IEEE ISIT 2016 Tutorial July 10, 2016

Energy Harvesting and Remotely Powered Wireless Networks

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& Networking Laboratory
WCAN@PSU

Ayfer Ozgur (Stanford)
Sennur Ulukus (UMD)
Aylin Yener(Penn State)

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Outline of This Tutorial

- Introduction to energy harvesting (EH)
- Single-user offline power/rate optimization [Aylin]
- Single-user online power/rate optimization [Ayfer]
- Multi-user offline power optimization [Sennur]
- Multi-user online power optimization [Sennur]
- Energy cooperation (EC) and optimization [Sennur]
- Information theory of EH, infinite/zero/unit battery [Aylin]
- Information theory w/ finite battery, connections to online & offline optimization; IT of EC [Ayfer]

Prerequisites for the Tutorial

Basic command of

- Optimization
- Communication Theory

Reasonable fluency in

- Shannon Theory

Fairly self-contained otherwise

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Energy Harvesting and Remotely Powered Wireless Networks- Part I

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Aylin Yener

yener@ee.psu.edu

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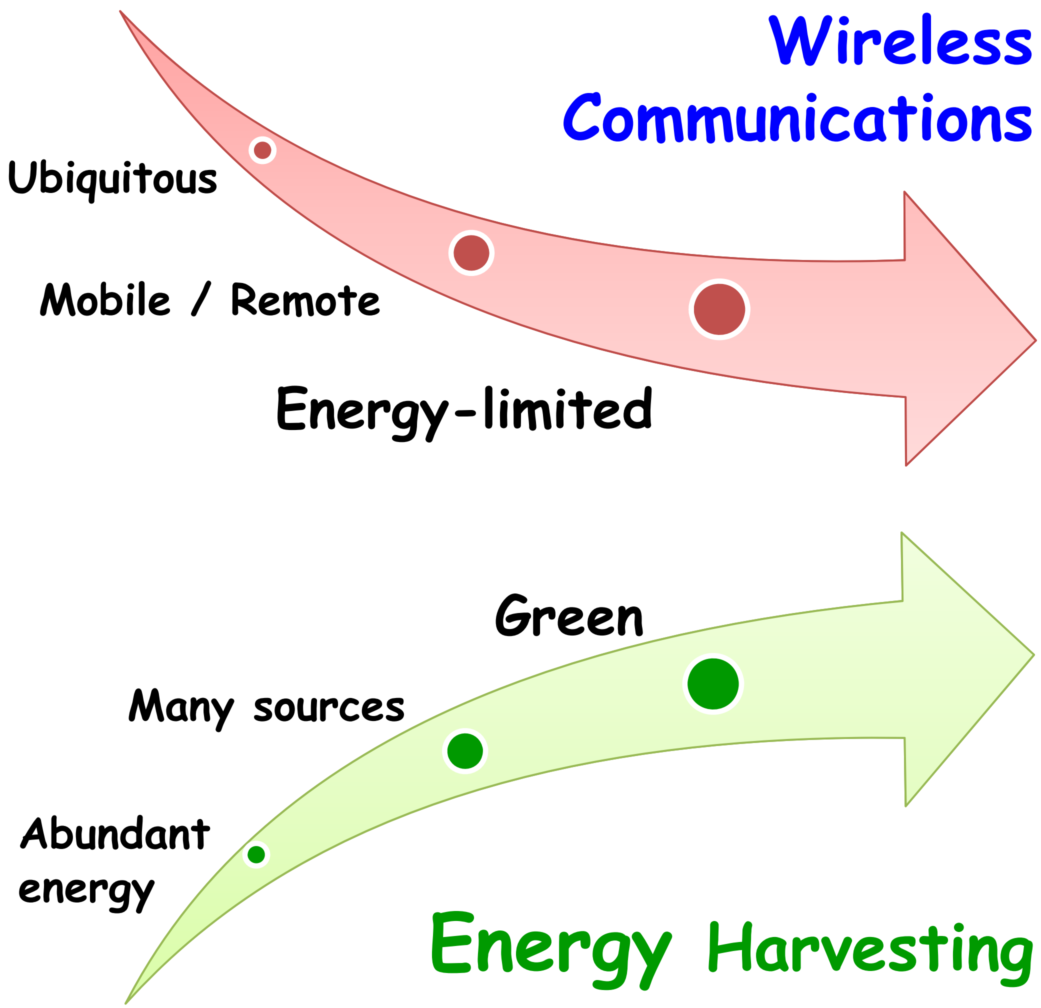
- Collaborators: Omur Ozel, Kaya Tutuncuoglu, Sennur Ulukus, Jing Yang.

Outline – Aylin- Part I

- Introduction to energy harvesting (EH)
- **Communication theory of EH – the optimization set up**
- Short term throughput maximization for single link with finite battery
- Transmission completion time minimization with finite battery
- Extension to fading channels
- Transmission policies for nodes with inefficient energy storage

Introduction

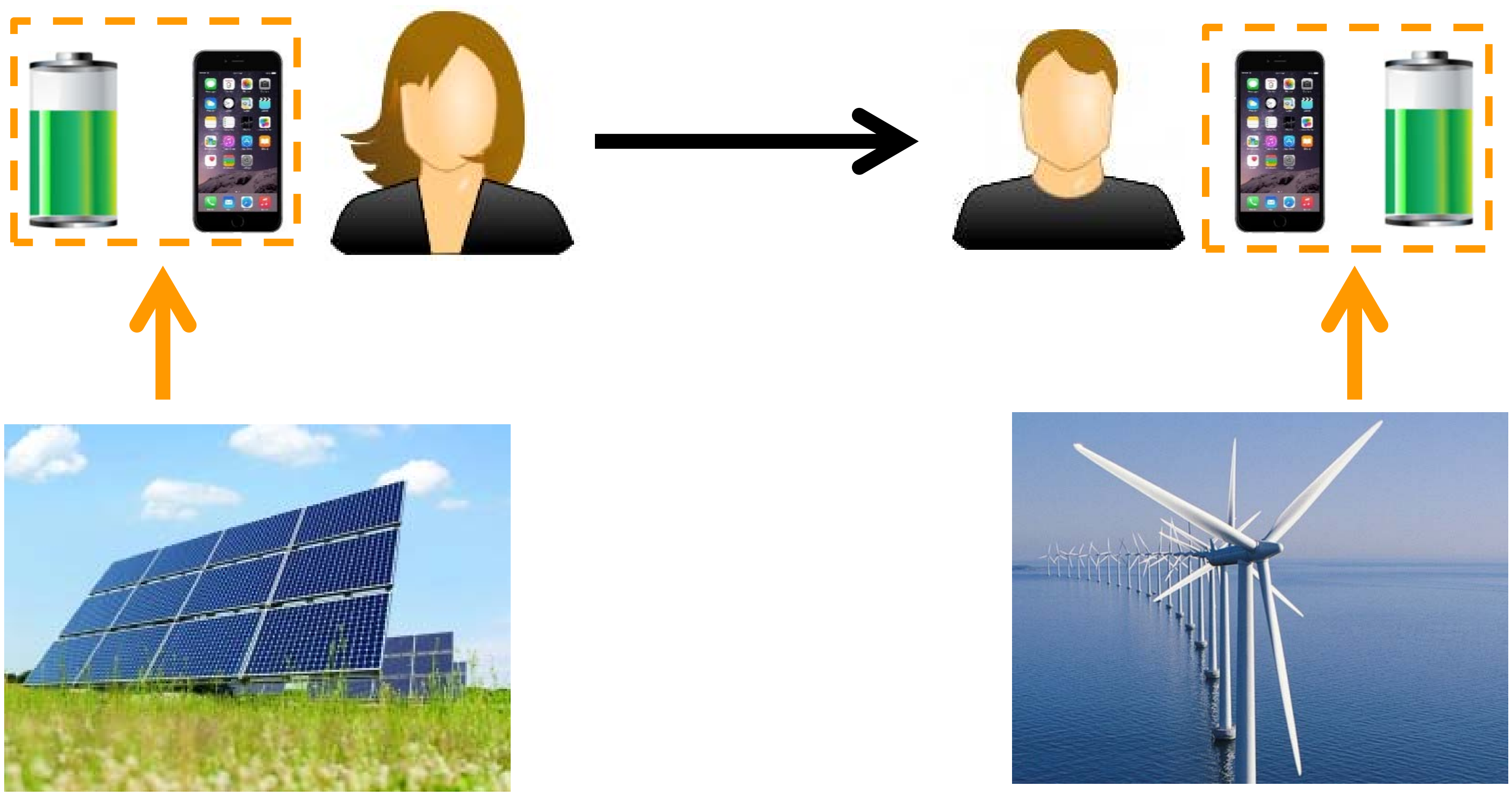


Energy
Harvesting
Wireless
Networks

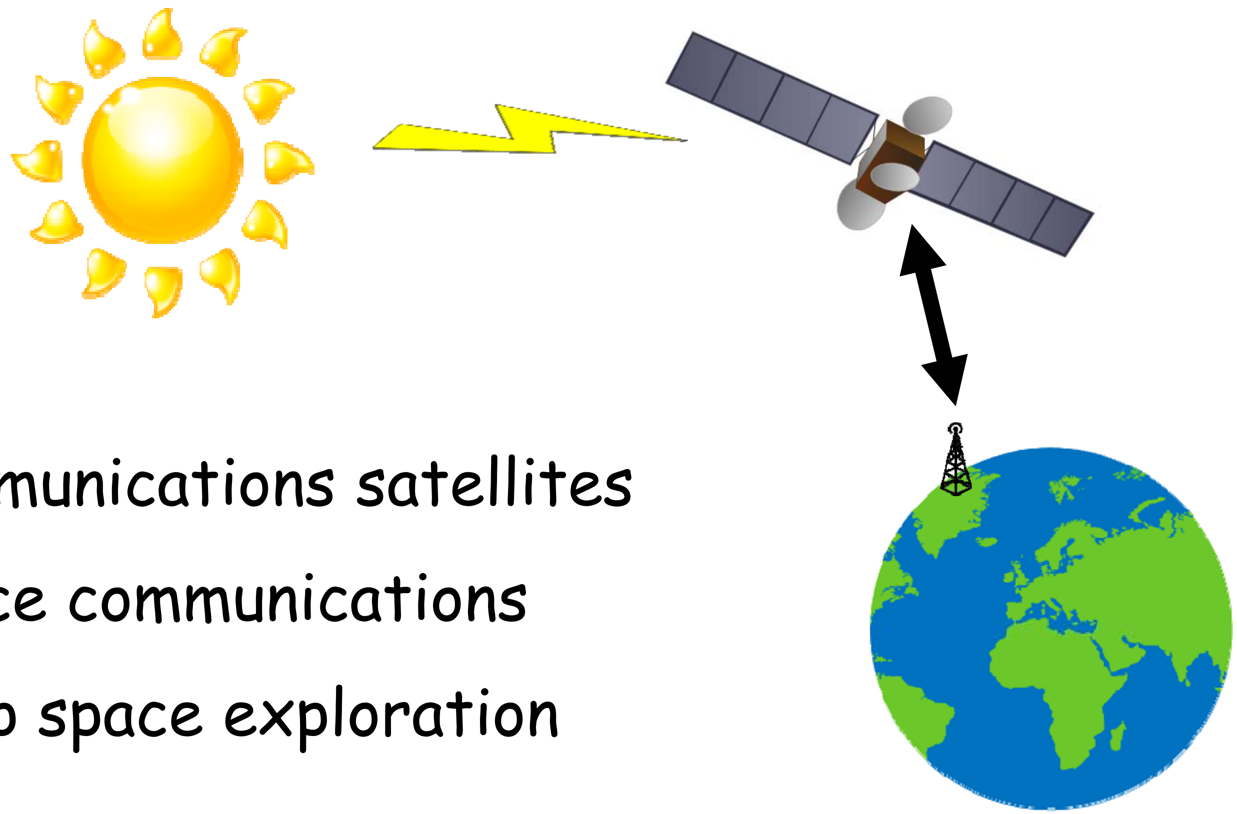
Energy Harvesting Networks

- **Wireless networking with rechargeable (energy harvesting) nodes:**
  - Green, self-sufficient nodes,
  - Extended network lifetime,
  - Smaller nodes with smaller batteries.

What could EH bring to communications?



Energy Harvesting Applications



- Communications satellites
- Space communications
- Deep space exploration

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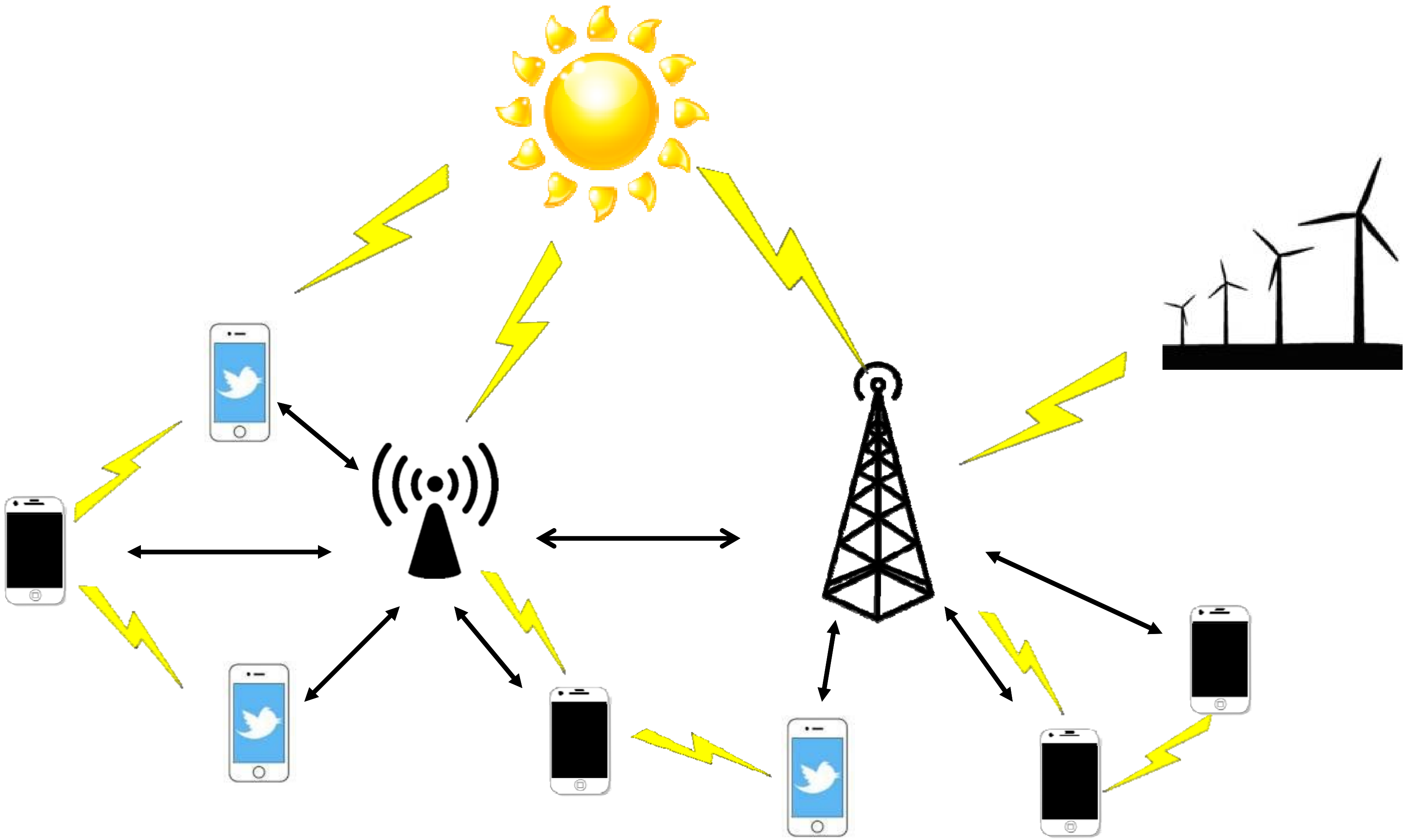
Wireless Energy Cooperation



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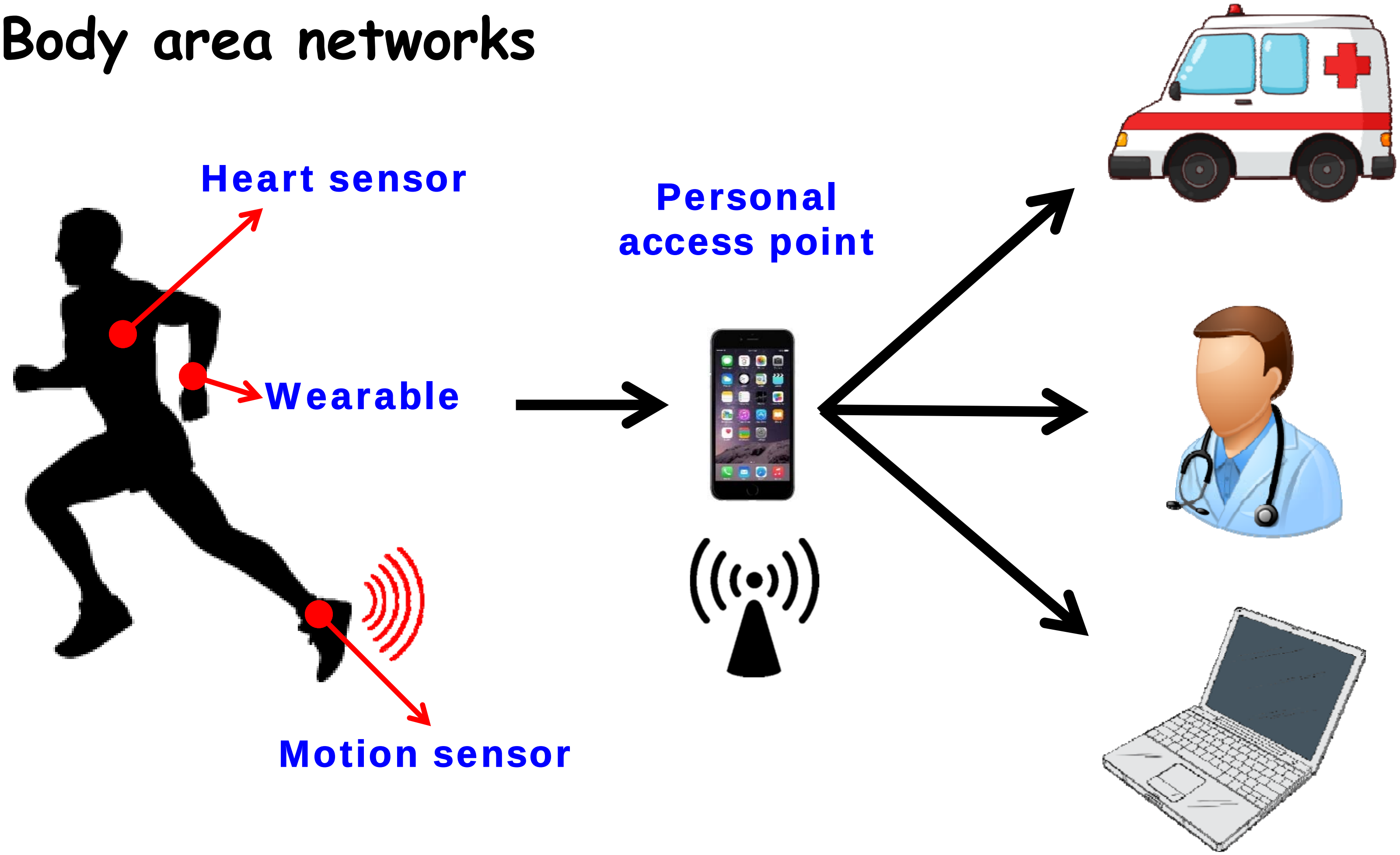
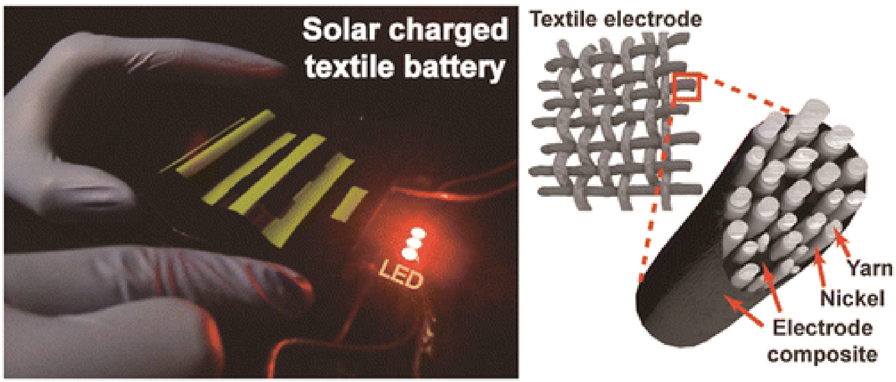
Energy Harvesting Applications

Body area networks



Energy Harvesting Applications



KAIST's Solar charged textile battery

MC10's **biostamps**

for medical monitoring,

powered **wirelessly**

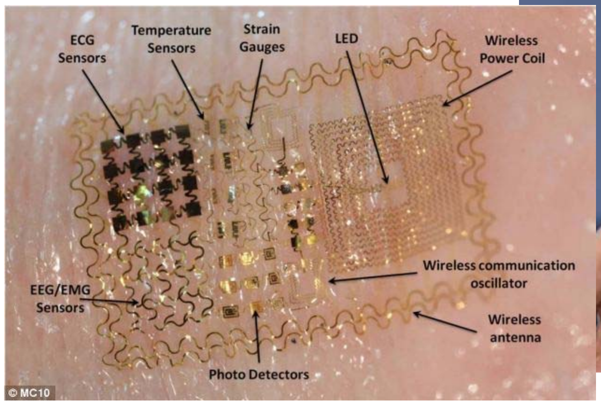
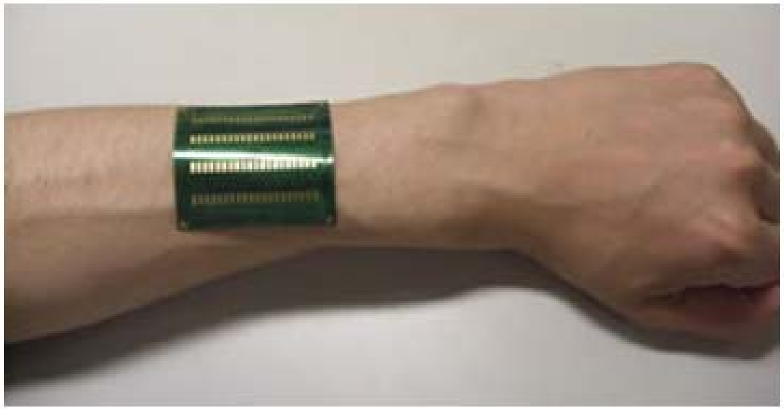
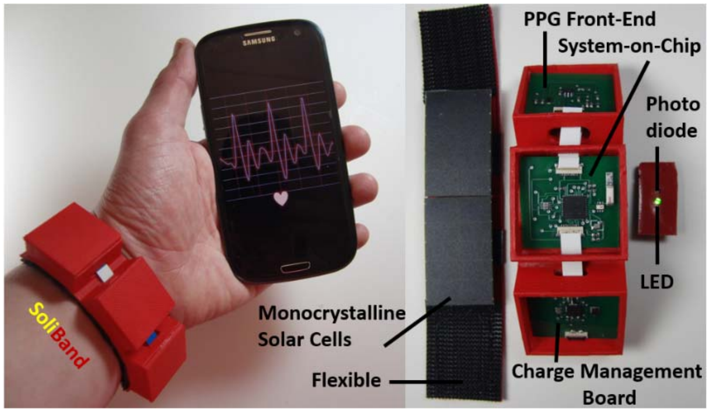


Image Credits: (top) http://pubs.acs.org/doi/abs/10.1021/nl403860k#aff1 (bottom)) http://www.dailymail.co.uk/sciencetech/article-2333203/Moto-X-Motorola-reveals-plans-ink-pills-replace-ALL-passwords.html

Energy Harvesting Applications

Fujitsu's hybrid device utilizing **heat or light.**





Health tracker built at at the ASSIST Center at North Carolina State University, utilizing **solar cells**

Image Credits: (top) http://www.fujitsu.com/global/news/pr/archives/month/2010/20101209-01.html
(bottom) https://assist.ncsu.edu/research/

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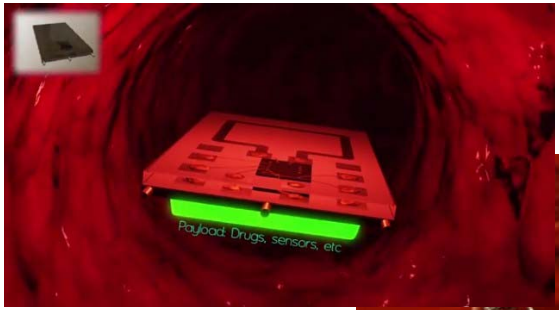
Energy Harvesting Applications



In-body (intravascular) wireless devices

Proteus Biomedical pills,
powered by **stomach acids**

Image Credits: (top) http://www.extremetech.com/extreme/119477-stanford-creates-wireless-implantable-innerspace-medical-device
(middle) http://www.imedicalapps.com/2012/03/robotic-medical-devices-controlled-wireless-technology-nanotechnology/
(bottom) http://scitechdaily.com/smart-pills-will-track-patients-from-the-inside-out/

What is in it for us?

- New: communication theory of EH nodes
- New: information theory of EH nodes

Key new ingredient:

A **set of energy feasibility constraints** based on harvests govern the communication resources.

Communications

- **New Wireless Network Design Challenge:**

  A **set of energy feasibility constraints** based on harvests govern the communication resources.

- **Design question:**

  When and at what rate/power should a "rechargeable" (energy harvesting) node transmit?

- **Optimality? Throughput; Delivery Delay**
- **Outcome: Optimal Transmission Schedules**

Two main metrics

- **Short-Term Throughput Maximization (STTM):**

  Given a deadline, maximize the number of bits sent before the end of transmission.

- **Transmission Completion Time Minimization (TCTM):**

  Given a number of bits to send, minimize the time at which all bits have departed the transmitter.

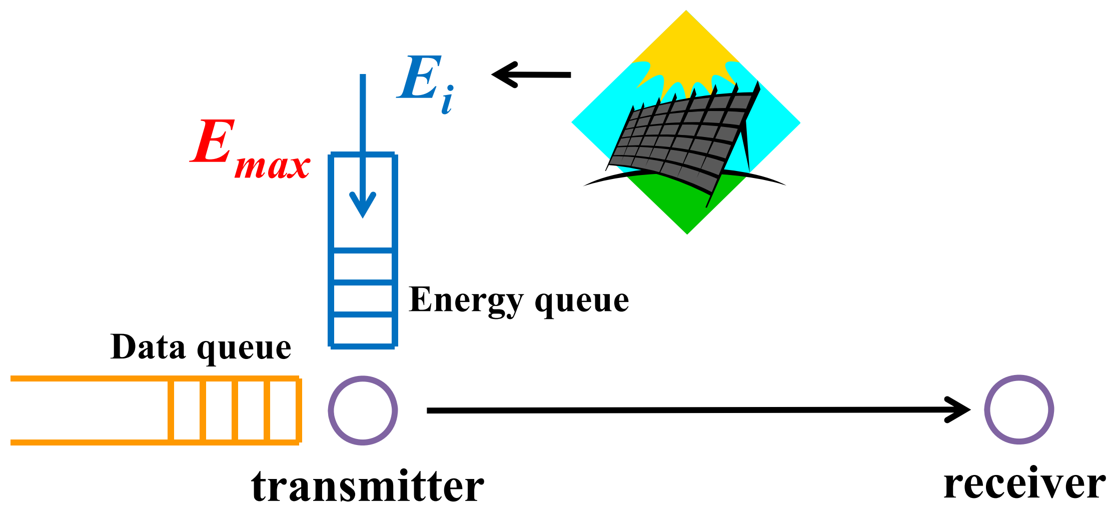
ST Throughput Maximization

[Tutuncuoglu-Yener'12]

- One Energy harvesting transmitter.
- Find optimal power allocation/transmission policy that departs maximum number of bits in a given duration **T**.
- Energy available intermittently.
- Up to a certain amount of energy can be stored by the transmitter → **BATTERY CAPACITY**.

System Model

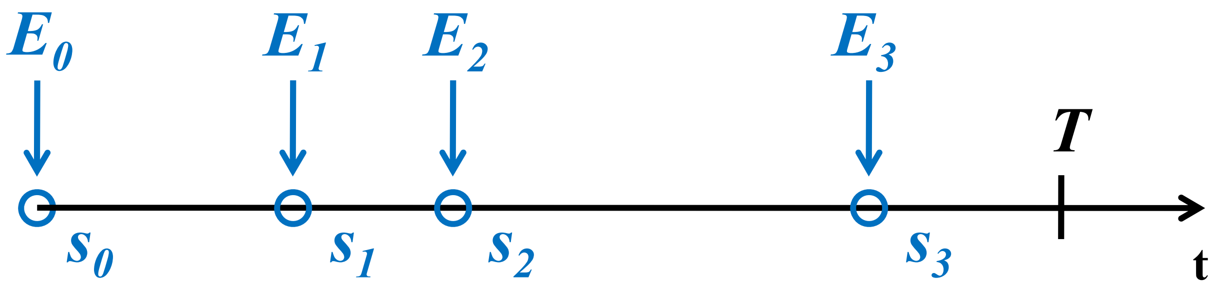
- Energy harvesting transmitter:



- Transmitter has **backlogged data** to send by deadline T
- Energy **arrives intermittently** from harvester
- Stored in a **finite battery** of capacity E_{max}

System Model

- Energy arrivals of energy E_i at times s_i



- Arrivals known **non-causally** by transmitter,
- Design parameter: **power** $\rightarrow$ **rate** $r(p)$.

Power-Rate Function

- Transmission with power p yields a rate of $r(p)$

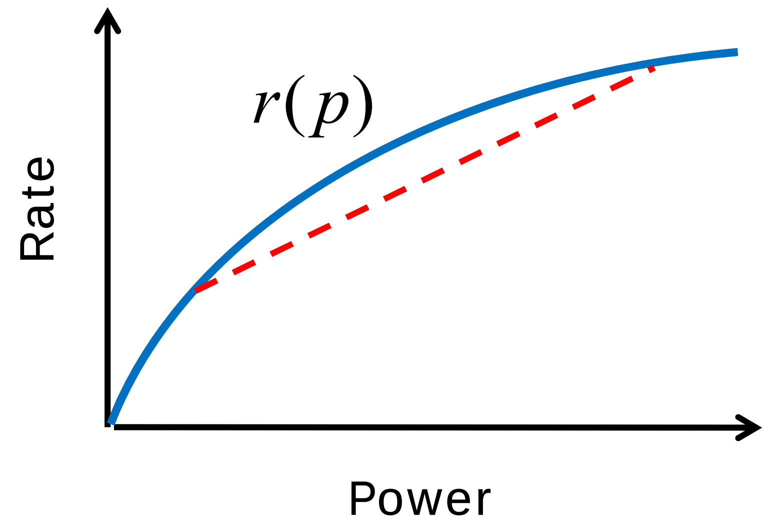
- **Assumptions on r(p):**

  i. $r(0)=0, r(p) \rightarrow \infty \text{ as } p \rightarrow \infty$

  ii. increases monotonically in p

  iii. strictly concave

  iv. $r(p)$ continuously differentiable



Example: AWGN Channel, $r(p) = \frac{1}{2}\log\left(1+\frac{p}{N}\right)$

Notations and Assumptions

- Power allocation function: $p(t)$
- Energy consumed: $\int_0^T p(t)\,dt$
- Short-term throughput: $\int_0^T r(p(t))\,dt$

Concave rate in power →Given a fixed energy, a longer transmission with lower power departs more bits.

