CENTRE FOR WIRELESS COMMUNICATIONS
University of Oulu

Wireless Information Theory Summer School
Oulu, Finland July 27, 2011

Afternoon Session- Part 1

Energy Harvesting Wireless Networks

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Aylin Yener

yener@ee.psu.edu

Introduction

- Energy efficient communications for "regular" nodes
  - Better signal processing techniques
  - Power efficiency
  - MIMO
- New Paradigm: Communication with "rechargeable nodes"

Introduction

- **Wireless networking with rechargeable (energy harvesting) nodes:**
  - Green, self-sufficient nodes,
  - Extended network lifetime,
  - Smaller nodes with smaller batteries.

A relatively new field with increasing interest.

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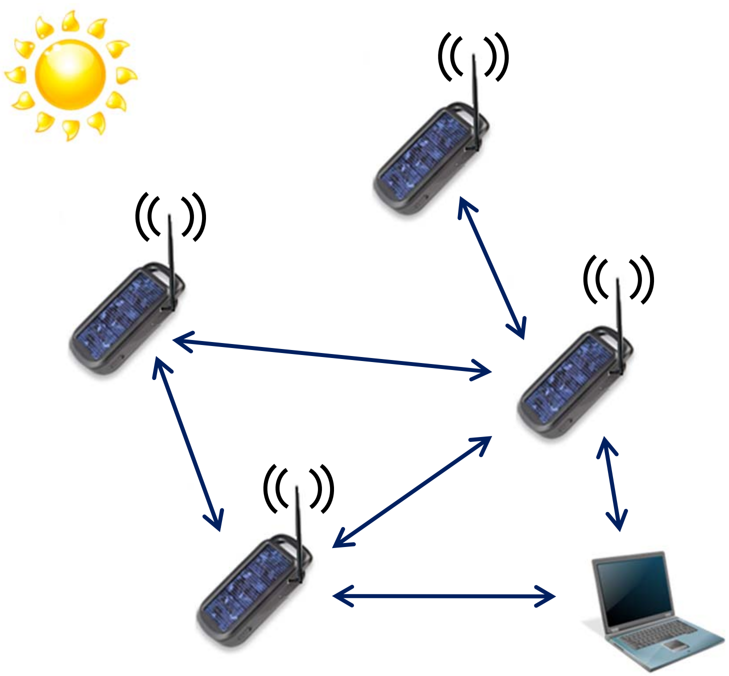
Energy Harvesting

- Conventional energy supply requires:
  - Electrical wiring
  - Battery replacement
- **Energy Harvesting:**
  - Generating electricity from surrounding environment
  - light, vibration, heat, radio waves…

Some Applications

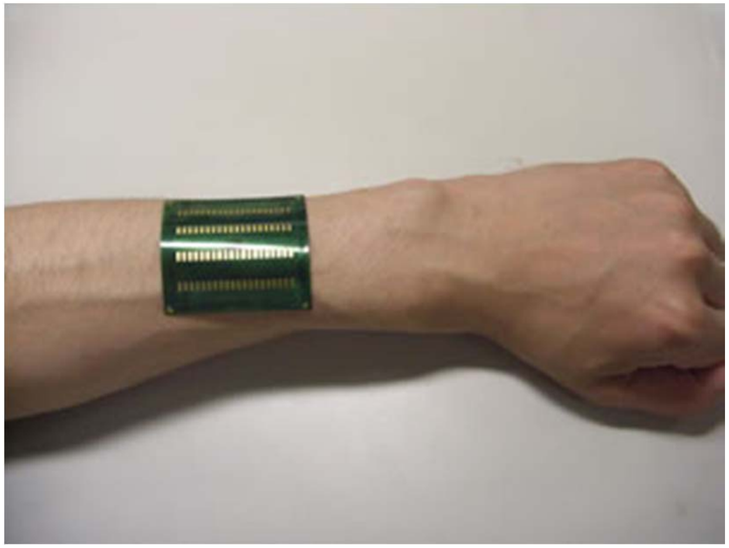
Wireless sensor networks

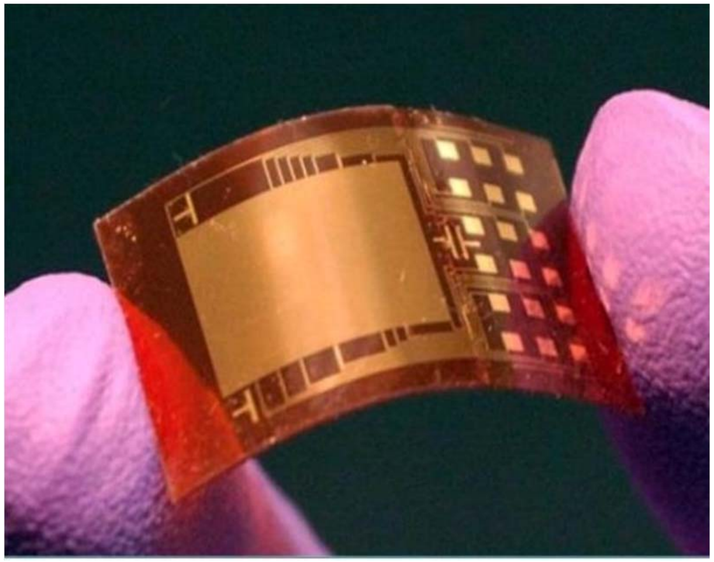


Green communications

Energy Harvesting

- Fujitsu's hybrid device utilizing **heat or light.**





- Nanogenerators built at Georgia Tech, utilizing **strain**

Image Credits:
(above) http://www.fujitsu.com/global/news/pr/archives/month/2010/20101209-01.html
(below) http://www.zeitnews.org/nanotechnology/squeeze-power-first-practical-nanogenerator-developed.html

Energy Harvesting

- Various practical applications

Image Credits:
(left) http://inhabitat.com/shoe-generator-harvests-power-from-walking/
(right) http://www.wafermaneuver.com/nick/energyharvesting.html

Motivation

- **New Wireless Network Design Challenge:**
  A **set of energy feasibility constraints** based on harvests govern the communication resources.
- **Design question:**
  When and at what rate/power should a "rechargeable" (energy harvesting) node transmit?
- **Optimality? Throughput; Delivery Delay**

Motivation

Many open problems related to all layers of the network design.

- Transmission scheduling
- Signal processing/PHY design
- MAC protocol design
- Channel capacity
- …

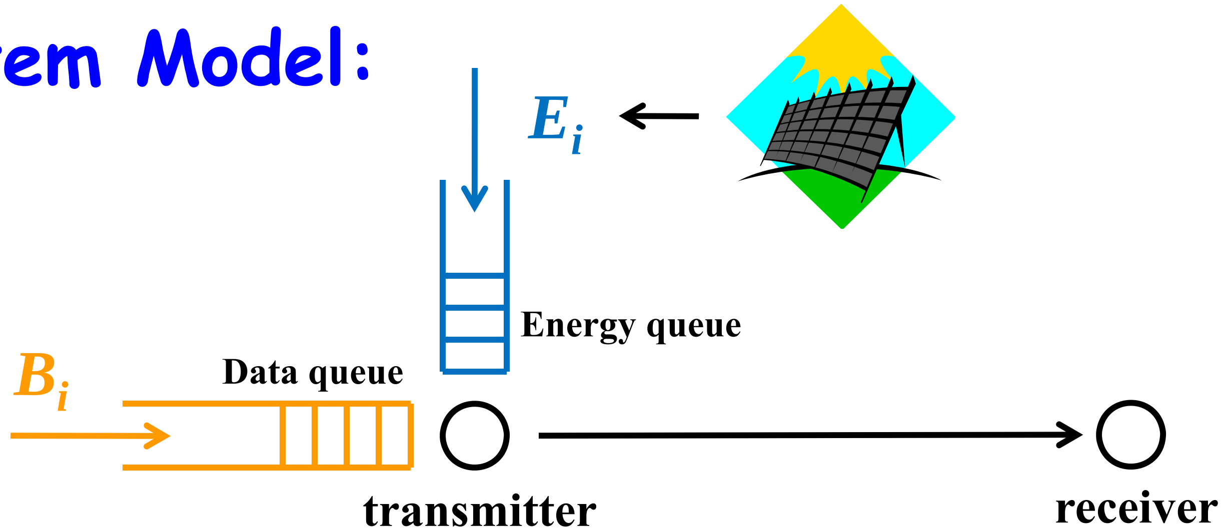
Remainder of this lecture

- Optimal Scheduling Policies for one Energy Harvesting Transmitter with the goal of maximizing throughput or minimizing transmission completion time for

1. Infinite energy storage
2. Finite Battery Capacity
3. Fading Channel

Optimal Scheduling [Yang, Ulukus 2010]

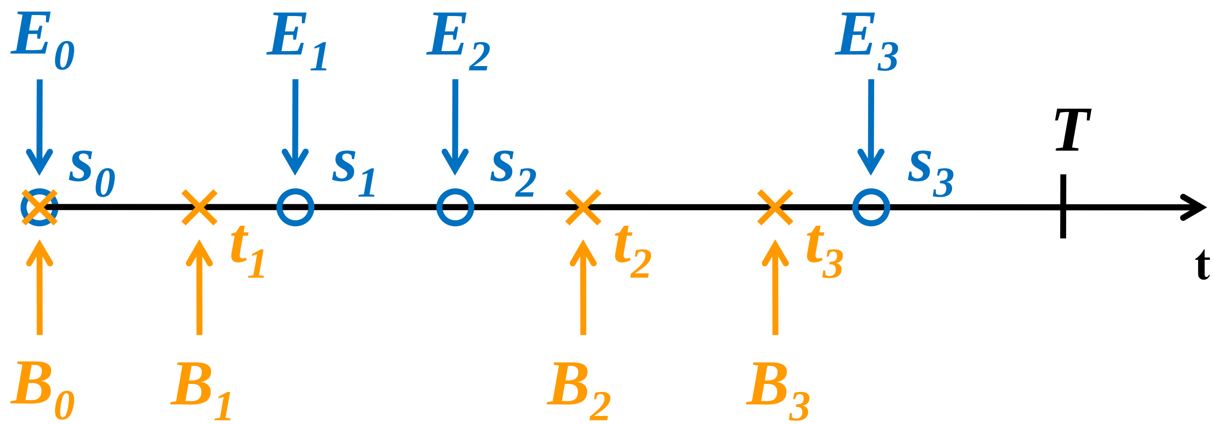
System Model:



- Single communication link, energy harvesting transmitter
- Energy and data arrivals to transmitter
- Transmitting with power ***p*** achieves rate ***r(p)***

Optimal Packet Scheduling

System Model:



- **Energy harvests:** Size E_i at time t_i
- **Data packet arrivals:** Size B_i at time s_i

All arrivals known by transmitter noncausally.

Optimal Packet Scheduling (TCTM)

- **Problem:**

  Find optimal transmission power/rate policy that minimizes transmission time for a known amount of arriving packets.

- What is the minimum T by which we can transmit all packets?: Transmission Completion Time Minimization (TCTM)

- **Constraints:**

  Cannot use energy not harvested yet

  Cannot transmit packets not received yet

Power-Rate Function

- Transmission with power ***p*** yields a rate of ***r(p)***

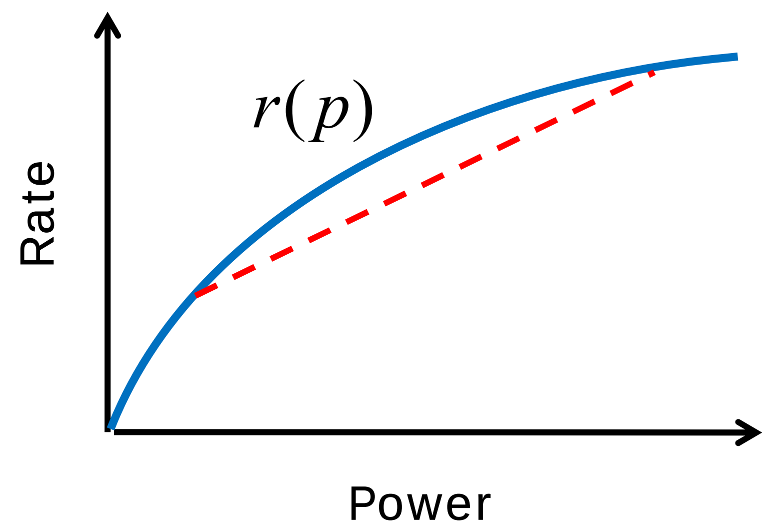
- **Assumptions on r(p):**

  i. $r(0)=0, r(p) \to \infty \text{ as } p \to \infty$

  ii. increases monotonically in p

  iii. strictly concave

  iv. $r(p)$ continuously differentiable



Example: AWGN Channel, $r(P) = \frac{1}{2}\log(1+\frac{P}{N})$

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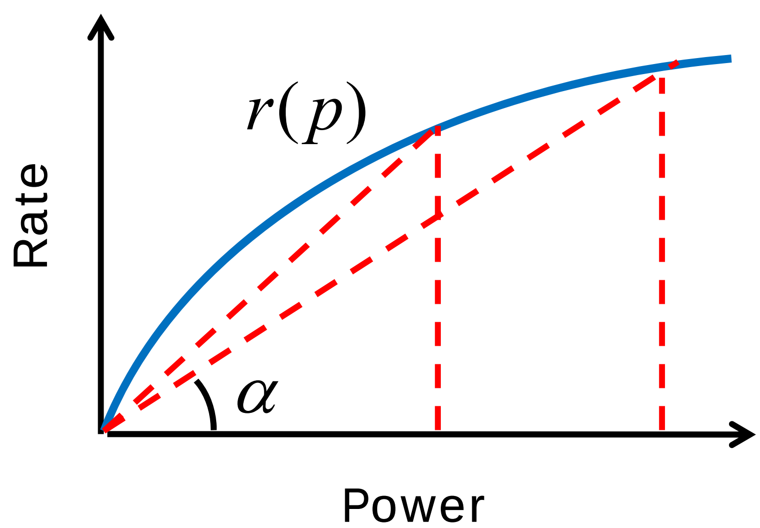
Power-Rate Function

