DERIVATION OF MOSFET $I_{DS}$ VS. $V_{DS} + V_{GS}$

Derive the current expressions in the MOSFET:

**Linear Region:**

$$I_D = \mu C_{ox} \frac{W}{L} [(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}]$$

**Saturation Region:**

$$I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_{TH})^2$$

1. **Linear Region**

![Image of a MOSFET with voltage labels](image)

**Figure 1.** Concentration Contours in Linear Region. A uniform narrow channel exists.

**KVL:**

$$V_G - V_S = V_G - V_C + V_C - V_S$$

$$V_G - V_S = V_{GS}$$

$$V_G - V_C = V_{GC}$$

$$V_C - V_S = V(x)$$

$$V_{GS} = V_{GC} + V(x) \text{ or } V_{GS} - V(x) = V_{GC}$$
Total charge density at x on capacitor($C_{OX}$) is $Q_T(x)$:

$$Q_T(x) = V_{GC}C_{OX} = (V_{GS} - V(x))C_{OX}$$

$$Q_T(x) = Q(x)_{mobile} + Q(x)_{depletion}$$

$Q(x)_{mobile}$=mobile electron charge in channel at x

$$Q(x)_{mobile} = [V_{GS} - V(x) - V_{TH}]C_{OX}$$

Use mobile charge to get current:

$$J_n = q\mu n E + qD_n \frac{dn}{dx} = q\mu n E \text{ (no diffusion current in the channel)}$$

$$qn(x) = Q(x)_{mobile} = Q_m(x)$$

$$J_n = Q_m(x)\mu E \text{, but } E = -\frac{dV}{dx}$$

$$J_n = -Q_m(x)\mu \frac{dV}{dx} \text{, substitute for } Q_m(x)$$

$$J_n = \mu C_{ox}(V_{GS} - V(x) - V_{TH}) \frac{dV}{dx} \text{, separate variables and neglect (-) sign. Consider only the magnitude.}$$

$$J_n dx = \mu C_{ox}[(V_{GS} - V_{TH}) - V(x)]dV$$

Due to continuity, $J_n = \text{constant}$ (no hole current or no generation, recombination). Integrating from source to drain or from x=0 to x=L, where L=gate length:

$$J_n \int_0^L dx = \mu C_{ox} \int_{V(0)}^{V(L)}[(V_{GS} - V_{TH}) - V(x)]dV$$

$$V(L) = V_{DS} \text{, } V(0)=0$$

$$J_n \int_0^L dx = \mu C_{ox} \int_0^{V_{DS}}[(V_{GS} - V_{TH}) - V(x)]dV$$

$$J_n L = \mu C_{ox}[(V_{GS} - V_{TH})V - \frac{V^2}{2}]_{0}^{V_{DS}}$$

$$J_n = \frac{\mu C_{ox}}{L}[(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}]$$
\[ I_D = J_n W \ (W=\text{Device Width}) \]

\[ J_n \ \text{for channel is Amp/cm since } Q_m = \text{Charge/cm}^2 \]

\[ I_D \ \text{for Linear Region: } I_D = \mu C_{ox} \frac{W}{L} [(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}] \]

2. Saturation Region

When \( V_{DS} \geq (V_{GS} - V_{TH}) \) channel pinches off. This means that the channel current near the drain spreads out and the channel near drain can be approximated as the depletion region. After this occurs, at \( V_{DS} = (V_{GS} - V_{TH}) \), if you make \( V_{DS} \) larger, the current \( I_D \) does not change (to zero approximation). This is because any additional \( V_{DS} \) you add will get dropped across the depletion region and won’t change the current \( I_D \).

So for \( V_{DS} \geq (V_{GS} - V_{TH}) \) we find \( I_D \) by setting \( V_{DS} = (V_{GS} - V_{TH}) \) substituting into the linear equation.

\[ I_D = \mu C_{ox} \frac{W}{L} [(V_{GS} - V_{TH})(V_{GS} - V_{TH}) - \frac{(V_{GS} - V_{TH})^2}{2}] \]

\[ I_D \ \text{for Saturation Region:} \]

\[ I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_{TH})^2 \]
Figure 2. Concentration Contours in Linear Region. A uniform narrow channel exists.

Figure 3. Concentration Contours in Saturation Region. Channel narrow near source and spreads out and widens near drain, said to be “pinched off”.