   (a) Let $X \sim \mathcal{N}(0, 1)$. Use the Chernoff bound to show that $P(X \geq c) \leq e^{-c^2/2}$. Then take $c = 2$ and compare the this estimate with the true value (please give the numbers for both ways of finding $P(X \geq 2)$).
   
   (b) Now let $X \sim \text{Poisson}(\lambda)$ (p.78 in the textbook). Use the Chernoff bound to estimate $P(X \geq c)$. For which $c$ does your calculation produce a trivial upper bound $P(X \geq c) \leq 1$?

2. A fair coin is tossed 6 times, yielding a sequence of Heads and Tails. Let $X$ be the parity of the number of Heads (the remainder of the division of the number of heads by 2) and let $Y$ be the remainder of the division of the number of Tails by 3.
   (a) Find the PMFs $p_X(k)$, $p_Y(k)$.
   (b) Are $X$ and $Y$ correlated? Are they independent?

3. Consider two continuous r.v.’s $X$ and $Y$ that are uniformly distributed in the region $\{|x + y| \leq 1, |x - y| \leq 1/2\}$ of the $(x,y)$ plane.
   (a) Find $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$ for all $x$ and $y$.
   (b) Find $\mathbb{E}[X|Y], \mathbb{E}[X]$.

4. Packets arrive at the server with an exponential wait time, so that the $k$th packet arrives after $X_1 + X_2 + \cdots + X_k$ minutes, where $X_k, k = 1, 2, \ldots$ are exponential random variables with $EX_k = 2$ for all $k$. Find the probability $P$ that the third packet arrives no earlier than 20 minutes after the start of the experiment. Estimate $P$ using the Markov inequality.

5. Let $X_1, X_2, \ldots$ be i.i.d. RVs with $EX = 2, \text{Var}(X) = 9$. Define $Y_i = X_i/2^i$ for all $i$. Finally let $T_n = \sum_{i=1}^{n} Y_i$ and $A_n = T_n/n$.
   (a) Find $E$ and $\text{Var}$ for $Y_n, T_n, A_n$.
   (b) Do the RVs $Y_n, T_n, A_n$ converge in probability to a number, and if yes, what are the limiting values?