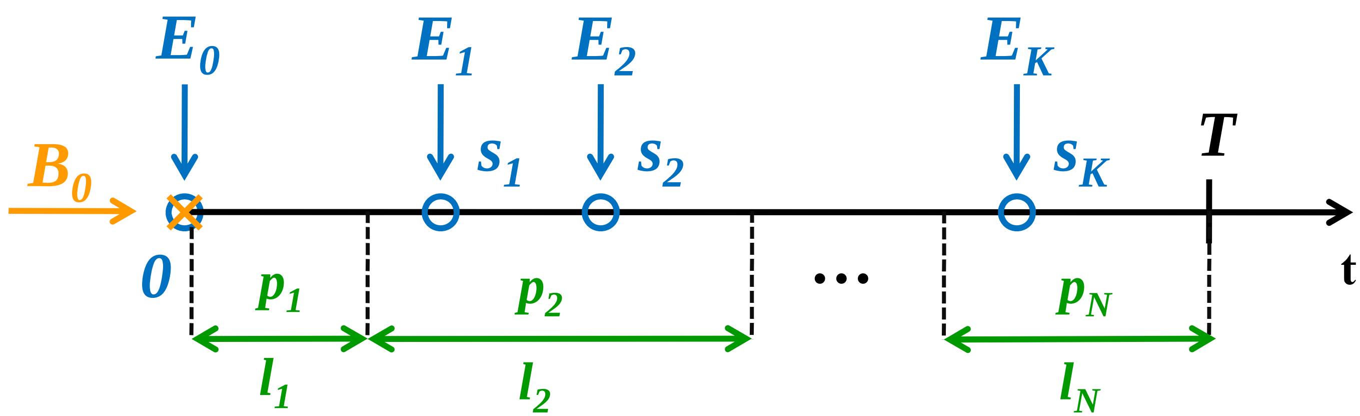
- $r(p)$ strictly concave, increasing, $r(0)=0$ implies

$$\tan(\alpha) = \frac{r(p)}{p} \text{ is monotonically decreasing in } p$$



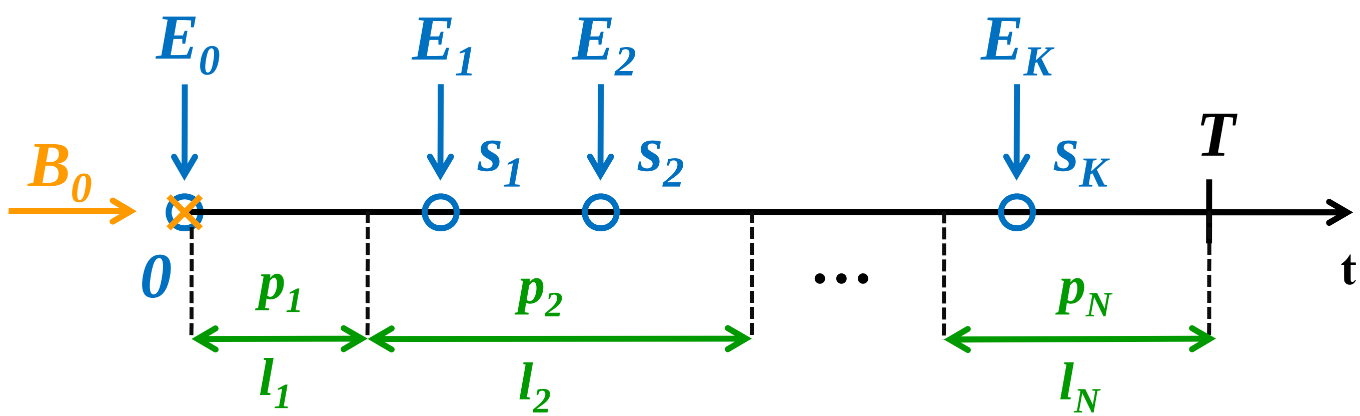
- Given a fixed energy, a longer transmission with lower power departs more bits (a la lazy scheduling)
- Also, $\mathbf{r^{-1}(p)}$ exists and is strictly convex

Scenario I: Packets Ready before Transmission



- **Transmission structure:** Power p_i for duration l_i
- **Harvested Energy:** $E(t)=\sum_{i=1}^{\bar{i}} p_i l_i + p_{\bar{i}+1}\left(t-\sum_{i=1}^{\bar{i}} l_i\right), \quad \bar{i}=\max\left\{i:\sum_{j=1}^{i} l_j \le t\right\}$
- **Departed bits:** $B(t)=\sum_{i=1}^{\bar{i}} r(p_i) l_i + r(p_{\bar{i}+1})\left(t-\sum_{i=1}^{\bar{i}} l_i\right)$

Scenario I: Packets Ready before Transmission



- **Problem Definition:** $\min T$

$$\text{s.t.} \quad E(t) \le \sum_{i:s_i<t} E_i \qquad 0 \le t \le T$$

$$B(T) = B_0$$

Necessary conditions for optimality

- **Lemma 1:** The transmit powers increase monotonically, i.e., $p_1 < p_2 < \ldots < p_N$

Proof: (by contradiction) assume not, i.e., $p_i > p_{i+1}$ for some i

Energy consumed in l_i and l_{i+1} is $p_i l_i + p_{i+1} l_{i+1}$

Consider the following constant power policy:

$$p_i' = p_{i+1}' = \frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}$$

which does not violate energy constraint since $p_i' < p_i$

Necessary conditions for optimality

- **Lemma 1:** The transmit powers increase monotonically, i.e., $p_1 < p_2 < \ldots < p_N$

Proof(cont'd): Transmitted bits then become

$$r_i' \cdot l_i + r_{i+1}' l_{i+1} = r\left(\frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}\right)(l_i + l_{i+1})$$

$$> r(p_i)\frac{l_i}{l_i + l_{i+1}}(l_i + l_{i+1}) + r(p_{i+1})\frac{l_{i+1}}{l_i + l_{i+1}}(l_i + l_{i+1})$$

$$= r(p_i) l_i + r(p_{i+1}) l_{i+1}$$

where inequality is due to **strict concavity of *r(p)***

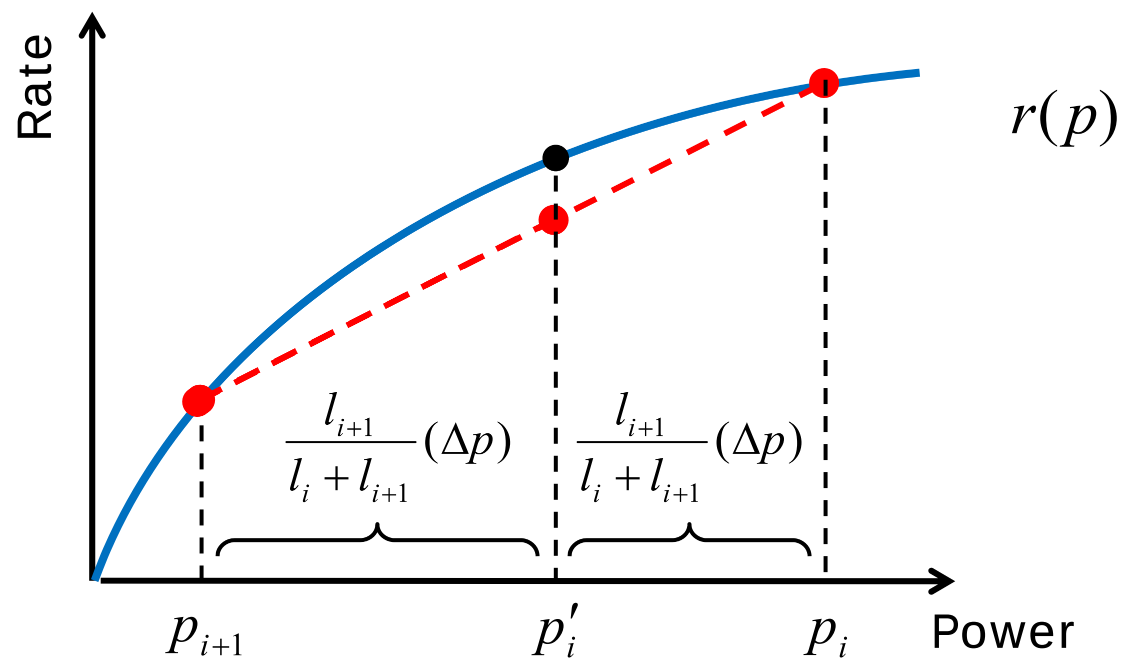
Therefore $p_i > p_{i+1}$ cannot be optimal

Necessary conditions for optimality

- **Lemma 1:** The transmit powers increase monotonically, i.e., $p_1 < p_2 < \ldots < p_N$

Proof(cont'd):

Time-sharing between any two points is strictly suboptimal for concave r(p)



Necessary conditions for optimality

- **Lemma 2:** The transmission power remains constant between energy harvests

Proof: (by contradiction) assume not

Let total consumed energy in epoch $[s_i, s_{i+1}]$ be E_{total}, which is available in energy queue at $t = s_i$

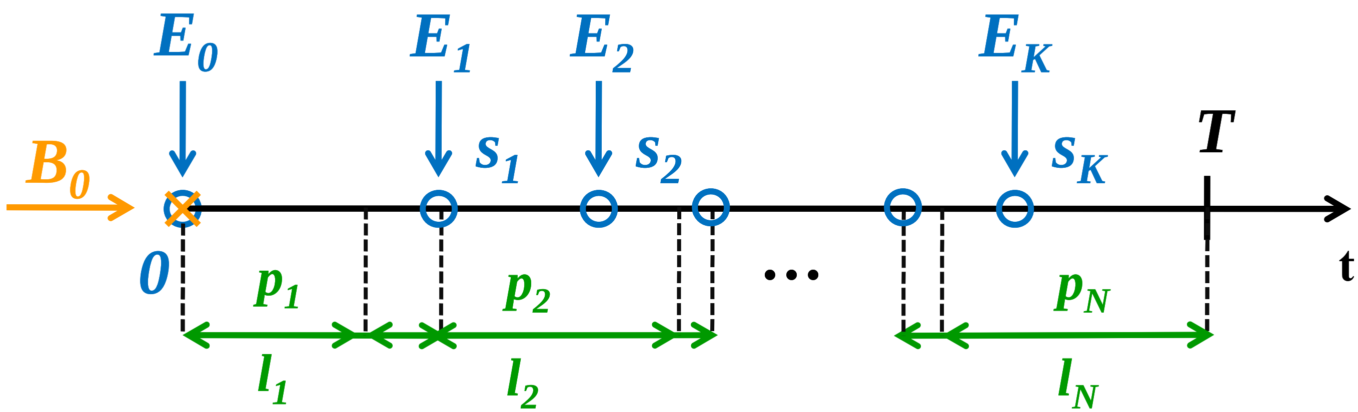
Then a constant power transmission

$$p' = \frac{E_{total}}{s_{i+1} - s_i}, \qquad t \in [s_i, s_{i+1}]$$

is feasible and strictly better than a non-constant transmission

Necessary conditions for optimality

- **Lemma 2:** The transmission power remains constant between energy harvests



Transmission power only changes on s_i

Necessary conditions for optimality

- **Lemma 3:** Whenever transmission rate changes, energy buffer is empty

Proof: (by contradiction) assume not, i.e., $p_i < p_{i+1}$ for some i and energy buffer has Δ energy remaining at time of change.

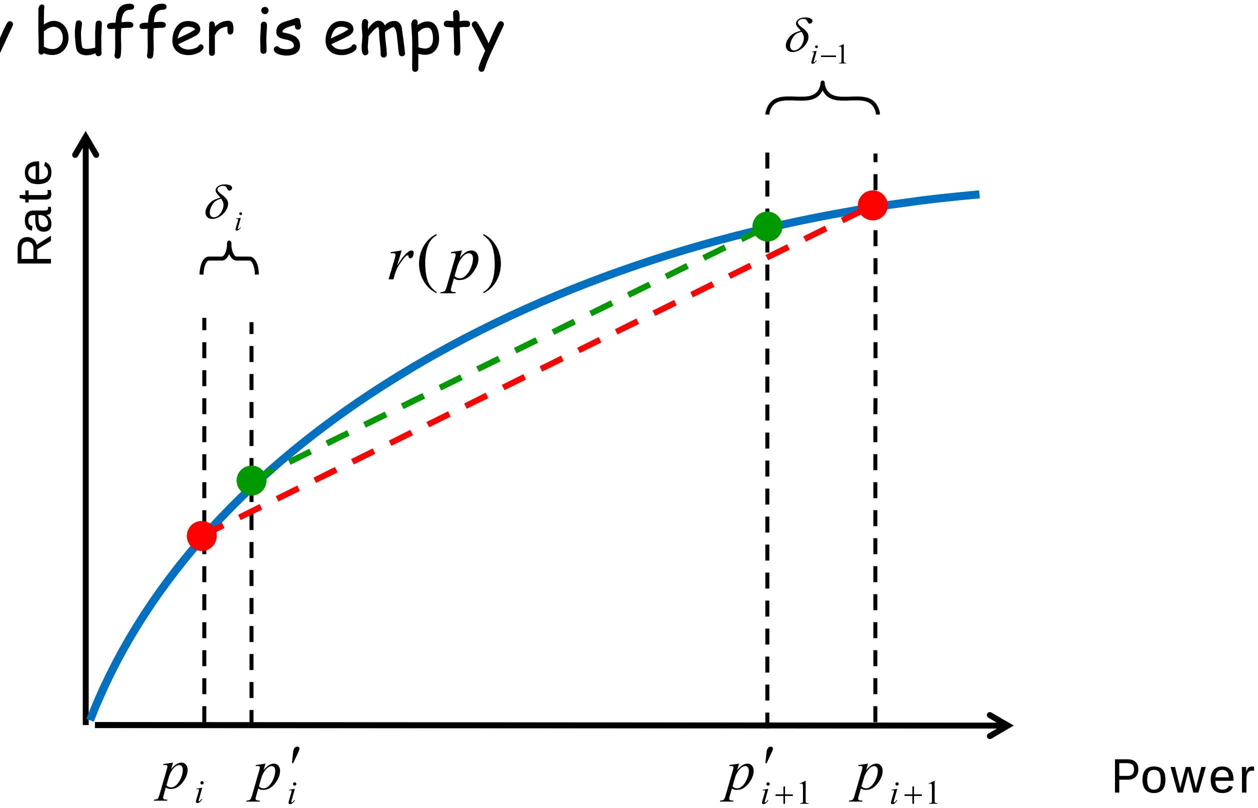
Choose δ_i and δ_{i+1} such that $\delta_i l_i = \delta_{i+1} l_{i+1} \leq \Delta$ and let

$$p_i' = p_i + \delta_i, \qquad p_{i+1}' = p_{i+1} + \delta_{i+1}$$

Since Δ amount of energy has moved from $i+1$ to i, and this was available at the buffer, this policy is feasible

Necessary conditions for optimality

- **Lemma 3:** Whenever transmission rate changes, energy buffer is empty



Necessary conditions for optimality

Summary:

- **L1:** Power only increases
- **L2:** Power constant between arrivals
- **L3:** At time of power change, energy buffer is empty

Conclusion:

For optimal policy, compare and sort (L1) power levels that deplete energy buffer (L3) at arrival instances (L2).