Energy Constraints

(Energy arrivals of E_i at times s_i)

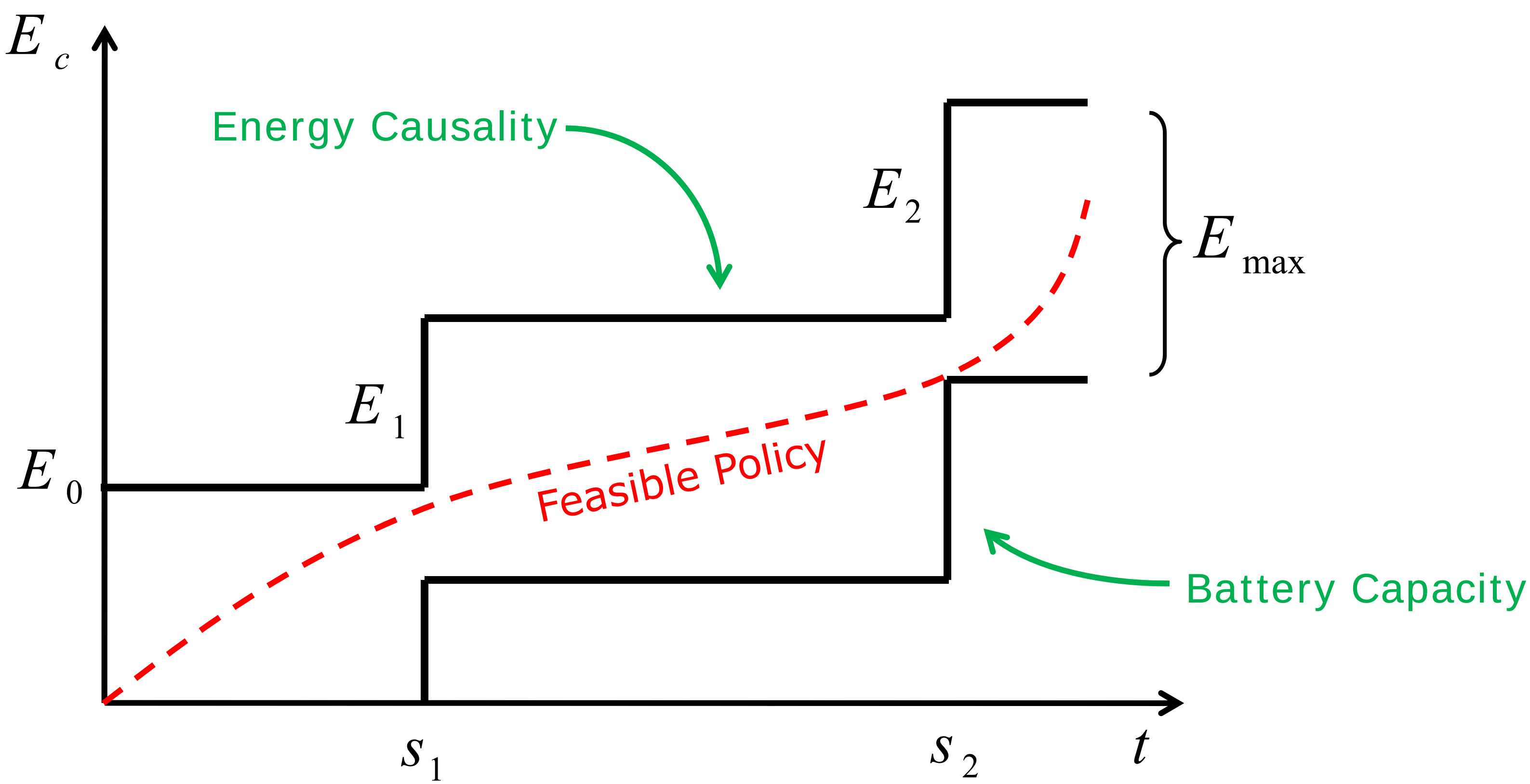
- **Energy Causality:** $\sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \geq 0 \qquad s_{n-1} \leq t' \leq s_n$

- **Battery Capacity:** $\sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \leq E_{\max} \qquad s_{n-1} \leq t' \leq s_n$

- **Set of energy-feasible power allocations**

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \leq \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \leq E_{\max}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\}$$

Energy "Tunnel"



Optimization Problem

- Maximize total number of transmitted bits by deadline T

$$\max_{p(t)} \int_0^T r(p(t))dt, \qquad s.t. \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

- Convex constraint set, concave maximization problem

Necessary conditions for optimality of a transmission policy

- **Property 1:** **Transmission power remains constant between energy arrivals.**

- Let the total consumed energy in epoch $[s_i, s_{i+1}]$ be E_{total} which is available at $t = s_i$. Then the power policy

$$p' = \frac{E_{total}}{s_{i+1} - s_i}, \qquad t \in [s_i, s_{i+1}]$$

is feasible and better than a variable power transmission; shown easily using concavity of $r(p)$

Necessary conditions for optimality

- **Property 2:** Battery never overflows.

Proof:

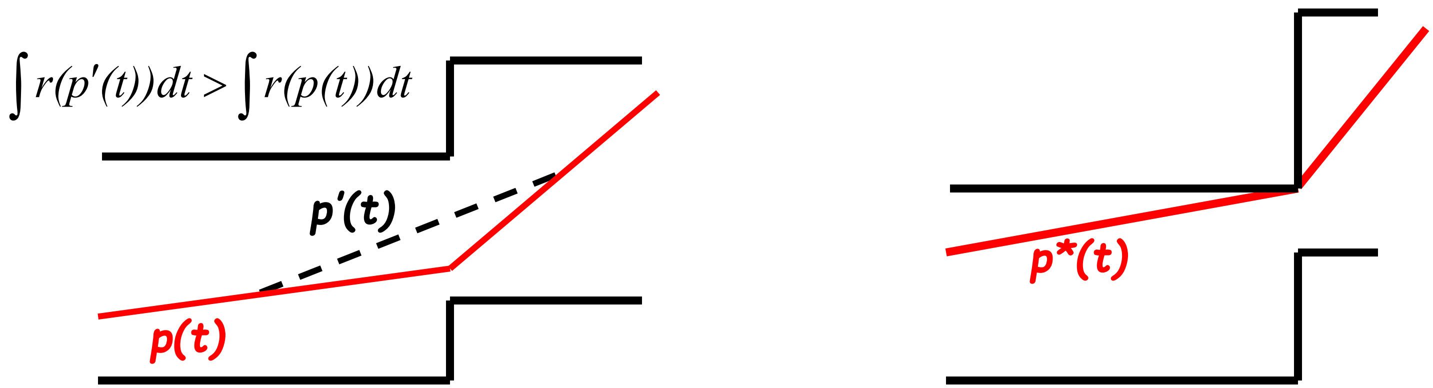
Assume an energy of Δ overflows at time τ

Define $p'(t) = \begin{cases} p(t) + \dfrac{\Delta}{\delta} & [\tau - \delta, \tau] \\ p(t) & else \end{cases}$

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))dt$ since $r(p)$ is increasing in p

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

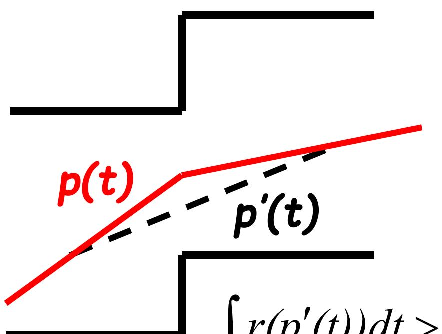


Policy can be improved

Policy cannot be improved

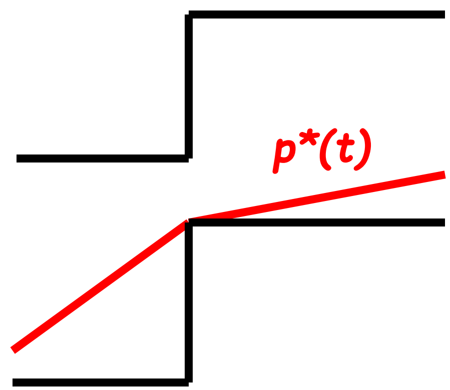
Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.



$$\int r(p'(t))dt > \int r(p(t))dt$$

Policy can be improved



Policy cannot be improved

Necessary conditions for optimality of a transmission policy

- **Property 4:** Battery is depleted at the end of transmission.

Proof: Assume an energy of Δ remains after p(t)

Define $$p'(t) = \begin{Bmatrix} p(t) + \dfrac{\Delta}{\delta} & [T-\delta, T] \\ p(t) & else \end{Bmatrix}$$

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\, dt$ since $r(p)$ is increasing

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Implications of the properties [Tutuncuoglu-Yener'12]

- **Structure of optimal policy is piece-wise linear.**

$$p(t) = \begin{cases} p_n & i_{n-1} < t < i_n \\ 0 & t > T \end{cases}, \qquad i_n \in \{s_n\}, \quad p_n \text{ constant}$$

- For power to increase or decrease, policy must meet the upper or lower boundary of the tunnel respectively.
- At termination step, battery is depleted.
- Utilizing this structure, a recursive algorithm emerges to find the unique optimum policy [Tutuncuoglu-Yener'12].

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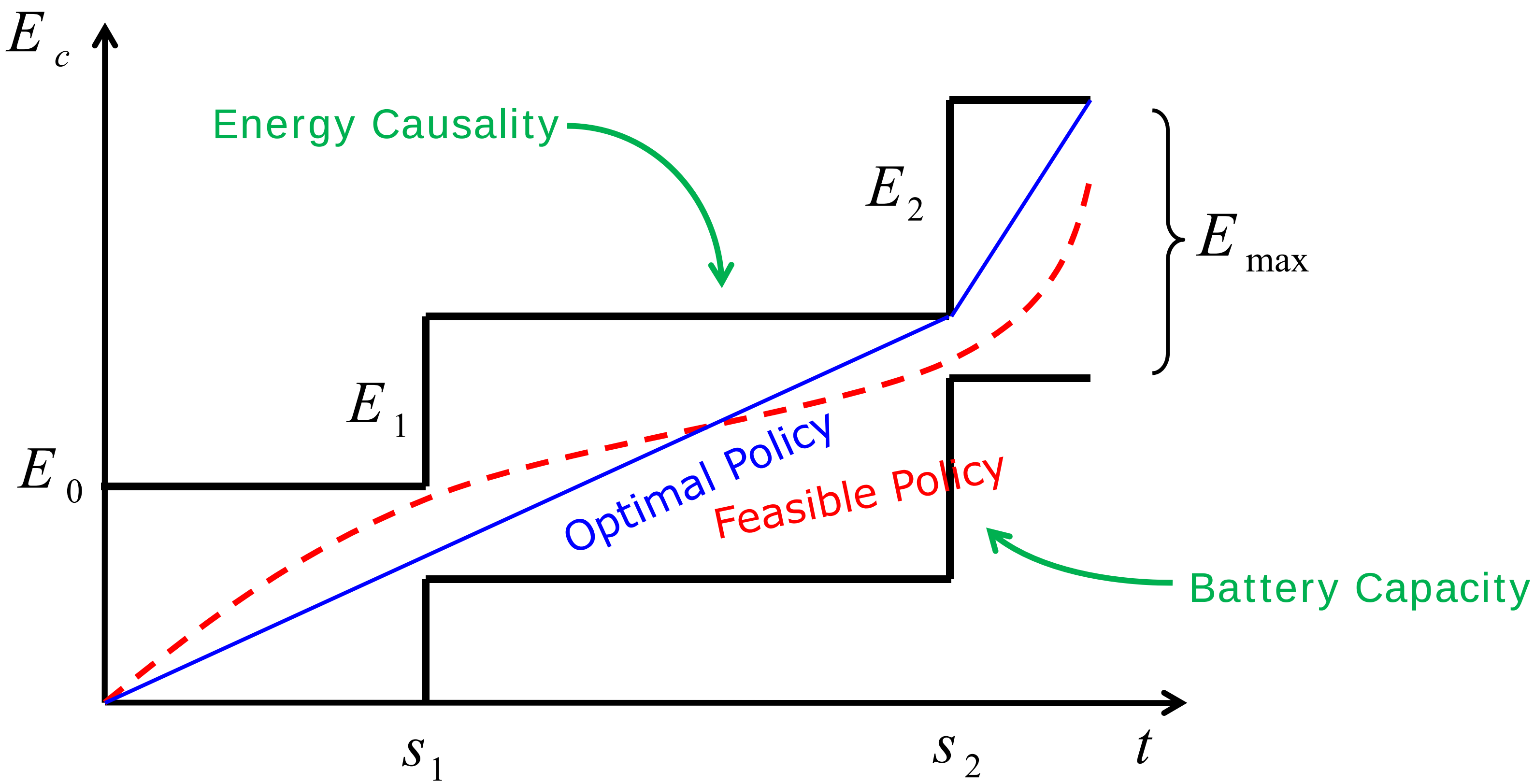
Energy “Tunnel”



Shortest Path Interpretation

- **Optimal policy is identical for any concave power-rate function!**
- Let $r(p) = -\sqrt{p^2+1}$, then the problem solved becomes:

$$\max_{p(t)} \int_0^T -\sqrt{p^2(t)+1}\, dt \qquad s.t. \;\; p(t) \in \mathfrak{P}$$

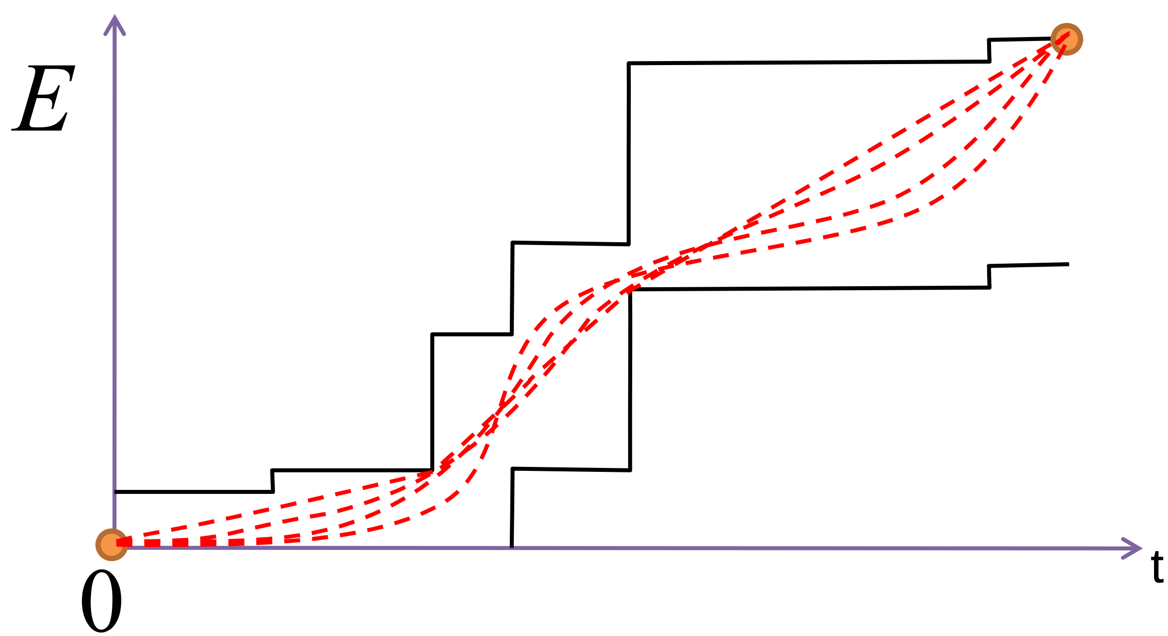
$$= \min_{p(t)} \underbrace{\int_0^T \sqrt{p^2(t)+1}\, dt}_{} \qquad s.t. \;\; p(t) \in \mathfrak{P}$$

length of policy path in energy tunnel

$\Rightarrow$ The **throughput maximizing policy** yields the **shortest path** through the energy tunnel for any concave power-rate function.

Shortest Path Interpretation

- **Property 1:** Constant power is better than any other alternative
- **Shortest path** between two points is a line (constant slope)



Alternative Solution (Using Property 1)

- Transmission power is constant within each epoch:

$$p(t) = \{ p_i, t \in epoch \ \ i, \ \ i = 1, \ldots, N \}$$

(N: Number of arrivals within [0,T])

$$\max_{p_i} \sum_{i=1}^{N} L_i . r(p_i)$$

(L_i: length of epoch i)

$$s.t. \quad 0 \leq \sum_{i=1}^{n} E_i - L_i p_i \leq E_{\max} \qquad n = 1, \ldots, N$$

- KKT conditions → optimum power policy.

Solution

- Complementary Slackness Conditions:

$$\lambda_n\left(\sum_{i=1}^{n} L_i p_i - E_i\right) = 0 \quad \forall n$$

$$\mu_n\left(\sum_{i=1}^{n} E_i - L_i p_i - E_{\max}\right) = 0 \quad \forall n$$

λ_n's are positive only when battery is empty $\left(\sum_{i=1}^{n} L_i p_i - E_i\right) = 0$

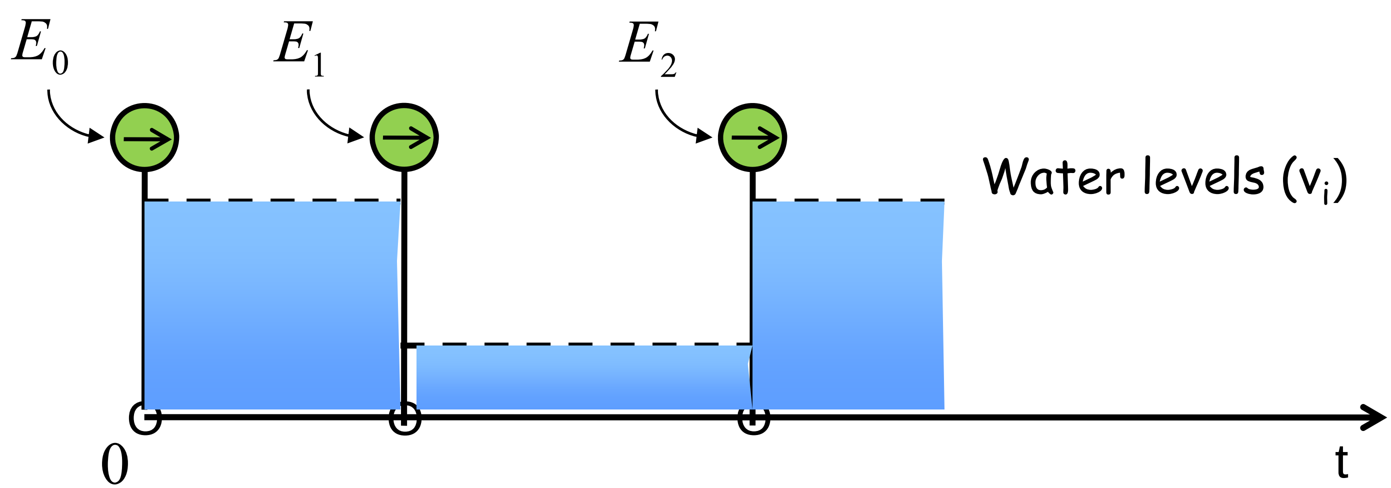
μ_n's only positive only when battery is full $\left(\sum_{i=1}^{n} E_i - L_i p_i - E_{\max}\right) = 0$

$$p_n^* = \left[\frac{1}{\sum_{j=n}^{N}(\lambda_j - \mu_j)} - 1\right]^+$$

increases with positive λ_n
decreases with positive μ_n

Directional Water-Filling

- [Ozel, Tutuncuoglu, Ulukus, Yener'11]
- Harvested energies filled into epochs individually



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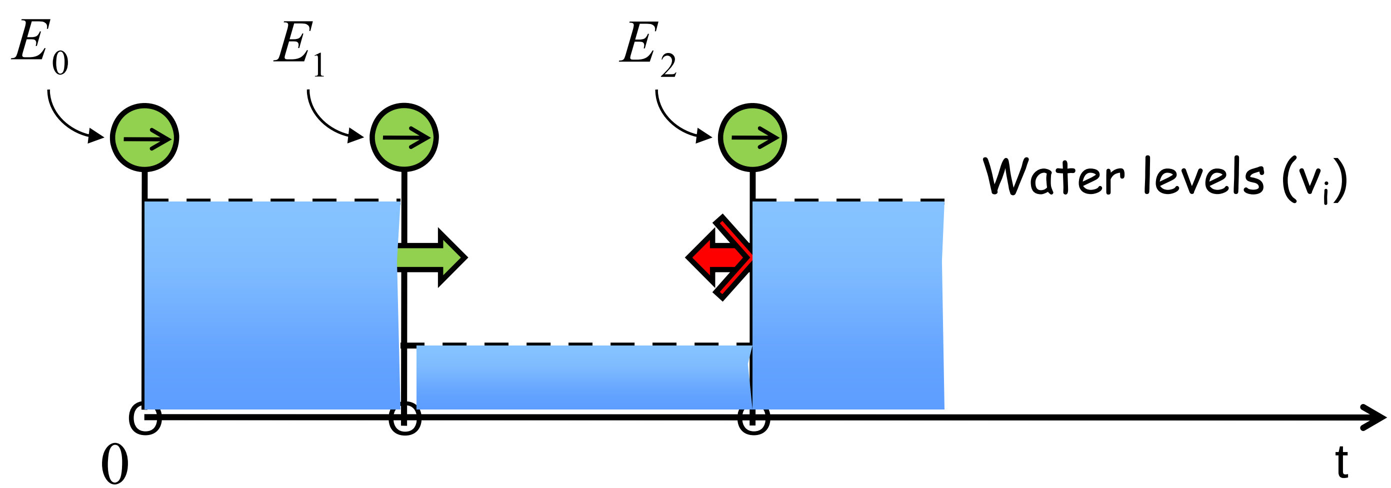
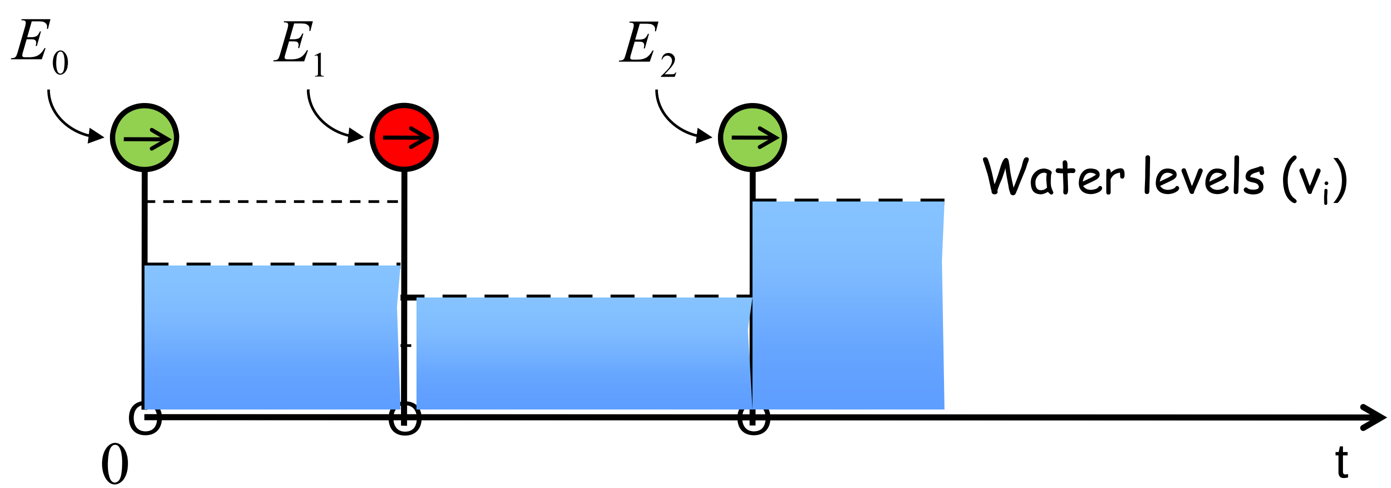
Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time



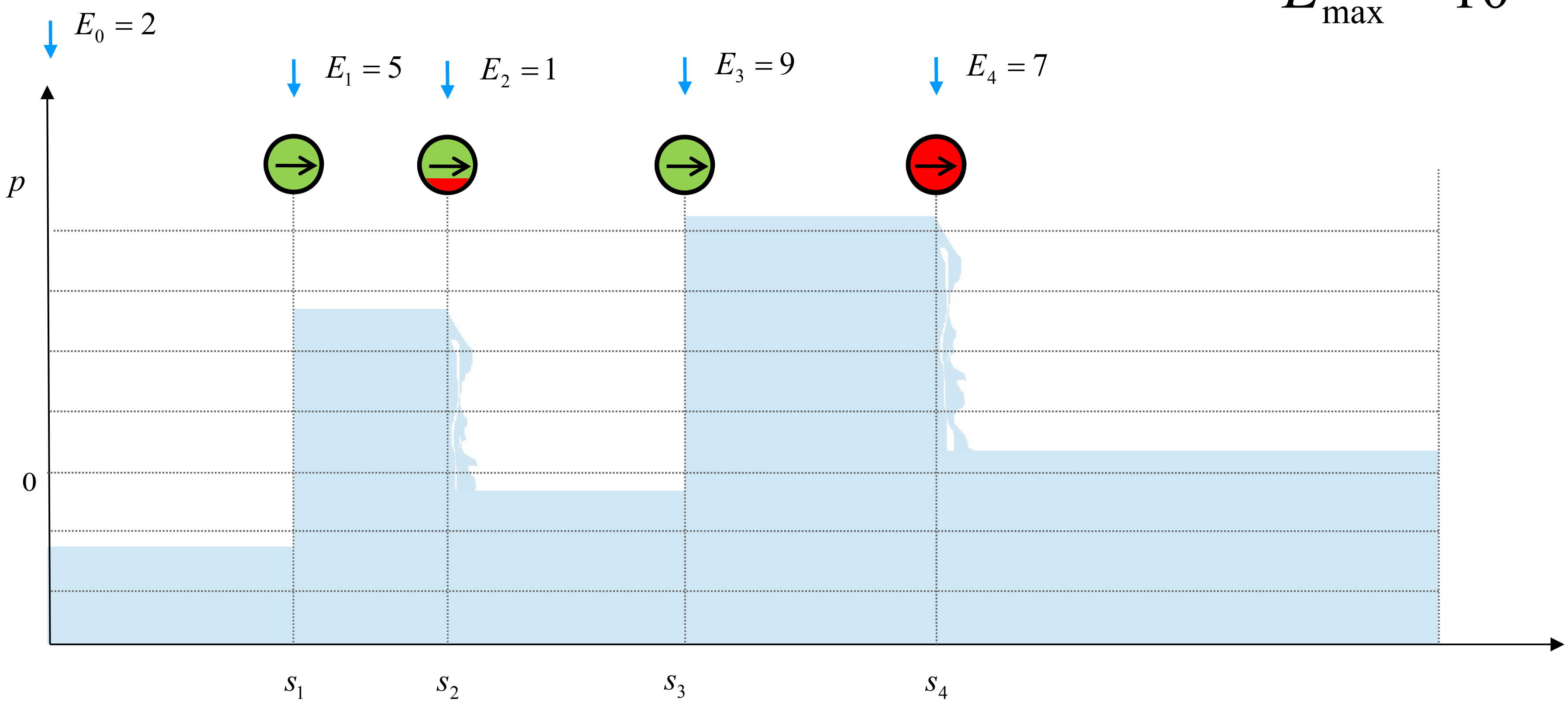
Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time
  - **Battery Capacity**: water-flow limited to E_{max} by taps

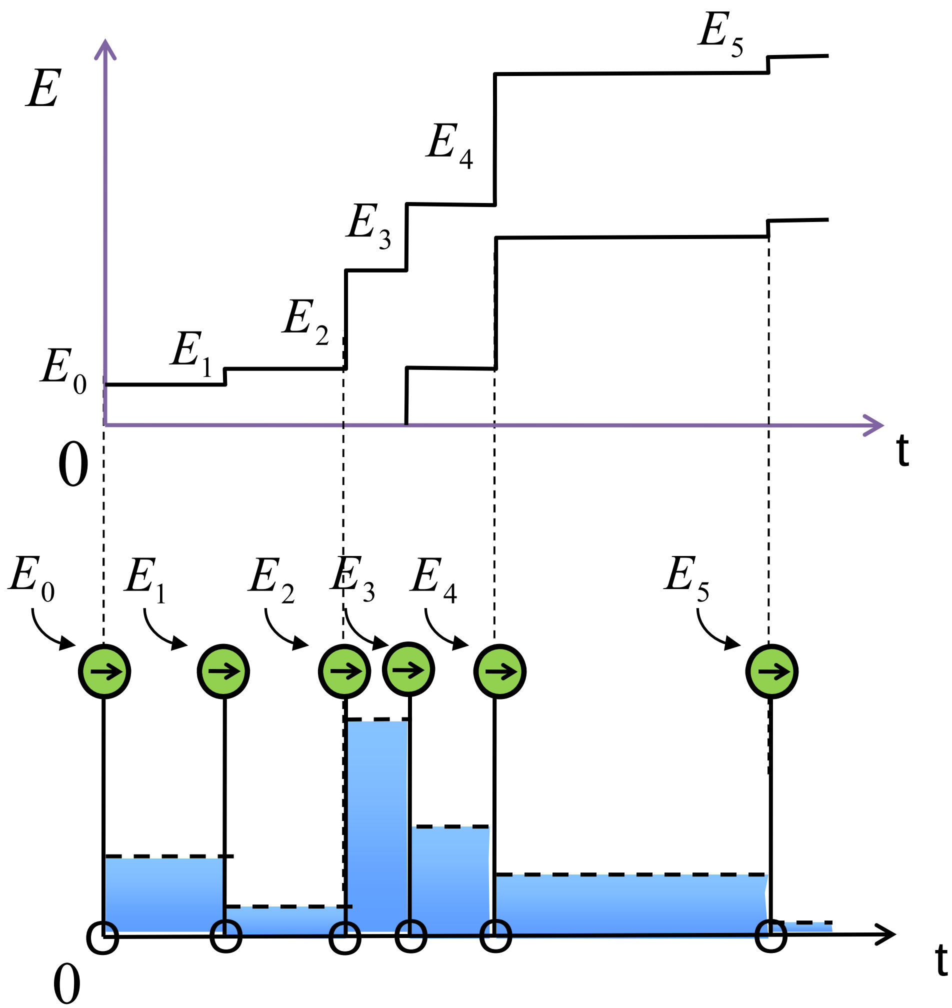


Example

$$E_{\max} = 10$$

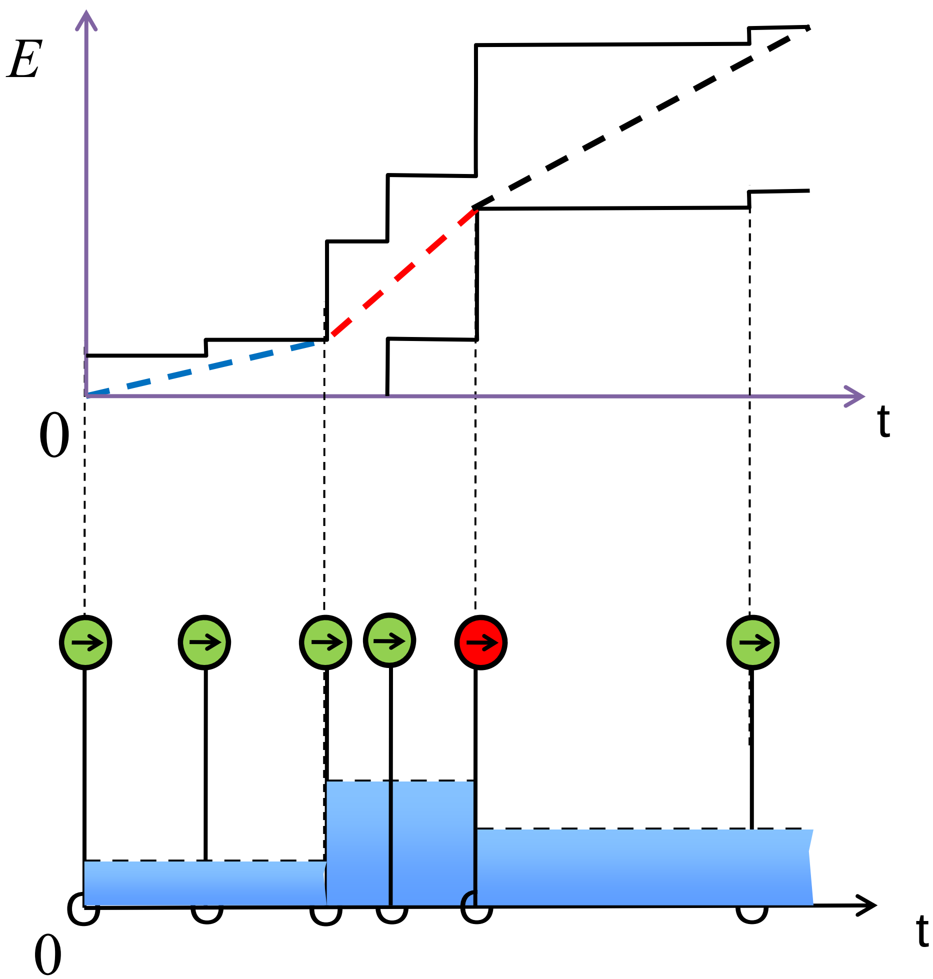


Directional Water-Filling



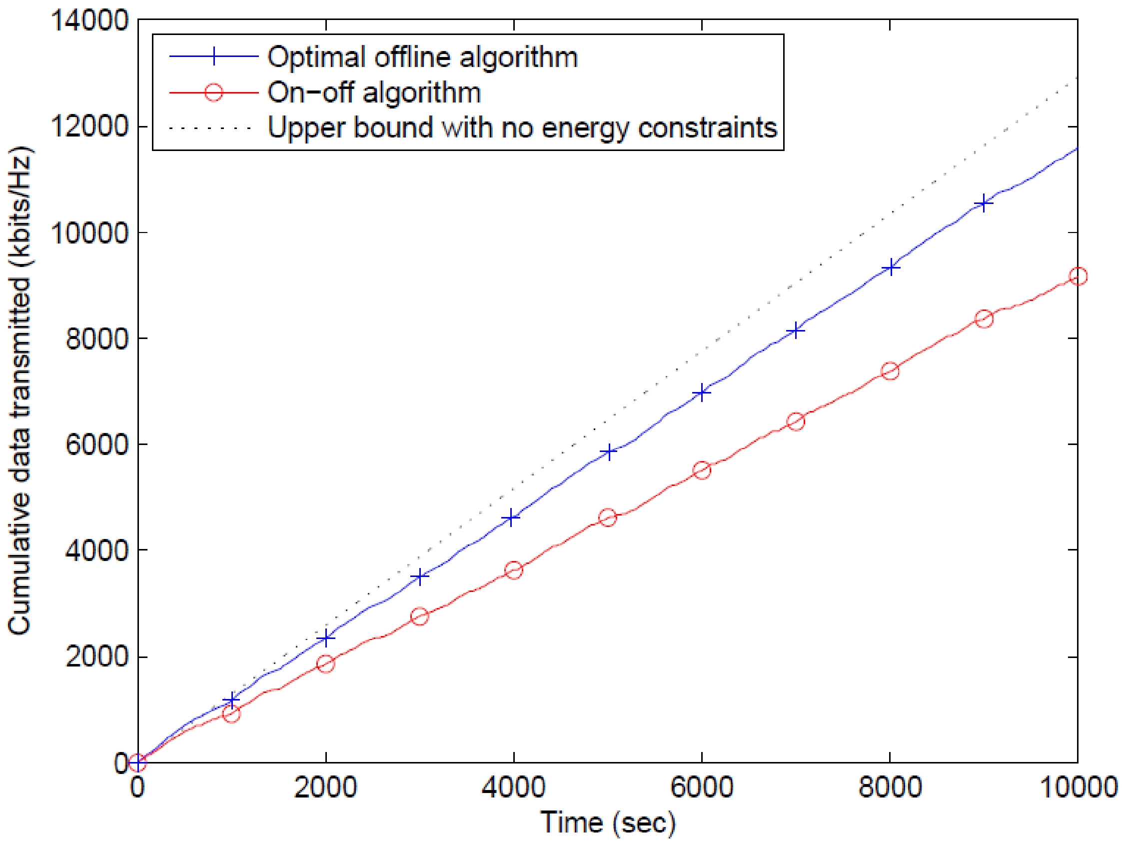
- Energy tunnel and directional water-filling approaches yield the same policy

