ENEE 630 Homework 7[1]

Material Covered: Parameters in LD Recursion and Prediction Error

Problem 1 Consider a wide-sense stationary process {u(n)} whose autocorrelation function has the following values for different lags:

$$r(0) = 1$$
$$r(1) = 0.8$$
$$r(2) = 0.6$$
$$r(3) = 0.4$$

(a) Use the Levinson-Durbin recursion to evaluate the reflection coefficients Γ_1, Γ_2 and Γ_3.

(b) Set up a three-stage lattice predictor for this process, using the values for the reflection coefficients found in part (a).

(c) Evaluate the average power of the prediction error produced at the output of each of the three stages in this lattice predictor. Hence, make a plot of prediction error power vs. prediction order. Comment on your results.

Problem 2 **(a)** A time series $\{u_1(n)\}$ consists of a single sinusoidal process of complex amplitude α and angular frequency w in additive white noise of zero mean and variance σ_v^2 as shown by

$$u_1(n) = \alpha e^{jwn} + v(n)$$

where

$$E[|\alpha|^2] = \sigma_\alpha^2$$
$$E[|v(n)|^2] = \sigma_v^2$$

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The time series {u(n)} is applied to a linear predictor of order M, optimized in the Wiener sense. Do the following:

(i) Determine the tap weights of the prediction-error filter of order M, and the final value of the prediction error power P_M.

(ii) Determine the reflection coefficients $\Gamma_1, \Gamma_2, \dots, \Gamma_M$ of the corresponding lattice predictor.

(iii) How are the results in part (i) and part (ii) modified when we let the noise variance σ_v^2 approach zero?

(b) Consider next an AR process $\{u_2(n)\}$ described by

$$u_2(n) = -\alpha e^{jw} u_2(n-1) + v(n)$$

where, as before, {v(n)} is an additive white noise process of zero mean and variance σ_v^2. Assume that $0 < |\alpha| < 1$ but very close to 1. The time series $\{u_2(n)\}$ is also applied to a linear predictor of order M, optimized in the Wiener sense.

(i) Determine the tap weights of the new prediction-error filter of order M.

(ii) Determine the reflection coefficients $\Gamma_1, \Gamma_2, \dots, \Gamma_M$ of the correspinding lattice predictor.

(c) Use your results in parts (a) and (b) to compare the similarities and differences between the linear prediction of the time series $\{u_1(n)\}$ and $\{u_2(n)\}$.

Problem 3 Starting with the definition of Δ_{m-1}, show that Δ_{m-1} equals the cross-correlation between the delayed backward prediction error b_{m-1}(n-1) and the forward prediction error f_{m-1}(n).

Problem 4 Consider an autoregressive process {u(n)} of order 2, described by the difference equation

$$u(n) = u(n-1) - 0.5u(n-2) + v(n)$$

where {v(n)} is a white noise process of zero mean and variance 0.5

(a) Find an average power of {u(n)}.

(b) Find the reflection coefficients Γ_1 and Γ_2.

(c) Find the average prediction-error powers P_1 and P_2.

Problem 5 **(a)** Construct the two-stage lattice predictor for the second-order autoregressive process {u(n)} considered in problem 4.

(b) Given a white noise process {v(n)}, construct the two-stage lattice synthesizer for generating the autoregressive process {u(n)}. Check your answer against the second-order difference equation for the process {u(n)} considered in problem 4.