Optimal Policy for Scenario I

For a given B_0 the optimal policy satisfies:

$$\sum_{n=1}^{N} r(p_n) l_n = B_0$$

and has the form

for $n = 1,2,\ldots, N$

$$i_n = \arg \min_{\substack{i: s_i \le T \\ s_i > s_{i_{n-1}}}} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\}$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}}, \qquad l_n = s_{i_n} - s_{i_{n-1}}$$

Algorithm for Scenario I

1. Find minimum number of energy arrivals required $i_{\min}$

   by comparing: $$A_i = r^{-1}\left(\frac{B_0}{s_i}\right)s_i \le \sum_{j=0}^{i-1} E_j$$

2. Find $s_{i_{\min}-1} < T_1 < s_{i_{\min}}$ satisfying $B_0 = r\underbrace{\left(\frac{\sum_{j=0}^{i-1} E_j}{T_1}\right)}_{\tilde{p}_1} s_i \cdot T_1$

3. Set $p_1 = \min\left\{\tilde{p}_1, \left\{\frac{\sum_{j=0}^{i-1} E_j}{s_i}, i = 1 \ldots i_{\min}\right\}\right\}$,

   $l_1 = s_{i_1}$ where i_1 is the minimizer of p_1

4. Repeat starting from s_{i_1}

Illustration – Step 1

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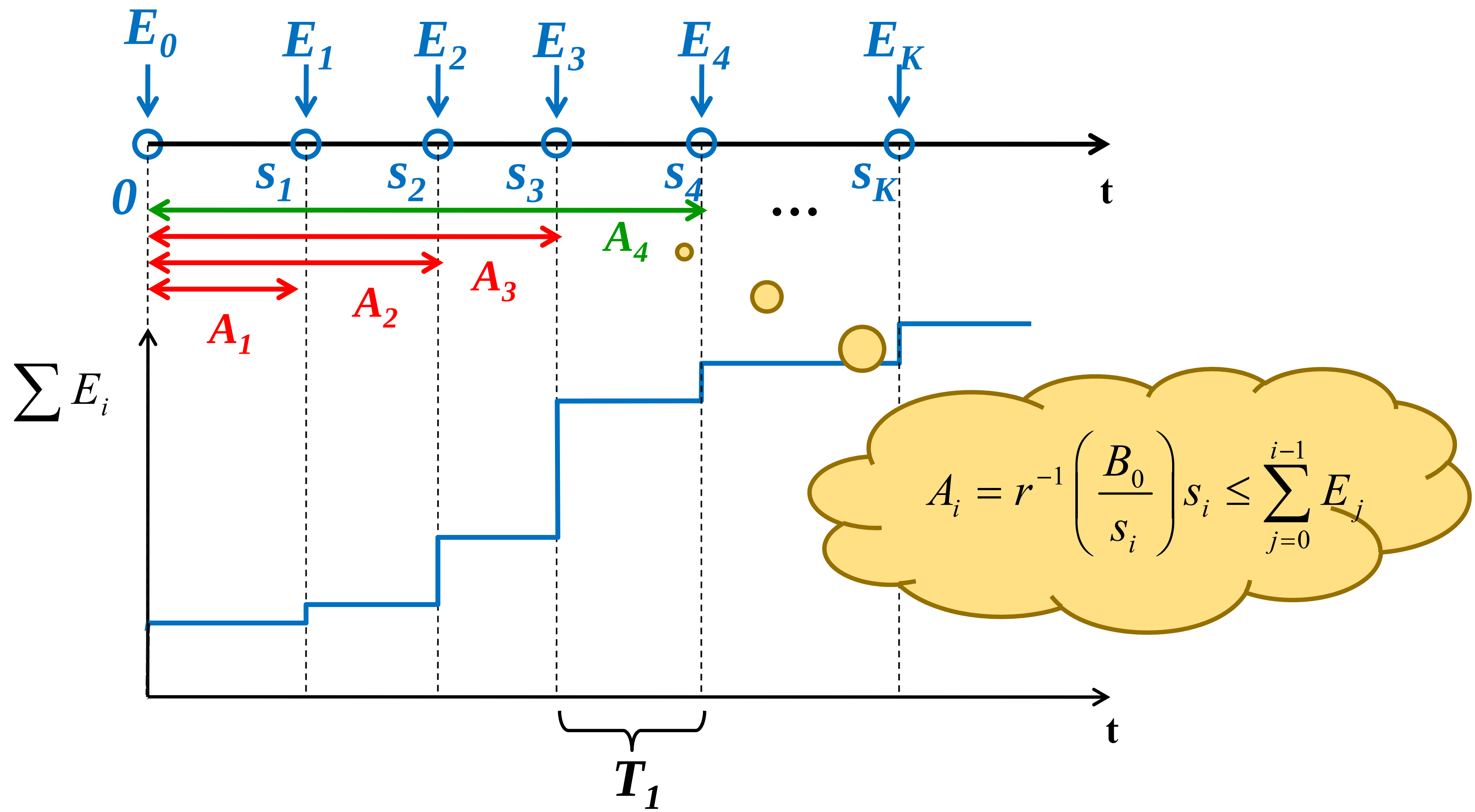

Illustration – Step 2

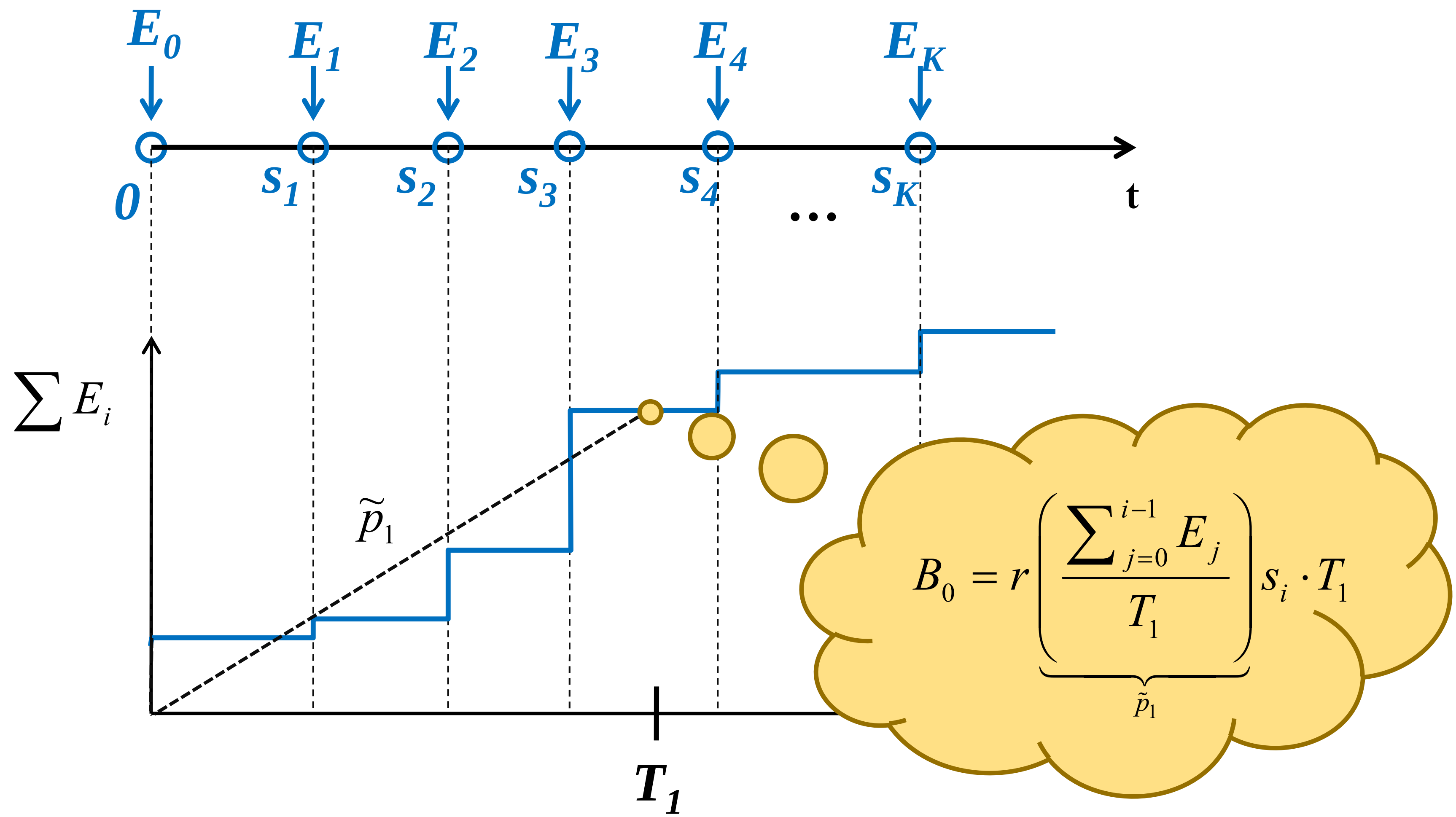

Illustration – Step 3

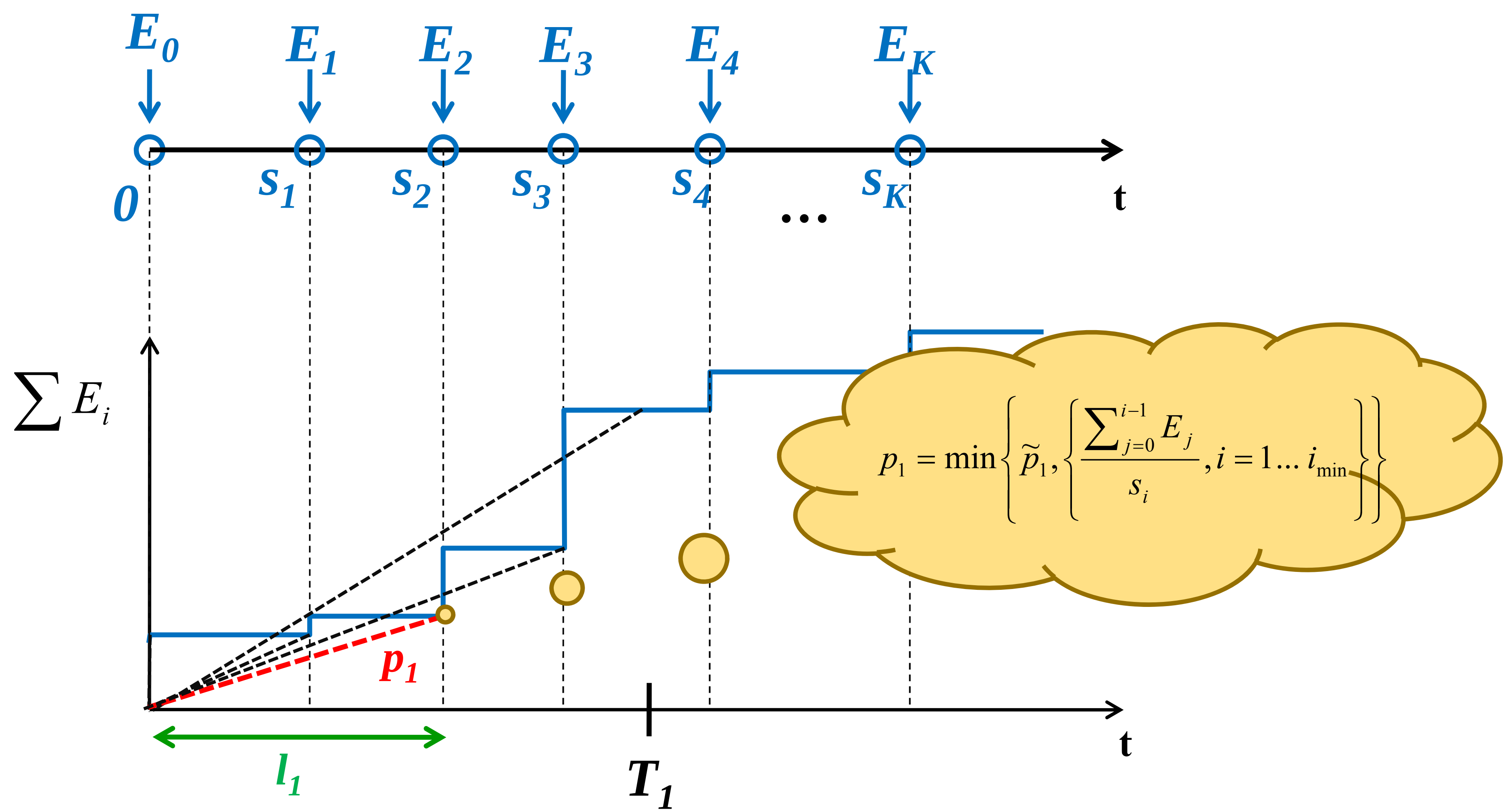

Illustration – Step 4

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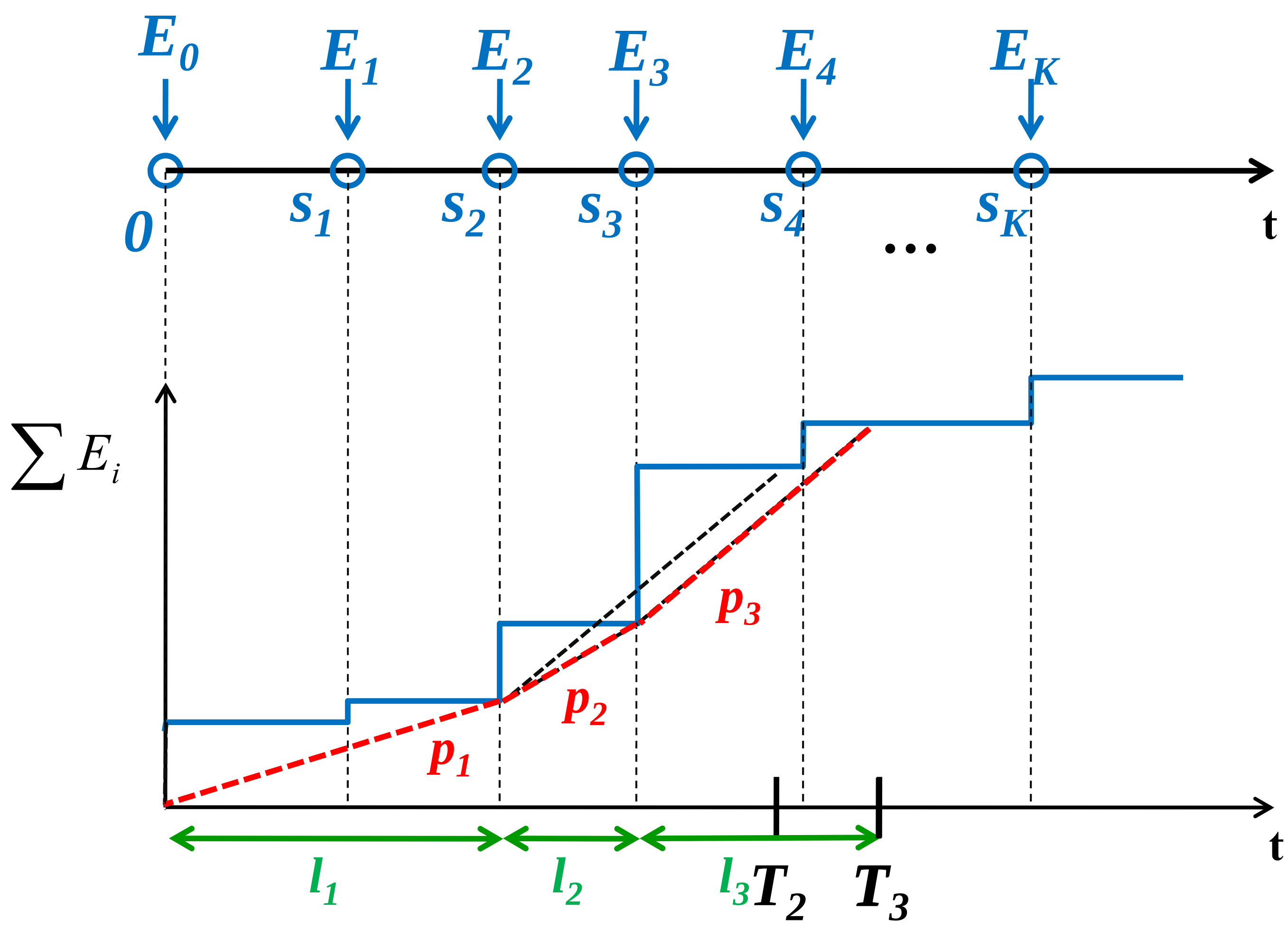
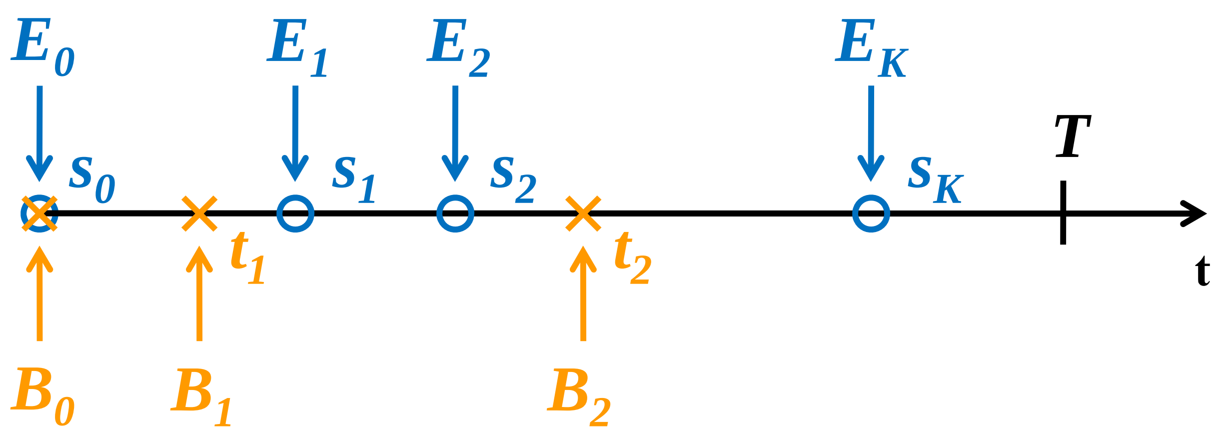