Directional Water-Filling



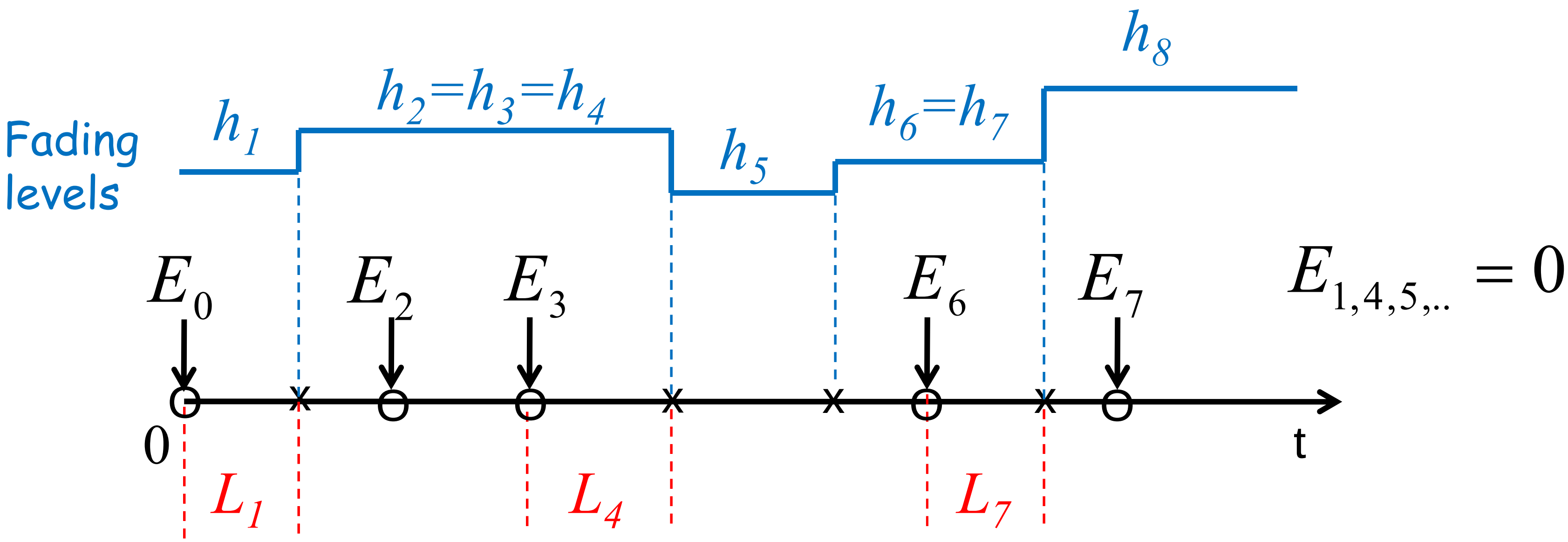
- Energy tunnel and directional water-filling approaches yield the same policy

Simulation Results



- Improvement of optimal algorithm over an *on-off transmitter* in a simulation with truncated Gaussian arrivals.

Fading Channels [Ozel-Tutuncuoglu-Ulukus-Yener'11]



- AWGN Channel with fading h: $r(p,h) = \frac{1}{2}\log(1+hp)$
- Each "epoch" defined as the interval between two "events".

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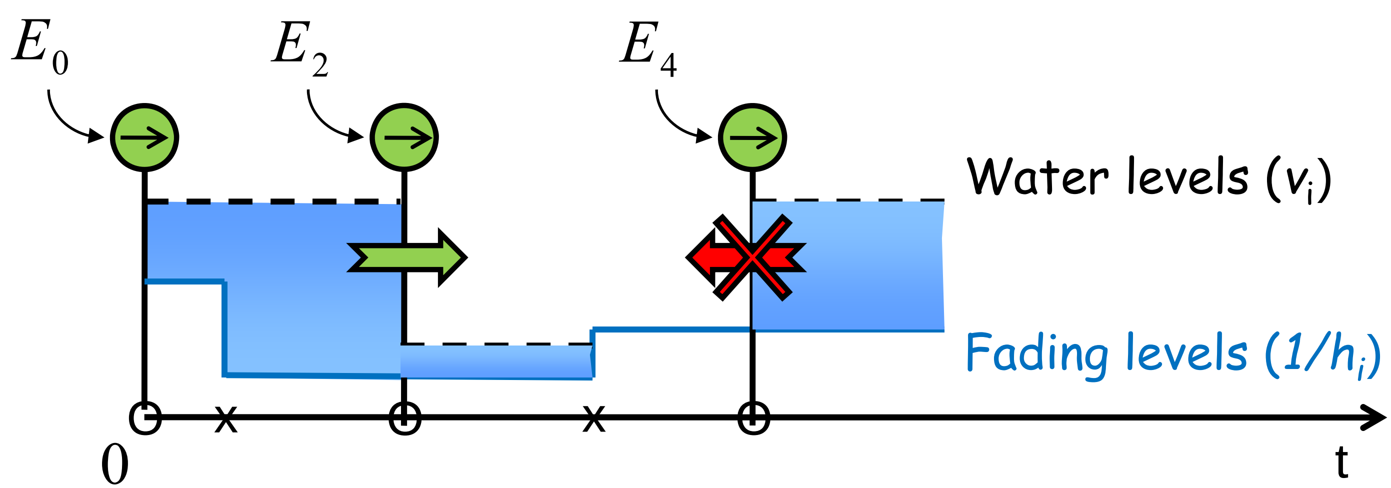
Directional Water-Filling for Fading Channels

- Same directional water filling with base levels adjusted according to channel quality.
  - Directional water flow **(Energy causality)**
  - Limited water flow **(Battery capacity)**



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Transmission Completion Time Minimization (TCTM) [Yang-Ulukus'12]

- Given the total number of bits to send as B, complete transmission in the shortest time possible.

$$\min_{p(t)} T \quad s.t. \quad B - \int_0^T r(p(t))dt \le 0, \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

Relationship of STTM and TCTM

- Lagrangian dual of TCTM problem becomes:

$$\max_{u\geq 0}\left(\min_{p(t)\in\mathfrak{P},T} T+u\left(B-\int_0^T r(p(t))dt\right)\right)$$

$$=\max_{u\geq 0}\left(\min_{T}\left(T+uB-u.\underbrace{\boxed{\max_{p(t)\in\mathfrak{P}}\int_0^T r(p(t))dt}}_{\text{STTM problem for deadline } T}\right)\right)$$

Relationship of STTM and TCTM

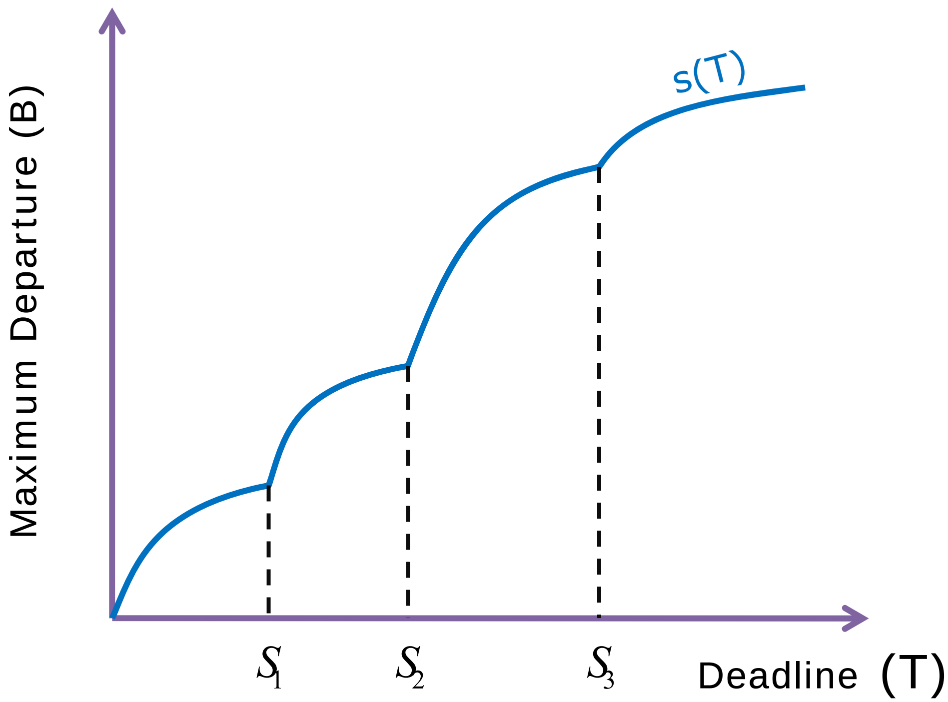
- Optimal allocations are identical:

STTM's solution for deadline T departing B bits	$\equiv$	TCTM's solution for departing B bits in time T

- STTM solution can be used to solve the TCTM problem

Maximum Service Curve

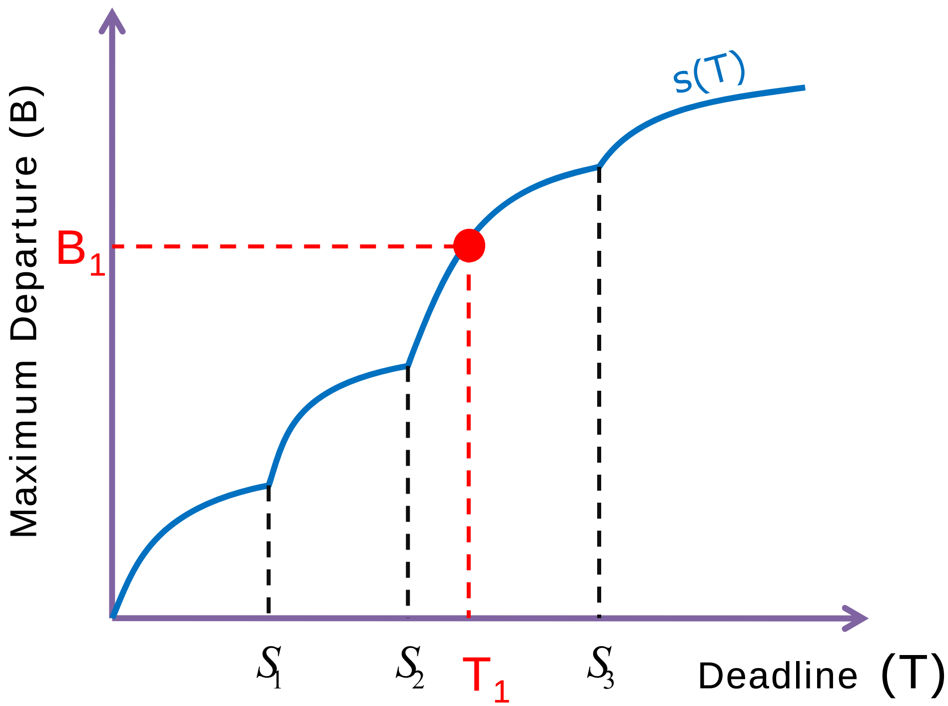
$$s(T) = \max_{p(t)} \int_0^T r(p(t))dt, \quad s.t. \quad p(t) \in \mathfrak{P}$$



- Maximum number of bits that can be sent in time T.
- Each point calculated by solving the corresponding STTM problem.

Maximum Service Curve

- Continuous, monotone increasing, **invertible**

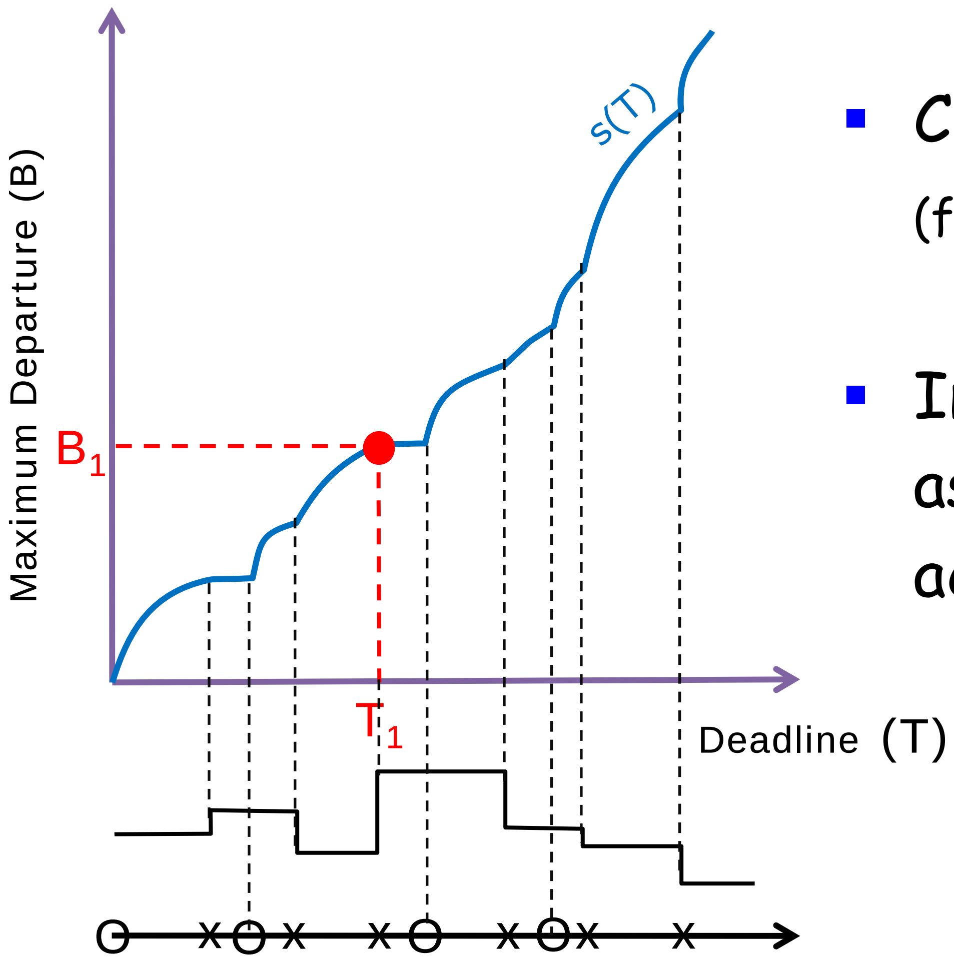


- Optimal allocation for TCTM with B_1 bits

$$=$$

Optimal allocation for STTM with deadline T_1

Maximum Service Curve: Fading



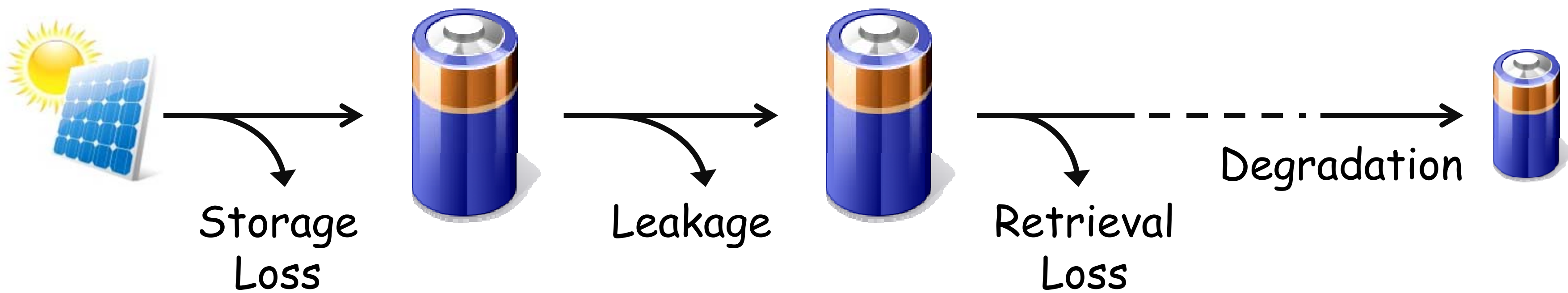
- Continuous, **non-decreasing**

  (flat regions when fading is severe)

- Inverse can be considered as the **smallest** T that achieves B_1

Transmission Policies with Inefficient Energy Storage

- Energy stored in a **battery**, **supercapacitor**, . . .
- **"Real life" issues:**



- [Devillers-Gunduz '12]: Leakage and Degradation
- [Tutuncuoglu-Yener-Ulukus '15]: Storage and Retrieval Losses

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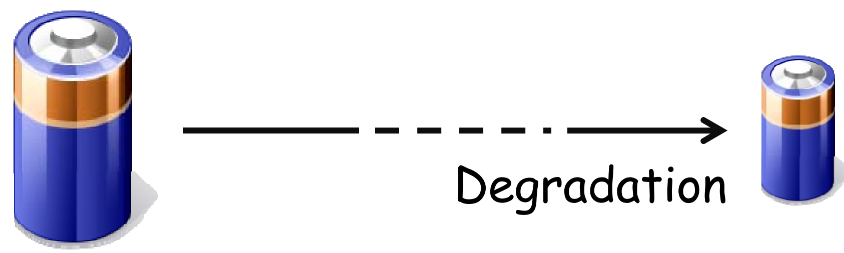
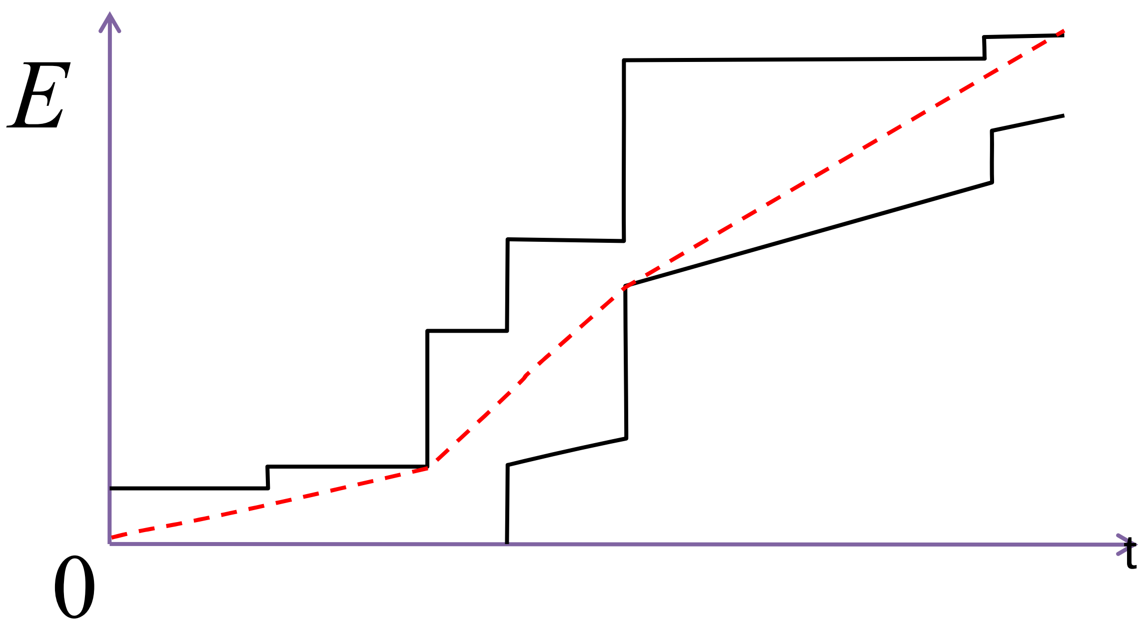
Battery Degradation

- [Devillers-Gunduz '12]



- **Optimal Policy:** Shortest path within **narrowing tunnel**



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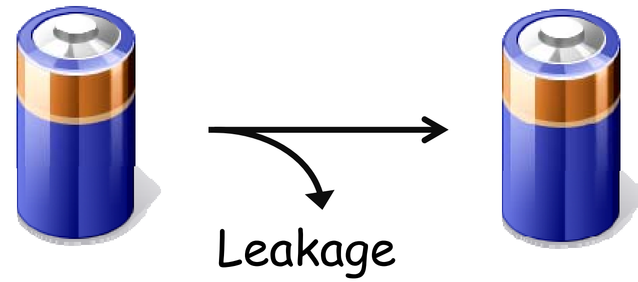
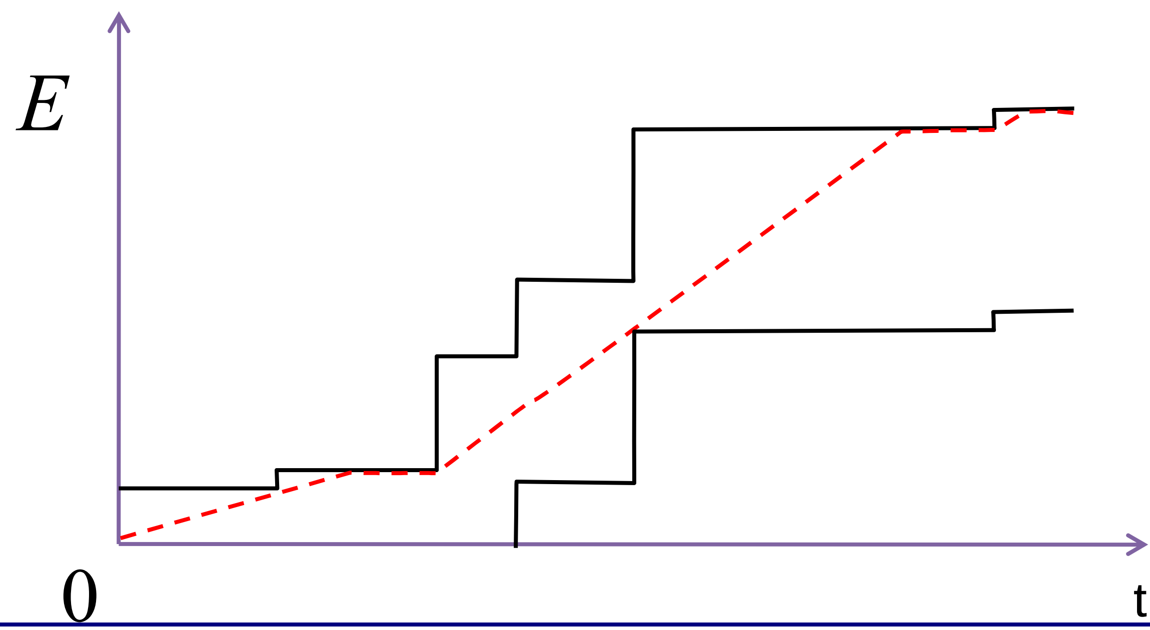
Battery Leakage

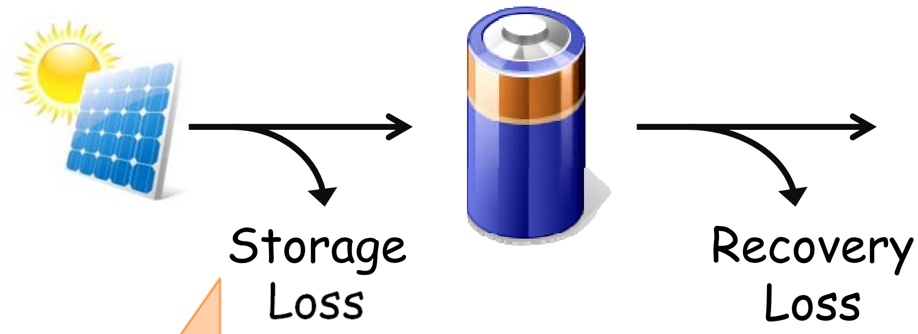


- [Devillers-Gunduz '12]
- **Optimal Policy:** When total energy in an epoch is low, deplete energy earlier to reduce leakage.



Storage/Recovery Losses

- [Tutuncuoglu-Yener-Ulukus '15]

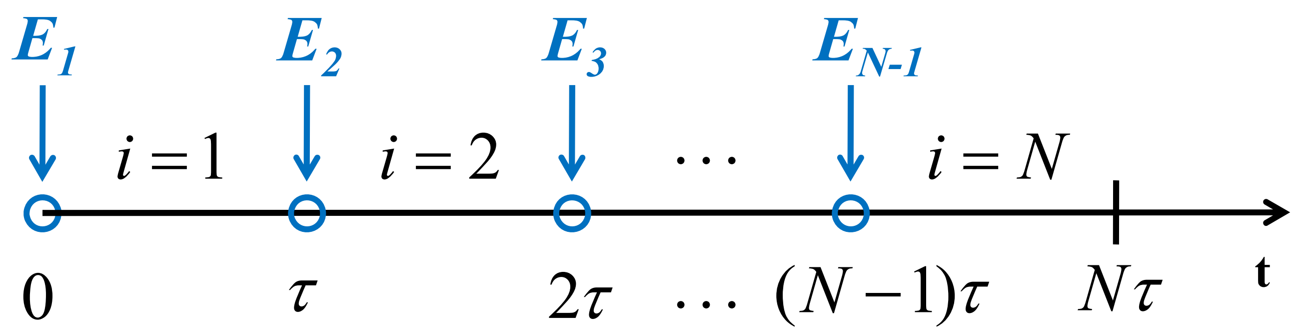


- **Main Tension:**

Concavity of r(p):
Use battery to maintain a constant power transmission

Battery inefficiency:
Storing energy in battery causes energy loss

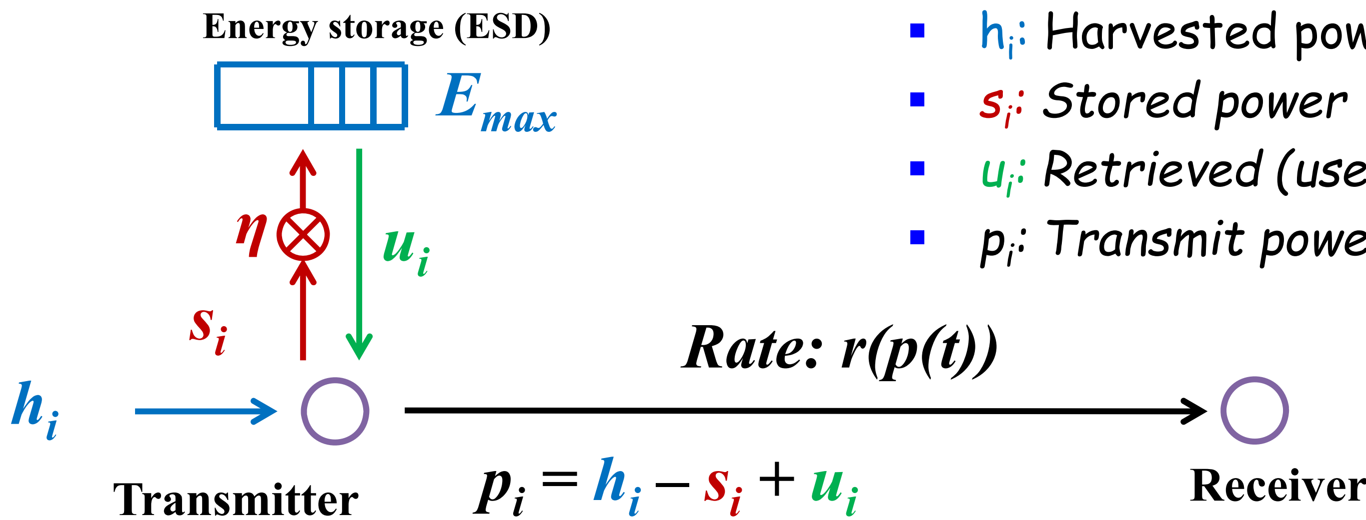
Time slotted model



- **Time slots** of duration $\tau = 1\,s$
- **Energy harvests:** Size E_i at the beginning of time slot i

All arrivals known by transmitter beforehand.

