1. Describe in detail a man-in-the-middle attack on the Diffie-Hellman key-exchange protocol whereby the adversary ends up sharing a key $k_A$ with Alice and a different key $k_B$ with Bob, and Alice and Bob cannot detect that anything has gone wrong.

What happens if Alice and Bob try to detect the presence of a man-in-the-middle adversary by sending each other (encrypted) questions that only the other party would know how to answer?

2. Consider the following key-exchange protocol:

   (a) Alice chooses $k, r \leftarrow \{0, 1\}^n$ at random, and sends $s := k \oplus r$ to Bob.
   (b) Bob chooses $t \leftarrow \{0, 1\}^n$ at random and sends $u := s \oplus t$ to Alice.
   (c) Alice computes $w := u \oplus r$ and sends $w$ to Bob.
   (d) Alice outputs $k$ and Bob outputs $w \oplus t$.

   Show that Alice and Bob output the same key. Analyze the security of the scheme (i.e., either prove its security or show a concrete attack).

3. Consider the following key-exchange protocol:

   Common input: The security parameter $1^n$.
   (a) Alice runs $G(1^n)$ to obtain $(G, q, g)$.
   (b) Alice chooses $x_1, x_2 \leftarrow \mathbb{Z}_q$ and sends $\alpha = x_1 + x_2$ to Bob.
   (c) Bob chooses $x_3 \leftarrow \mathbb{Z}_q$ and sends $h_2 = g^{x_3}$ to Alice.
   (d) Alice sends $h_3 = g^{x_2x_3}$ to Bob.
   (e) Alice outputs $h_2^{x_1}$. Bob outputs $(g^\alpha)^{x_3} \cdot (h_3)^{-1}$.

   Show that Alice and Bob output the same key. Analyze the security of the scheme (i.e., either prove its security or show a concrete attack).

4. Show that any 2-round key-exchange protocol (that is, where each party sends a single message) can be converted into a CPA-secure public-key encryption scheme.

5. Fix an RSA public key $(N, e)$ and assume we have an algorithm $A$ that always correctly computes $\text{lsb}(x)$ given $[x^e \mod N]$. Write full pseudocode for an algorithm $A'$ that computes $x$ from $[x^e \mod N]$.