ANSWERS TO EXAM QUESTIONS
(BY OLCAY KOROL, TA/FALL 1999)

Problem 1

a) \(527 = (\ ?)_{6}\)

\[6^x - 1 > 527\]
\[6^x > 528\]

Smallest \(x\) satisfying this condition is 4.

b) \((F)_{16} = (33)_{4}\)

\((E)_{16} = (32)_{4}\)

\((A)_{16} = (22)_{4}\)

\((\text{FFAAFFEE})_{16} = (\text{3333222233333232})_{4}\)

Problem 2

a) \(111011110\) 2's complement representation of \((-34)_{10}\)

\(101010101\) 2's complement representation of \((-171)_{10}\)

Complementation gives \((205)_{10}\). So, the result is -205 decimal. And there is no overflow because the 1 at the beginning is inherent in the addition of two negative numbers. (Or because the result is in the range -256 to +255)

b) \(001100111\) = \((103)_{10}\)

\(000001001\) = \((9)_{10}\) (2's complement of 111110111 which is the subtrahend.)

\(001110000\) = \((112)_{10}\) No overflow

Problem 3
a) \( F = 1 \) iff \( a = 1, b = 0 \) or \( a = 0, b = 1 \)

\[
F(a, b, c, d) = ab' + a'b
\]

\[
= ab'(c + c')(d + d') + a'b(c + c')(d + d')
\]

\[
= ab'(cd + cd' + c'd + c'd') + a'b(cd + cd' + c'd + c'd')
\]

\[
= ab'cd + ab'cd' + ab'c'd + ab'c'd' + a'bcd + a'bcd' + a'bc'd + a'bc'd'
\]

So, \( F(a, b, c, d) = \sum m_{4} + m_{5} + m_{6} + m_{7} + m_{8} + m_{9} + m_{10} + m_{11} \)

\[
\text{There are two ways of solving that problem:}
\]

1) \( F = \sum m_{4} + m_{5} + m_{6} + m_{7} + m_{8} + m_{9} + m_{10} + m_{11} \)

\[
= (a' + b + c' + d')' + (a' + b + c' + d)' + (a' + b + c + d')' + (a' + b + c + d)'
\]

\[
= (a + b' + c + d) (a + b' + c + d') (a + b' + c + d) (a + b' + c + d')
\]

\[
\text{directly from SOP form of } F
\]

II) \( F' = \prod M_{4} M_{5} M_{6} M_{7} M_{8} M_{9} M_{10} M_{11} \)

\[
= (a + b' + c + d) (a + b' + c + d') (a + b' + c + d) (a + b' + c + d')
\]

\[
(\overline{F'})' = F = (a + b' + c + d)' + (a + b' + c + d')' + (a + b' + c' + d)' + (a + b' + c' + d)'
\]

\[
= (a'bc'd')' (a'bc'd')' (a'bcd)' (a'bcd)' (ab'c'd')' (ab'c'd')' (ab'cd)' (ab'cd)' (ab'cd)'
\]

\[
\text{directly from POS form of } F'
\]

b) \( F'(a, b, c, d) = \sum m_{0} + m_{1} + m_{2} + m_{3} + m_{12} + m_{13} + m_{14} + m_{15} = \prod M_{4} M_{5} M_{6} M_{7} M_{8} M_{9} M_{10} M_{11} \)

\[
\text{Again there are two ways to solve this problem:}
\]

I) \( F' = \prod M_{4} M_{5} M_{6} M_{7} M_{8} M_{9} M_{10} M_{11} \)

\[
= (a + b' + c + d) (a + b' + c + d') (a + b' + c' + d) (a + b' + c' + d')
\]

\[
= (a' + b + c + d) (a' + b + c + d') (a' + b + c' + d) (a' + b + c' + d')
\]

\[
= (a'bc'd')' (a'bc'd')' (a'bcd)' (a'bcd)' (ab'c'd')' (ab'c'd')' (ab'cd)' (ab'cd)' (ab'cd)'
\]

\[
\text{directly from POS form of } F'
\]
II) \[ F = m_4 + m_5 + m_6 + m_7 + m_8 + m_9 + m_{10} + m_{11} \]
\[ = a'bc'd' + a'bc'd + ab'cd' + ab'c'd + ab'cd + ab'cd' + ab'cd + ab'cd \]
\[ F' = (a'bc'd')' (a'bc'd)' (a'bc'd')' (a'bc'd')' (ab'c'd')' (ab'c'd)' (ab'cd')' (ab'cd)' \]

c) Karnaugh Map for F:

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c
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\[ F(a, b, c, d) = ab' + a'b \] by collecting the two four-square terms in the map

In order to have a NOR-NOR implementation,

\[ F' = (ab + a'b') \]
\[ = ((a' + b')' + (a + b'))' \]
\[ F = (F')' = ((a' + b')' + (a + b'))' \]

3 NOR gates are needed to implement this function
Problem 4

a) \( Y = 1 \) iff \( 2 < (abcd)_2 < 14 \)

Truth table of function \( Y \):
b) 

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<tr>
<th>a</th>
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Prime Implicants: \( a'b \)

\[
\begin{array}{c|cccc}
\text{cd} & 00 & 01 & 11 & 10 \\
\hline
\text{ab} & 1 & 1 & 1 & 1 \\
\text{00} & 1 & 1 & 1 & 1 \\
\text{01} & 1 & 1 & 1 & 1 \\
\text{11} & 1 & 1 & 1 & 1 \\
\text{10} & 1 & 1 & 1 & 1 \\
\end{array}
\]

Essential Prime Implicants: \( a'b \)
Problem 5 (Note the solutions we did in class are slightly different from the ones given here!)

a)

\[ Y(a, b, c, d, e, f, g) = (a + bc)(c + de)(f + g) \]
\[ = (a + b)(a + c)(c + d)(c + e)(f + g) \]
\[ = (a'b')(a'c')(c'd')(c'e')(f'g')' \]
\[ = \left[ [(a'b')(a'c')(c'd')(c'e')(f'g')]' \right]' \]

\[ a' \]
\[ b' \]
\[ \]
\[ a' \]
\[ c' \]
\[ \]
\[ c' \]
\[ d' \]
\[ \]
\[ c' \]
\[ e' \]
\[ \]
\[ f' \]
\[ g' \]

b)
Y (a, b, c, d, e, f, g) = (a + bc) (c + de) (f + g)

= (a + b) (a + c) (c + d) (c + e) (f + g)

= [(a + b)' + (a + c)' + (c + d)' + (c + e)'+ (f + g)']'

c) NAND-NAND realization uses 7 of these gates and is a 3-level implementation while NOR-NOR realization uses 6 of those gates and is a 2-level implementation. So, the latter is obviously better considering the issues of cost and propagation time.