System Model



- h_i: Harvested power
- s_i: Stored power
- u_i: Retrieved (used) power
- p_i: Transmit power

- ESD has finite capacity E_{max} and storage efficiency η.
- **Energy Causality:** $\sum_{n=1}^{i} \eta s_n - u_n \geq 0, \qquad i = 1, \ldots, N$
- **Storage Capacity:** $\sum_{n=1}^{i} \eta s_n - u_n \leq E_{\max}, \qquad i = 1, \ldots, N$

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Throughput Maximization

- Find optimal energy storage policy that maximizes the average throughput of an energy harvesting transmitter within a deadline of N time slots.

$$
\begin{aligned}
&\max_{\{s_i, r_i\}} \sum_{i=1}^{N} r(E_i - s_i + u_i) \\
&s.t. \quad 0 \le E_0 + \sum_{n=1}^{i} (\eta s_i - u_i) \le E_{\max}, \qquad i = 1, \ldots, N, \\
&\qquad\quad E_i - s_i + u_i \ge 0, \; s_i \ge 0, \; u_i \ge 0, \qquad i = 1, \ldots, N.
\end{aligned}
$$

Throughput Maximization

Old problem:

$$\max_{\{p_i\}} \sum_{i=1}^{N} r(p_i)$$

$$s.t.\quad 0 \le \sum_{n=1}^{i} (E_i - p_i) \le E_{\max}, \quad i = 1, \ldots, N,$$

$$p_i \ge 0, \quad i = 1, \ldots, N.$$

$$\max_{\{s_i, r_i\}} \sum_{i=1}^{N} r(E_i \underline{- s_i + u_i})$$

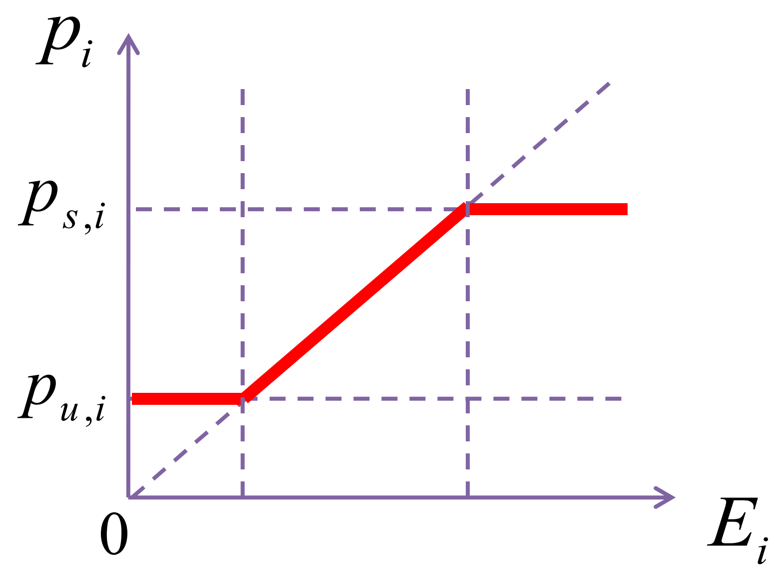
$$s.t.\quad 0 \le \sum_{n=1}^{i} (\underline{\eta s_i - u_i}) \le E_{\max}, \quad i = 1, \ldots, N,$$

$$\underline{E_i - s_i + u_i \ge 0}, \; s_i \ge 0, \; u_i \ge 0, \quad i = 1, \ldots, N.$$

Optimal Power Policy

- **Structure of optimal policy:**

$$p_i = \begin{cases} [p_{s,i}]^+ & E_i \geq p_{s,i} \\ E_i & p_{u,i} \leq E_i \leq p_{s,i} \\ p_{u,i} & E_i \leq p_{u,i} \end{cases}$$



"Double Threshold Policy"

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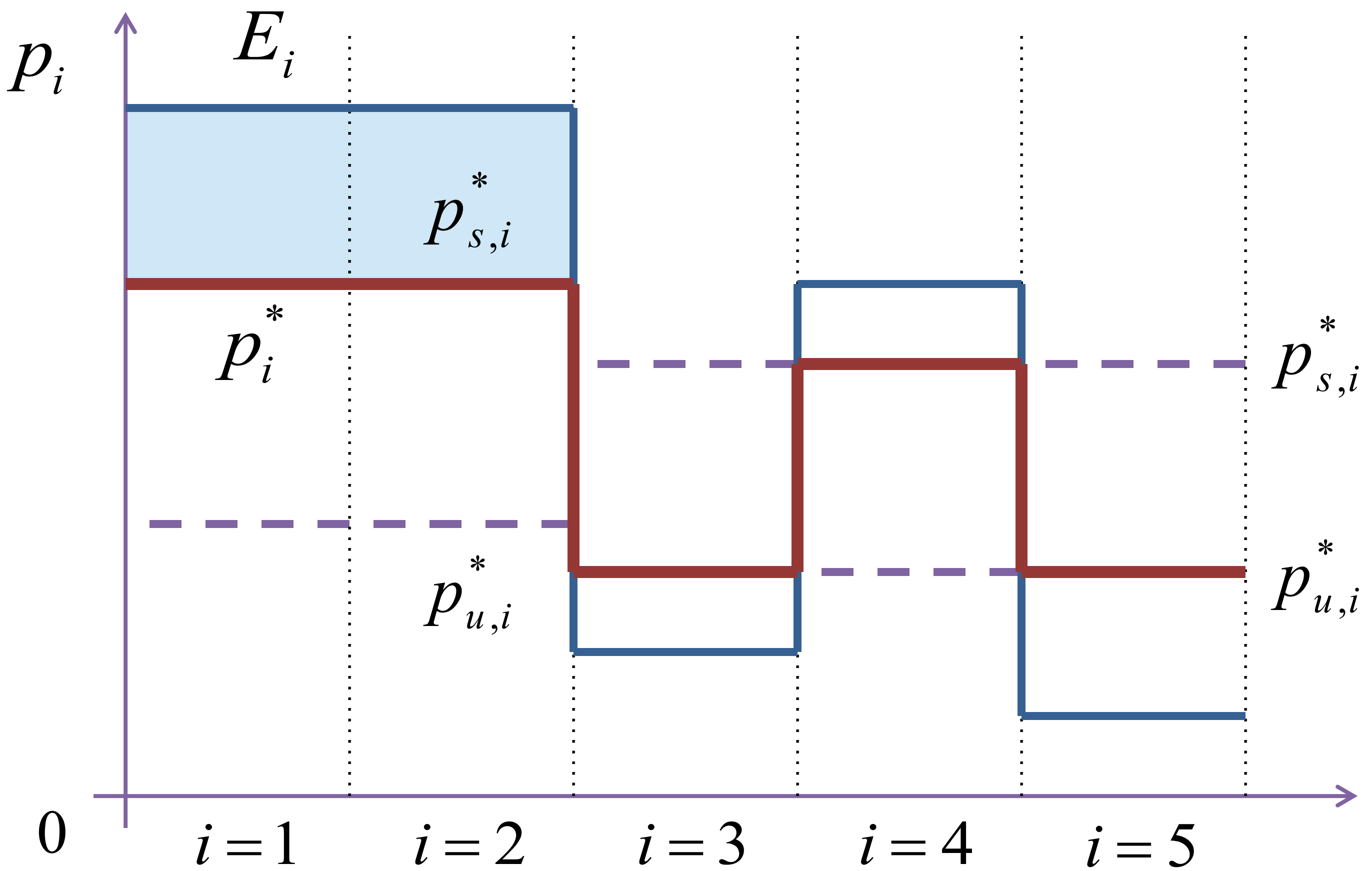
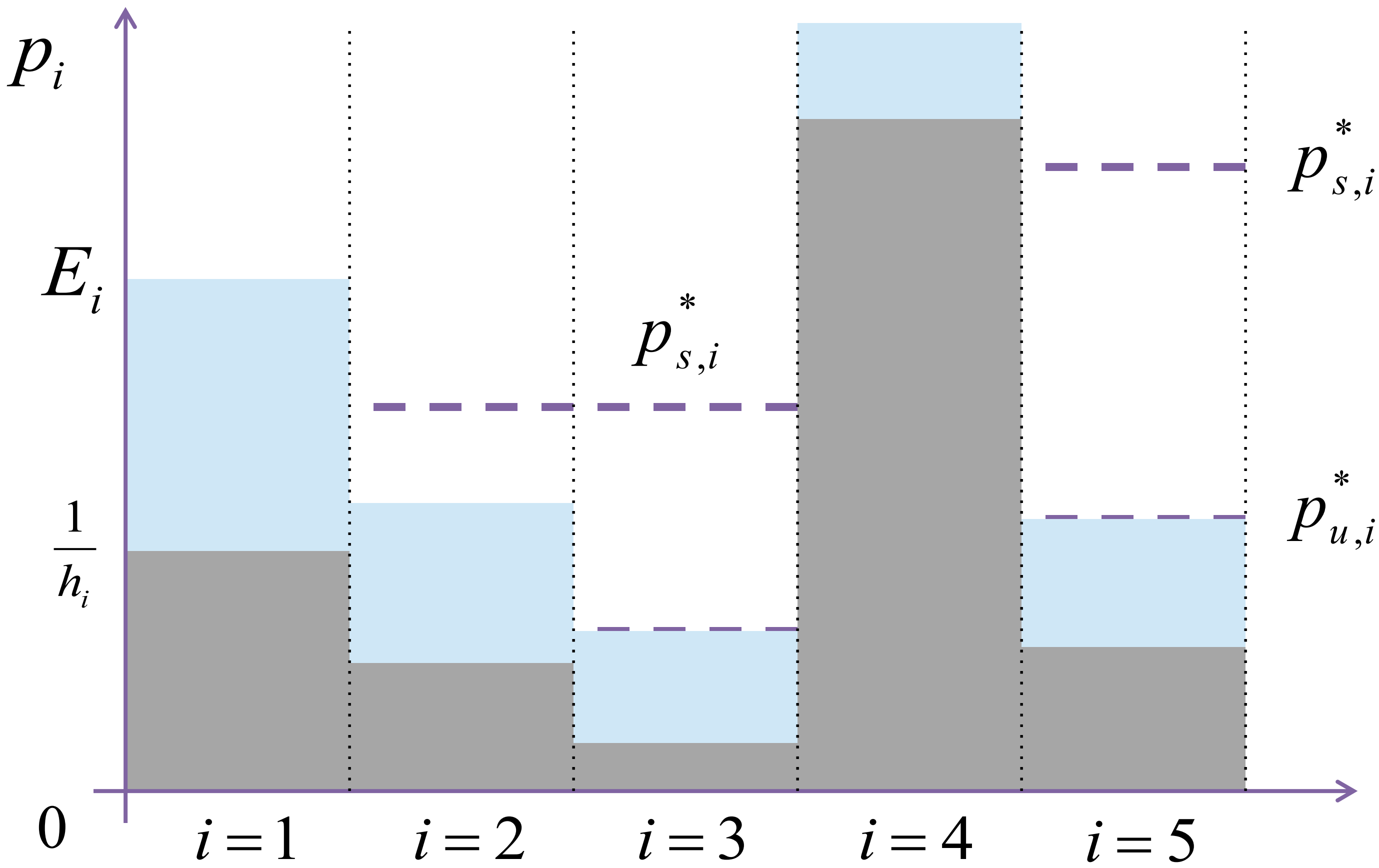
Optimal Power Policy



Optimal Power Policy
(Fading channel)



Simulations

$N = 10^4$ *time slots*

$\tau = 10\ ms$

$E_{\max} = 1\ mJ$

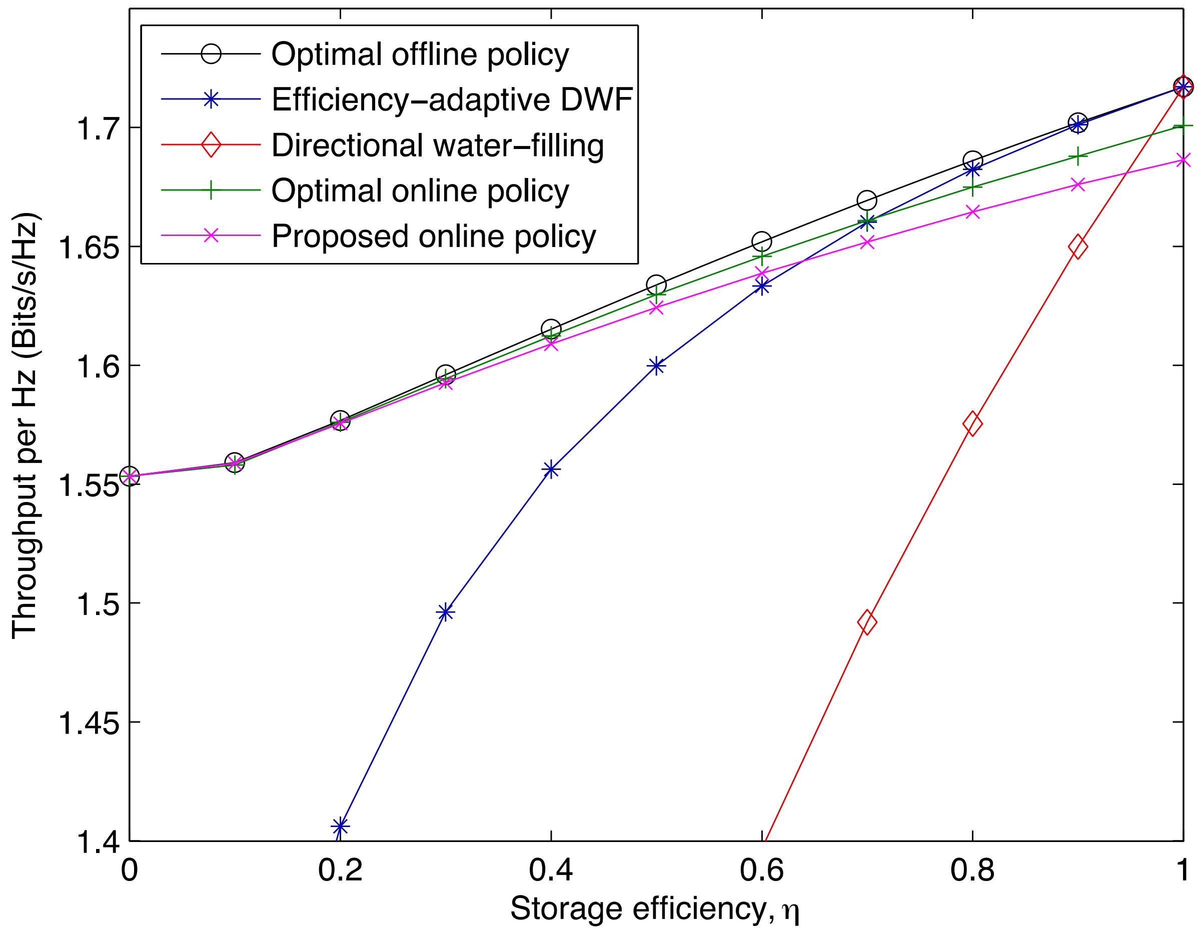
$E_0 = 0$

$E_i \sim i.i.d.\ U[0{,}200]\ \mu J$

$h = -100\ dB$

$B = 1\ MHz$

$N_0 = 10^{-19}\ W/Hz$



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Online Power Control for Energy Harvesting Nodes

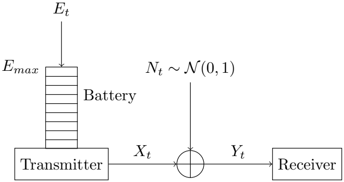
Ayfer Özgür

Tutorial on Energy Harvesing and Remotely Powered Communication

ISIT 2016, Barcelona, Spain

Offline Power Control for Energy Harvesting Nodes

An Alternative Formulation



E_t: i.i.d. energy harvesting process, can be continuous or discrete, its realization is known ahead of time.

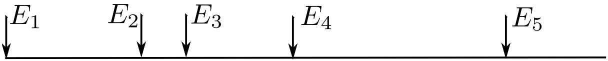
Power Control Problem:

$$T = \sup_{g} \liminf_{n\to\infty} \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} \frac{1}{2}\log(1+\gamma g_t)\right],$$

where $g_t : \mathcal{E}^n \to \mathbb{R}_+, \quad t = 1, \dots n$ is a power control policy that satisfies :

$$0 \le g_t \le b_t$$
$$b_{t+1} = \min(b_t - g_t + e_{t+1}, E_{max})$$

Offline Setting



Optimal Solution:

- Ensure battery never overflows.
- Allocate energy as equally as possible over time.

Online Setting

Energy arrivals are known causally:

$$g_t : \mathcal{E}^t \to \mathbb{R}_+, \quad t = 1, \dots n$$

Easy to observe that this a Markov Decision Process:

state	b_t	state space	$[0, E_{max}]$
action	g_t	action space	$[0, b_t]$
disturbance	E_t	disturbance distribution	$p(e)$ or $f(e)$

state evolution $\quad b_{t+1} = \min(b_t - g_t + e_{t+1}, E_{max})$

stage reward $\quad \frac{1}{2}\log(1 + \gamma g_t)$

Markov Decision Processes

$$s_{t+1} = f(s_t, u_t, w_t)$$

state	s_t	state space	$\mathcal{S}$
action	u_t	action space	$\mathcal{U}(s_t)$
disturbance	w_t	disturbance distribution	$p(w\|s, u)$

history	$h_t = (s_1, w_1, w_2, \ldots, w_{t-1})$
policy	$\pi = \{\mu_1, \mu_2, \ldots\}, \quad u_t = \mu_t(h_t)$
reward	$g(s_t, u_t)$

Goal: maximize average reward

$$J = \sup_{\pi} \liminf_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \mathbb{E}\left[g\left(S_t, \mu_t(H_t)\right)\right]$$

A dynamic programming approach

Bellman Equation

If there exists a scalar $\lambda \in \mathbb{R}_+$ and a bounded function $h : [0, E_{max}] \to \mathbb{R}_+$ that satisfy

$$\lambda + h(b) = \sup_{0 \leq g \leq b} \left\{ \frac{1}{2} \log(1 + \gamma g) + \mathbb{E}[h(\min\{b - g + E_t, E_{max}\})] \right\}$$

for all $0 \leq b \leq E_{max}$, then the optimal policy is given by $g_t^\star(E^t) = g^\star(b_t(E^t))$.

Limitations:

- can be computationally demanding;
- solution depends on the exact statistical model of energy arrivals;
- no insight on the structure of the optimal policy and the qualitative behavior of the resultant throughput;

Heuristic Online Policies

- Either no or only asymptotic guarantees on performance.
- Two natural heuristics widely considered: greedy policy and constant policy.

Greedy policy:

- instantenously uses all the incoming energy;
- ensures no battery overflow;
- becomes optimal when SNR$\to$ 0:

$$\frac{1}{n}\sum_{t=1}^{n}\frac{1}{2}\log(1+\gamma g_t) \approx \frac{\gamma}{2}\frac{1}{n}\sum_{t=1}^{n} g_t$$

Constant Policy

- keep power allocation as constant as possible over time;

$$g_t = \begin{cases} \mu = E[E_t] & \text{if } b_t \geq \mu \\ b_t & \text{if } b_t < \mu. \end{cases}$$

- becomes optimal when $E_{max} \to \infty$:

$$T = \frac{1}{2} \log(1 + \gamma\mu).$$

For finite parameter values

- these schemes can be arbitrarily away from optimality.
- asymptotic results provide no insights about the gap to optimality.
- which of the previous two policies is a better choice for a given problem?

A constant gap approach

> Look for policies that are provably close to optimal across all parameter regimes and any distribution of the energy arrivals.

Universal near-optimal policies:

- have minimal dependence on the distribution of the energy arrivals, e.g depend only on the mean.
- achieve the optimal throughput simultaneously within a constant additive and multiplicative gap for all parameter values and distributions of energy arrivals.

Wireless information theory over the last 15 years

Degrees of Freedom

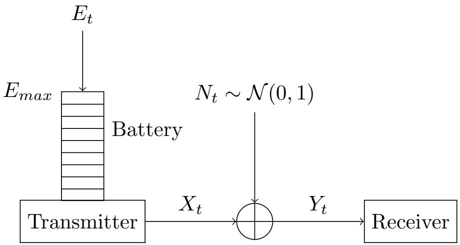
$\Downarrow$

Generalized Degrees of Freedom

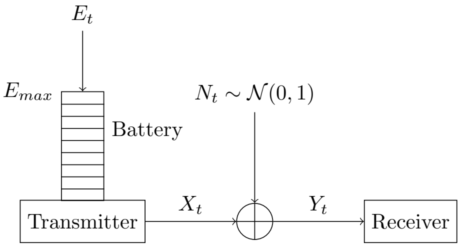
$\Downarrow$

Constant Gap Approximations

Starting Point: Bernoulli Arrivals



Starting Point: Bernoulli Arrivals



First, we focus on i.i.d. Bernoulli energy arrival process:

$$E_t = \begin{cases} E_{max} & \text{w.p. } p \\ 0 & \text{w.p. } 1-p, \end{cases}$$

Bernoulli Battery Recharges

Law of large numbers for regenerative processes:

$$\sup_g \liminf_{n\to\infty} \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} \frac{1}{2}\log(1+g(t))\right] = \sup_g \frac{1}{\mathbb{E}L}\mathbb{E}\left[\sum_{t=1}^{L} \tfrac{1}{2}\log(1+\gamma g(b_t))\right]$$

$$= \max_{\substack{\{\mathcal{E}_i\}_{i=1}^{\infty}: \\ \mathcal{E}_i \geq 0 \ \forall i \\ \sum_{i=1}^{\infty} \mathcal{E}_i \leq E_{max}}} \sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{1}{2}\log(1+\mathcal{E}_i)$$

The optimal power control policy:

$$\mathcal{E}_i = \begin{cases} \frac{(N+E_{max})}{1-(1-p)^N} p(1-p)^{i-1} - 1 & , i = 1, \ldots, N \\ 0 & , i > N \end{cases}$$

where N is the smallest positive integer satisfying

$$1 > (1-p)^N[1 + p(E_{max} + N)].$$

Bernoulli Battery Recharges

Law of large numbers for regenerative processes:

$$\sup_g \liminf_{n\to\infty} \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} \frac{1}{2}\log(1+g(t))\right] = \sup_g \frac{1}{\mathbb{E}L}\mathbb{E}\left[\sum_{t=1}^{L} \tfrac{1}{2}\log(1+\gamma g(b_t))\right]$$

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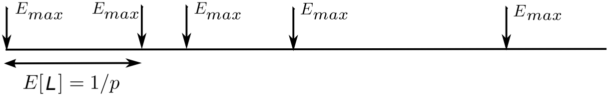
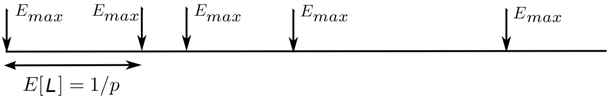
Exponentially decreasing power allocations



- Because rate is a concave function of energy/power, allocate the energy as equally as possible across time.
- Use p fraction of the available energy at each time slot:

$$g_t = pB_t$$

Exponentially decreasing power allocations



- Because rate is a concave function of energy/power, allocate the energy as equally as possible across time.
- Use p fraction of the available energy at each time slot:

$$g_t = pB_t$$

$$\Downarrow$$

$$g_t = p(1-p)^j\, E_{max}$$

where $j = t - \max\{t' \leq t : E_{t'} = E_{max}\}$.

Simplified policy for Bernoulli Arrivals

Fixed Fraction Policy:

$$g_t = pB_t$$

Theorem

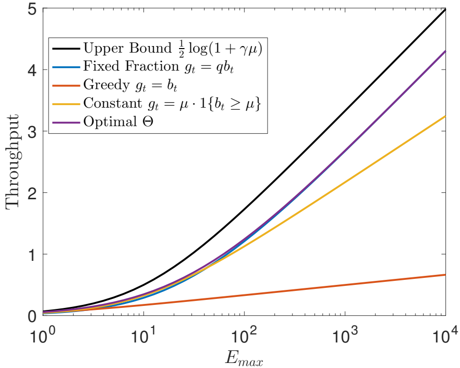
Let E_t be i.i.d Bernoulli(p, E_{max}) as before. The throughput T_{FF} achieved by the constant fraction policy satisfies

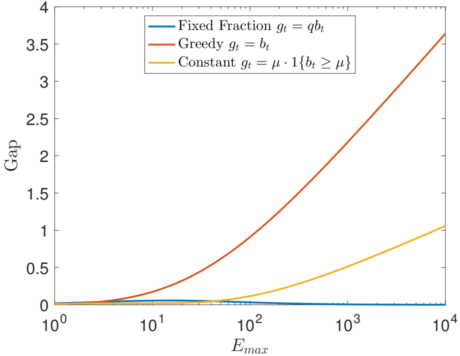
$$T_{FF} \geq \frac{1}{2}\log(1 + \gamma\, pE_{max}) - 0.72,$$

and

$$T_{FF} \geq \frac{1}{2}\frac{1}{2}\log(1 + \gamma\, pE_{max}).$$

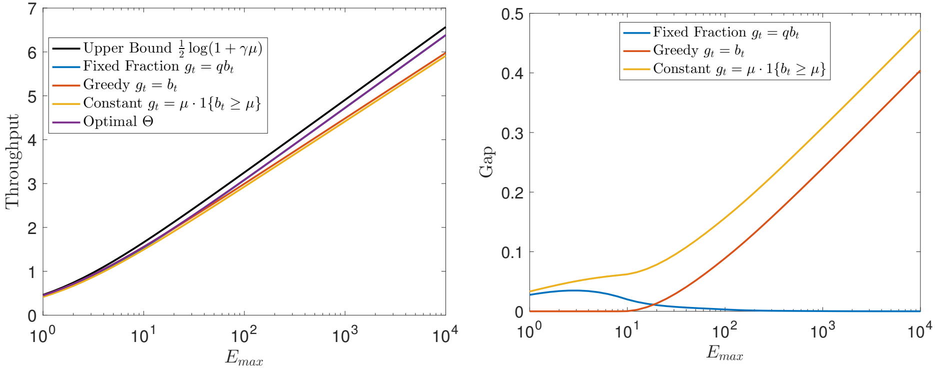
Simulation





E_t is Bernoulli$(0, E_{max})$ with $p = 0.1$.

Simulation



E_t is Bernoulli$(0, E_{max})$ with $p = 0.9$.

General i.i.d. energy arrivals

Fixed Fraction Policy:

$$g_t = qB_t, \qquad \text{where} \quad q = \mu/E_{max}$$

Theorem

The throughput T_{FF} achieved by the constant fraction policy satisfies

$$T_{FF} \geq \frac{1}{2}\log(1+\gamma\mu) - 0.72,$$

and

$$T_{FF} \geq \frac{1}{2}\frac{1}{2}\log(1+\gamma\mu).$$

Proof idea

Theorem

Bernoulli$(0, E_{max})$ is the worst case among all distributions with the same mean.

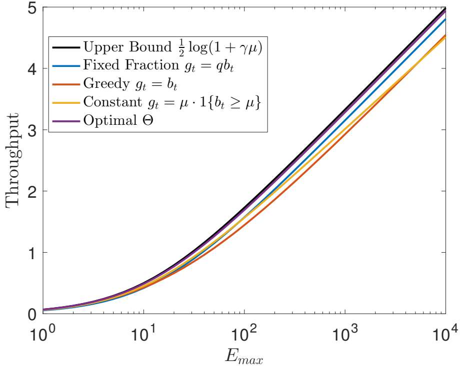
Previous heuristics: the greedy policy and the constant policy

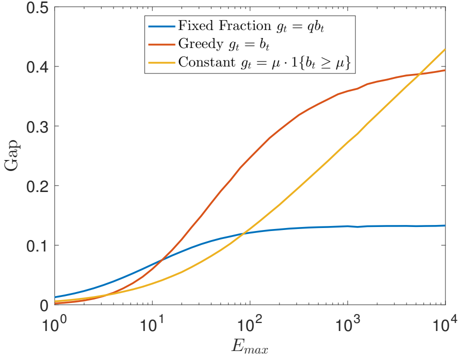
- build on the insights from the best case scenario: $E_t = \mu$ deterministically.

The fixed fraction policy

- builds on the insights from the worst case scenario: Bernoulli arrivals.

Simulation





E_t is Exponential$(1/0.1\,E_{max})$.

Open Questions and Directions

- Is Bernoulli the worst case for the optimal policy?
- Non i.i.d. energy arrivals.
- Fading Channels.
- Battery Imperfections.
- Multi-user Settings (to be discussed in the next part).

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- CCF-1618278, Center for Science of Information (CSoI), an NSF Science and Technology Center, under grant agreement CCF-0939370, and Stanford SystemX.
- Slides: Dor Shaviv.

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Offline Multi-user Energy Harvesting Settings

Online Multi-user Energy Harvesting Settings

Energy Cooperation in Energy Harvesting Networks

Şennur Ulukuş

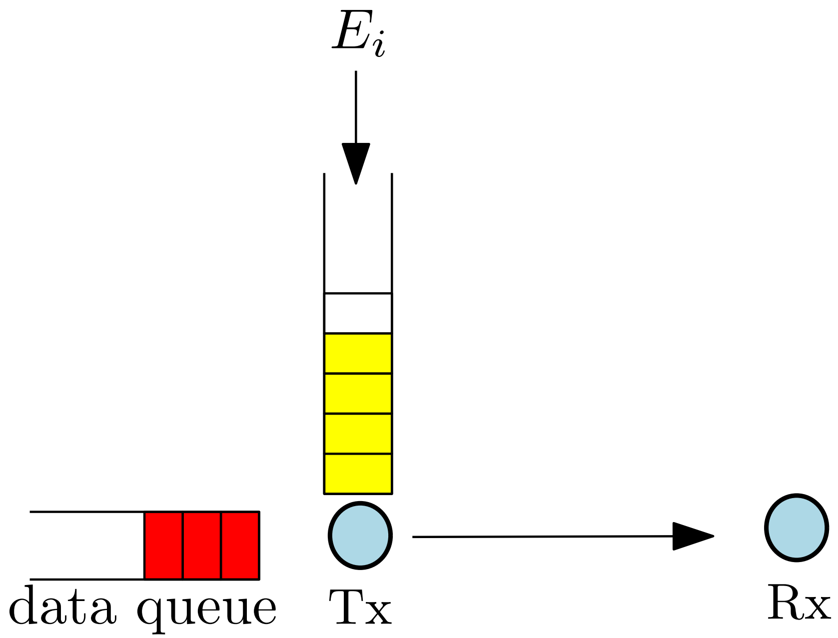
Department of ECE

University of Maryland

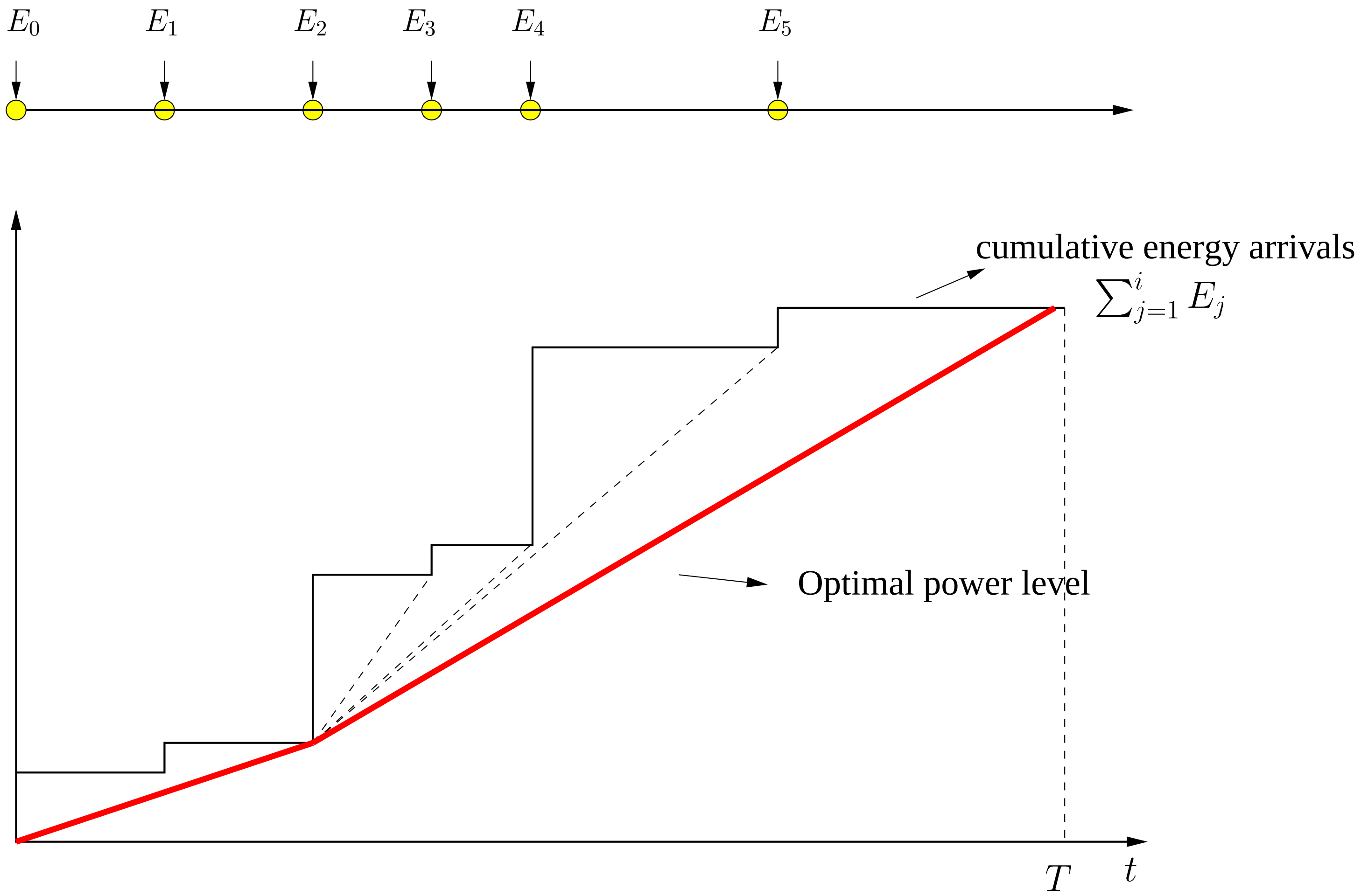
ulukus@umd.edu

So Far, We Learned...

