1. (10 pts.) Consider a bar of semiconductor, which is n-type with non-uniform doping (i.e. Nd is not constant throughout the sample.) In the bar, at equilibrium, there is an electric field pointing to the right. Sketch a possible Nd(x), which is the donor concentration in terms of position, that would result in this direction for a built-in electric field at equilibrium. Indicate the electron particle drift and diffusion directions and drift and diffusion current directions. If we double Nd(x) throughout the sample, what would happen to the drift and diffusion currents? Why? If we add a constant Nd0 to the Nd(x) throughout the sample, what would happen to the currents? Why?

Both cases (double Nd(x) and add a constant Nd0), initially n(x) doubles thus drift current increases as much as doping increases (n(x) = Nd(x)) but built-in electric field changes. When Nd(x) is doubled, diffusion current becomes doubled.

\[ J_{\text{drift}} = q\mu_n n(x) E(x), \quad J_{\text{diffusion}} = qD_n \frac{dn(x)}{dx} \]

When a constant Nd0 is added, diffusion current stays same.

\[ J'_{\text{diffusion}} = qD_n \frac{d(Nd0 + n(x))}{dx} = qD_n \frac{dn(x)}{dx} = J_{\text{diffusion}} \]

At equilibrium (after enough time passes), built-in electric field changes and drift current comes back to the same amount with diffusion current which are twice for the 1'st case and same for the 2'nd case.

2. A direct-recombination semiconductor sample has \( N_A - N_D = 10^{16} \) 1/cm^3 and \( \tau = \tau_n = \tau_p = 1\mu s \). It is illuminated until a uniform excess minority density of \( 10^5 \) 1/cm^3 has been generated. The light is turned off at \( t = 0 \).

(a) (5 pts.) What is the recombination rate immediately after the light is turned off for the excess minority carriers?
This is a p-type (\(p_0 \gg n_d, p_0 = 10^{16}, n_0 = \frac{n_d^2}{p_0} = n_i^2 \times 10^{-16}\))

\[
\frac{dn(t)}{dt} = G(t) - R(t) = \alpha_p n_i^2 - \alpha_p n(t)p(t) - \alpha_p (n_u + \delta n(t)) (p_u + \delta p(t))
\]

Becomes

\[
\frac{d\delta n(t)}{dt} = \alpha_p p_0 \delta p(t)
\]

The solution to this equation is

\[
\delta n(t) = \Delta n e^{-t/\tau_n}, \tau_n = (\alpha_p p_0)^{-1}, \Delta n = 10^6
\]

Recombination rate is

\[
R(t) = \alpha_p n(t)p(t) = \frac{1}{\tau_n p_0} (n_u + \delta n(t)) (p_u + \delta p(t)) = \frac{n_i^2 + (n_u + p_0) \delta n(t) + \delta n(t)^2}{\tau_n p_0}
\]

\[
R(t) = \frac{n_i^2 + n_i^2 \times 10^{-10} + 10^{12} \times e^{-t/10^8} + 10^{12} \times e^{-2t/10^8}}{10^{-5} \times 10^{16}}
\]

\[
= n_i^2 \times 10^{-10} \times e^{-t/10^8} + n_i^2 \times 10^{-20} + n_i^2 \times 10^{-20} \times e^{-2t/10^8} + 10^{12} \times e^{-t/10^8} \approx 10^{12} \times e^{-t/10^8}
\]

When the light is turned off (\(t=0\)), \(R(0) = 10^{12}\)

(b) (5 pts.) What is this rate for the excess majority carriers?

As the electrons and holes recombine in pairs, this rate for both is same as result of (a) above.

(c) (5 pts.) What is this rate at \(t = \tau\) for the excess minority carriers?

\(R(\tau) = 10^{12} \times e^{-1} = 3.68 \times 10^{11}\)

(d) (5 pts.) What is this rate at \(t = 10\ \tau\) for the excess minority carriers?

\(R(10\tau) = 10^{12} \times e^{-10} = 4.54 \times 10^{7}\)

3. (10 pts.) Remember that for a pn-junction with the p-side doped with an acceptor density of Na and n-side doped with a donor density of Nd, we showed that the net charge on either side of the depletion region would be given by \(Q_n = qANdx_n\) and \(Q_p = -qANAx_p\), where \(x_n\) and \(x_p\) are the depletion region penetration into the n-side and p-side, respectively. At equilibrium, the magnitude of these charges are equal to each other, and this is similar to the
situation of charges in a parallel-plate capacitor: \(|Q_{\text{dep}}| = |Q_n| = |Q_p|\). When an external bias \(V_A\) is applied, the depletion region width changes, and therefore so does this charge. Derive an expression for the depletion (or junction) capacitance, \(C_j\), by using the definition of capacitance as \(C_j = \frac{dQ_{\text{dep}}}{dV_A}\). (Hint: Use the expression that links the junction potential to depletion region width; remember what happens to the junction potential with applied bias.)

The width of the transition region in terms of the electrostatic potential barrier \((V_0 - V)\) from Eq. (5-57)

\[
W = \frac{2\varepsilon(V_0 - V)}{q} \left( \frac{N_A + N_D}{N_A N_D} \right)^{1/2}
\]

The value of the charge \((Q)\) can be written in terms of the doping concentration and transition region width on each side of the junction

\[
|Q| = qA x_n N_D = qA x_p N_A
\]

And each side of the junction can be written in terms of the total width of the transition region

\[
x_n = \frac{N_A}{N_A + N_D} W, x_p = \frac{N_D}{N_A + N_D} W
\]

Therefore the charge on each side

\[
|Q| = qA \frac{N_A N_D}{N_A + N_D} W = A \left[ 2q \varepsilon (V_0 - V) \frac{N_A N_D}{N_A + N_D} \right]^{1/2}
\]

The junction capacitance

\[
C_j = \left| \frac{dQ}{d(V_0 - V)} \right| = \frac{A q}{2} \frac{2q \varepsilon}{(V_0 - V) N_A N_D} \left[ \frac{N_A N_D}{N_A + N_D} \right]^{1/2}
\]

4. (60 pts.) Streetman and Banerjee, problems 4.5, 5.10, 5.11, 5.13(a), 5.19, 5.30.

