat{P}_{PER}(0) = \frac{1}{N}\left(\sum_{n=0}^{N-1} x[n]\right)^2$$

If $x[n]$ is a real white Gaussian noise process with PSD

$$P_{xx}(f) = \sigma_x^2$$

find the mean and variance of $\hat{P}_{PER}(0)$. Does the variance converge to zero as $N \to \infty$?

Hint: Note that

$$\hat{P}_{PER}(0) = \sigma_x^2 \left(\sum_{n=0}^{N-1} \frac{x[n]}{\sigma_x\sqrt{N}}\right)^2$$

where the quantity inside the parentheses is $\sim N(0,1)$.

Problem 4 Consider the estimator

$$\hat{P}_{AVPER}(0) = \frac{1}{N} \sum_{m=1}^{N-1} \hat{P}_{PER}^{(m)}(0)$$

where

$$\hat{P}_{PER}^{(m)}(0) = x^2[m]$$

for the process from Problem 3. This estimator may be viewed as an *averaged periodogram.* In essence the data record is sectioned into blocks(in this case, of length 1) and the periodograms for each block are averaged. Find the mean and variance of $\hat{P}_{AVPER}(0)$. Compare this result to that obtained in Problem 3.

Problem 5 For the real data set $\{x[0], x[1], x[2]\}$ the unbiased ACF estimator yields

$$\hat{r}'_{xx}[0] = \frac{1}{3}(x^2[0] + x^2[1] + x^2[2])$$

$$\hat{r}'_{xx}[1] = \frac{1}{2}(x[0]x[1] + x[1]x[2])$$

Show that the autocorrelation sequence is not positive semidefinite by giving a numerical example where the matrix

$$\begin{bmatrix} \hat{r}'_{xx}[0] & \hat{r}'_{xx}[1] \\ \hat{r}'_{xx}[1] & \hat{r}'_{xx}[0] \end{bmatrix}$$

is not positive semidefinite.