- Wireless nodes **harvesting energy** from nature.
- **Single-user** communication with an energy harvesting transmitter.
- Energy arrives (is harvested) **during the communication session.**
- A non-trivial shift from the conventional battery powered systems.
- Transmission policy is **adapted to energy arrivals.**
- Objective: maximize throughput, minimize delay.

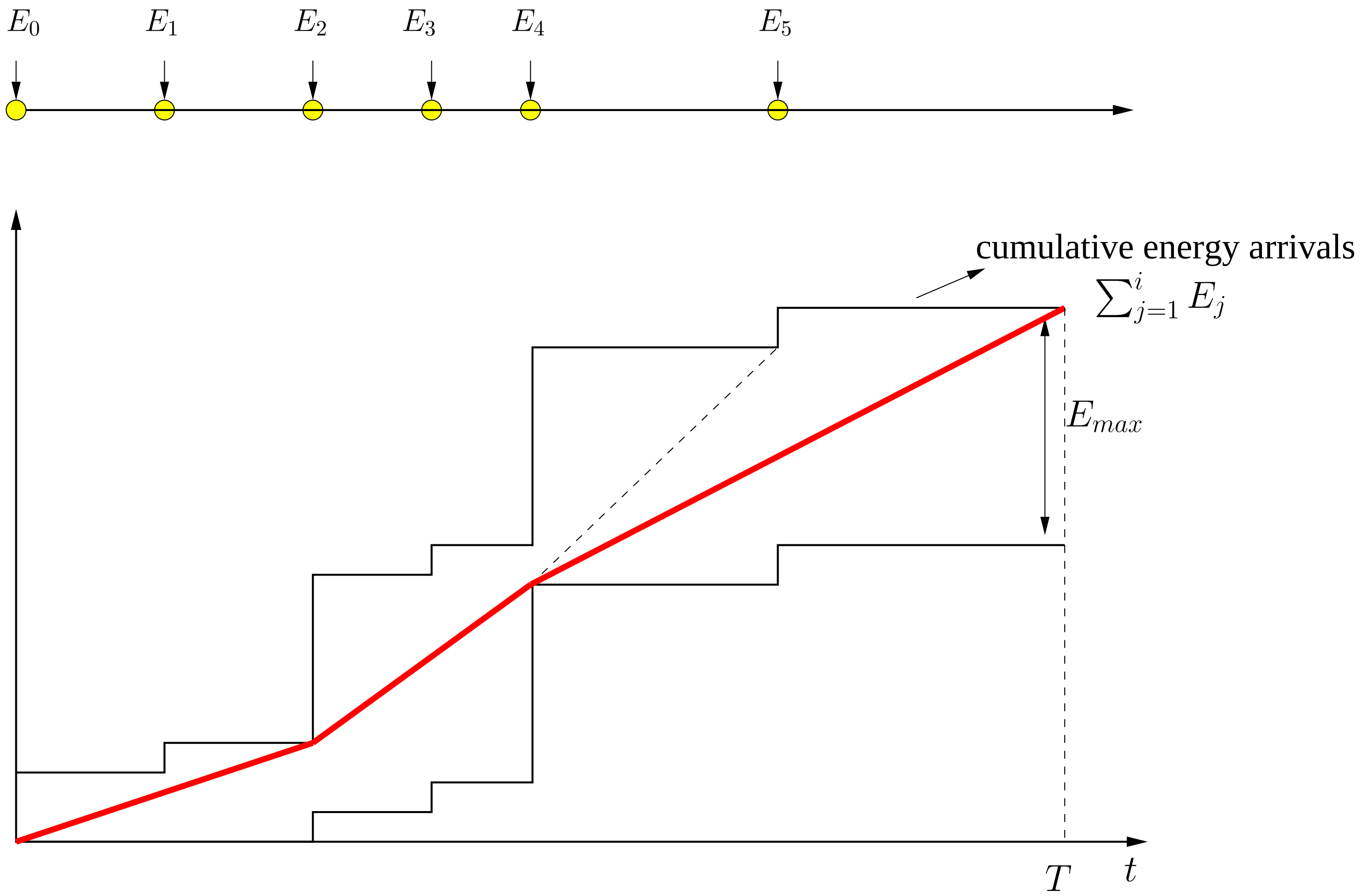


Single-User Optimal Policy for $E_{max} = \infty$



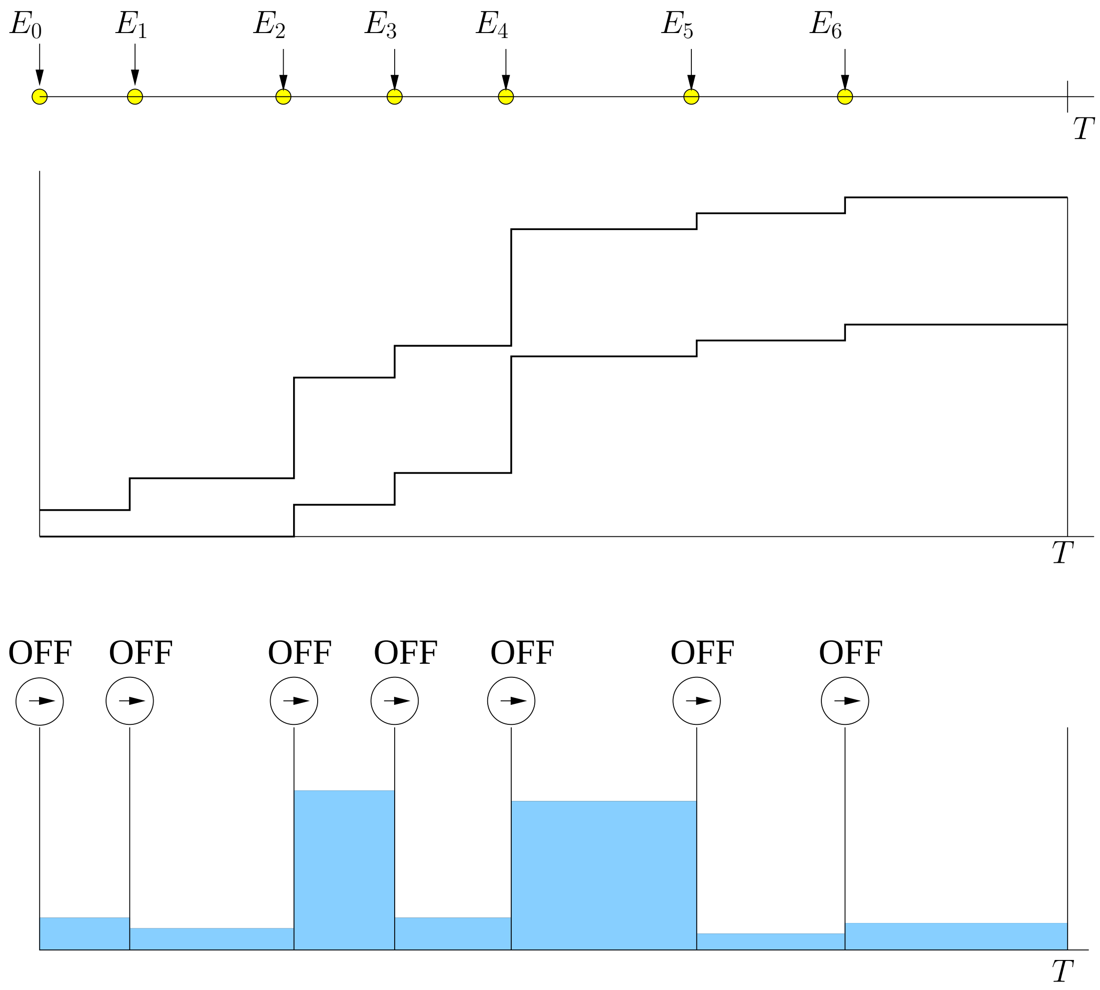
- Find the tightest curve under the cumulative energy arrival staircase.

Single-User Optimal Policy for $E_{max} < \infty$

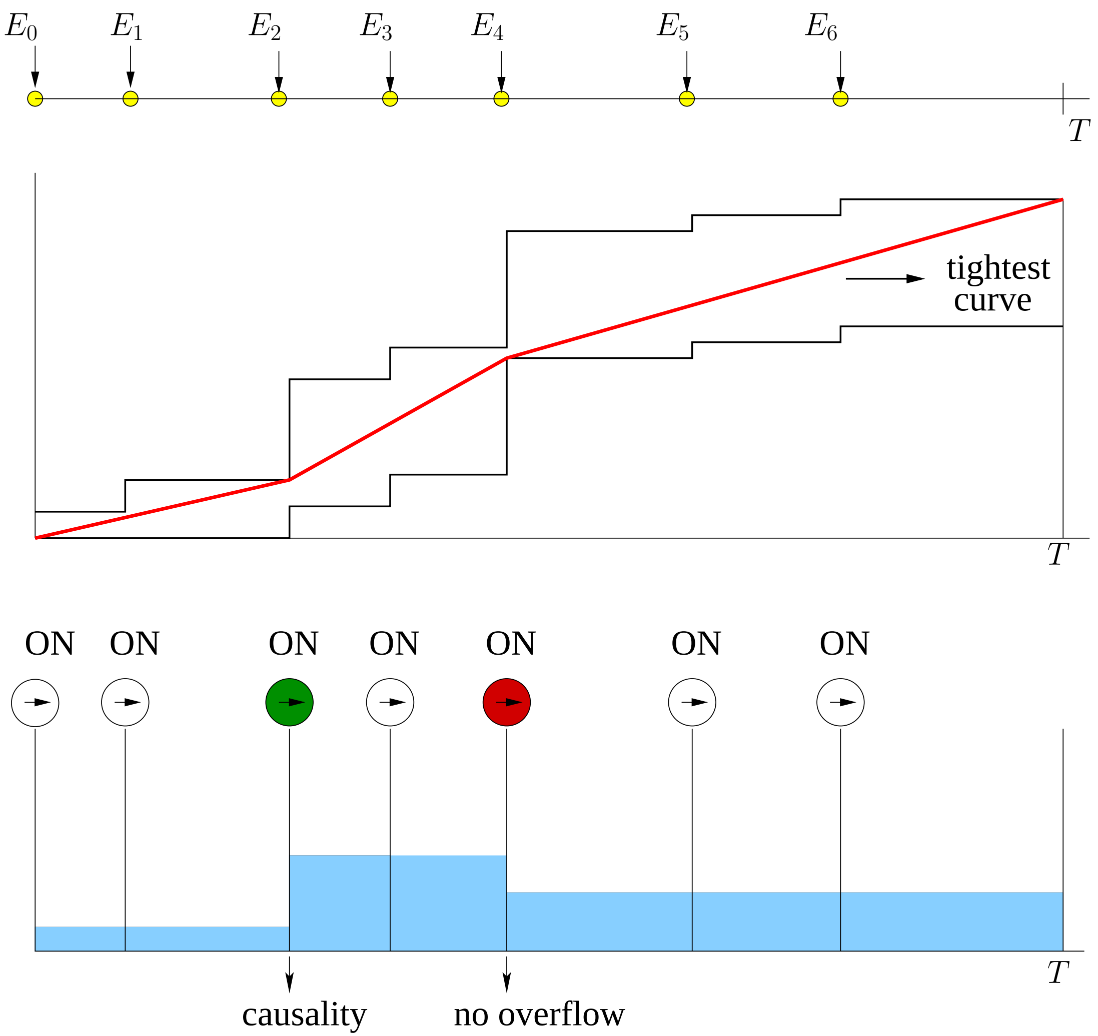


- Find the tightest curve in the energy feasibility tunnel.

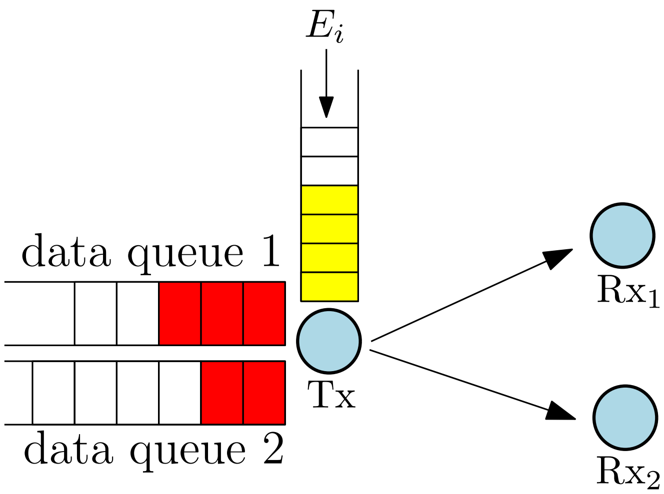
Equivalence of Feasibility Tunnel and Directional Water-filling



Equivalence of Feasibility Tunnel and Directional Water-filling

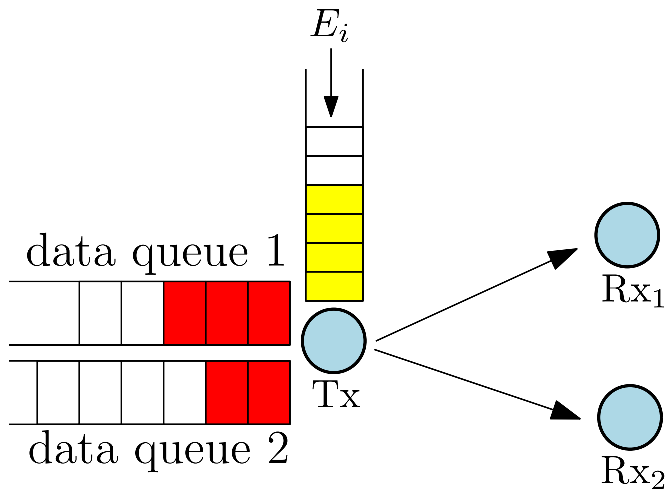


Optimal Packet Scheduling: Broadcast Channel



- Energy arrives (is harvested) **during the communication session**
- Assume battery has infinite storage capacity: $E_{max} = \infty$
- Broadcasting data to two users by **adapting to energy arrivals**
- Objective: maximize the data departure region

Broadcast Channel Model: $E_{max} = \infty$



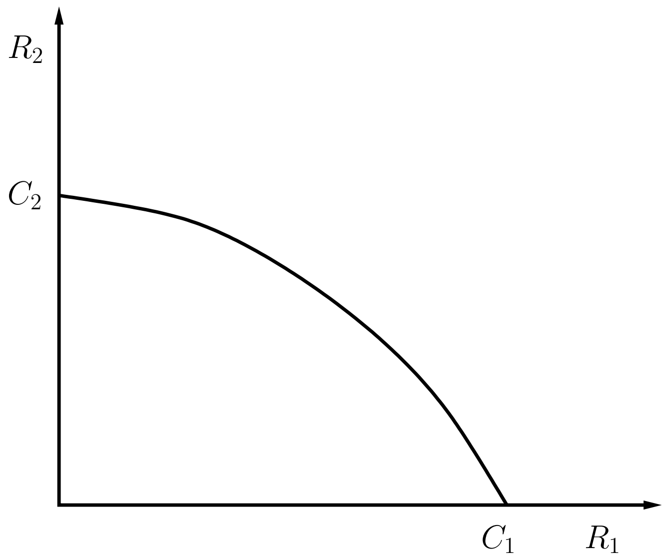
- AWGN broadcast channel:

$$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

  where $N_1 \sim \mathcal{N}(0, \sigma_1^2)$, $N_2 \sim \mathcal{N}(0, \sigma_2^2)$

- $\sigma_2^2 > \sigma_1^2$: 2nd user is degraded; we call 1st user stronger and 2nd user weaker

Broadcast Channel Model



- Broadcast capacity region:

$$r_1 \leq \frac{1}{2}\log_2\left(1+\frac{\alpha P}{\sigma_1^2}\right), \qquad r_2 \leq \frac{1}{2}\log_2\left(1+\frac{(1-\alpha)P}{\alpha P+\sigma_2^2}\right)$$

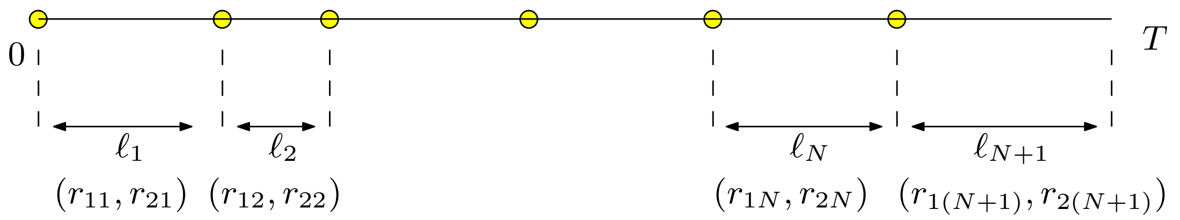
- We work in the (r_1, r_2) domain:

$$P = \sigma_1^2 2^{2(r_1+r_2)} + (\sigma_2^2 - \sigma_1^2)2^{2r_2} - \sigma_2^2 \triangleq g(r_1, r_2)$$

- $g(r_1, r_2)$ is the minimum power required to send at rates (r_1, r_2)

Finding the Maximum Departure Region

- The maximum departure region $\mathcal{D}(T)$: union of (B_1, B_2) pairs achievable by some rate allocation policy that satisfies the energy causality constraint.



- Transmission rates, and power, remain constant between energy harvests
- The energy causality constraint reduces to constraints on (r_{1i}, r_{2i}):

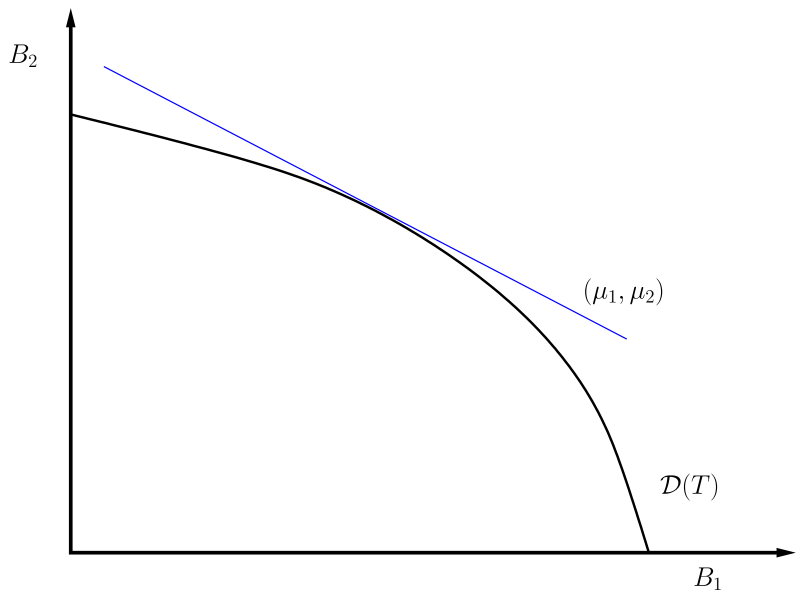
$$\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, N+1$$

Finding the Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.
- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\max_{\mathbf{r}_1,\mathbf{r}_2} \ \mu_1 \sum_{i=1}^{N+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{N+1} r_{2i}\ell_i$$

$$\text{s.t.} \ \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, N+1$$



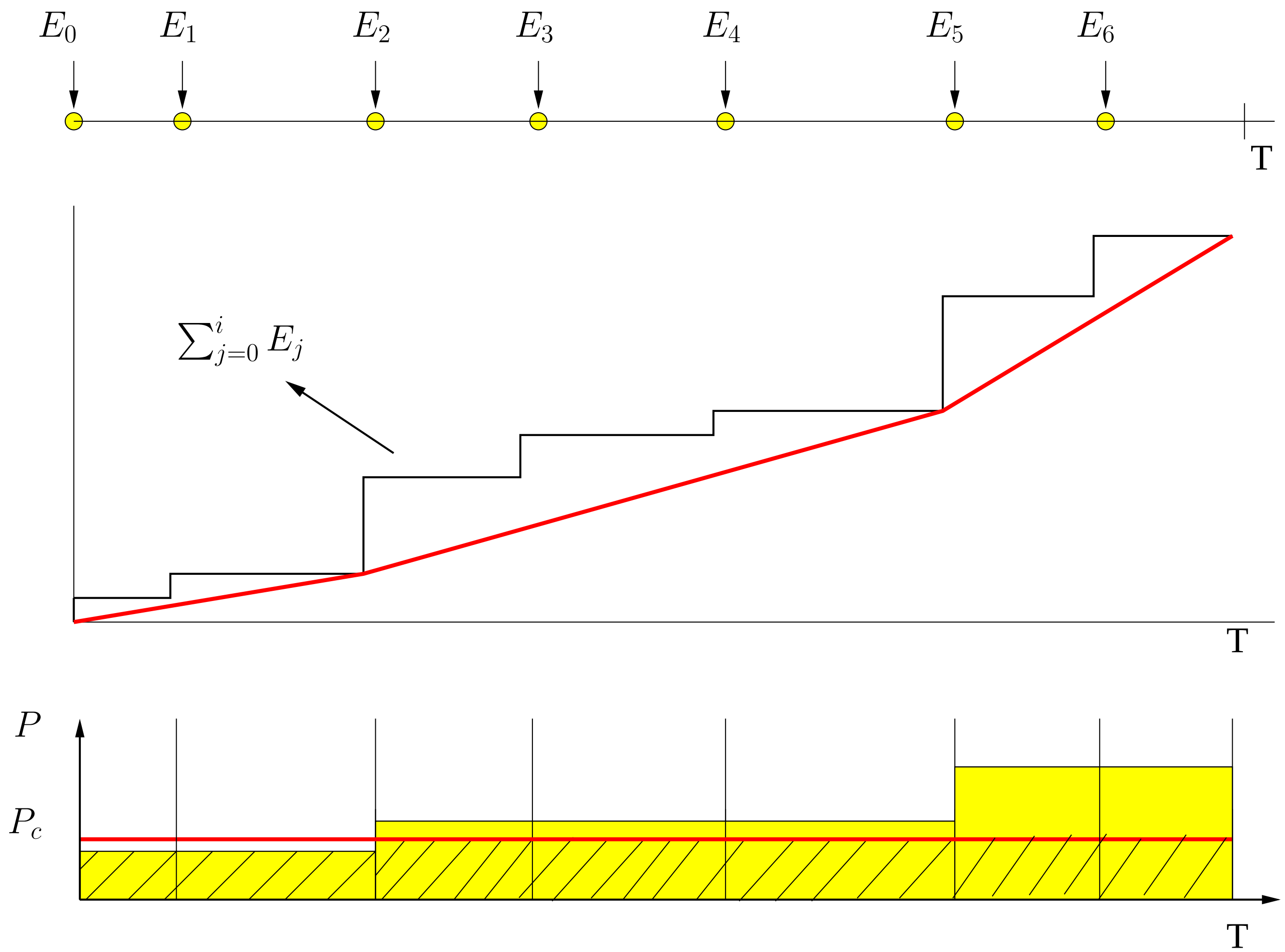
Structure of the Optimal Policy

- Total transmit power is the same as the single-user case.
- The power shares follow a cut-off structure.
- Cut-off level P_c

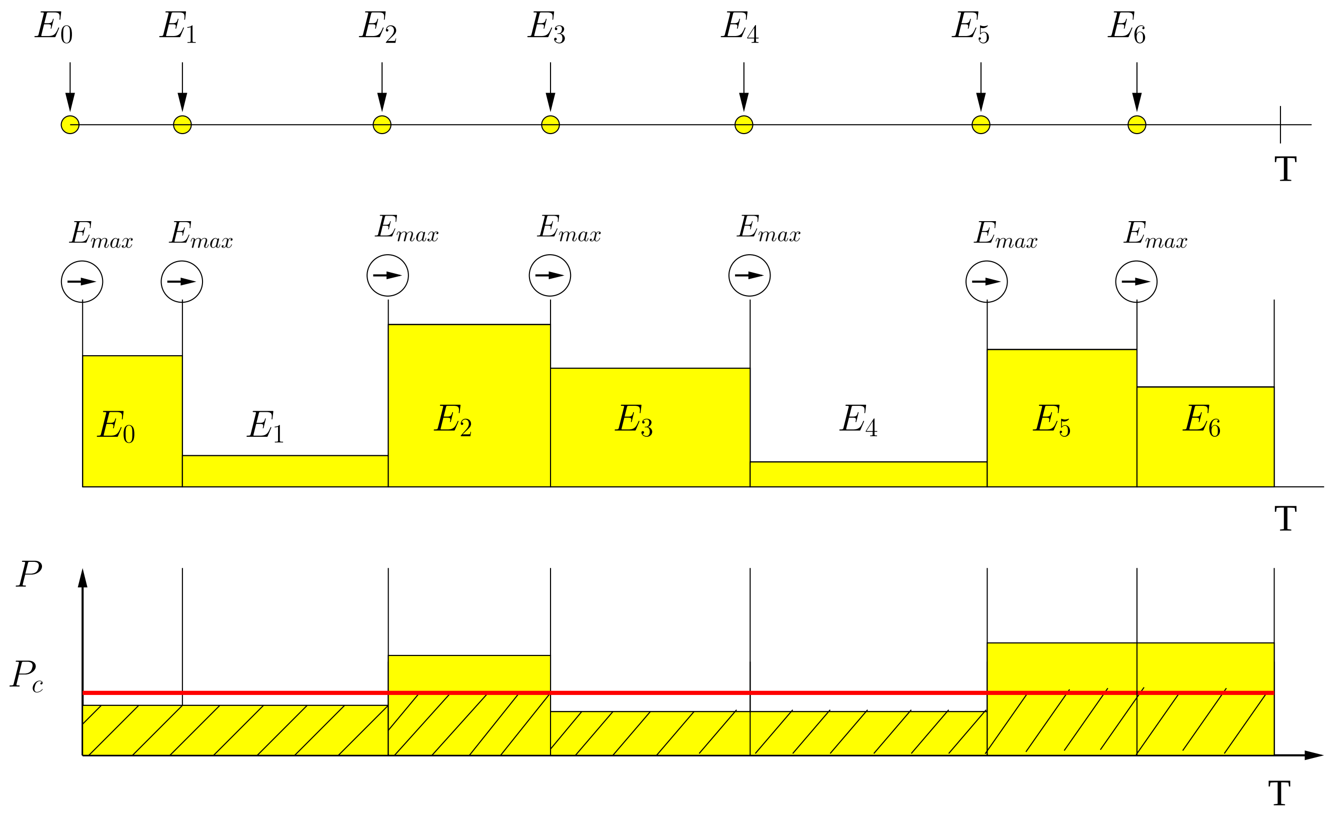
$$P_c = \left(\frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+$$

- If total power is below P_c, then, only transmit to the stronger user.
- Otherwise, stronger user's power share is P_c.
- P_c (share of the stronger user) decreases with μ_2, the priority of the weaker user.

The Structure of the Optimal Policy for $E_{max} = \infty$



The Structure of the Optimal Policy for $E_{max} < \infty$



Conclusions for the Offline Broadcasting Scenario

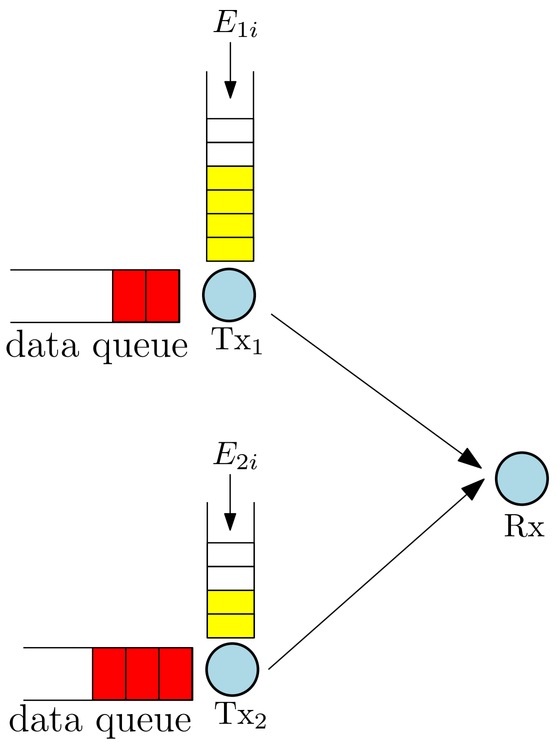
- Energy harvesting transmitter with infinite and finite capacity battery
- Maximize the departure region.
- Obtain the structure of the solution, such as:
  - the monotonicity of the transmit power
  - the cut-off power property

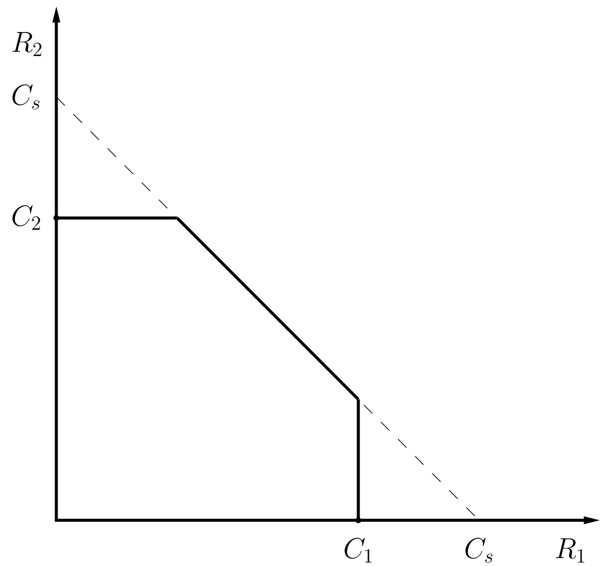
Optimal Packet Scheduling: Multiple Access Channel

- AWGN MAC channel $Y = X_1 + X_2 + Z$, $Z \sim N(0,1)$.
- The capacity region is a pentagon denoted as $\mathcal{C}(P_1, P_2)$:

$$R_1 \leq f(P_1), \quad R_2 \leq f(P_2), \quad R_1 + R_2 \leq f(P_1 + P_2)$$

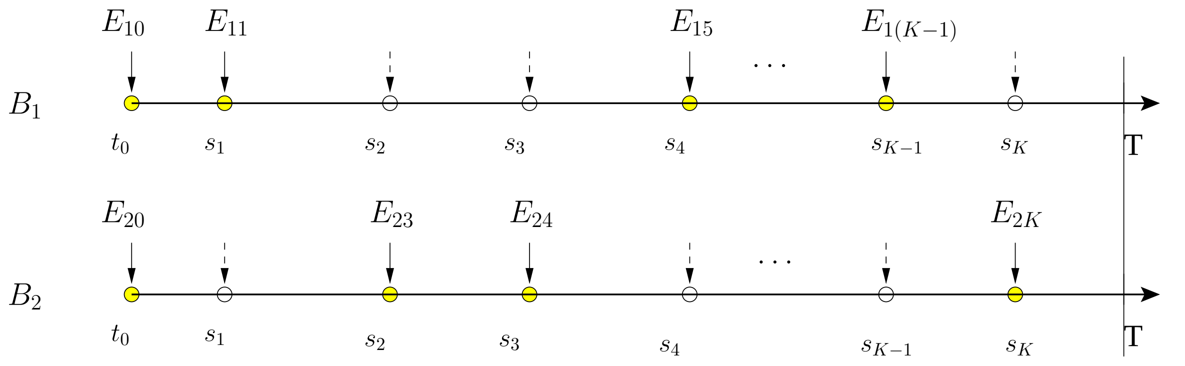
where $f(p) = \frac{1}{2}\log(1+p)$.

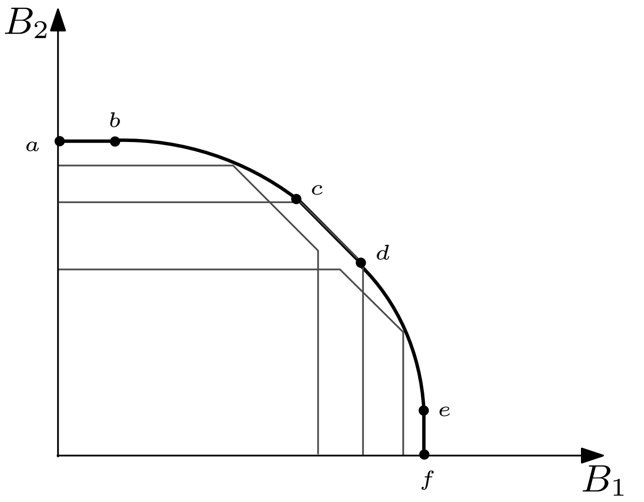




Problem Formulation

- Maximize the departure region $\mathcal{D}(T)$ by time T.