4.5

(a) \(s(x) = -\frac{\partial n}{\partial x} = -\frac{kT}{q} \frac{3}{N_A N_D} \exp \left( -\frac{x}{N_A N_D} \right) \exp \left( -ax \right) = \frac{kT e}{q}
\]

(b) \(E(1\ \mu m^{-1}) = 0.0259 \ V/\mu m = 0.0259 \times 10^6 \ V/cm
\]

(c)
5.10

\[ V_0 = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2} = 0.0259 \ln \frac{4 \times 10^{18} \cdot 10^{18}}{(1.5 \times 10^{10})^2} = 0.8498 \text{ V} \]

\[ W = \sqrt{\frac{2eV_0}{q} \left( \frac{1}{N_{i}} - \frac{1}{N_d} \right)} = \sqrt{\frac{2 \cdot 11.8 \times 8.85 \times 10^{-14} \cdot 0.8498}{1.6 \times 10^{-19} \left( \frac{1}{4 \times 10^{18}} + \frac{1}{10^{18}} \right)}} = 0.335 \mu m \]

\[ \chi_{n0} = \frac{W}{1 + N_d/N_a} \approx \frac{0.333}{1 + 0.0025} = 0.332 \mu m \]

\[ \chi_{p0} = \frac{W}{1 + N_a/N_d} \text{ or } W = \chi_{n0} = 0.83 \text{ nm} \]

\[ Q_+ = qA \chi_{n0} N_d = 0.106 nC \]

\[ E_0 = \frac{-qN_d \chi_{n0}}{\varepsilon_0} = -5.1 \times 10^4 \text{ V/cm} \]
5.11

\[ L_p = \sqrt{D_p \tau_p} = \sqrt{20 \times 10^{-12}} = 10^{-3} \text{ cm} = 10 \mu m \]

\[ \delta p = \frac{n_i^2}{N_d} \left( \exp \left( \frac{qV}{kT} \right) - 1 \right) \exp \left( -\frac{x}{L_p} \right) \]

\[ \frac{d \delta p}{dx} = -\frac{1}{L_p} \cdot \frac{n_i^2}{N_d} \left( \exp \left( \frac{qV}{kT} \right) - 1 \right) \exp \left( \frac{x}{L_p} \right) = -\frac{1}{10^{-3}} \left( \frac{(10^{10})^2}{10^{22}} \right) \left( \exp \left( \frac{0.6}{0.125} \right) - 1 \right) \exp \left( -\frac{2}{10} \right) \]

\[ = -8.62 \times 10^{16} \text{ } 1/cm^4 \]

\[ J_p = -qD_p \frac{d \delta p}{dx} = 1.6 \times 10^{-19} \times 20 \times 6.62 \times 10^{16} = 0.276 \text{ A/cm}^2 \]
This is independent of the doping; there will be no change after doubling the p+ doping.

5.13(a)

For reverse bias more than a few tenths of a volt, changes in the reverse bias do not appreciably alter the excess hole distribution. The primary variation is in the width of the depletion region, giving rise to the junction capacitance.

5.19

(a)

\[ V_0 = \frac{RT}{q} \ln \frac{N_a N_d}{n_T^2} = 0.0259 \times \ln \left( \frac{10^{15} \times 10^{17}}{(1.5 \times 10^{12})^2} \right) = 0.695 \text{ V} \]

(b)

\[ W = \sqrt{\frac{2e_0 V_0}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)} = \sqrt{\frac{2 \times 11.8 \times 8.85 \times 10^{-14} \times 0.695}{1.6 \times 10^{-19} \left( \frac{1}{10^{14}} + \frac{1}{10^{17}} \right)} = 0.96 \text{ \mu m} \]

(c)

\[ D_n = \mu_n \frac{RT}{q} = 1500 \times 0.0259 = 36.9 \text{ , } D_p = \mu_p \frac{RT}{q} = 450 \times 0.0259 = 11.7 \]

\[ L_n = \sqrt{D_n \tau} = 0.31 \text{ , } L_p = \sqrt{D_p \tau} = 0.17 \]

\[ J_0 = q\tau^2 \left( \frac{D_p}{N_d L_p} + \frac{D_n}{N_a L_n} \right) = 4.5 \times 10^{-12} \text{ C/cm}^2 \text{s} \]

\[ I = A \cdot J_0 \left( \exp \left( \frac{qV}{kT} \right) - 1 \right) = 0.001 \times 4.5 \times 10^{-12} \times \left( \exp \left( \frac{0.5}{0.0259} \right) - 1 \right) = 1.09 \times 10^{-7} \text{ A} \]

Most of the current is carried by electrons because Na is less than Nd.

To double the electron current, reduce the acceptor doping in half.
Taking the Laplace transform

\[ I_{F_2} = \frac{Q_p(s)}{\tau_p} + sQ_p(s) - I_{F_1} \tau_p \]

\[ Q_p(s) = \left( \frac{I_{F_2}}{s} + I_{F_1} \tau_p \right) \frac{1}{s + \frac{1}{\tau_p}} \]

Transforming back to the time domain

\[ Q_p(t) = I_{F_2} \tau_p \left( 1 - \exp \left( \frac{-t}{\tau_p} \right) \right) + I_{F_1} \tau_p \exp \left( \frac{-t}{\tau_p} \right) \]