Scenario II: Packets Arrive During Transmission



- **Transmitter cannot depart packets not received yet!**
- ***Additional packet constraints apply***

Scenario II: Packets Arrive During Transmission

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Harvested Energy: $E(t)=\sum_{i=1}^{\bar{i}} p_i l_i + p_{\bar{i}+1}\left(t-\sum_{i=1}^{\bar{i}} l_i\right), \quad \bar{i}=\max\left\{i:\sum_{j=1}^{i} l_j \le t\right\}$

Departed bits: $B(t)=\sum_{i=1}^{\bar{i}} r(p_i) l_i + r(p_{\bar{i}+1})\left(t-\sum_{i=1}^{\bar{i}} l_i\right)$

Problem Definition:

$$\min \quad T$$

$$\text{s.t.} \quad E(t) \le \sum_{i:s_i<t} E_i \qquad 0 \le t \le T$$

Energy Causality

$$B(t) \le \sum_{i:t_i<t} B_i \qquad 0 \le t \le T$$

Packet Causality

$$B(T) = \sum_{i=0}^{M} B_i$$

Necessary conditions for optimality

- **Lemma 4:** Power only increases
- **Lemma 5:** Power constant between 2 arrivals of any kind
- **Lemma 6:** At time of power change

  if $t = s_i$ (energy arrival), energy buffer is empty

  if $t = t_i$ (packet arrival), packet buffer is empty

(Proofs are similar to Lemmas 1-3)

Optimal Policy for Scenario II

The optimal policy satisfies $\sum_{n=1}^{N} r(p_n) l_n = \sum_{n=1}^{M} B_i$

and has the form

$$r(p_1) = \min_{i:u_i \le T} \left\{ g\left(\frac{\sum_{j:s_j < u_i} E_j}{u_i} \right), \frac{\sum_{j:t_j < u_i} B_j}{u_i} \right\}$$

where $\{u_i\}$ is the ordered combination of $\{s_i\}$ and $\{t_i\}$ and subsequent rates are found iteratively

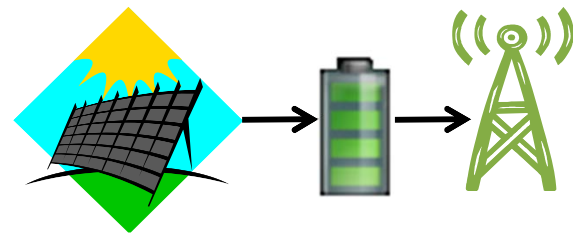
Short-term Throughput Maximization (STTM) [Tutuncuoglu-Yener 2010]

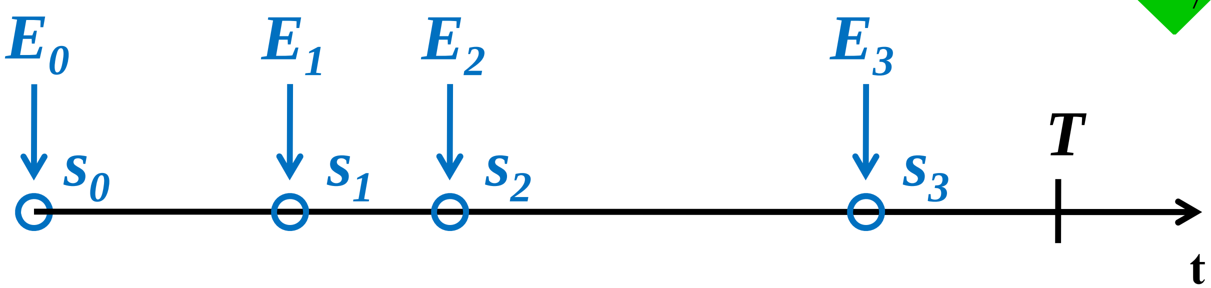
- Maximize the throughput of an energy harvesting transmitter by **deadline** T.
- Find optimal power allocation/transmission policy that departs maximum number of bits in a given duration.

- Up to a certain amount of energy can be stored by the transmitter → **BATTERY CAPACITY**

System Model

- Energy arrivals of energy E_i at times s_i





- Arrivals known **non-causally** by transmitter,
- Stored in a **finite battery** of capacity $E_{\max}$,
- Design parameter: **power** $\rightarrow$ **rate** $r(p)$.

Notations and Assumptions

- Power allocation function: $p(t)$
- Energy consumed: $\int_0^T p(t)dt$
- Short-term throughput: $\int_0^T r(p(t))dt$

- Power-rate function $r(p)$: Strictly concave in p
- Overflowing energy is lost (not optimal)

Energy Constraints

(Energy arrivals of E_i at times s_i)

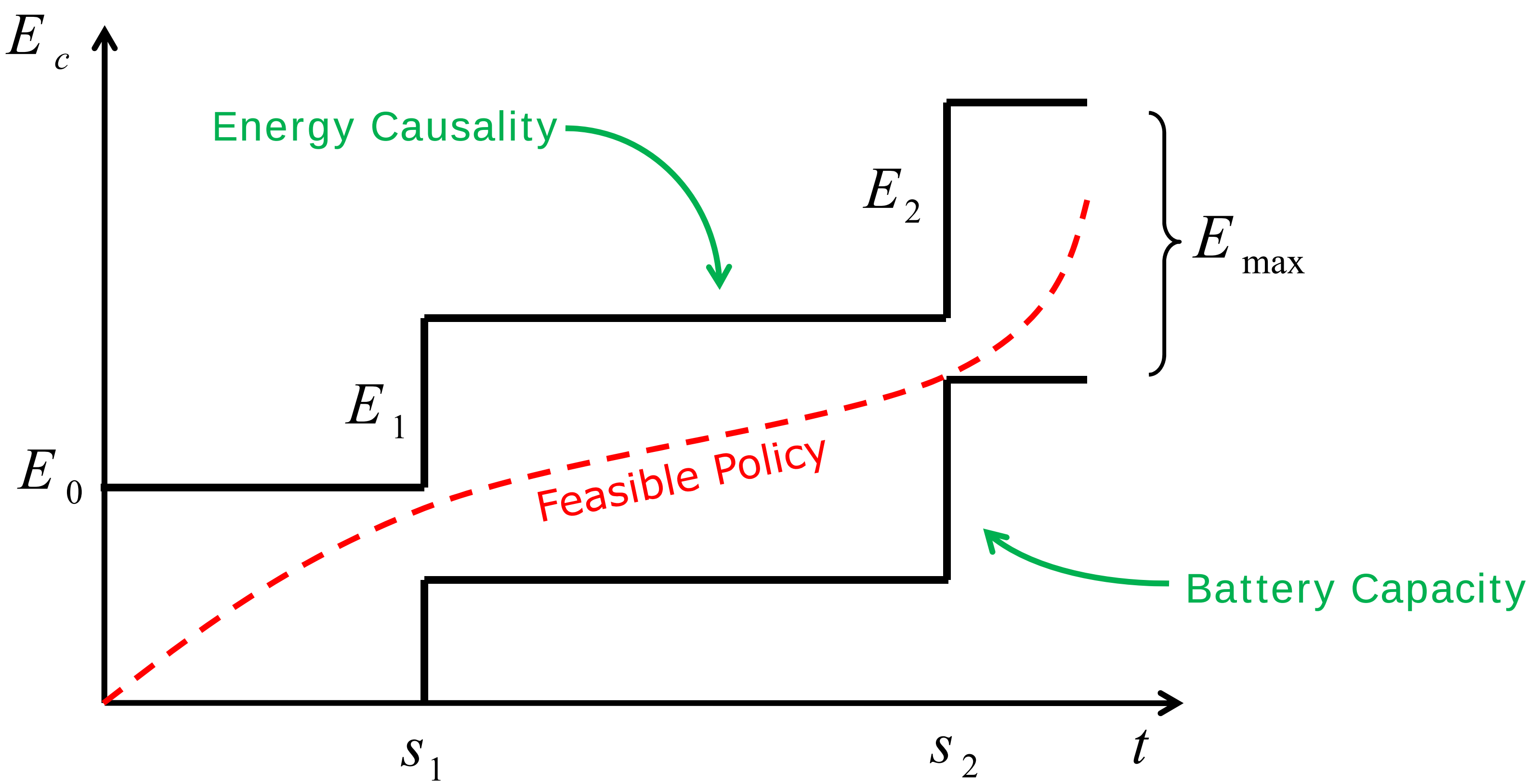
- **Energy Causality:** $\sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \geq 0 \qquad s_{n-1} \leq t' \leq s_n$

- **Battery Capacity:** $\sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{\max} \qquad s_{n-1} \leq t' \leq s_n$

- **Set of energy-feasible power allocations**

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \leq \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{\max}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\}$$

Energy "Tunnel"



Short-Term Throughput Maximization Problem

- Maximize total number of transmitted bits by deadline **T**

$$\max_{p(t)} \int_0^T r(p(t))dt, \quad s.t. \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

- Convex constraint set, concave maximization problem

Necessary conditions for optimality of a transmission policy

- **Property 1:** Transmission power remains constant between arrivals.
- **Property 2:** Battery never overflows.

Proof: Assume an energy of Δ overflows at time τ

Define $$p'(t) = \begin{cases} p(t) + \dfrac{\Delta}{\delta} & [\tau - \delta, \tau] \\ p(t) & else \end{cases}$$

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\,dt$ since $r(p)$ is increasing in p

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

Proof: Let $p(\tau^-) < p(\tau^+)$

Define $p'(t) = \begin{cases} p(t) - \varepsilon & [\tau, \tau + \delta] \\ p(t) + \varepsilon & [\tau - \delta, \tau] \\ p(t) & else \end{cases}$ (Feasible unless battery is depleted)

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\,dt$ due to strict concavity of $r(p)$

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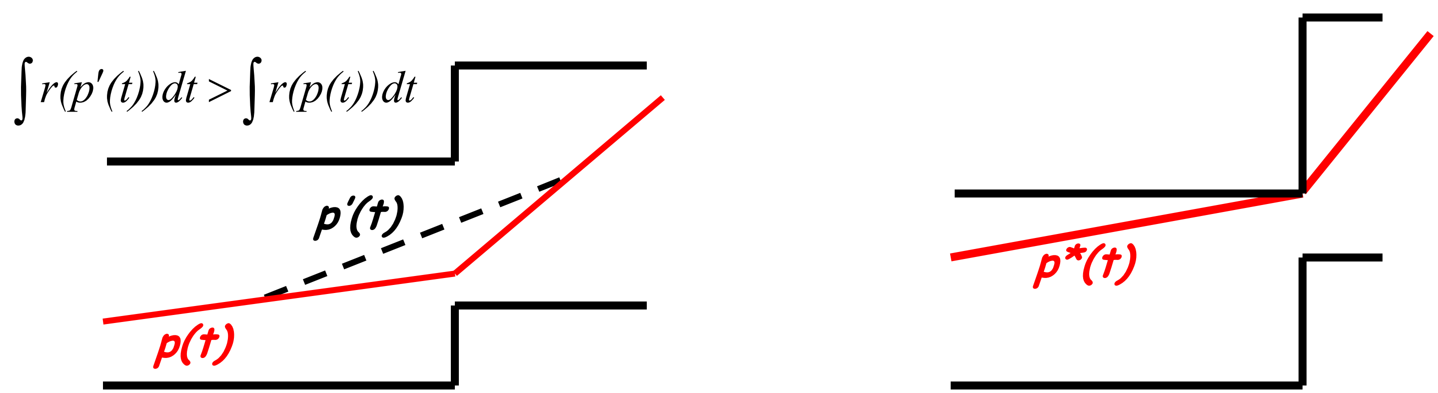
Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.



Policy can be improved **Policy cannot be improved**

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

Proof: Let $p(\tau^-) > p(\tau^+)$

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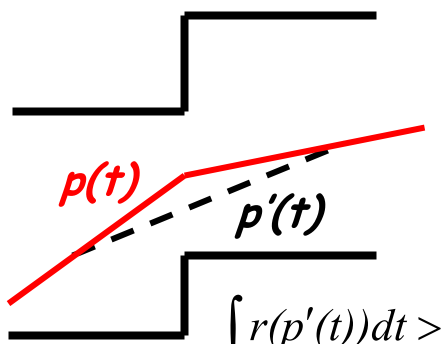
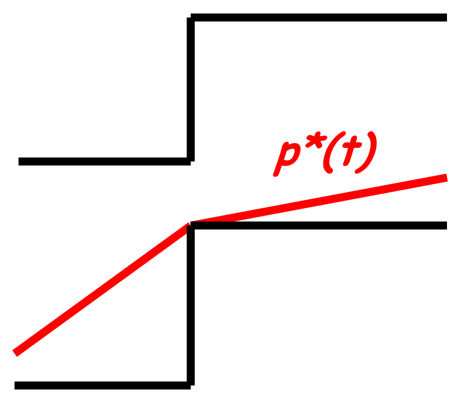
Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. <u>Conversely, power level decreases at an energy arrival instant only if battery is full.</u>



$$\int r(p'(t))dt > \int r(p(t))dt$$

Policy can be improved



Policy cannot be improved

Necessary conditions for optimality of a transmission policy

- **Property 4:** Battery is depleted at the end of transmission.