- Each feasible policy gives a pentagon.
- Union of all feasible policies gives $\mathcal{D}(T)$.

Characterizing $\mathcal{D}(T)$

- Transmission rate remains constant between energy harvests.
- For any feasible transmit power sequences $\mathbf{p}_1$, $\mathbf{p}_2$, the departure region is a pentagon

$$B_1 \leq \sum_{n=1}^{N} f(p_{1n}) l_n$$

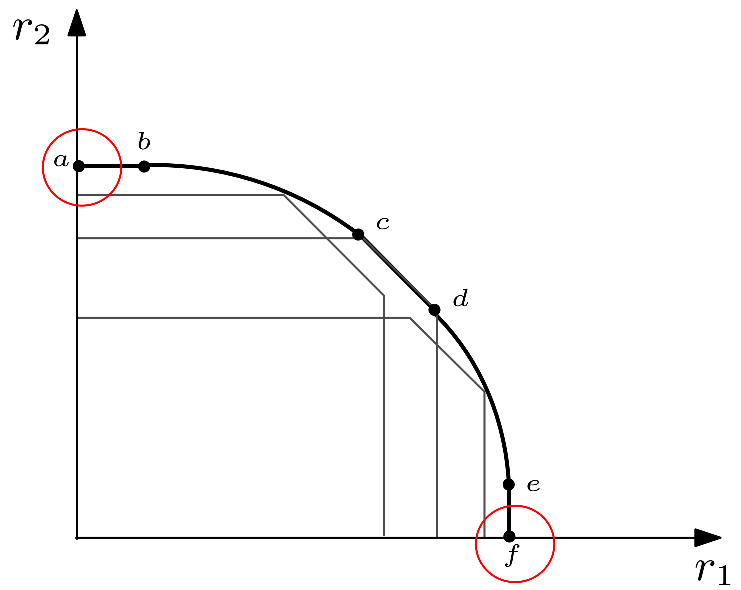
$$B_2 \leq \sum_{n=1}^{N} f(p_{2n}) l_n$$

$$B_1 + B_2 \leq \sum_{n=1}^{N} f(p_{1n} + p_{2n}) l_n$$

- $\mathcal{D}(T)$ is a union of (B_1, B_2) and convex.
- The boundary points maximize $\mu_1 B_1 + \mu_2 B_2$ for some $\mu_1, \mu_2 \geq 0$.

Point a

- Single-user power allocation.

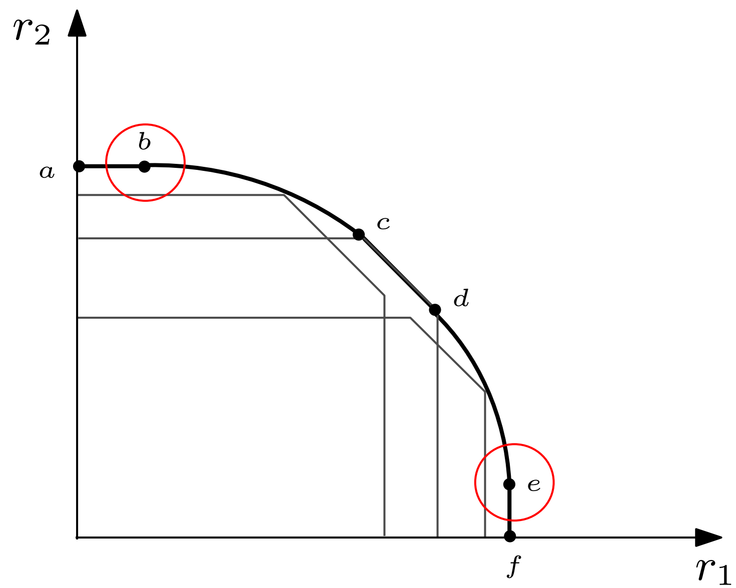


$$\max_{\mathbf{p}_1} \quad \sum_n f(p_{1n}) l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \leq N$$

Point b

- User 2 power is fixed to its single-user power allocation.

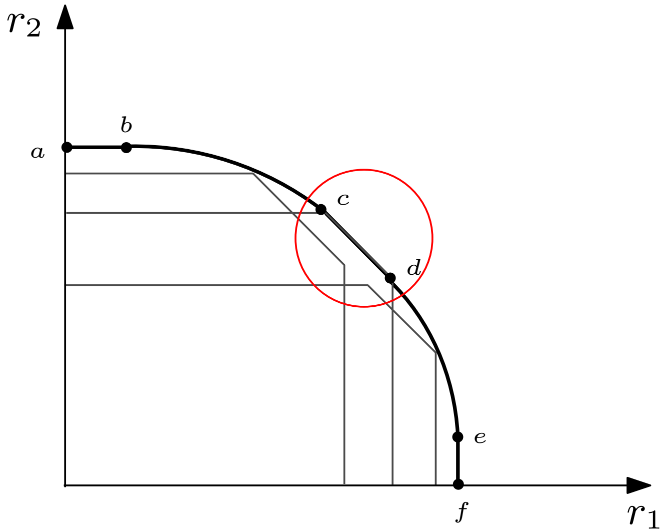


$$\max_{\mathbf{p}_1} \quad \sum_n f(p_{1n} + p_{2n}^*) l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \leq N$$

Sum-rate: Points between c and d

- Maximize the sum-rate of the users.



$$\max_{\mathbf{p}_1,\mathbf{p}_2} \quad \sum_n f(p_{1n}+p_{2n})l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n}l_n \le \sum_{n=0}^{j-1} E_{1n}, \quad \forall j: 0<j\le N$$

$$\sum_{n=1}^{j} p_{2n}l_n \le \sum_{n=0}^{j-1} E_{2n}, \quad \forall j: 0<j\le N$$

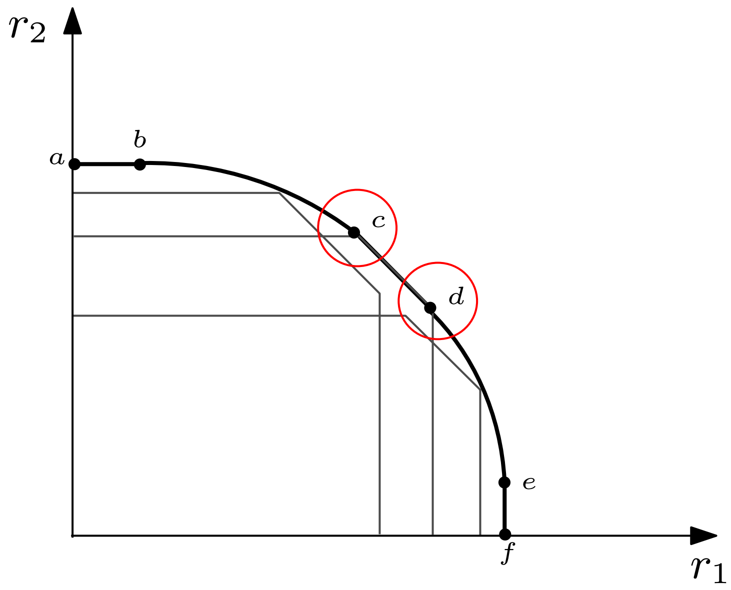
Sum-Rate: Points between c and d, Equivalent Problem

- Equivalent problem:

$$\max_{\mathbf{p}_1+\mathbf{p}_2} \quad \sum_n f(p_{1n}+p_{2n})l_n$$

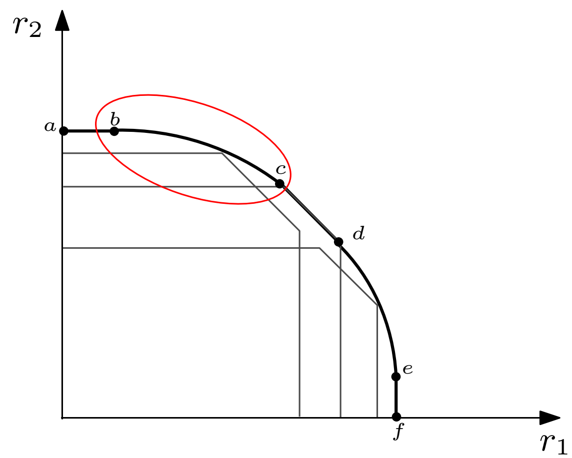
$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n}l_n + p_{2n}l_n \le \sum_{n=0}^{j-1} E_{1n}+E_{2n}, \quad \forall j: 0<j\le N$$

- Power can be divided back to $\mathbf{p}_1, \mathbf{p}_2$ in infinite number of ways.



Points between b and c: Arbitrary μ_1, μ_2

- Each boundary point corresponds to a corner point of some pentagon.
- $\mu_2 > \mu_1 \Rightarrow$ achieving points between point b and point c:



$$\max_{\mathbf{p}_1,\mathbf{p}_2} \quad (\mu_2-\mu_1)\sum_n f(p_{2n})l_n + \mu_1 \sum_n f(p_{1n}+p_{2n})l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n}l_n \le \sum_{n=0}^{j-1} E_{1n}, \quad \forall j: 0<j\le N$$

$$\sum_{n=1}^{j} p_{2n}l_n \le \sum_{n=0}^{j-1} E_{2n}, \quad \forall j: 0<j\le N$$

Generalized Iterative Backward Waterfilling

- Solve the problem via generalized iterative backward waterfilling:
- Given $\mathbf{p}_1^*$, solve for $\mathbf{p}_2$:

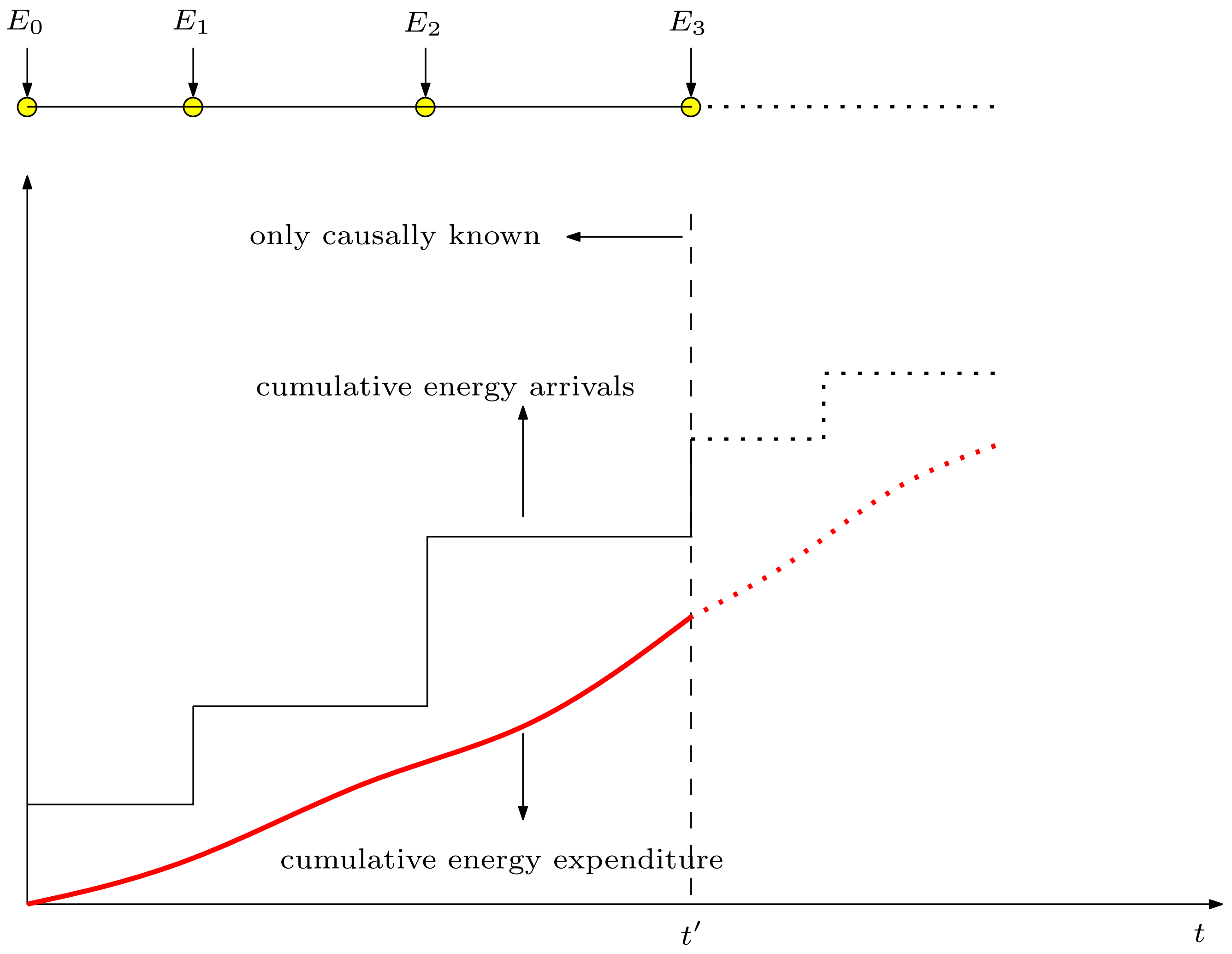
$$\begin{aligned} \max_{\mathbf{p}_2} \quad & (\mu_2 - \mu_1) \sum_{n=1}^{N} f(p_{2n}) l_n + \mu_1 \sum_{n=1}^{N} f(p_{1n}^* + p_{2n}) l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{2n} l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad 0 < j \leq N \end{aligned}$$

- Once $\mathbf{p}_2^*$ is obtained, we do a backward waterfilling for the second user.
- We perform the optimization for both users in an alternating way.
- The iterative algorithm converges to the global optimal solution.

Conclusions for the Offline Multiple Access Scenario

- Energy harvesting transmitters sending messages to a single access point.
- The problem: maximization of the departure region.
- Obtain the structure using generalized iterative waterfilling.

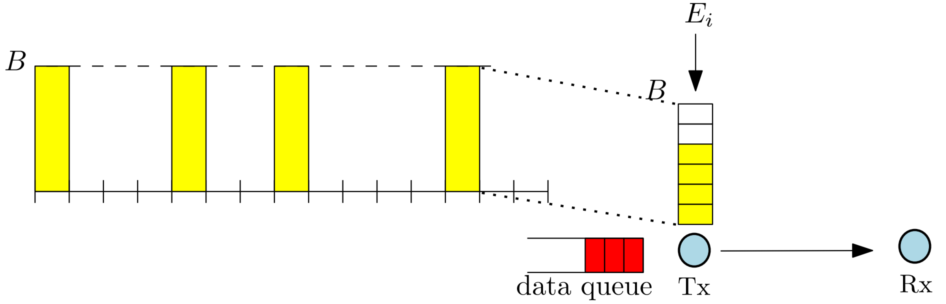
So Far, We Learned...

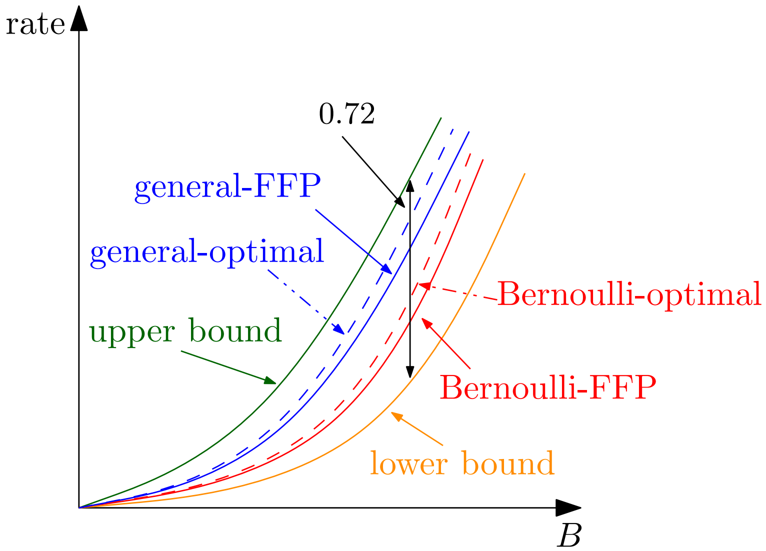


- So far, mostly: dynamic programming, learning algorithms, heuristics.

A Unique Approach: Online Power Scheduling

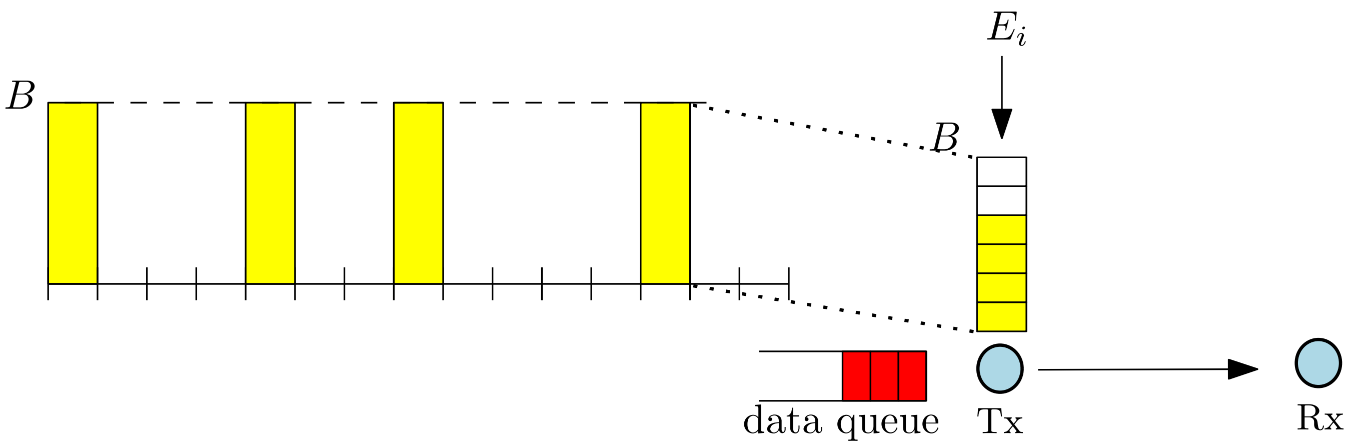
- Feel the Bernoulli.
- Steps of the approach:
  - Study Bernoulli energy arrivals with $\{0, B\}$ support.
  - Propose a simple sub-optimal policy for Bernoulli arrivals.
  - Bound its performance.
  - Extend this sub-optimal policy for general energy arrivals.
  - Bernoulli is the worst energy arrival for the proposed algorithm.
  - Obtain a near-optimal online power policy.





Online Policy for the Single-User Channel

- Bernoulli energy arrivals:



- $\mathbb{P}[E_i = B] = 1 - \mathbb{P}[E_i = 0] = p$
- When an energy arrives, a **renewal** occurs.

Long-Term Average Throughput Using Renewal Theory

- Long-term average throughput, under Bernoulli energy arrivals:

$$\begin{aligned}
\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}\log\left(1+P_i\right)\right] &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{1}{2}\log\left(1+P_i\right)\right] \\
&= p\sum_{k=1}^{\infty}p(1-p)^{k-1}\sum_{i=1}^{k}\frac{1}{2}\log\left(1+P_i\right) \\
&= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty}p^2(1-p)^{k-1}\frac{1}{2}\log\left(1+P_i\right) \\
&= \sum_{i=1}^{\infty}p(1-p)^{i-1}\frac{1}{2}\log\left(1+P_i\right)
\end{aligned}$$

- L is inter-energy arrival time, geometric with $\mathbb{E}[L]=\frac{1}{p}$.

Resulting Optimization Problem

- To characterize $\{P_i\}$ which achieves the maximum, we solve

$$\begin{aligned} \max_{\{P_i\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1} \frac{1}{2} \log\left(1+P_i\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} P_i \leq B \\ & P_i \geq 0, \quad \forall i \end{aligned}$$

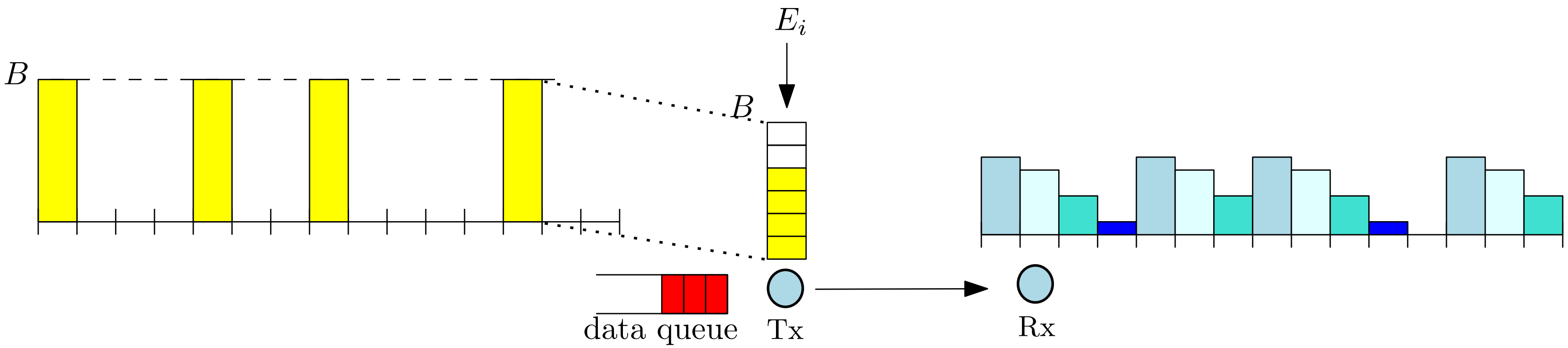
- Solution:

$$P_i = \frac{p(1-p)^{i-1}}{\lambda} - 1, \qquad i = 1, \ldots, \tilde{N}$$

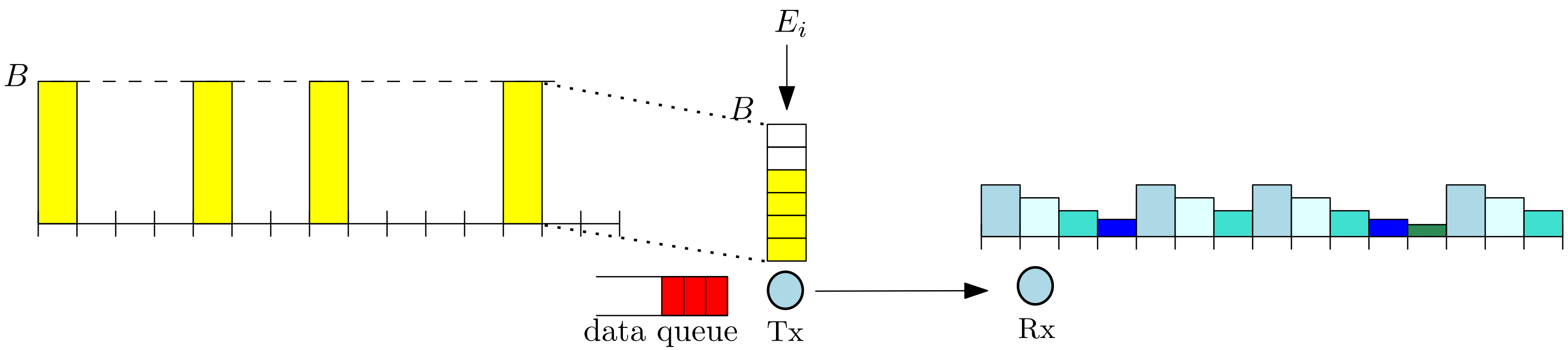
- Decreasing power for a finite duration $\tilde{N}$ that depends on B.

Online Policy for the Single-User Channel

- Bernoulli energy arrivals:
  - Optimal power allocation with $\tilde{N} = 4$:



  - Sub-optimal fractional power allocation, $P_i = Bp(1-p)^{i-1}$:



Bounds on the Online Policies

- Upper bound from offline policy:

$$r \leq \frac{1}{2}\log(1+\mu)$$

- Lower bound algebraically for Bernoulli arrivals:

$$r \geq \frac{1}{2}\log(1+\mu) - 0.72$$

Sketch of the proof:

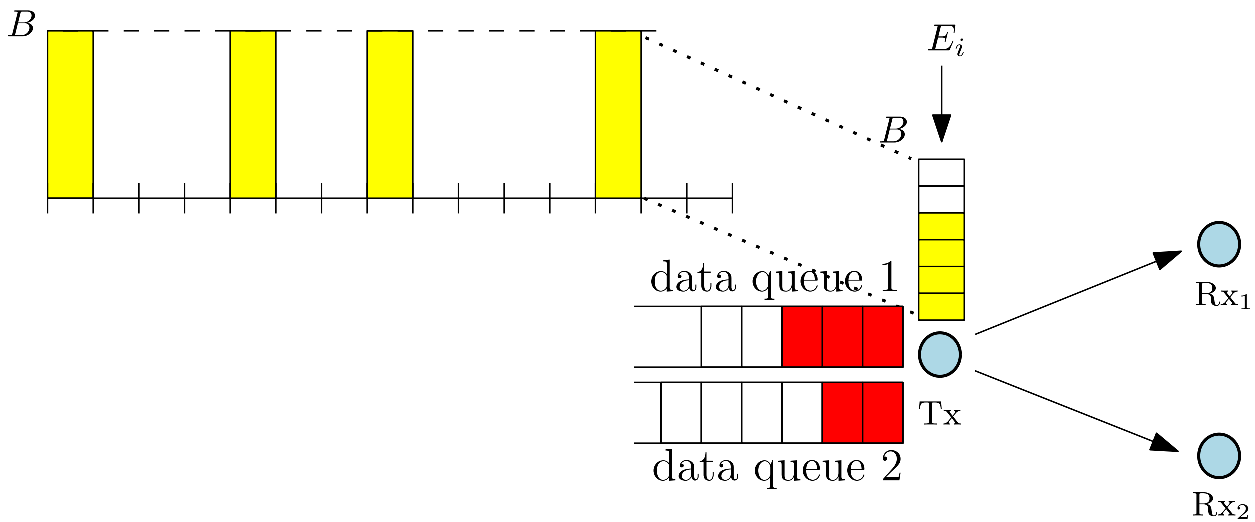
$$\begin{aligned} r &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{1}{2}\log\left(1+Bp(1-p)^{i-1}\right)\right] \\ &\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{1}{2}\log(1+Bp)+\frac{1}{2}\log\left((1-p)^{i-1}\right)\right] \geq \frac{1}{2}\log(1+\mu)-0.72 \end{aligned}$$

- Bernoulli is the worst energy arrival for the fractional policy:

$$\boxed{T_{upper} - 0.72 \;\leq\; T_{Bern} \;\leq\; T_{any} \;\leq\; T_{upper}}$$

Online Policies for the Broadcast Channel

- Bernoulli energy arrivals:



- $\mathbb{P}[E_i = B] = 1 - \mathbb{P}[E_i = 0] = p$
- When an energy arrives, a **renewal** occurs.

Long-Term Weighted Average Throughput

- Long-term weighted average throughput, under Bernoulli energy arrivals:

$$\lim_{n\to\infty}\mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(\mu_1 r_{1i}+\mu_2 r_{2i})\right] = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(\mu_1 r_{1i}+\mu_2 r_{2i})\right]$$

$$= p\sum_{k=1}^{\infty} p(1-p)^{k-1}\sum_{i=1}^{k}(\mu_1 r_{1i}+\mu_2 r_{2i})$$

$$= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} p^2(1-p)^{k-1}(\mu_1 r_{1i}+\mu_2 r_{2i})$$

$$= \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i}+\mu_2 r_{2i})$$

Resulting Optimization Problem for the Broadcast Channel

- Problem becomes

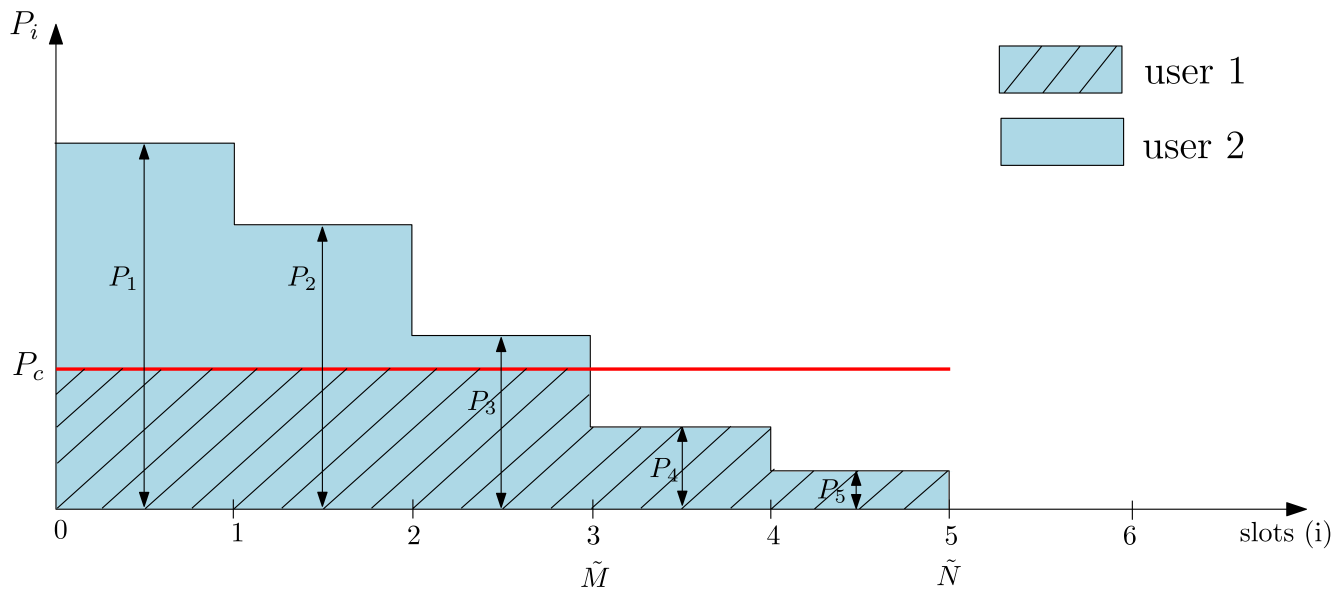
$$\begin{aligned} \max_{\{r_{1i}, r_{2i}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i} + \mu_2 r_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} g(r_{1i}, r_{2i}) \leq B \\ & r_{1i}, r_{2i} \geq 0, \quad \forall i \end{aligned}$$

  where

$$P_i = \sigma_1^2 e^{2(r_{1i}+r_{2i})} + (\sigma_2^2 - \sigma_1^2) e^{2r_{2i}} - \sigma_2^2 \triangleq g(r_{1i}, r_{2i})$$

- Modified offline problem:
  - One energy arrival.
  - Generalized fading due to $p(1-p)^{i-1}$

