1) a) Consider a sample of n-type silicon L cm long, W cm wide, and T cm thick that is nonuniformly doped in such a manner that the equilibrium majority carrier population varies throughout its thickness as \( n_0(x) \). Show that if the mobility \( \mu_e \) is constant, independent of the doping level, then the end-to-end resistance of this sample depends only on the integral of \( n_0(x) \) over the thickness of the sample (i.e. from \( x=0 \) at the top surface to \( x=T \) at the bottom surface) and not on the actual shape of \( n_0(x) \).

\[ \text{Hint: Mentally divide the sample into slabs of material } dx \text{ wide, and add the conductances of these slabs connected in parallel.} \]

b) In an integrated circuit, dopants are introduced to the to surface of a silicon wafer (slab) to produce nonuniformly doped regions like the sample described in (a) and resistors are formed by putting contacts at the ends of rectangularly shaped regions doped in this manner. Suppose that the doping profile of such a resistive region is such that

\[ n_0(x) = 10^{18} e^{-x/X} \text{ cm}^{-3} \]

where \( X \) is 2 \( \mu \) m. What resistance would a square region with this profile, L units by L units, have? This resistance is called the sheet resistance \( R_s \) of the region.

c) The dopant profile in part (b) is introduced in a pattern like that illustrated in the figure below. What is the approximate resistance between points A and B of this resistor?
2) The conductivity of copper is approximately $6 \times 10^5$ mho/cm at room temperature and is due to the mobility of electrons (one per atom) free to move under the influence of an electric field. The concentration of these conduction electrons is approximately $10^{23}$ cm$^{-3}$.
   
   a) Calculate the electron mobility in copper at room temperature. How does this compare with Si and Ge?
   
   b) Calculate the net average velocity of the electrons in the direction of the current flow (assume it is the x-direction) in a 0.1 mm$^2$ cross-sectional area wire carrying a current of 1A. Assume that the current is due to the cooperative motion of the electrons (“drift”) superimposed on their random thermal velocity.

3) In a silicon region with donor concentration $N_d = 10^{15}$ cm$^{-3}$, we apply an electric field in the +x-direction with a magnitude of $10^3$ V/cm.
   
   a) What is the electron drift velocity (magnitude and sign)?
   
   b) What is the electron drift current density (magnitude and sign)?
   
   c) How long is required for an electron to drift 1 µm, on average?
   
   d) How many collisions occur while it is drifting? You can assume that the collision time for the drifting electron is 0.1 ps.

4) The gradient in minority hole concentration in a silicon sample over the region $x \geq 0$ is given by
   
   $$ p(x) = 10^{13} \left(1 - e^{-\frac{x}{\mu \mu m}}\right) \text{cm}^{-3} $$
   
   The donor concentration is $N_d = 5 \times 10^{15}$ cm$^{-3}$.
   
   a) What is the hole concentration at $x=0$?
   
   b) What is the hole diffusion current density at $x=0$? Provide an explanation for any apparent discrepancy between parts (a) and (b).
   
   c) Plot the hole diffusion current density over the interval $5 \mu m \geq x \geq 0$.

5) When a gradient in minority carrier concentration is set up in a semiconductor, an identical gradient in majority carrier concentration is quickly established, in order to keep the silicon electrically neutral. This situation is shown in the figure below.
   
   a) Find the minority electron current density.
   
   b) Find the majority hole current density.
   
   c) Since the silicon region is open-circuited, the total current density must be zero. Find the required drift current density.
   
   d) Assuming that the contribution of the minority carriers to the drift current is negligible (it is), what electric field is required in the sample to generate the drift current density needed from part (c).
Research question:
The ratio of charge $q$ to mass $m$ for an electron was first measured by Sir J. J. Thomson more than a century ago. For this and other basic work in physics he was awarded the Nobel prize in physics. Describe Thomson’s experiment and present the simple theoretical analysis that relates the charge to mass ratio to the experimental observations.