Proof: Assume an energy of Δ remains after p(t)

Define $$p'(t) = \begin{cases} p(t) + \dfrac{\Delta}{\delta} & [T-\delta, T] \\ p(t) & else \end{cases}$$

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\,dt$ since $r(p)$ is increasing

Necessary Conditions for Optimality

Implications of Properties 1-4:

- **Structure of optimal policy:** (Property 1)

$$p(t)=\begin{cases} p_n & i_{n-1}<t<i_n \\ 0 & t>T \end{cases}, \qquad i_n \in \{s_n\}, \quad p_n \text{ constant}$$

- For power to increase or decrease, policy must meet the upper or lower boundary of the tunnel respectively (Property 3)
- At termination step, battery is depleted (Property 4).

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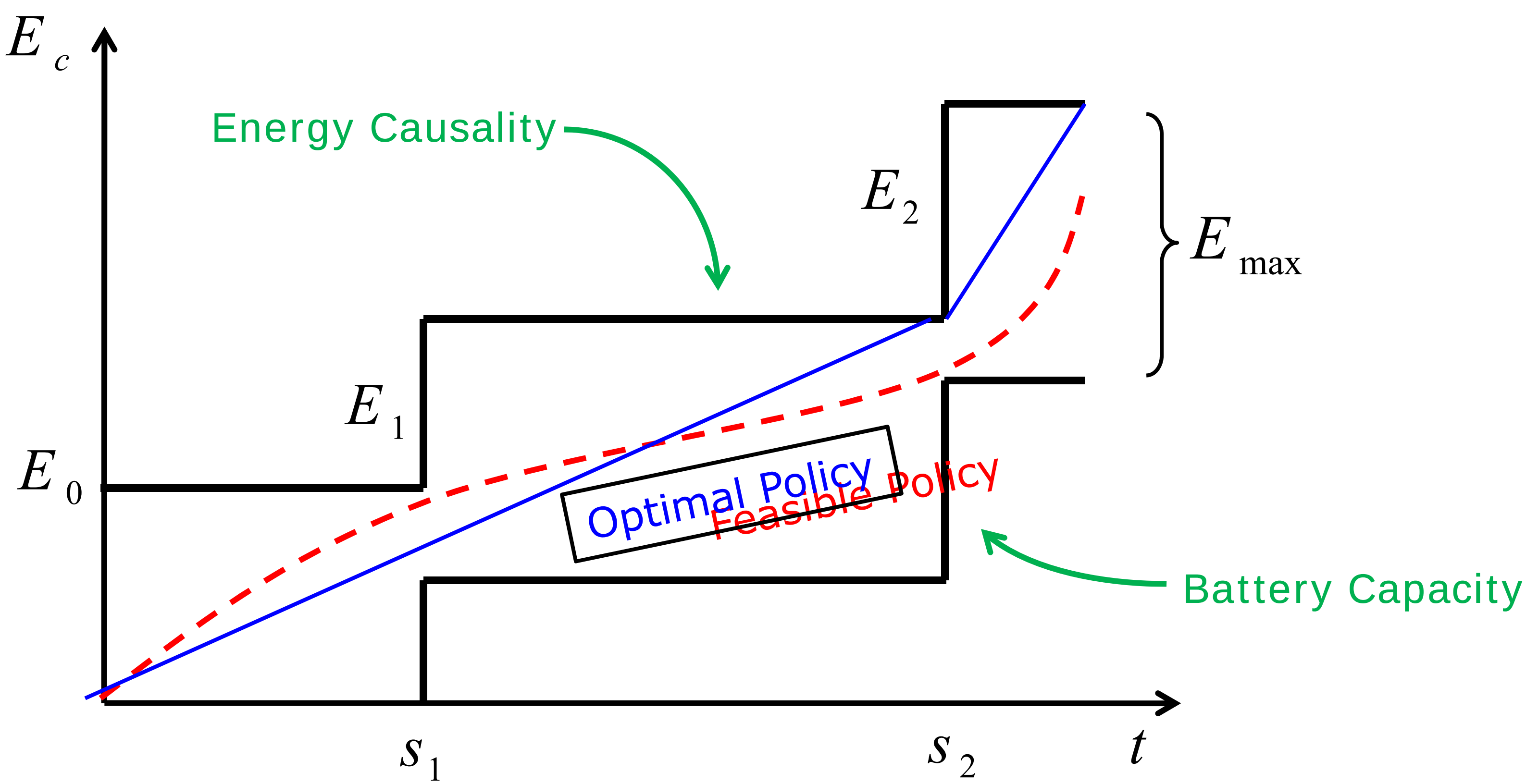
Energy "Tunnel"



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Shortest Path Interpretation

- **Optimal policy is identical for any concave power-rate function!**
- Let $r(p) = -\sqrt{p^2+1}$, then the problem solved becomes:

$$\max_{p(t)} \int_0^T -\sqrt{p^2(t)+1}\, dt \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

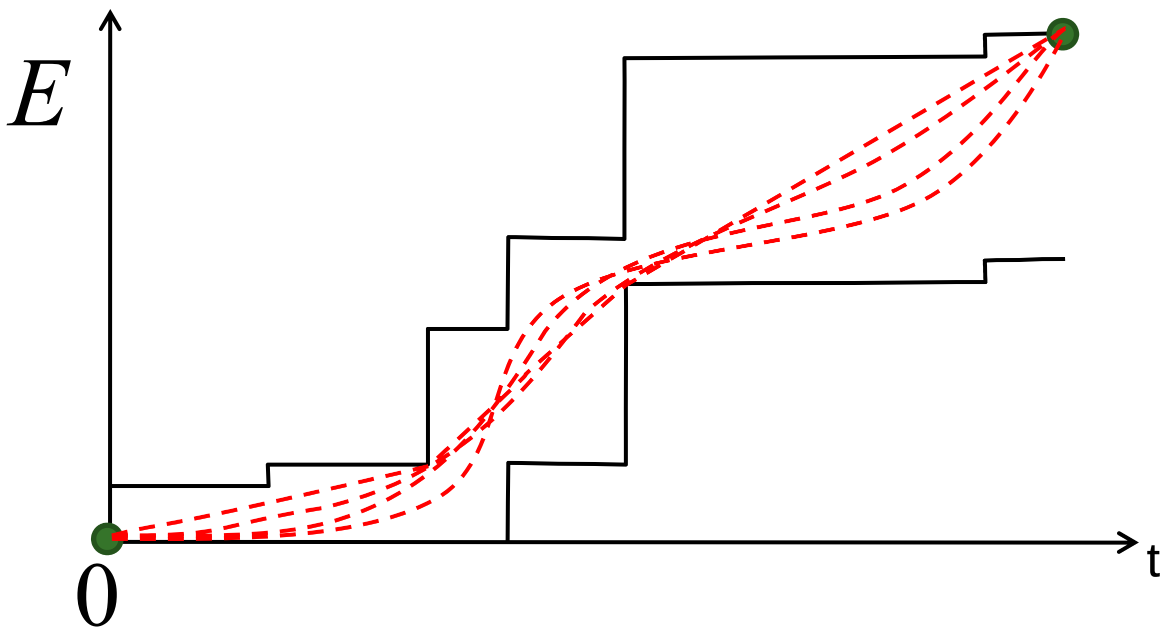
$$= \min_{p(t)} \underbrace{\int_0^T \sqrt{p^2(t)+1}\, dt}_{} \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

length of policy path in energy tunnel

$\Rightarrow$ The **throughput maximizing policy** yields the **shortest path** through the energy tunnel for any concave power-rate function.

Shortest Path Interpretation

- **Property 1:** Constant power is better than any other alternative
- **Shortest path** between two points is a line (constant slope)



Throughput Maximizing Algorithm (TMA)

- Knowing the structure of the policy, we can construct an iterative algorithm to get the tightest string in the tunnel.
- Note: After a step (p_1, i_1) is determined, the rest of the policy is the solution to a ***shifted problem*** with shifted arrivals and deadline:

$$E_0' = \sum_{k=0}^{n_1} E_k - i_1.p_1, \quad T' = T - i_1, \quad n'_{\max} = n_{\max} - n_1,$$

$$E_n' = E_{n+n_1}, \quad s_n' = s_{n+n_1} - i_1, \quad \text{for } n = 0, \ldots, n'_{\max}$$

- **Essentially, the algorithm compares and find the tightest segment that hits the upper or lower wall staying feasible all along.**

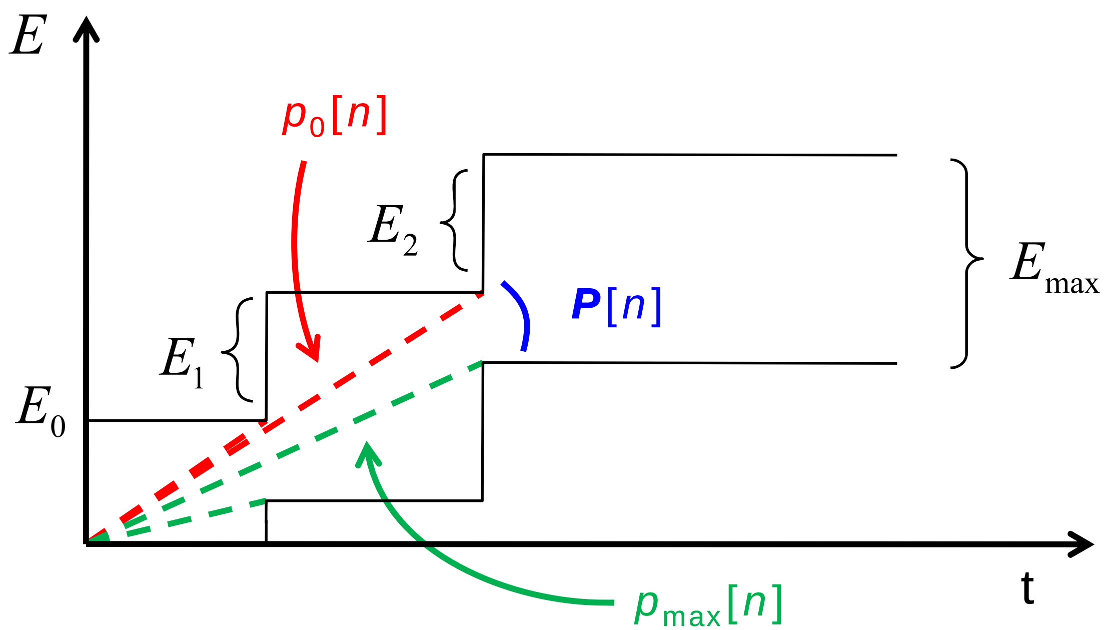
Throughput Maximizing Algorithm (TMA)

$$p_{\max}[n] = \max\left\{\frac{\sum_{k=0}^{n} E_k - E_{\max}}{s_n}, 0\right\}$$

$$p_0[n] = \frac{\sum_{k=0}^{n-1} E_k}{s_n}$$

$$\mathbf{P}[n] = [p_{\max}[n], p_0[n]]$$

$$\mathbf{P}[n_{\max}] = \{p_0[n_{\max}]\}$$



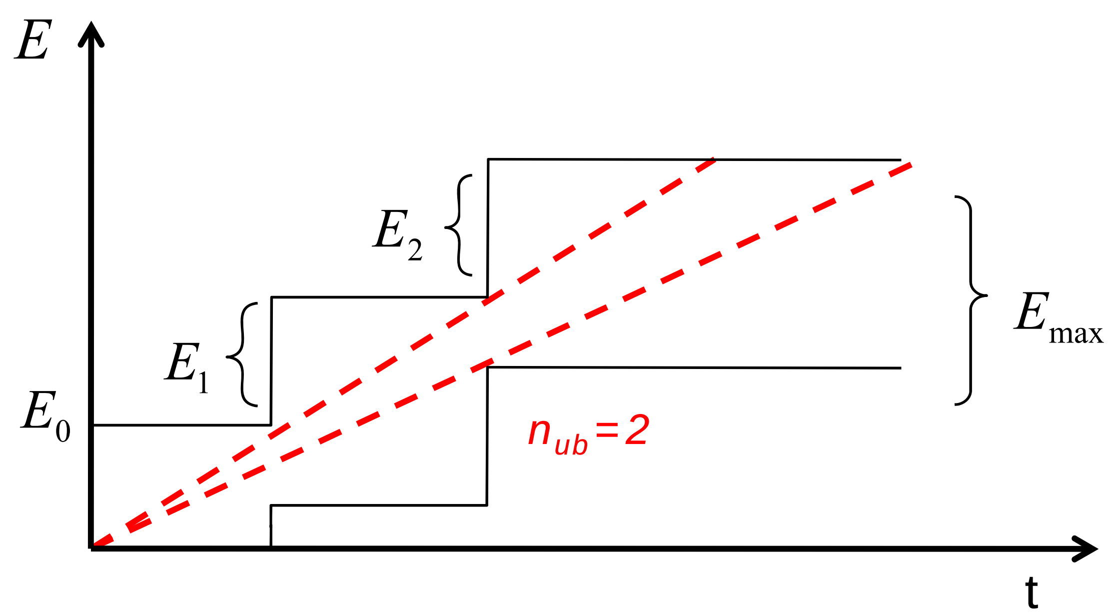
Throughput Maximizing Algorithm (TMA)

$$n_{ub} = \max\{n \mid \bigcap_{k=1}^{n} \mathbf{P}[k] \neq \varnothing, n = 1,2,\ldots,n_{\max}\}$$

(Upper bound for the duration of the first step)

- The **transmission power must change** before arrival n_{ub+1} to stay in the feasible tunnel

$\Longrightarrow$ At or before n_{ub}, battery must be **empty or full** to allow the necessary change. (Prop. 3)



Throughput Maximizing Algorithm (TMA)

1. Find n_{ub} . If $n_{ub} = n_{\max}$ terminate with power $(\sum_{k=0}^{n_{\max}} E_k \,)/T$
2. Determine relation between $\mathbf{P}[n_{ub}+1]$ and $\bigcap_{k=0}^{n_{ub}} \mathbf{P}[k]$
3. Transmit based on the outcome of step 2 with:

$$n_1 = \max\{n \mid p_0[n] \in \textstyle\bigcap_{k=0}^{n} \mathbf{P}[k]\}$$
$$p_1 = p_0[n_1]$$
$$i_1 = s_{n_1}$$

$$n_1 = \max\{n \mid p_{\max}[n] \in \textstyle\bigcap_{k=0}^{n} \mathbf{P}[k]\}$$
$$p_1 = p_{\max}[n_1]$$
$$i_1 = s_{n_1}$$

4. Repeat for shifted problem with updated parameters:

$$E_0' = \sum_{k=0}^{n_1} E_k - i_1.p_1, \quad T' = T - i_1, \quad n'_{\max} = n_{\max} - n_1,$$
$$E_n' = E_{n+n_1}, \quad s_n' = s_{n+n_1} - i_1, \quad \text{for } n = 0,\ldots,n'_{\max}$$