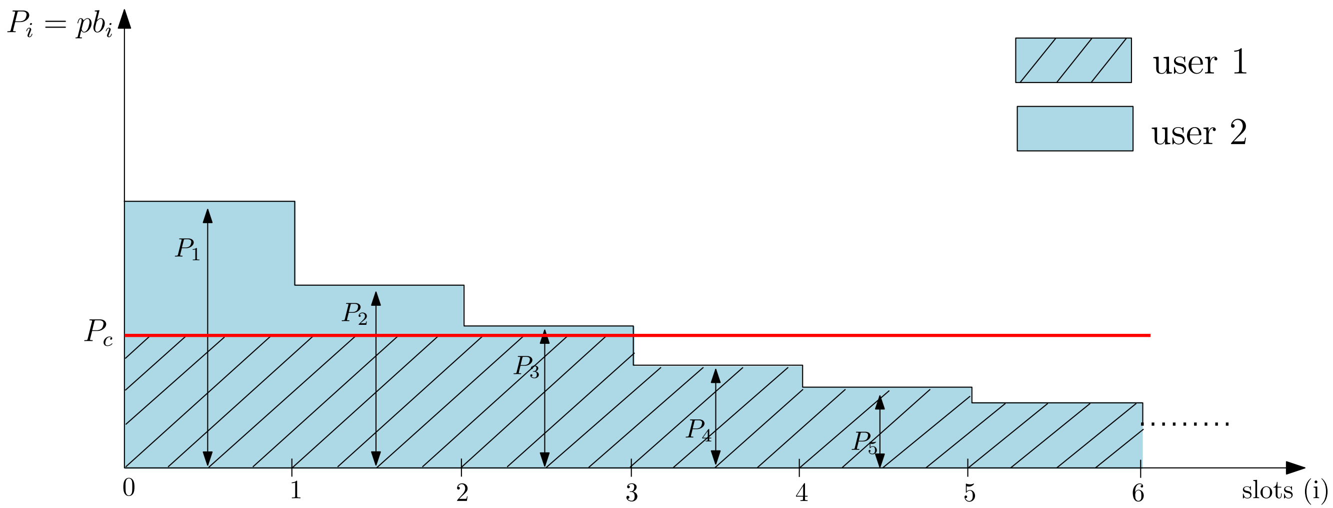
Structure of the Optimal Online Policy



- User 1 is served for a time no shorter than user 2.
- Both users' powers are decreasing.
- Cut-off level P_c:

$$P_c = \left(\frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+$$

Proposed Sub-optimal Policy for Bernoulli Energy Arrivals



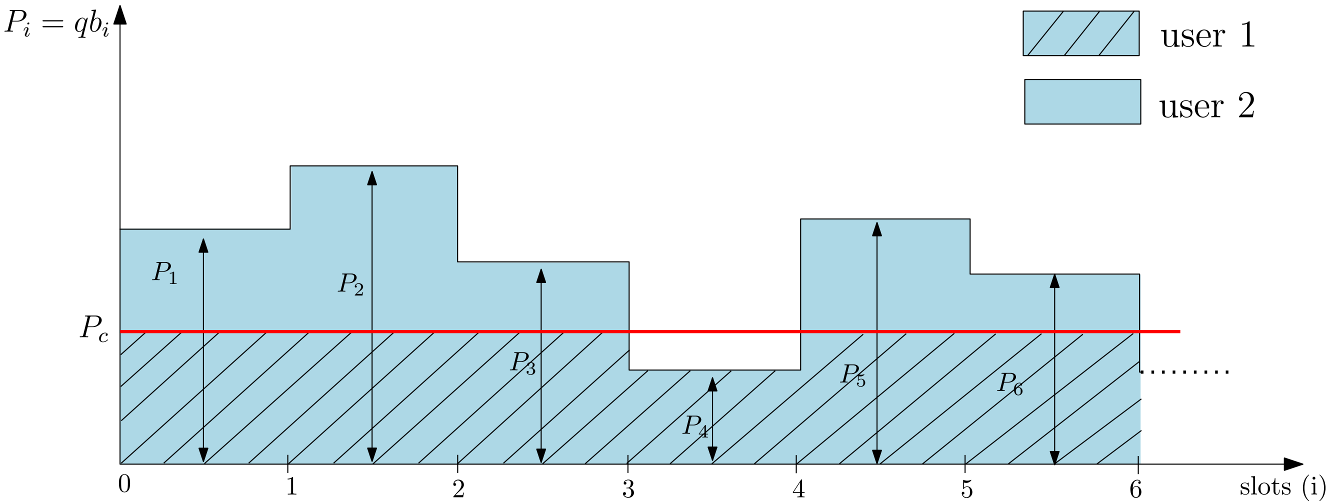
- Sub-optimal fractional total power policy:
  - Total power per slot:

$$P_i = P_{1i} + P_{2i} = pb_i = Bp(1-p)^{i-1}$$

  - Optimally divided power according to cut-off:

$$P_{1i} = \min\{P_c, Bp(1-p)^{i-1}\}$$
$$P_{2i} = Bp(1-p)^{i-1} - P_{1i}$$

Proposed Sub-optimal Policy for General Energy Arrivals



- Defining $q = \mu/B$.
- Total power per slot:

$$P_i = qb_i$$

- Optimally divided power according to cut-off:

$$P_{1i} = \min\{P_c, qb_i\}$$
$$P_{2i} = qb_i - P_{1i}$$

Bounds on the Online Policies

- Bernoulli energy arrivals gives a lower bound for general energy arrivals.
- Lower bound:

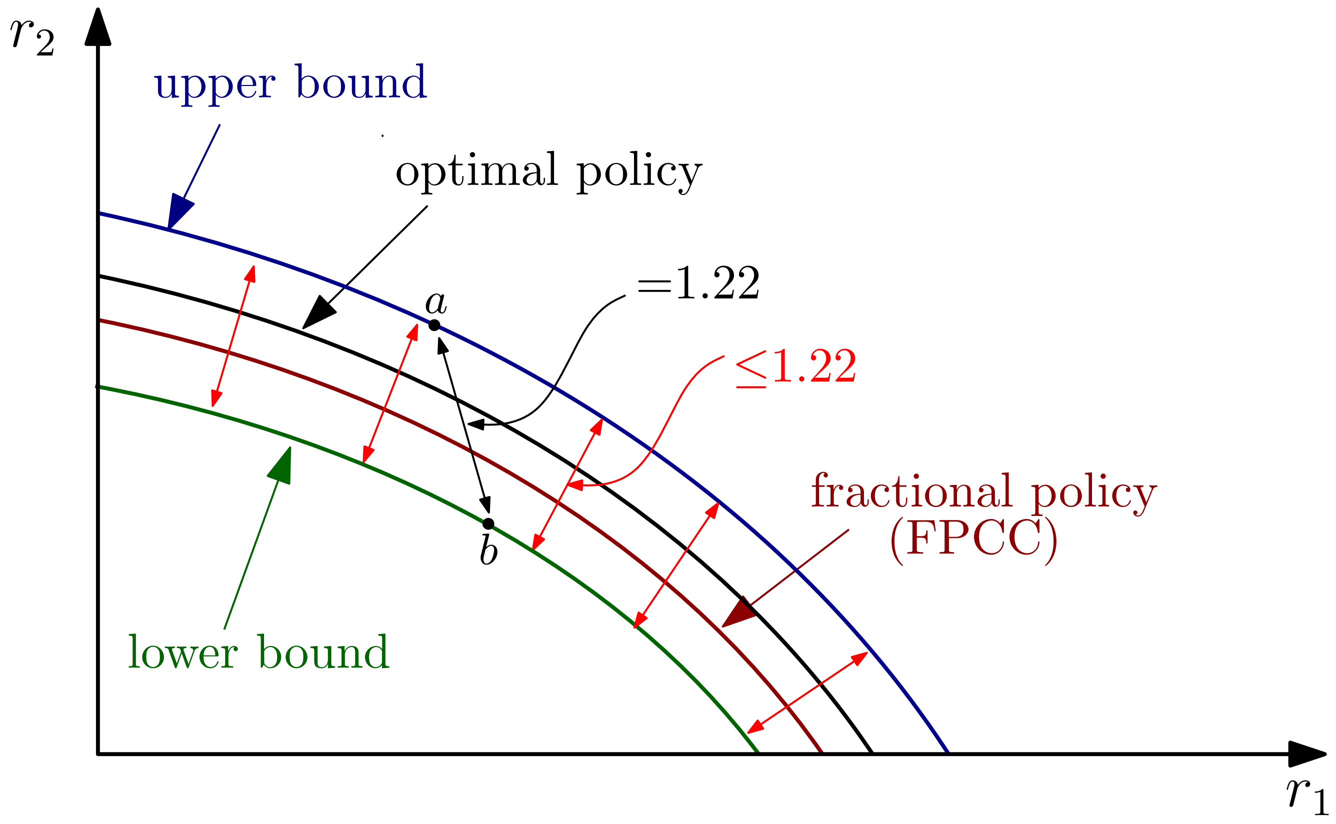
$$r_1 \geq \frac{1}{2}\log\left(1+\frac{\alpha\mu}{\sigma_1^2}\right)-0.72$$
$$r_2 \geq \frac{1}{2}\log\left(1+\frac{(1-\alpha)\mu}{\alpha\mu+\sigma_2^2}\right)-0.99$$

- Upper bound:

$$r_1 \leq \frac{1}{2}\log\left(1+\frac{\alpha\mu}{\sigma_1^2}\right)$$
$$r_2 \leq \frac{1}{2}\log\left(1+\frac{(1-\alpha)\mu}{\alpha\mu+\sigma_2^2}\right)$$

for some $\alpha \in [0,1]$, where $\mu = \mathbb{E}[E_i]$ is the average recharge rate.

Illustration of Bounds



- Distance between any two points with the same α on the upper and lower bounds is equal to:

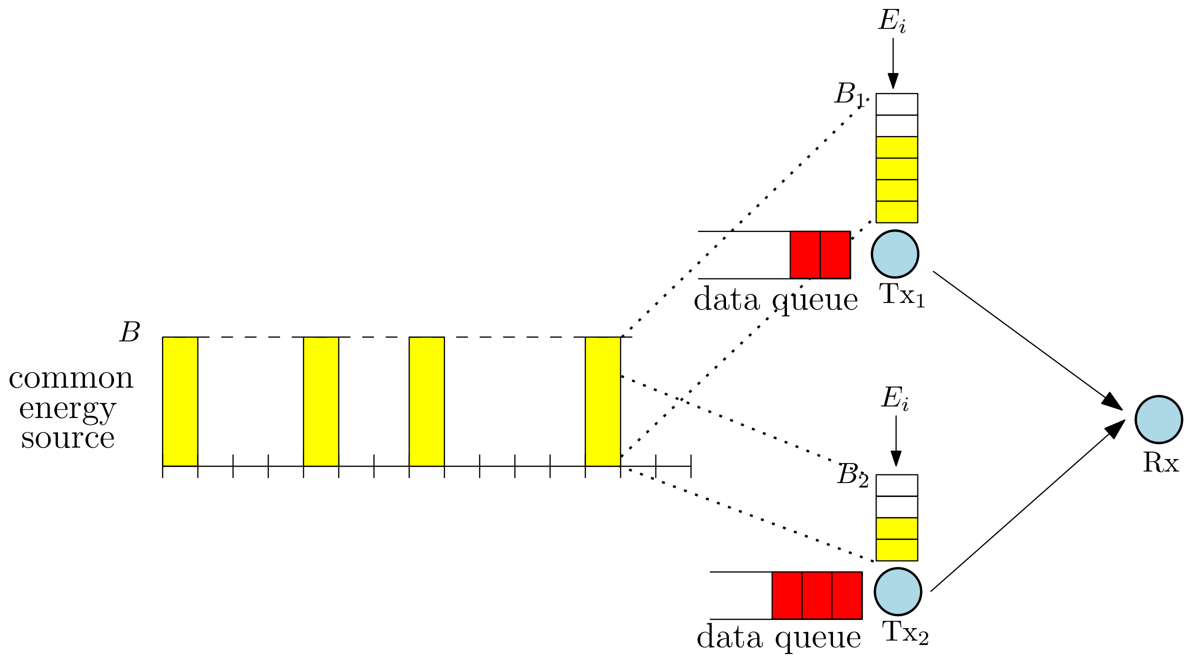
$$\sqrt{0.72^2 + 0.99^2} = 1.22$$

Conclusions for the Online Broadcasting Scenario

- Energy harvesting transmitter with finite capacity battery
- Maximize the departure region.
- Obtain the structure of the solution, such as:
  - the monotonicity of the transmit power
  - the cut-off power property
- Near-optimal policy.

Multiple Access Channel with Common Source

- Bernoulli energy arrivals:



- $\mathbb{P}[E_i = B] = 1 - \mathbb{P}[E_i = 0] = p$, where $B \geq \max\{B_1, B_2\}$.
- Average admitted energies at the two users are not the same.
- When an energy arrives, a **renewal** occurs.

Long-Term Weighted Average Throughput

- Long-term weighted average throughput, under Bernoulli energy arrivals:

$$
\begin{aligned}
\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(\mu_1 r_{1i}+\mu_2 r_{2i})\right] &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(\mu_1 r_{1i}+\mu_2 r_{2i})\right] \\
&= p\sum_{k=1}^{\infty} p(1-p)^{k-1}\sum_{i=1}^{k}(\mu_1 r_{1i}+\mu_2 r_{2i}) \\
&= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} p^2(1-p)^{k-1}(\mu_1 r_{1i}+\mu_2 r_{2i}) \\
&= \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i}+\mu_2 r_{2i})
\end{aligned}
$$

Online Policies for the Multiple Access Channel with Common Source

- For Bernoulli energy arrivals:

$$
\begin{aligned}
\max_{\{P_{1i},P_{2i}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1} (\mu_1 r_{1i} + \mu_2 r_{2i}) \\
\text{s.t.} \quad & (r_{1i}, r_{2i}) \in \mathcal{C}(P_{1i}, P_{2i}) \\
& \sum_{i=1}^{\infty} P_{1i} \leq B_1, \ \sum_{i=1}^{\infty} P_{2i} \leq B_2
\end{aligned}
$$

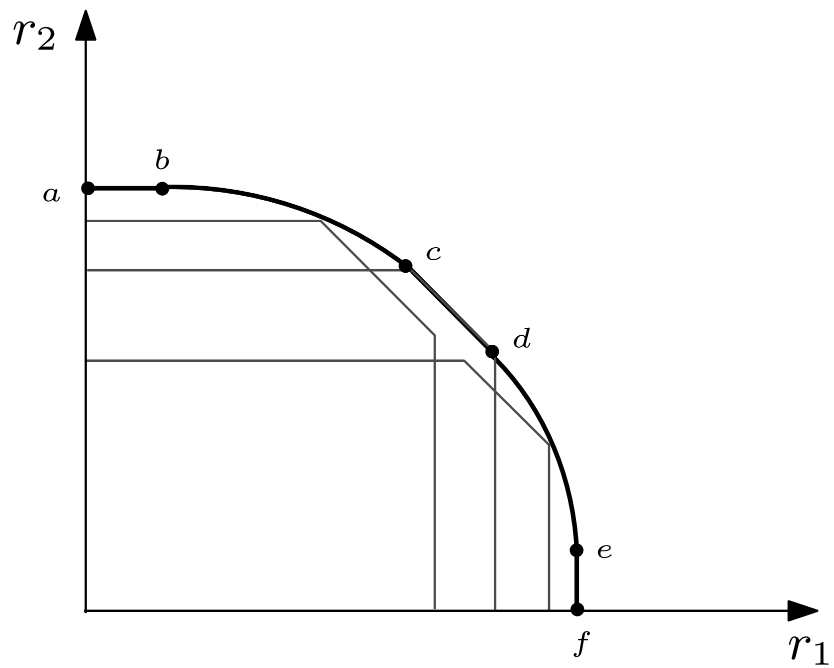
where $\mathcal{C}(P_{1i},P_{2i})$ of this channel in slot i is:

$$
\begin{aligned}
r_{1i} &\leq \frac{1}{2}\log\left(1+\frac{P_{1i}}{\sigma^2}\right) \\
r_{2i} &\leq \frac{1}{2}\log\left(1+\frac{P_{2i}}{\sigma^2}\right) \\
r_{1i}+r_{2i} &\leq \frac{1}{2}\log\left(1+\frac{P_{1i}+P_{2i}}{\sigma^2}\right)
\end{aligned}
$$

- Modified offline problem:
  - One energy arrival.
  - Generalized fading due to $p(1-p)^{i-1}$

Online Policies for the Multiple Access Channel with Common Source

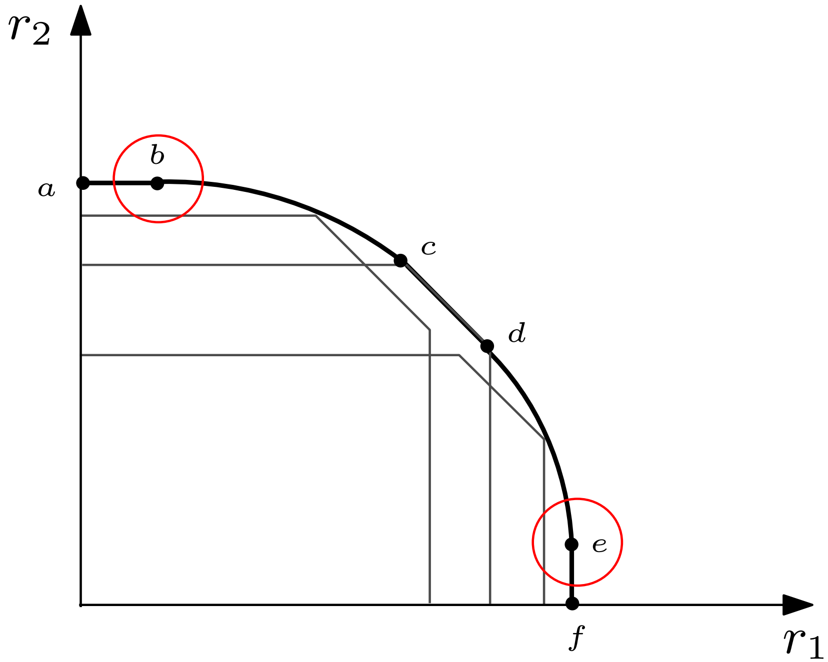
- Achievable rate region



- Each feasible policy achieves a pentagon
- Rate region is the union of all such pentagons
- Points a and f are single-user rates

Online Policies for the Multiple Access Channel with Common Source

- Achievable rate region



- Each feasible policy achieves a pentagon
- Rate region is the union of all such pentagons
- Points a and f are single-user rates

Point b

- User 2 power is fixed to:

$$P_{2i}^* = \frac{p(1-p)^{i-1}}{\lambda_2} - \sigma^2, \qquad i = 1, \ldots, \tilde{N}_2$$

- Optimization problem becomes:

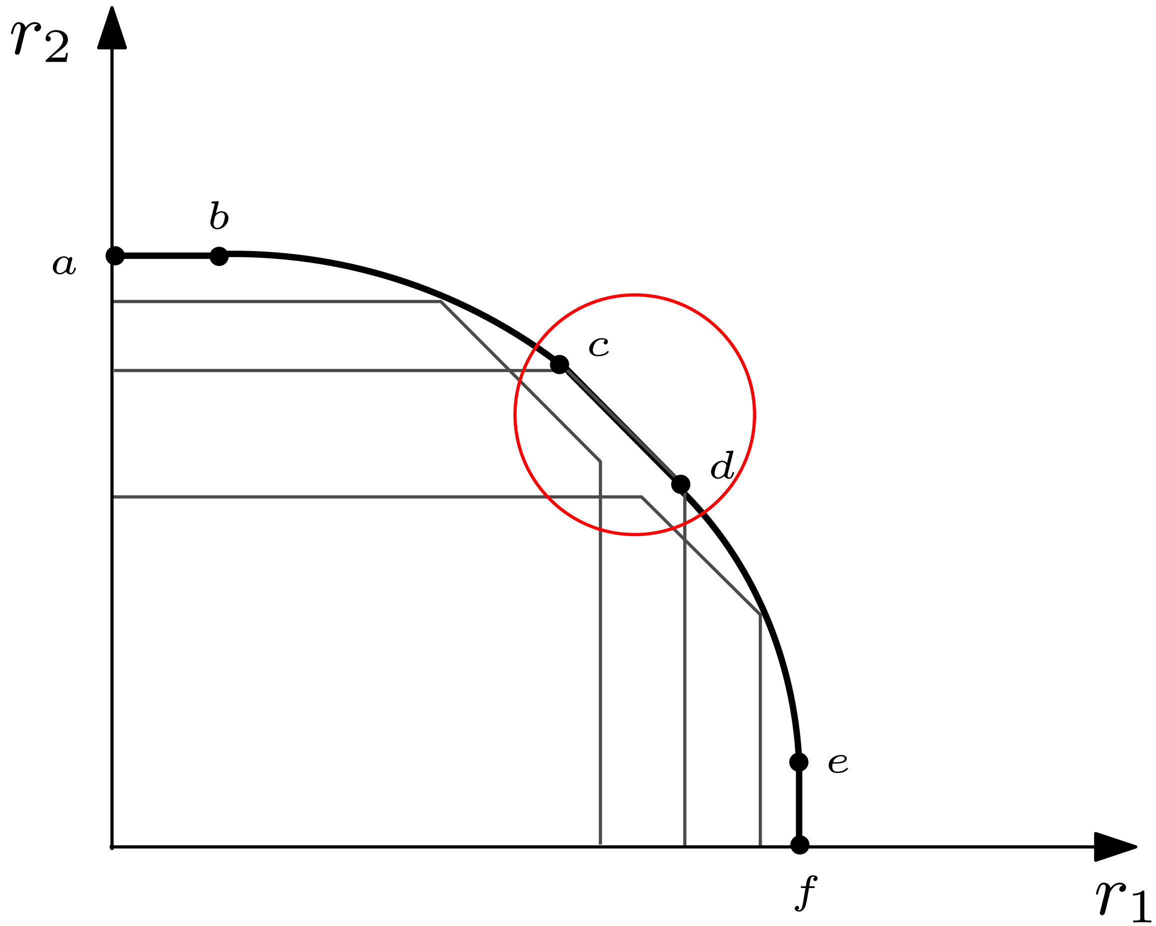
$$\begin{aligned} \max_{\{P_{1i}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1} r_{1i} \\ \text{s.t.} \quad & r_{1i} \in \mathcal{C}(P_{1i}, P_{2i}^*), \quad \sum_{i=1}^{\infty} P_{1i} \le B_1 \end{aligned}$$

- The optimal power:

$$P_{1i} = \frac{p(1-p)^{i-1}}{\lambda_1 - \nu_{1i}} - \sigma^2 - P_{2i}^*$$

- At point b, user 1 transmits for a duration no shorter than user 2.
- Power of both users are monotonically decreasing.

Sum-Rate



Sum-Rate

- $\mu_1 = \mu_2 = 1$
- The optimization problem becomes:

$$\begin{aligned} \max_{\{P_{1i},P_{2i}\}} \quad & \frac{1}{2}\sum_{i=1}^{\infty} p(1-p)^{i-1}\log\left(1+\frac{P_{1i}+P_{2i}}{\sigma^2}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} P_{1i} \le B_1, \ \sum_{i=1}^{\infty} P_{2i} \le B_2 \end{aligned}$$

- A relaxed problem:

$$\begin{aligned} \max_{\{P_{1i},P_{2i}\}} \quad & \frac{1}{2}\sum_{i=1}^{\infty} p(1-p)^{i-1}\log\left(1+\frac{P_{1i}+P_{2i}}{\sigma^2}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} P_{1i}+P_{2i} \le B_1+B_2 \end{aligned}$$

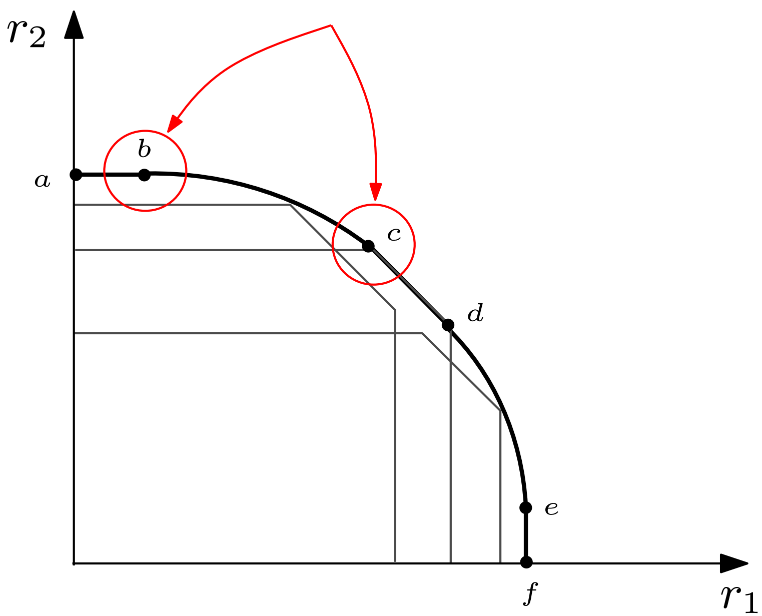
- Equivalent problems.
  - Use $P_{1i} = (P_{1i}^* + P_{2i}^*)\frac{B_1}{B_1+B_2}$
- Hence, solve a single-user problem for $(P_{1i}+P_{2i})$.

Sum-Rate

- $(P_{1i} + P_{2i})^*$ is positive for a duration $\tilde{N}_s \geq \max\{\tilde{N}_1, \tilde{N}_2\}$
- It is sufficient to show that:

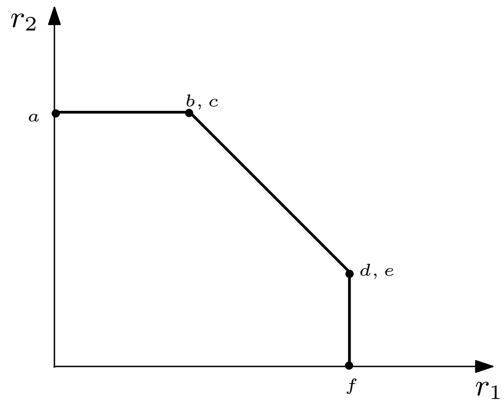
$$(P_{1i} + P_{2i})^* - P_{2i}^* \geq 0$$

- Implies that the single-user power allocation is feasible



Online Policies for the Multiple Access Channel with Common Source

- Optimal capacity region with Bernoulli arrivals is a single pentagon



- Distributed sub-optimal policy, let $q_k \triangleq \frac{\bar{P}_k}{B_k}$:
  - For Bernoulli energy arrivals:

$$P_{1i} = B_1 p(1-p)^{i-1}$$
$$P_{2i} = B_2 p(1-p)^{i-1}$$

  - For general energy arrivals:

$$P_{1i} = q_1 b_{1i}$$
$$P_{2i} = q_2 b_{2i}$$

Bounds for the Multiple Access Channel with Common Source

- Bernoulli energy arrivals gives a lower bound for general energy arrivals.
- Lower bound:

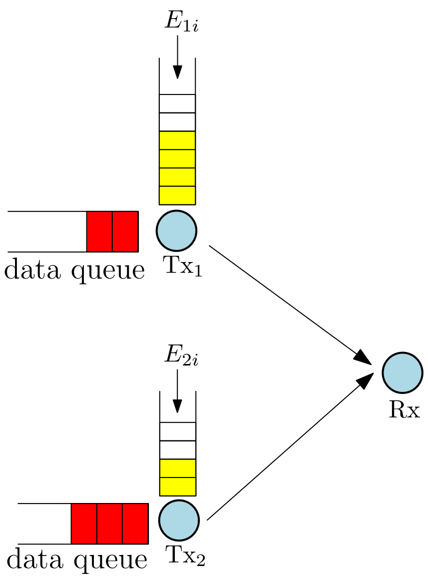
$$
\begin{aligned}
r_1 &\geq \frac{1}{2}\log\left(1+\frac{\bar{P}_1}{\sigma^2}\right)-0.72 \\
r_2 &\geq \frac{1}{2}\log\left(1+\frac{\bar{P}_2}{\sigma^2}\right)-0.72 \\
r_1+r_2 &\geq \frac{1}{2}\log\left(1+\frac{\bar{P}_1+\bar{P}_2}{\sigma^2}\right)-0.72
\end{aligned}
$$

- Upper bound for any energy arrival:

$$
\begin{aligned}
r_1 &\leq \frac{1}{2}\log\left(1+\frac{\bar{P}_1}{\sigma^2}\right) \\
r_2 &\leq \frac{1}{2}\log\left(1+\frac{\bar{P}_2}{\sigma^2}\right) \\
r_1+r_2 &\leq \frac{1}{2}\log\left(1+\frac{\bar{P}_1+\bar{P}_2}{\sigma^2}\right)
\end{aligned}
$$

Multiple Access Channel with General (Arbitrarily Correlated) Arrivals

- Bounds are the same for any arbitrary energy arrivals.



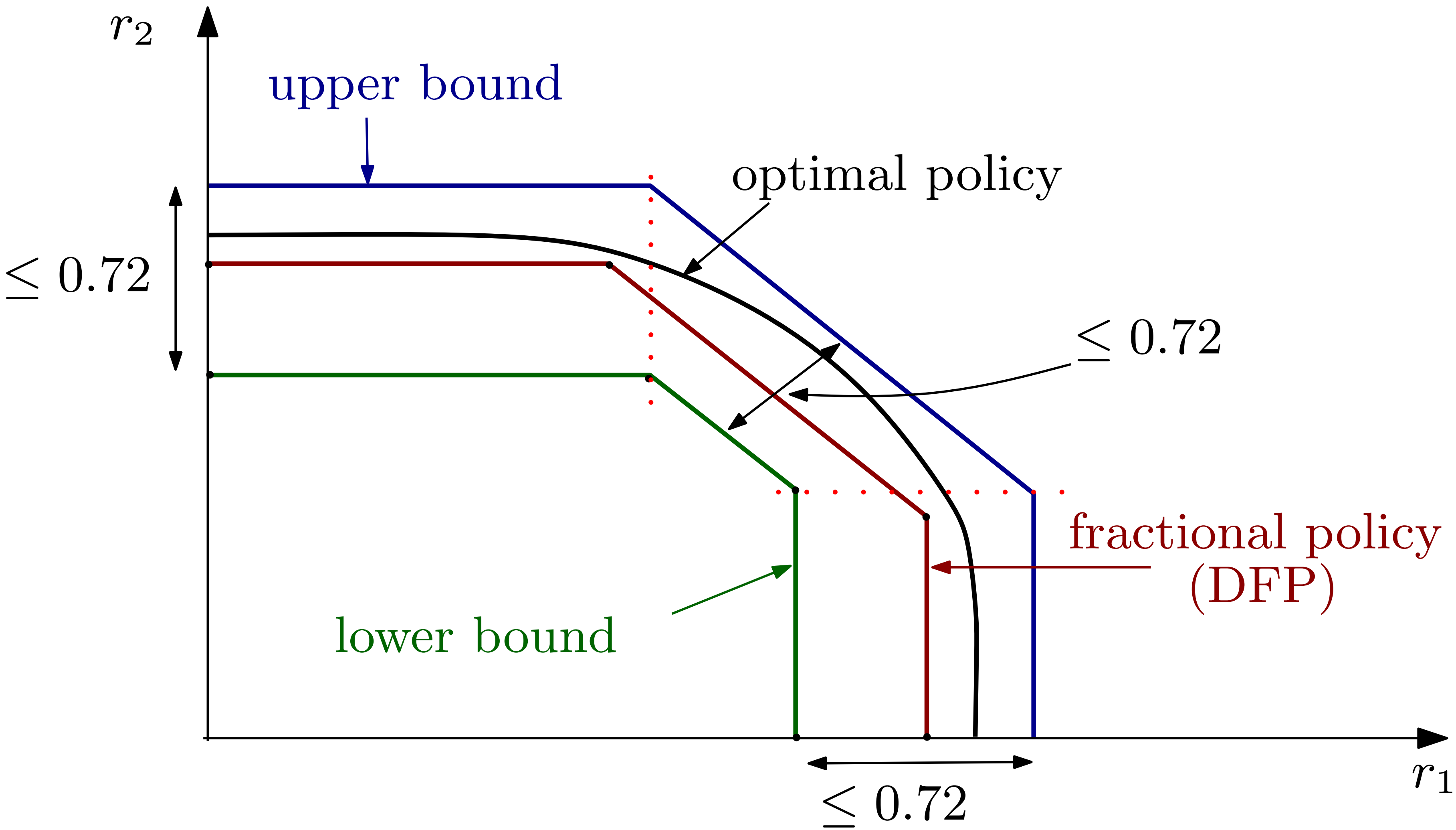
  - Using $q_k = \frac{\bar{P}_k}{B_k}$, the lower bound is:

$$r_1 \geq \frac{1}{2}\log\left(1+\frac{\bar{P}_1}{\sigma^2}\right) - 0.72$$

$$r_2 \geq \frac{1}{2}\log\left(1+\frac{\bar{P}_2}{\sigma^2}\right) - 0.72$$

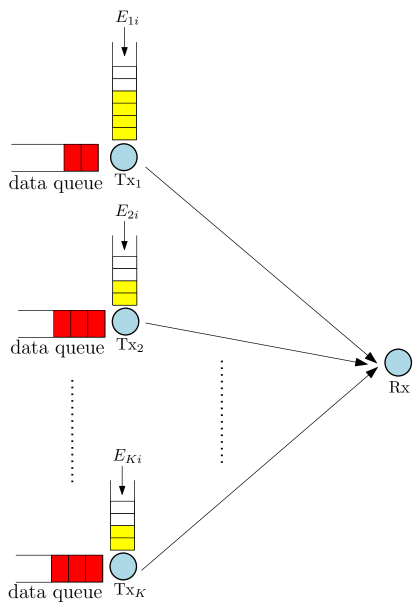
$$r_1 + r_2 \geq \frac{1}{2}\log\left(1+\frac{\bar{P}_1+\bar{P}_2}{\sigma^2}\right) - 0.72$$

Illustration of Bounds for the Multiple Access Channel



Multiple Access Channel with Arbitrary Number of Users

- Bounds are the same for any arbitrary number of users.