Alternative Solution

- Transmission power constant within each epoch:

$$p(t) = \{ p_i \qquad t \in \text{epoch } i, \; i = 1,\ldots, N + M + 1\}$$

- STTM problem expressed with above notation

$$\max_{p_i} \sum_{i=1}^{M+N+1} L_i . r(p_i)$$

(L_i: length of epoch i)

$$s.t. \quad 0 \le \sum_{i=1}^{l} E_i - L_i p_i \le E_{\max} \quad \forall l$$

Energy constraints: sufficient to check arrivals only

Water-filling approach

- Lagrangian function for STTM

$$\max_{p_i} \sum_{i=1}^{M+N+1} L_i.r(p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right)$$

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0 \quad \forall j$$

$$\mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0 \quad \forall j$$

(Complementary slackness conditions)

- KKT

$$\nabla \left(\sum_{i=1}^{M+N+1} L_i.r(p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) \right) = 0 \text{ at } p = p^*$$

Water-filling approach

- Gradient for kth component

$$\nabla_k\left(\sum_{i=1}^{M+N+1} L_i.r(p_i)-\sum_{j=1}^{M+N+1}\lambda_j\left(\sum_{i=1}^{j} L_i p_i - E_i\right)-\sum_{j=1}^{M+N+1}\mu_j\left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max}\right)\right)=0 \quad \forall n$$

$$=\sum_{i=1}^{M+N+1} L_i.\underbrace{\nabla_k r(p_i)}_{\frac{\delta_{(i-k)}}{1+p_i}}-\sum_{j=1}^{M+N+1}\lambda_j\underbrace{\left(\sum_{i=1}^{j} L_i(\nabla_k p_i)\right)}_{\left\{\begin{array}{ll} L_k & if\ j>k \\ 0 & if\ j<k\end{array}\right\}}-\sum_{j=1}^{M+N+1}\mu_j\underbrace{\left(\sum_{i=1}^{j} L_i(\nabla_k p_i)\right)}_{\left\{\begin{array}{ll} L_k & if\ j>k \\ 0 & if\ j<k\end{array}\right\}}=0$$

$$=L_k\cdot\frac{1}{1+p_k^*}-L_k\sum_{j=k}^{M+N+1}\lambda_j-L_k\sum_{j=k}^{M+N+1}\mu_j=0$$

$$=>\frac{1}{1+p_k^*}=\sum_{j=k}^{M+N+1}(\lambda_j-\mu_j)$$

$$\boxed{=>p_k^*=\frac{1}{\sum_{j=k}^{M+N+1}(\lambda_j-\mu_j)}-1 \qquad \text{(Water Filling)}}$$

Water-filling approach

- Complementary Slackness Conditions:

$$\lambda_j\left(\sum_{i=1}^{j} L_i p_i - E_i\right) = 0 \quad \forall j$$

$$\mu_j\left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max}\right) = 0 \quad \forall j$$

λ_j's are positive only when battery is empty $\left(\sum_{i=1}^{j} L_i p_i - E_i\right) = 0$

μ_j's only positive only when battery is full $\left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max}\right) = 0$

$$p_k^* = \frac{1}{\sum_{j=k}^{M+N+1}(\lambda_j - \mu_j)} - 1$$

decreases at a positive μ_j
increases at a positive λ_j

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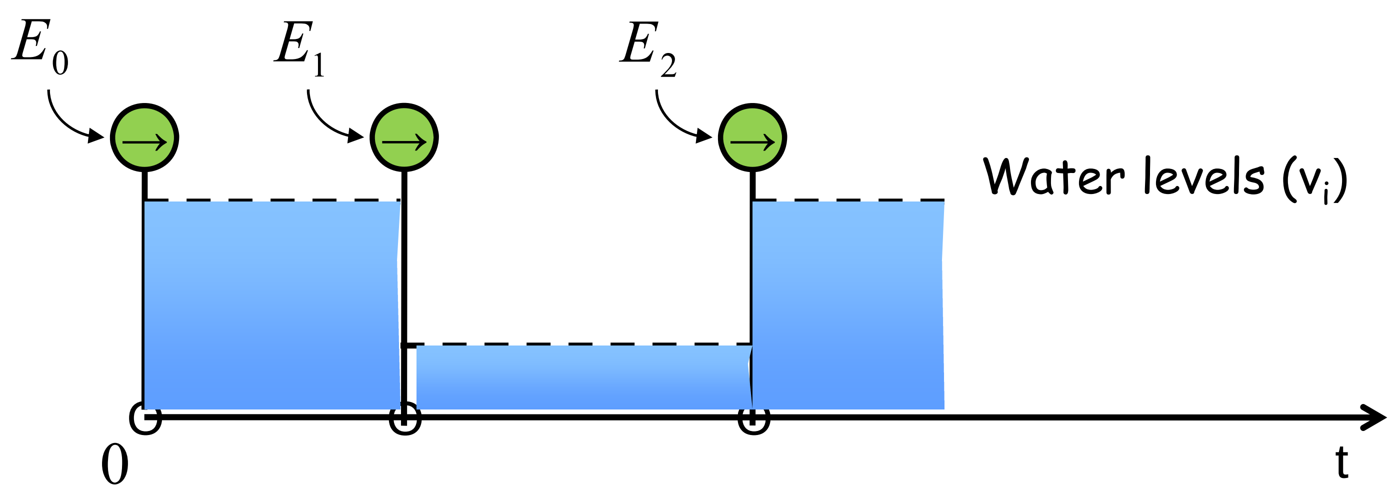
Directional Water-Filling

- Harvested energies filled into epochs individually



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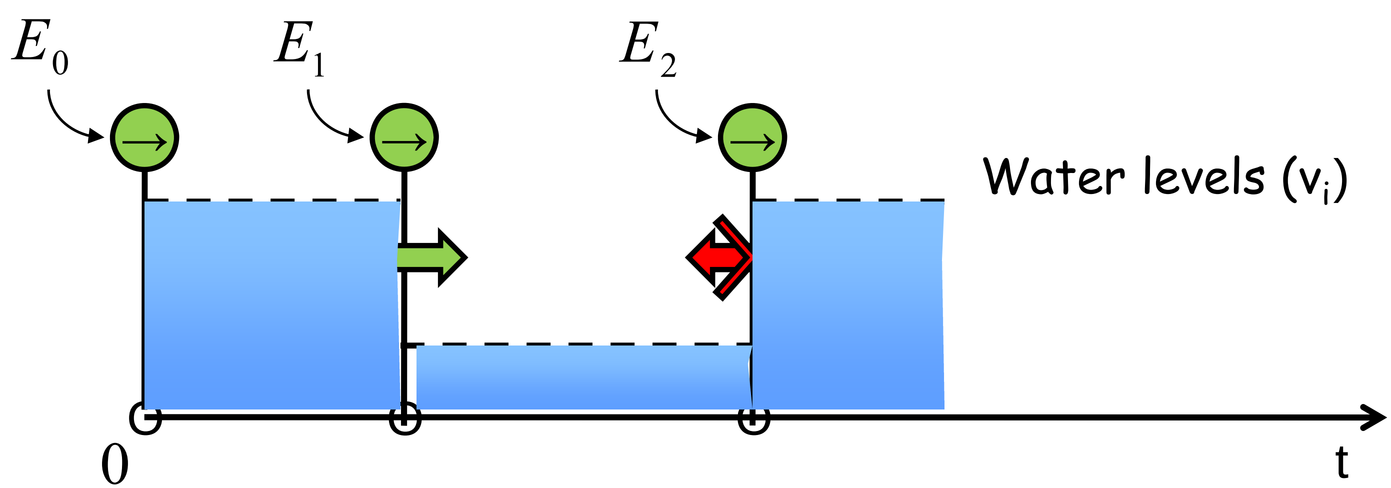
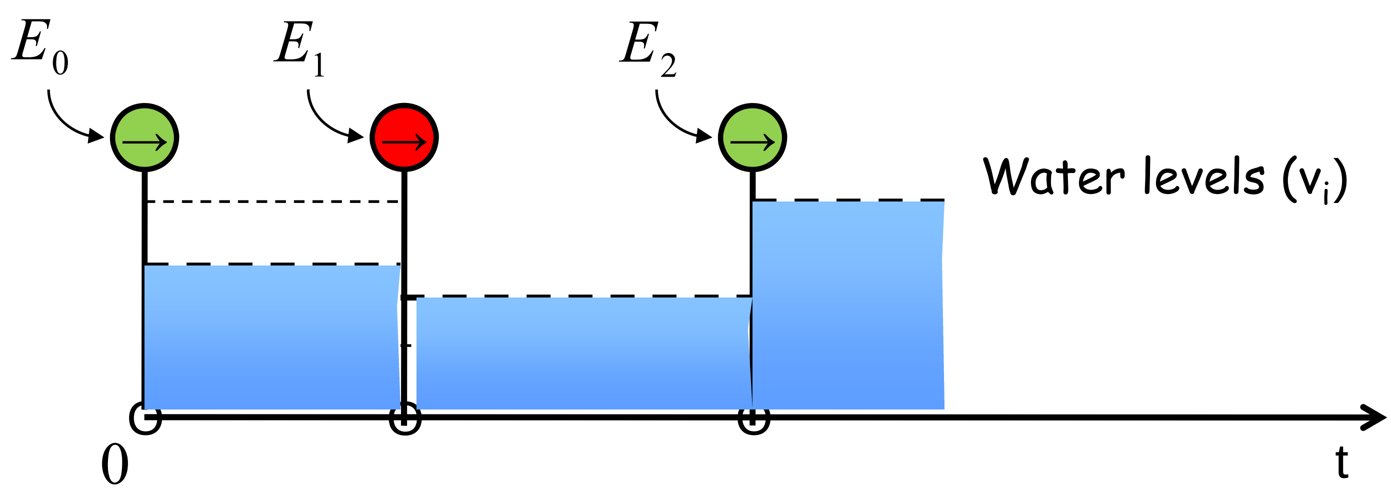
Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time



Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time
  - **Battery Capacity**: water-flow limited to E_{max} by taps



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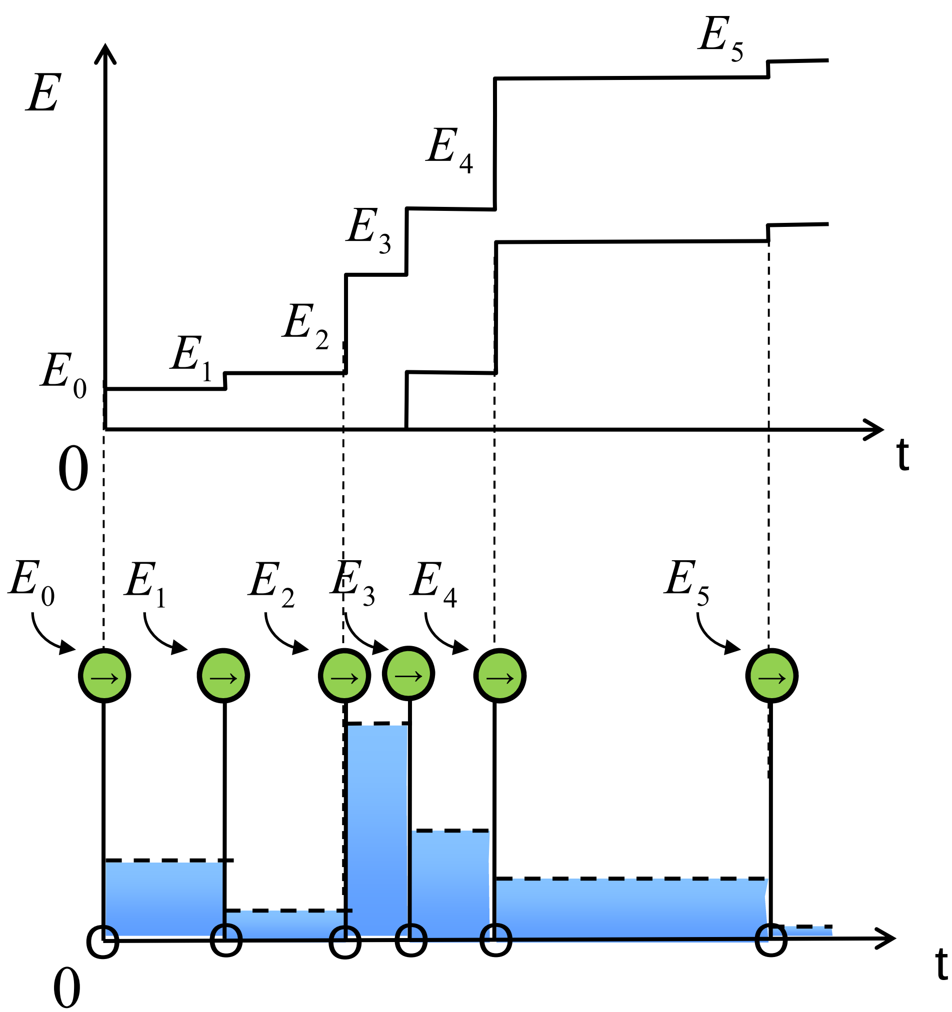
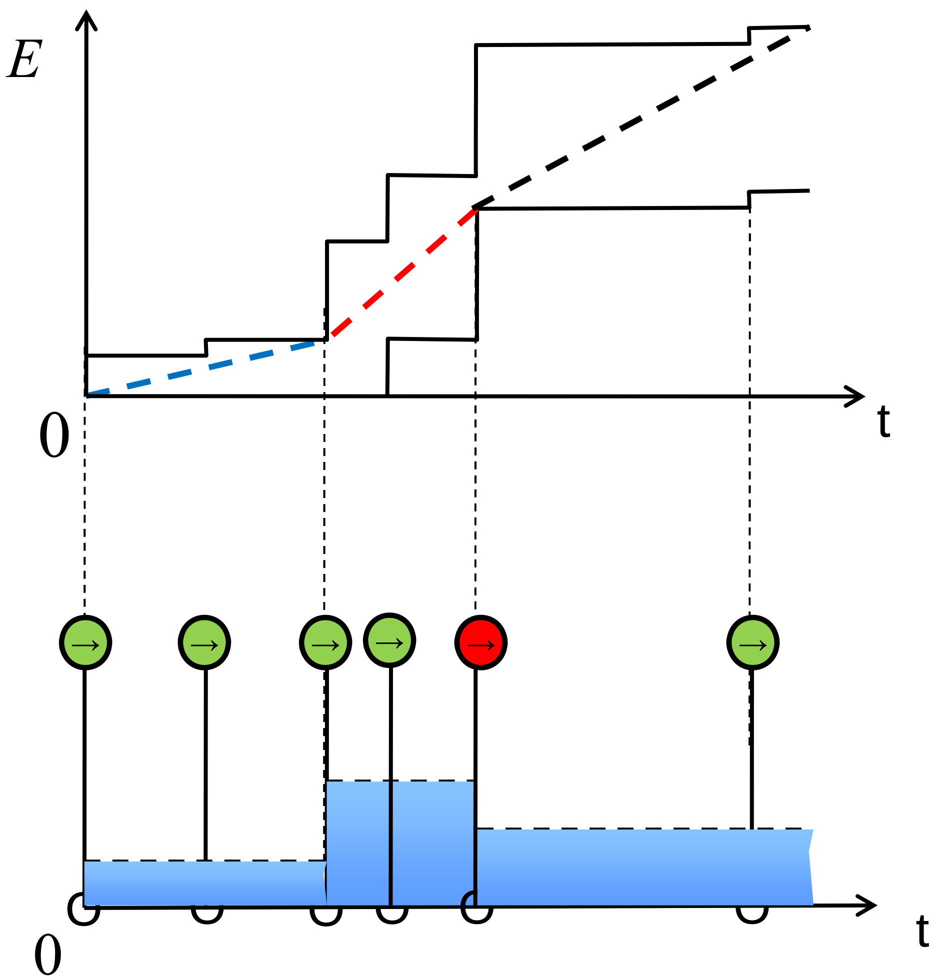
Directional Water-Filling



- **Energy tunnel** and **directional water-filling** approaches yield the same policy

Directional Water-Filling



- Energy tunnel and directional water-filling approaches yield the same policy

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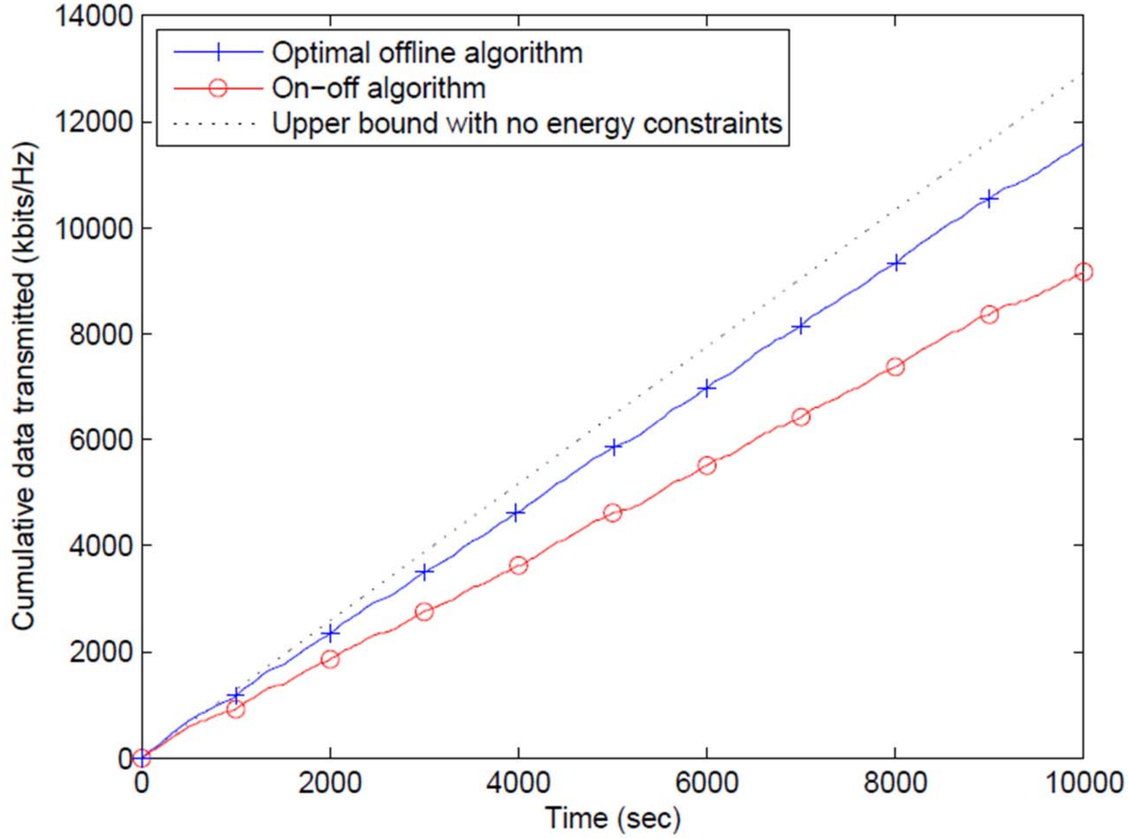
Simulation Results



- Improvement of optimal algorithm over an *on-off transmitter* in a simulation with truncated Gaussian arrivals.

Transmission Completion Time Minimization with Finite Battery [Tutuncuoglu, Yener 2010]

- Given the total number of bits to send as B, finalize the transmission in the shortest time possible.

$$\min_{p(t)} T \quad s.t. \quad B - \int_0^T r(p(t))dt \le 0, \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

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Relationship of STTM and TCTM problems

- Lagrangian dual of TCTM problem becomes:

$$\max_{u\geq 0}\left(\min_{p(t)\in\mathfrak{P},T} T+u\left(B-\int_0^T r(p(t))dt\right)\right)$$

$$=\max_{u\geq 0}\left(\min_{T}\left(T+uB-u.\underbrace{\boxed{\max_{p(t)\in\mathfrak{P}}\int_0^T r(p(t))dt}}_{\text{STTM problem for deadline } T}\right)\right)$$

Relationship of STTM and TCTM problems

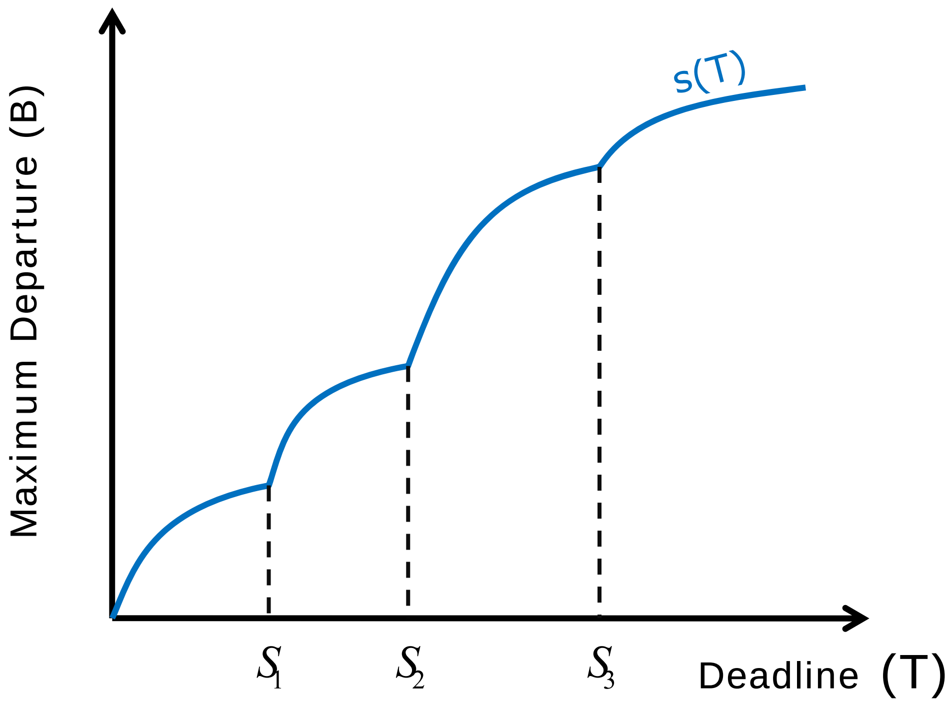
- Optimal allocations are identical:

STTM's solution for deadline T departing B bits	$\equiv$	TCTM's solution for departing B bits in time T

- STTM solution can be used to solve the TCTM problem

Maximum Service Curve

$$s(T) = \max_{p(t)} \int_0^T r(p(t))\,dt, \quad s.t. \quad p(t) \in \mathfrak{P}$$



- Maximum number of bits that can be sent in time T.
- Each point calculated by solving the corresponding STTM problem.

Maximum Service Curve

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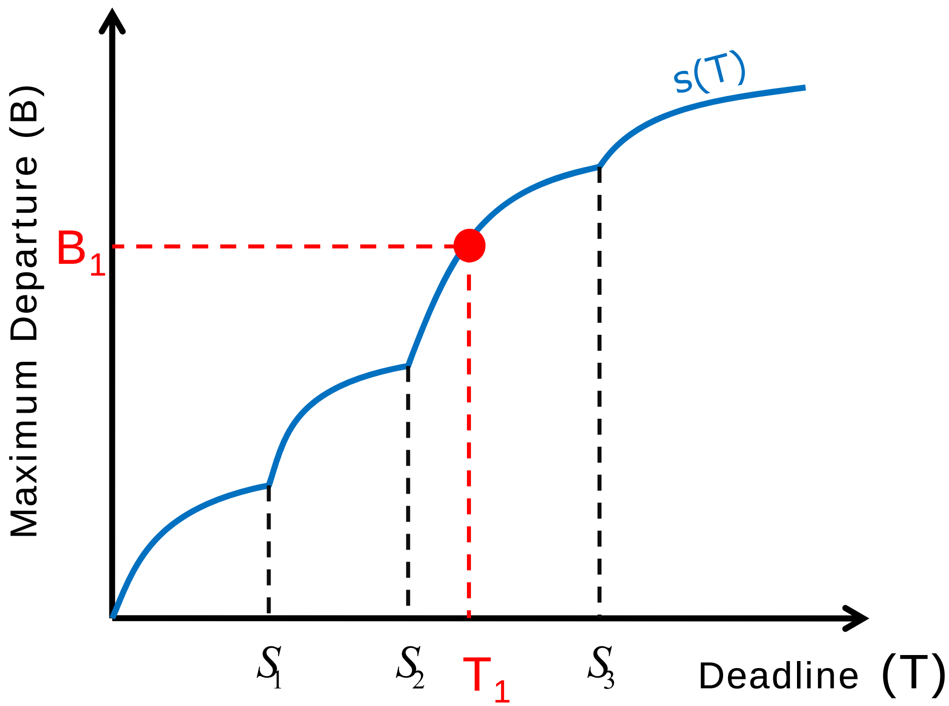
- Continuous, monotone increasing, **invertible**



- Optimal allocation for TCTM with B_1 bits

  $=$

  Optimal allocation for STTM with deadline T_1

Extension to Fading Channels [Ozel et al 2010]

- Find the short-term throughput maximizing and transmission completion time minimizing power allocations in a fading channel with non-causally known channel states.

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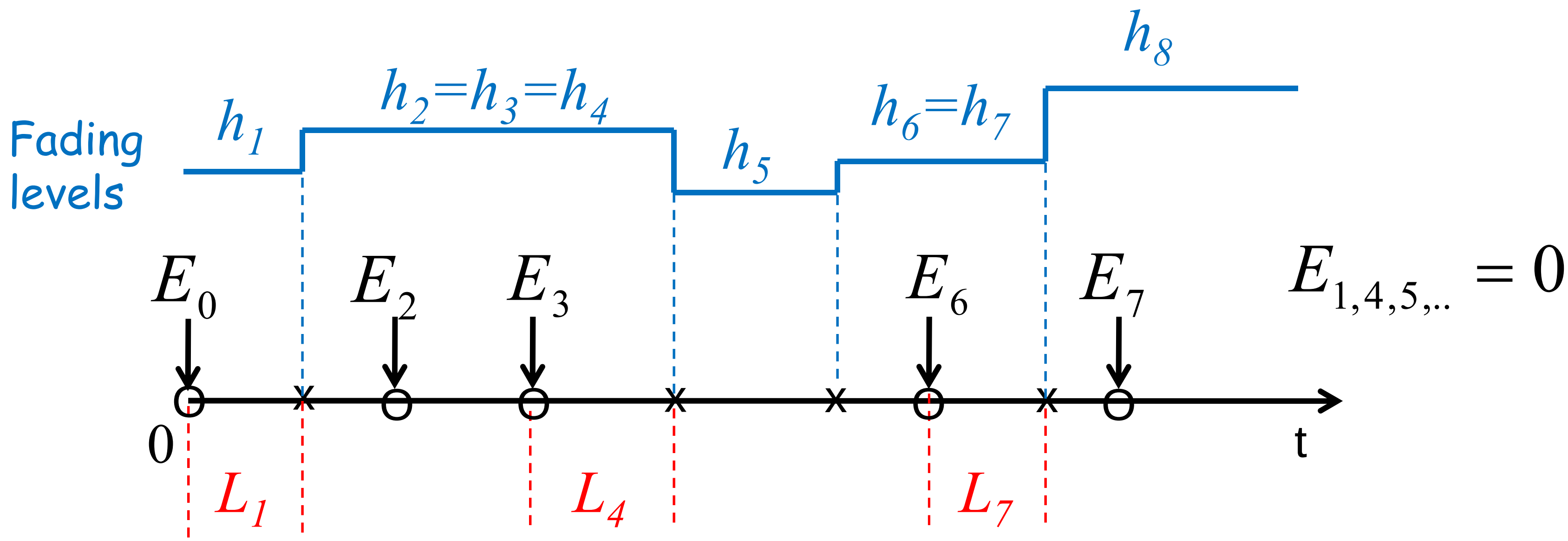
System Model



- AWGN Channel with fading h: $R(P,h)=\frac{1}{2}\log(1+h.P)$
- Each "epoch" defined as the interval between two "events".
- Fading states and harvests known **non-causally**

STTM Problem with Fading

- Transmission power constant within each epoch:

$$p(t) = \{ p_i \quad t \in \text{epoch } i, \; i = 1, \ldots, N + M + 1 \}$$

- Maximize total number of transmitted bits by a deadline T

$$\max_{p_i} \sum_{i=1}^{M+N+1} \frac{L_i}{2} \log(1 + h_i p_i)$$

$$s.t. \quad 0 \le \sum_{i=1}^{l} E_i - L_i p_i \le E_{\max} \quad \forall l$$

STTM Problem with Fading

- Lagrangian of the STTM problem

$$\max_{p_i} \sum_{i=1}^{M+N+1} \frac{L_i}{2}\log(1+h_i p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) \boxed{+ \sum_{i=1}^{M+N+1} \eta_i p_i}$$

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0 \quad \forall j$$

$$\mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0 \quad \forall j$$

$$\eta_j p_j = 0 \quad \forall j$$

(Complementary slackness conditions)

- **Solution:** *constrained* water-filling with fading levels:

$$p_i^* = \left[v_i - \frac{1}{h_i} \right]^+, \qquad v_i = \frac{1}{\sum_{j=i}^{M+N+1} \lambda_j - \mu_j}$$

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STTM Problem with Fading

$$\nabla_k\left(\sum_{i=1}^{M+N+1} L_i . r(p_i) - \sum_{j=1}^{M+N+1}\lambda_j\left(\sum_{i=1}^{j} L_i p_i - E_i\right) - \sum_{j=1}^{M+N+1}\mu_j\left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max}\right) + \sum_{i=1}^{M+N+1}\eta_i p_i\right) = 0 \quad \forall n$$

$$= \sum_{i=1}^{M+N+1} L_i . \underbrace{\nabla_k r(p_i)}_{\frac{h_k \cdot \delta_{(i-k)}}{1+h_k p_i}} - \sum_{j=1}^{M+N+1}\lambda_j \underbrace{\left(\sum_{i=1}^{j} L_i (\nabla_k p_i)\right)}_{\begin{cases} L_k & if\ j>k \\ 0 & if\ j<k \end{cases}} - \sum_{j=1}^{M+N+1}\mu_j \underbrace{\left(\sum_{i=1}^{j} L_i (\nabla_k p_i)\right)}_{\begin{cases} L_k & if\ j>k \\ 0 & if\ j<k \end{cases}} + \underbrace{\sum_{i=1}^{M+N+1}\eta_i (\nabla_k p_i)}_{\eta_k} = 0$$

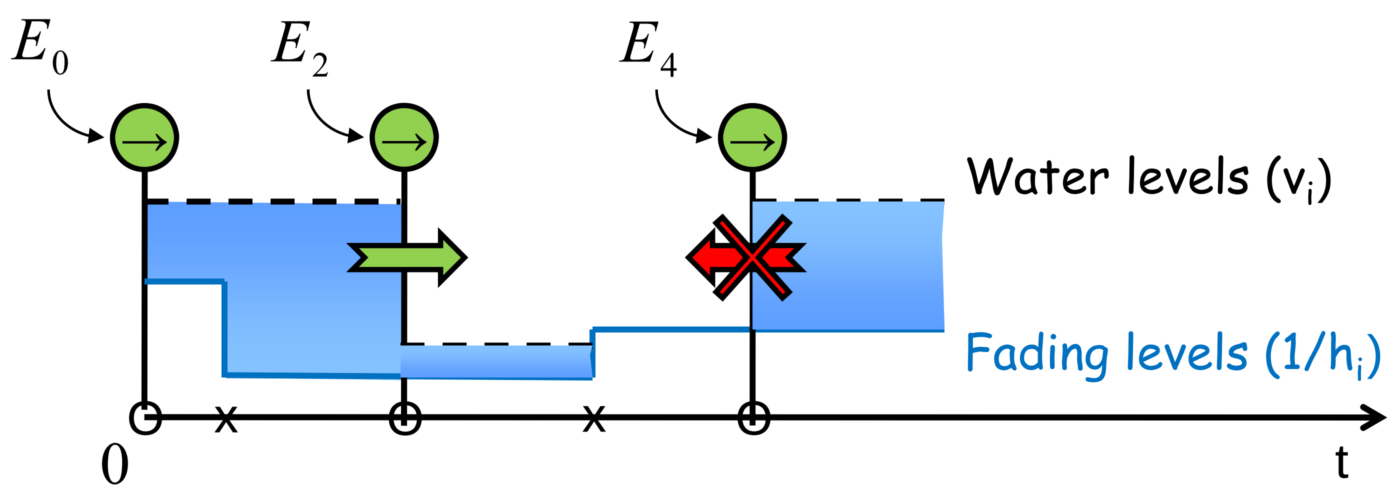
$$= L_k \cdot \frac{h_k}{1+h_k p_k^*} - L_k \sum_{j=k}^{M+N+1}\lambda_j - L_k \sum_{j=k}^{M+N+1}\mu_j + \eta_k = 0$$

$$=> \frac{h_k}{1+h_k p_k^*} = \sum_{j=k}^{M+N+1}(\lambda_j - \mu_j)$$ (if $p_k^* > 0$ is satisfied. Otherwise $p_k^* = 0$ and $\eta_k > 0$)

$$=> p_k^* = \left[\frac{1}{\sum_{j=k}^{M+N+1}(\lambda_j - \mu_j)} - \frac{1}{h_k}\right]^+$$ **(Water Filling)**

Directional Water-Filling

- Same directional water filling model with added fading levels.
  - Directional water flow **(Energy causality)**
  - Limited water flow **(Battery capacity)**



Directional Water-Filling

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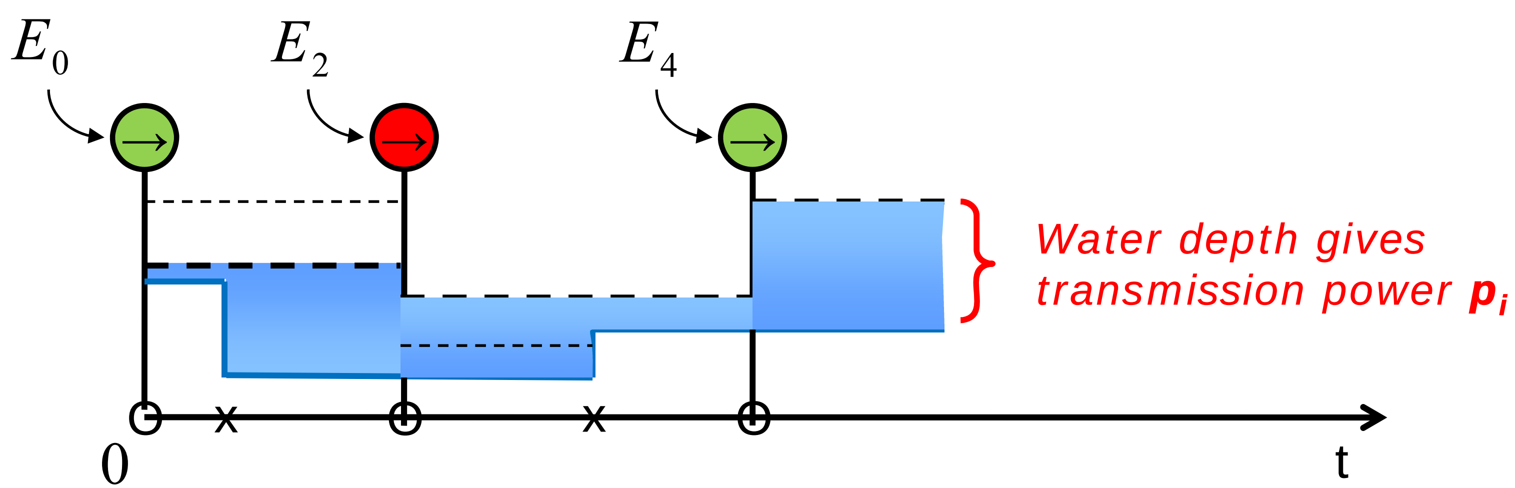

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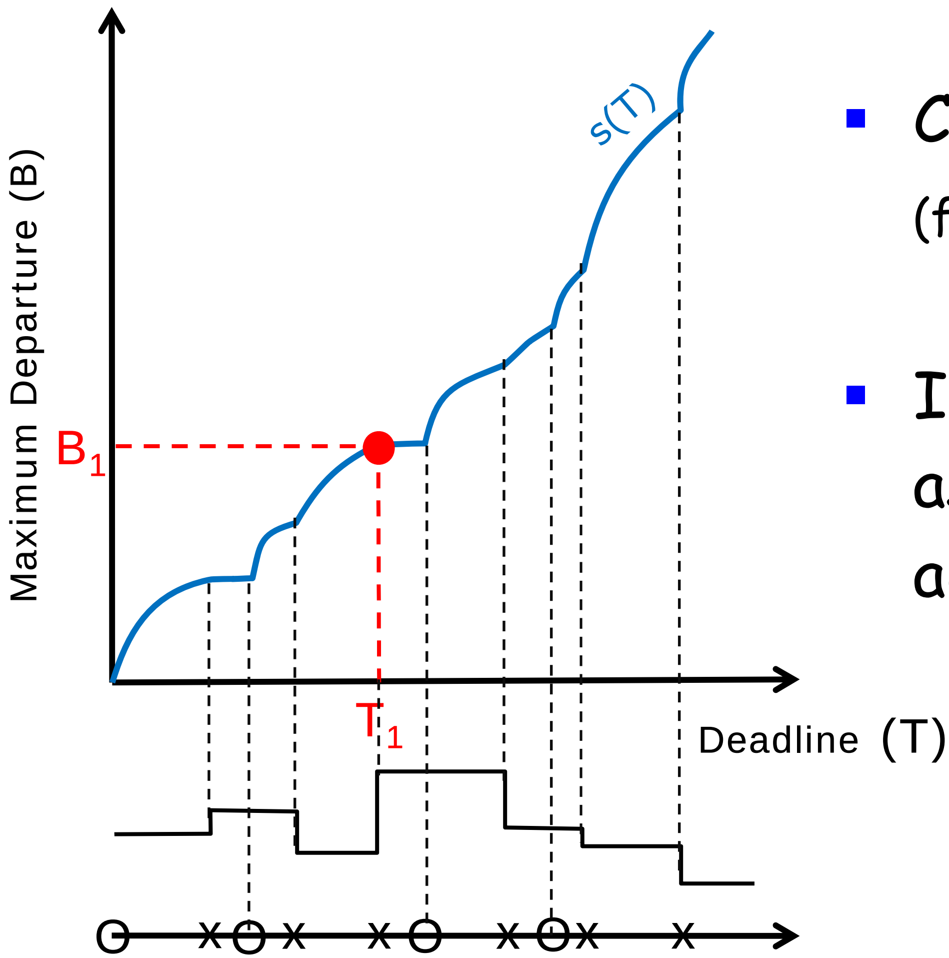
Maximum Service Curve



- Continuous, **non-decreasing** (flat regions when fading is severe)
- Inverse can be considered as the **smallest** T that achieves B_1

Online Algorithms [Ozel et al 2010]

Optimal online policy can be found using dynamic programming

- States of the system: fade level: ***h***, battery energy: ***e***

$$J_g(e,h,t) = E\left[\int_t^T \frac{1}{2}\log(1+h(\tau)g(e,h,\tau))d\tau\right]$$

$$J(e,h,t) = \sup_g J_g$$

- Quantizing time by δ, $g^*(e,h,k\delta)$ can be found by iteratively solving

$$\max_{g(e,h,t)}\left(\frac{\delta}{2}\log(1+h.g(e,h,t)) + J(e',h',t+\delta)\right)$$

$$\begin{pmatrix} e' = e+\delta(-g(e,h,t)+P_{avg}) \\ h' = E[h(t+\delta)\,|\,h(t)] \end{pmatrix}$$

Online Algorithms

Constant Water Level

- A cutoff fading level h_0 is determined by the **average harvested power P_{avg}** as:

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh = P_{avg}$$

$f(h)$: Fading distribution

- Transmitter uses the corresponding water-filling power if available, goes silent otherwise

$$p_i = \left(\frac{1}{h_0}-\frac{1}{h_i}\right)^+$$

Online Algorithms

Energy Adaptive Water-Filling

- Cutoff fade level h_0 determined from **current energy** as:

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh = E_{current}$$

- Transmission power determined by water-filling expression:

$$p_i = \left(\frac{1}{h_0}-\frac{1}{h_i}\right)^+$$

- Sub-optimal, but requires **only** fading statistics.

Online Algorithms

Time-Energy Adaptive Water-filling

- h_0 determined by **remaining energy scaled by remaining time** as

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh=\frac{E_{current}}{T-t}$$

Hybrid Adaptive Water-filling

- h_0 determined similarly but by **adding average received power**

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh=\frac{E_{current}}{T-t}+P_{avg}$$

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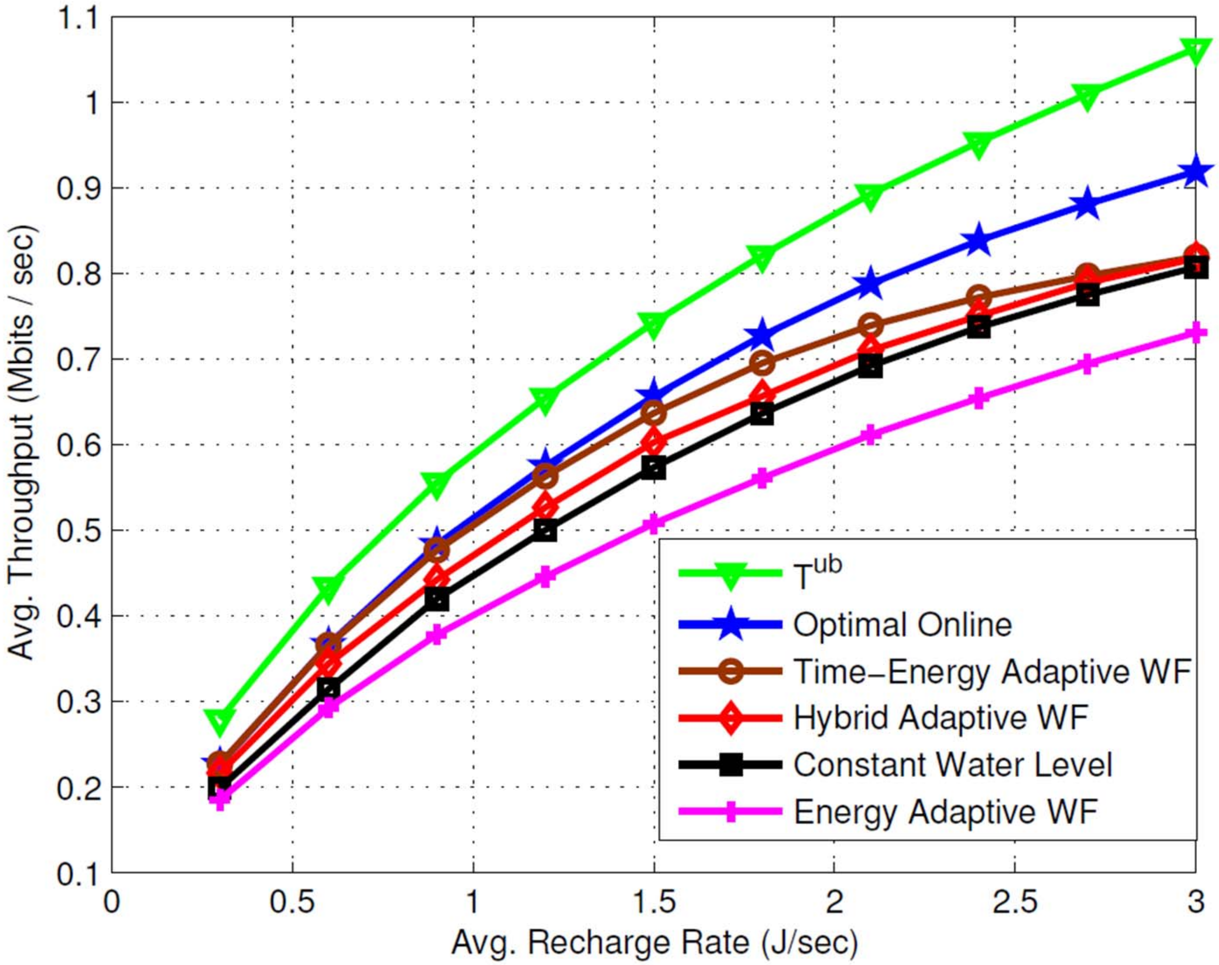
Simulations



Performances of the policies w.r.t. **energy arrival rates** under:

- unit mean Rayleigh fading
- T = 10 sec
- E_{max} = 10 J.

Conclusion

- New paradigm: Networking with energy harvesting nodes
- New design insights arising from new energy constraints
- Lots of open problems in this area!

- In this presentation, we covered optimal scheduling policies for one EH transmitter.
- **Next: Multiuser scenarios**

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- Omur Ozel, Kaya Tutuncuoglu, Jing Yang, Sennur Ulukus and Aylin Yener, Transmission with Energy Harvesting Nodes in Fading Wireless Channels: Optimal Policies, *to appear in IEEE Journal on Selected Areas in Communications: Energy-Efficient Wireless Communications*, ArXiv (CISS 11, INFOCOM 11).