  - Using $q_k = \frac{\bar{P}_k}{B_k}$, the lower bound is:

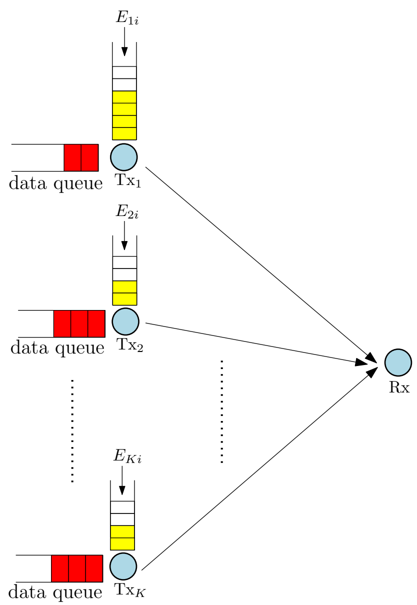
$$\sum_{i \in \mathcal{S}} r_i \geq \frac{1}{2} \log \left(1 + \frac{\sum_{i \in \mathcal{S}} \bar{P}_i}{\sigma^2} \right) - 0.72, \quad \forall \mathcal{S} \subset \{1, \ldots, K\}$$

  - Upper bound:

$$\sum_{i \in \mathcal{S}} r_i \leq \frac{1}{2} \log \left(1 + \frac{\sum_{i \in \mathcal{S}} \bar{P}_i}{\sigma^2} \right), \qquad \forall \mathcal{S} \subset \{1, \ldots, K\}$$

Multiple Access Channel with Large Number of Users

- Sum-rate approaches the capacity for very large number of users.



  - Using $q_k = \frac{\bar{P}_k}{B_k}$, the lower bound is:

$$\sum_{i=1}^{K} r_i \geq \frac{1}{2} \log \left(1 + \frac{\sum_{i=1}^{K} \bar{P}_i}{\sigma^2} \right) - 0.72$$

  - Upper bound:

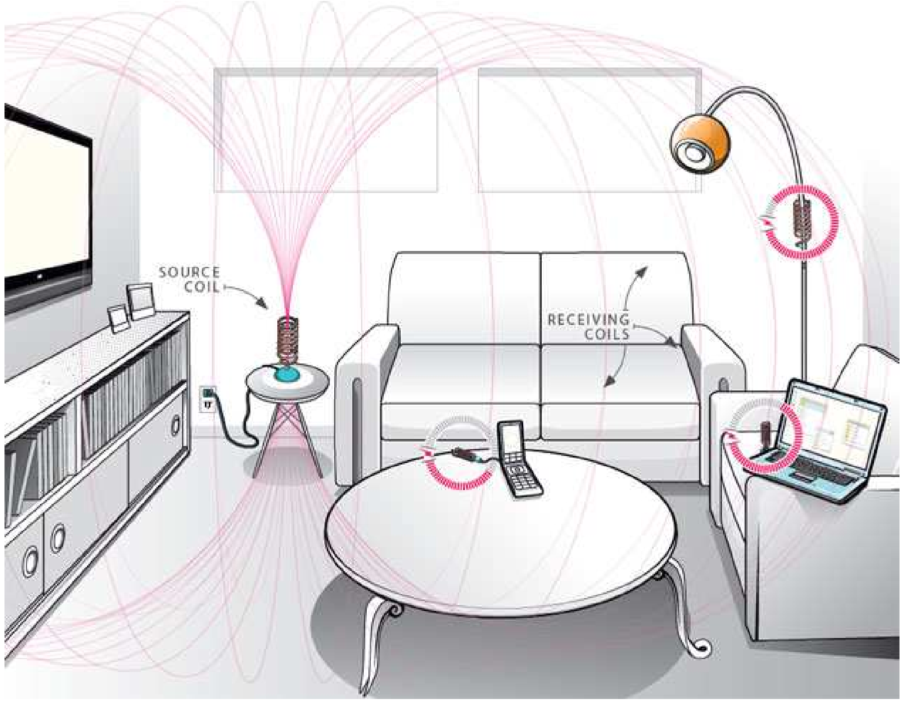
$$\sum_{i=1}^{K} r_i \leq \frac{1}{2} \log \left(1 + \frac{\sum_{i=1}^{K} \bar{P}_i}{\sigma^2} \right)$$

Conclusions for the Online Multiple Access Scenario

- Energy harvesting transmitters sending messages to a single access point.
- The problem: maximization of the departure region.
- Obtain the structure of the solution, such as:
  - Monotonicity of the power.
  - Synchronous multiple access capacity region is a pentagon.
- Near-optimal policy.

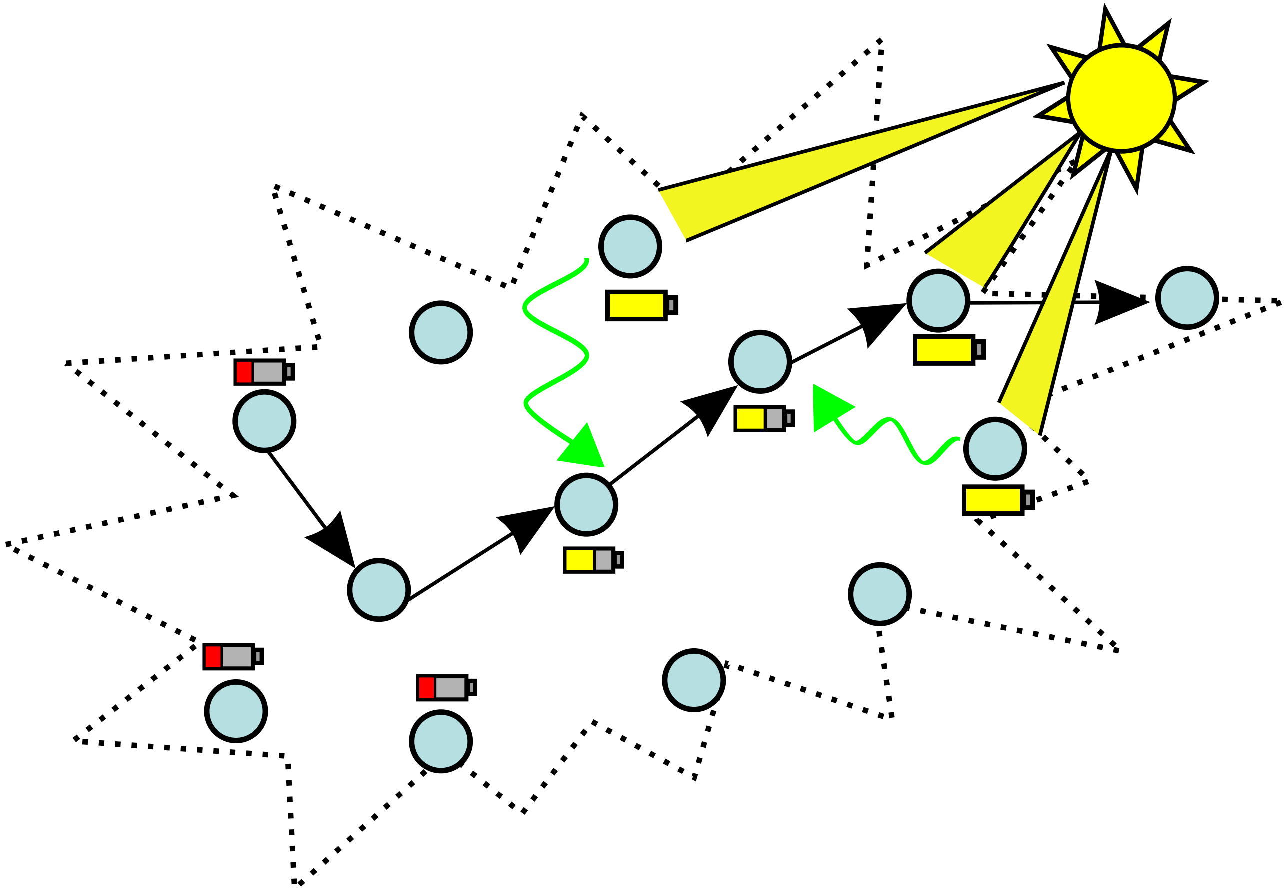
Wireless Energy Transfer

- Newly emerging technologies have enabled us to perform wireless energy transfer efficiently.
- Inductive coupling can be used to wirelessly transfer energy.



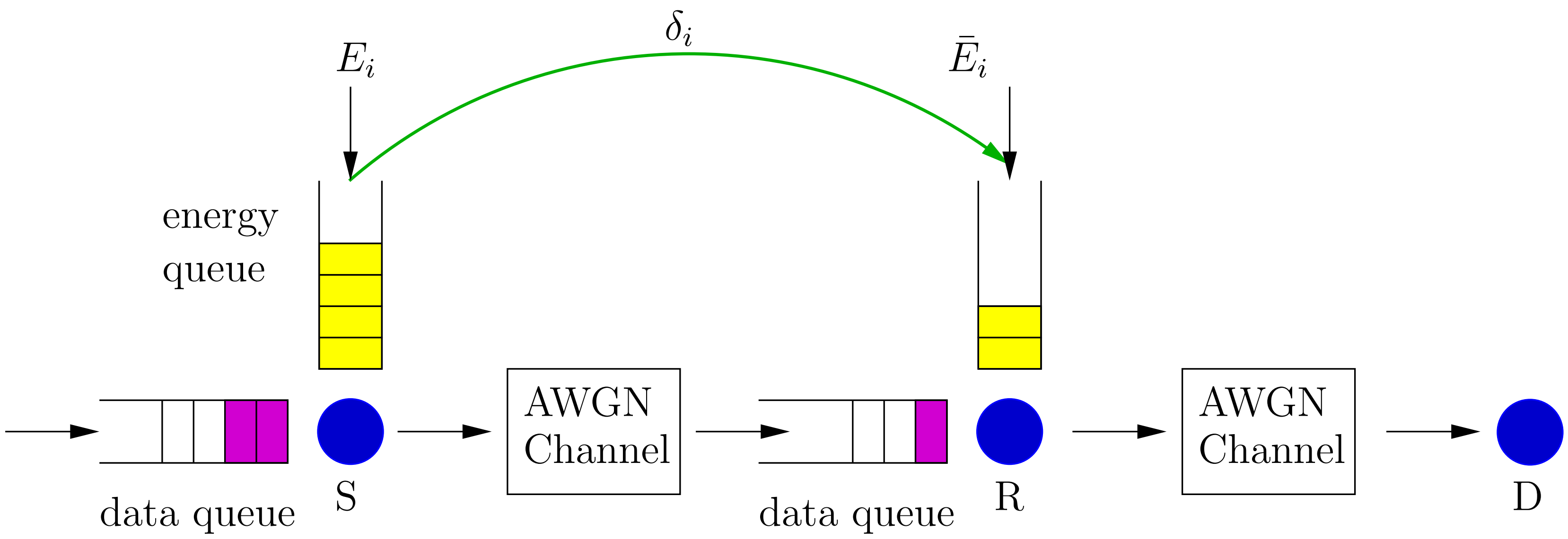
Energy Cooperation in Multi-user Energy Harvesting Communications

- Wireless energy transfer is a **new cooperation paradigm.**



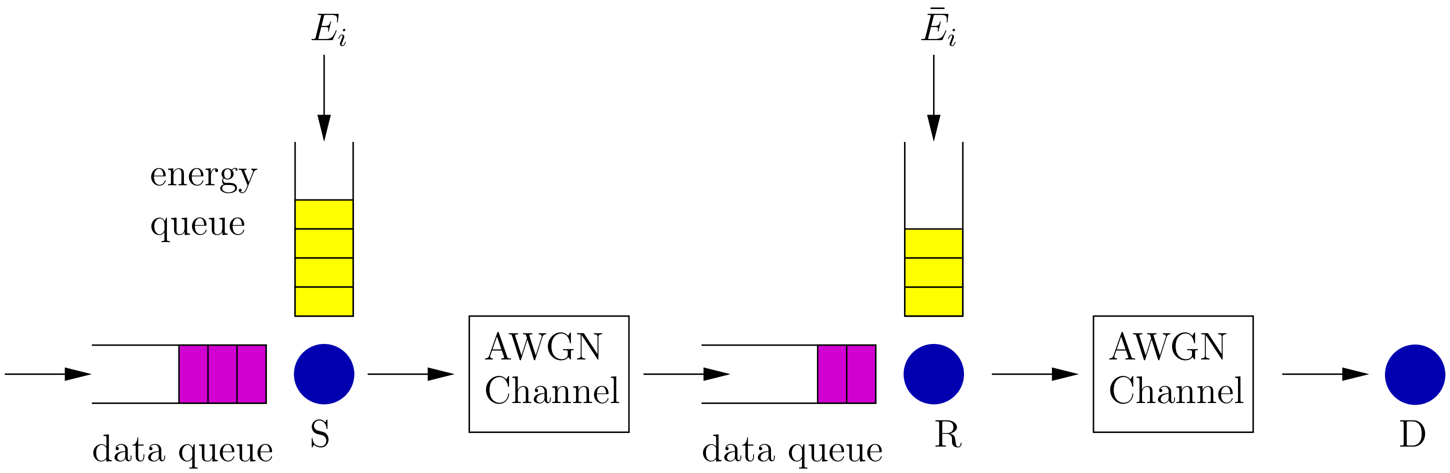
- **Energy cooperation:** Nodes share their energy as well as their information.

Gaussian Two-Hop Relay Channel with Energy Cooperation



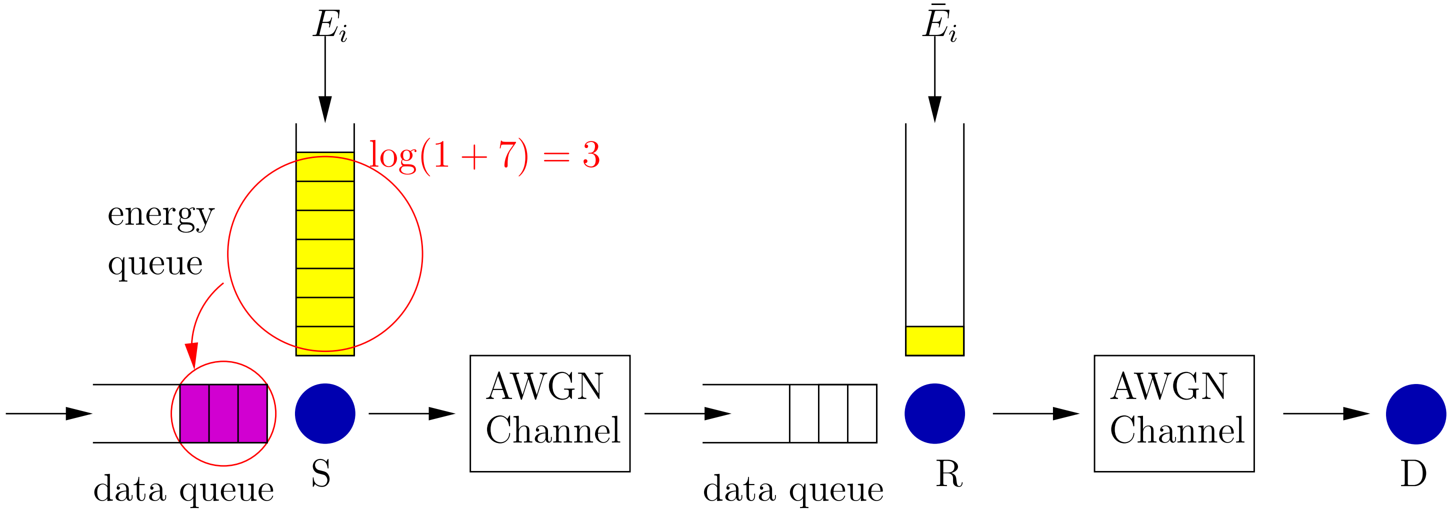
- Energy harvesting source and relay with deterministic energy arrivals E_i, $\bar{E}_i$.
- Wireless energy transfer unit that allows the source to transfer some of its energy to the relay (with $0 \leq \alpha \leq 1$ efficiency).
- Unlimited data and energy buffers at the source and the relay.
- New energy arrivals at every slot i, $1 \leq i \leq T$.
- The source transfers δ_i energy to the relay at slot i.
- Relay receives $\alpha\delta_i$ of this transferred energy at the next slot.

Two Hop Relay Channel without Energy Cooperation



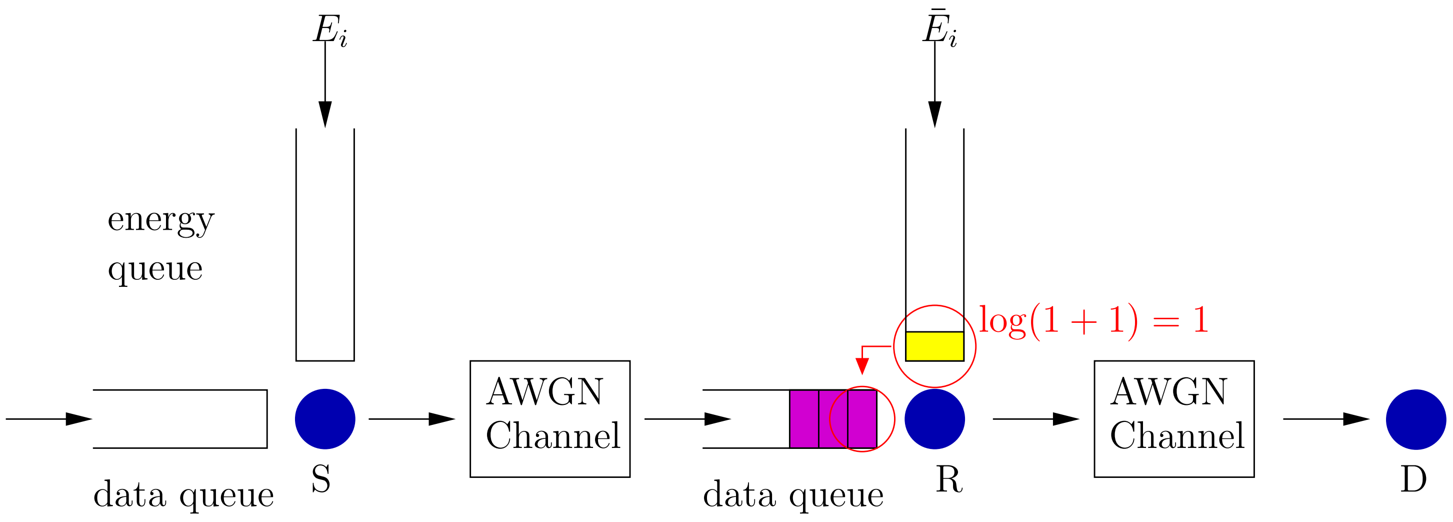
- Optimal source/relay profile is a separable policy.
- Source performs single-user throughput maximization with respect to its own energy arrivals.
- Relay forwards as many of the received bits as possible, satisfying data causality and energy causality.

Two Hop Relay Channel without Energy Cooperation



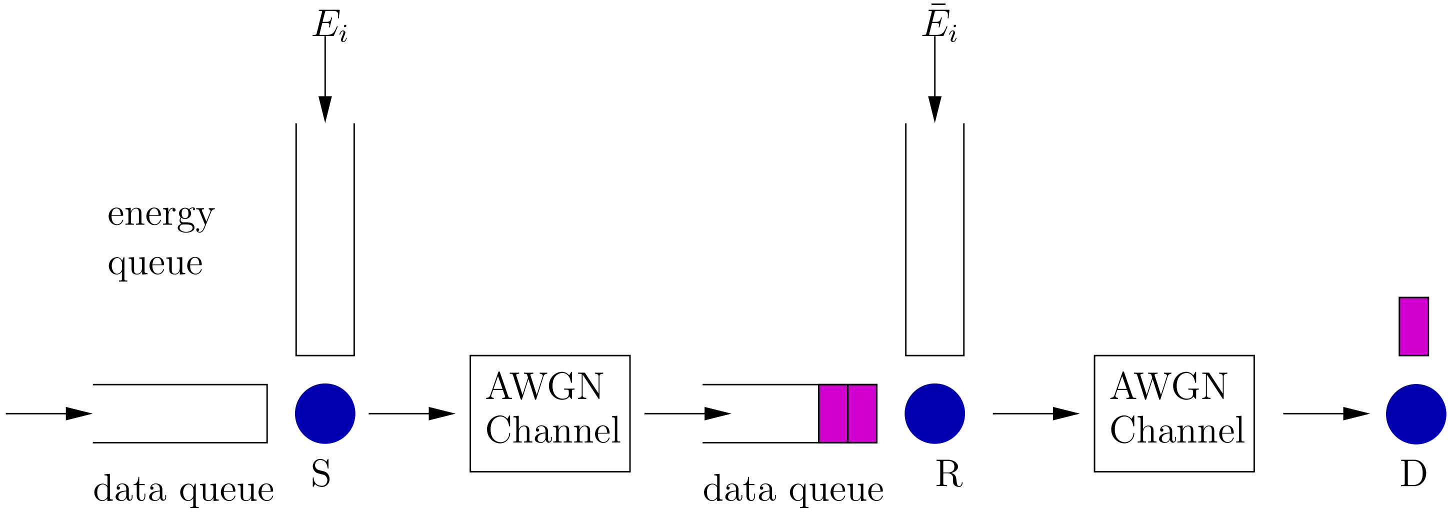
- Separable policy, source maximizes its own throughput.

Two Hop Relay Channel without Energy Cooperation



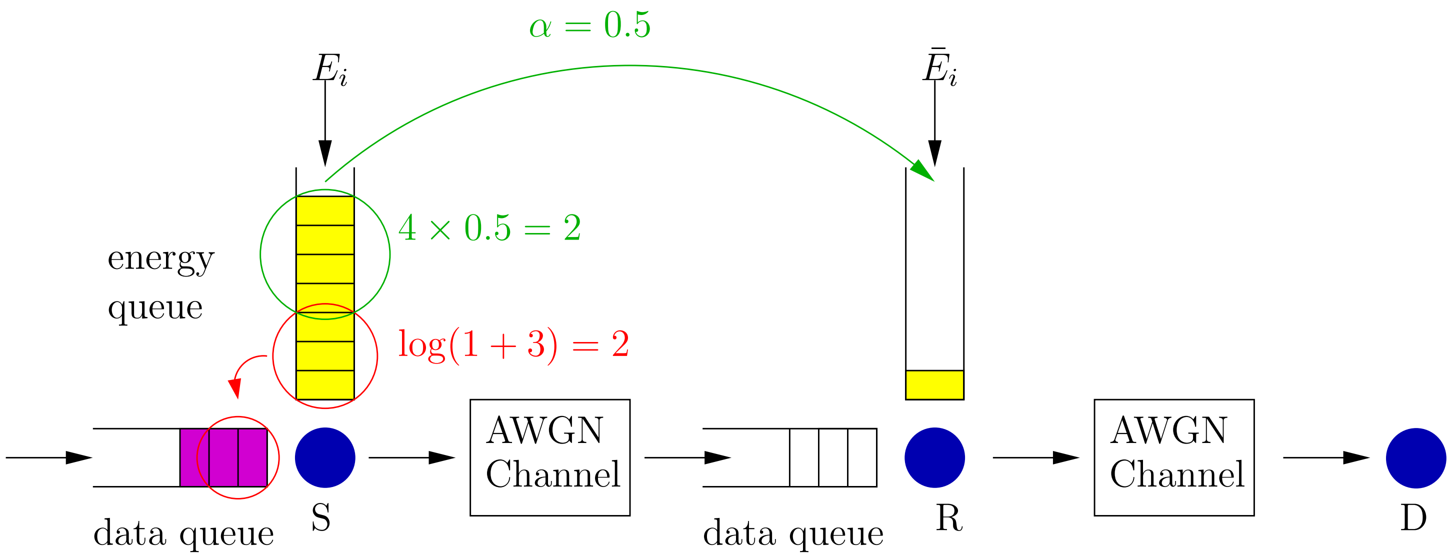
- Separable policy, source maximizes its own throughput.
- Relay tries to send as much as it can.

Two Hop Relay Channel without Energy Cooperation



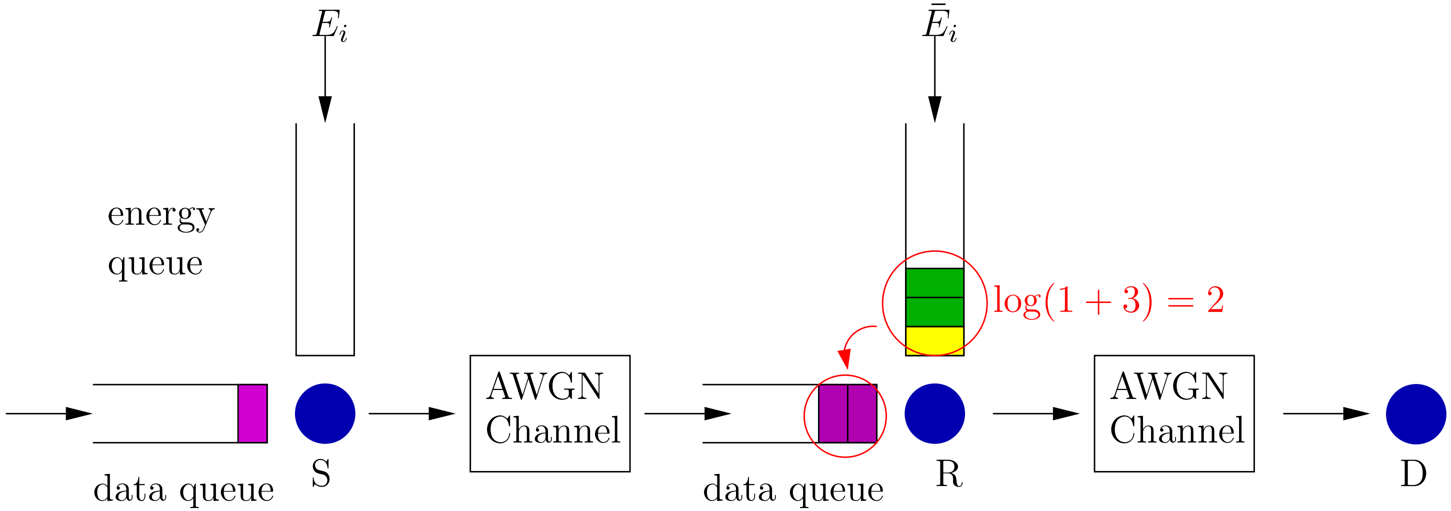
- Separable policy, source maximizes its own throughput.
- Relay tries to send as much as it can.
- 1 bit sent to destination, 2 bits remaining at the relay.
- End-to-end throughput is 1 bit.

Two Hop Relay Channel with Energy Cooperation



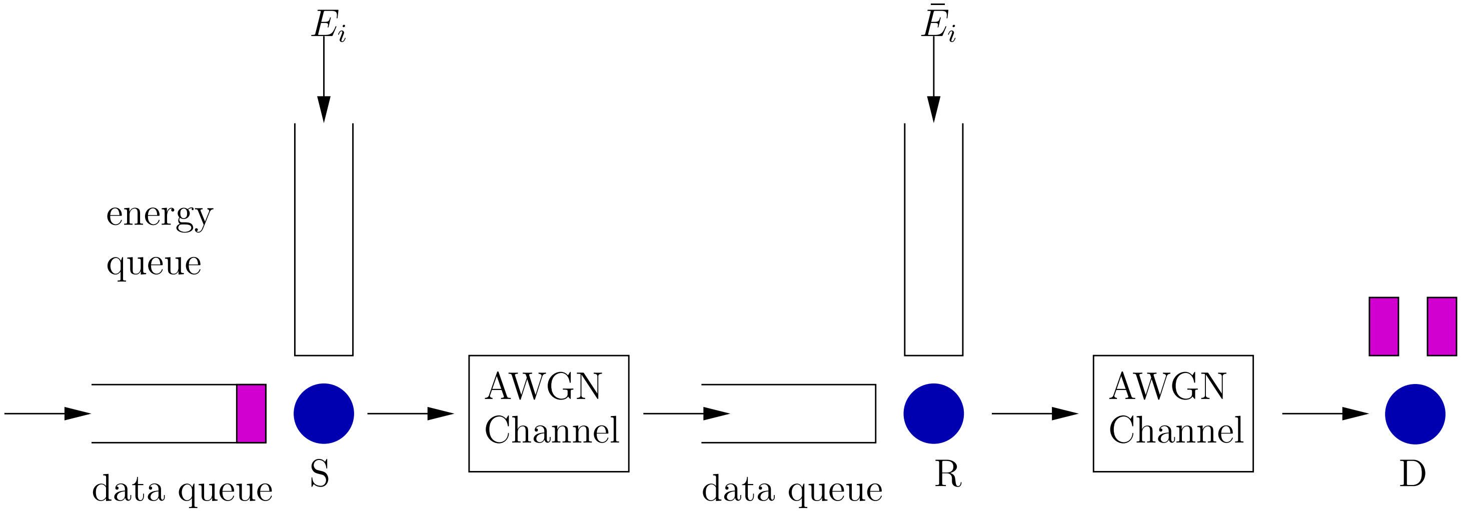
- Source sends less data, but some energy to assist the relay.

Two Hop Relay Channel with Energy Cooperation



- Source sends less data, but some energy to assist the relay.
- Relay uses this extra energy to forward more data.

Two Hop Relay Channel with Energy Cooperation



- Source sends less data, but some energy to assist the relay.
- Relay uses this extra energy to forward more data.
- 2 bits sent to destination, 0 bits remaining at the relay.
- End-to-end throughput is 2 bits.

End-to-end Throughput Maximization

- Maximize end-to-end throughput

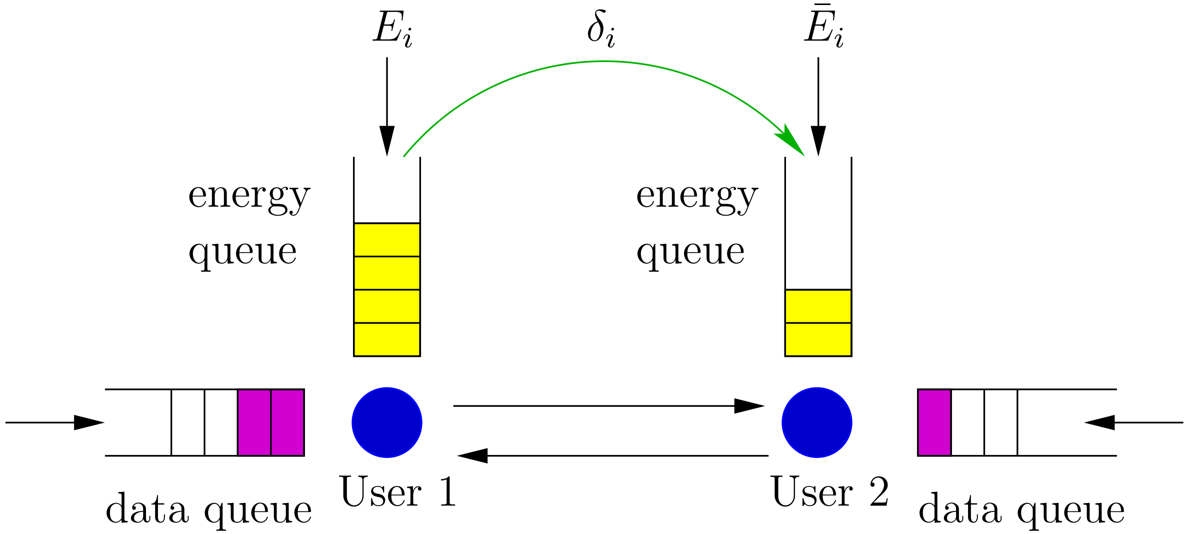
$$
\begin{aligned}
\max \quad & \sum_{i=1}^{T} \frac{1}{2} \log\left(1 + \bar{P}_i\right) \\
\text{s.t.} \quad & \sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} (E_i - \delta_i), \quad \forall k \\
& \sum_{i=1}^{k} \bar{P}_i \leq \sum_{i=1}^{k} (\bar{E}_i + \alpha\delta_i), \quad \forall k \\
& \sum_{i=1}^{k} \frac{1}{2} \log\left(1 + \bar{P}_i\right) \leq \sum_{i=1}^{k} \frac{1}{2} \log\left(1 + P_i\right), \quad \forall k
\end{aligned}
$$

subject to:

- Data causality at the relay node
- Energy causality at both nodes
- (Possibly) non-zero energy transfers

Gaussian Two Way Channel with Energy Cooperation

- Energy harvesting users with deterministic energy arrivals E_i, $\bar{E}_i$
- One-way wireless energy transfer with efficiency $0 < \alpha < 1$.



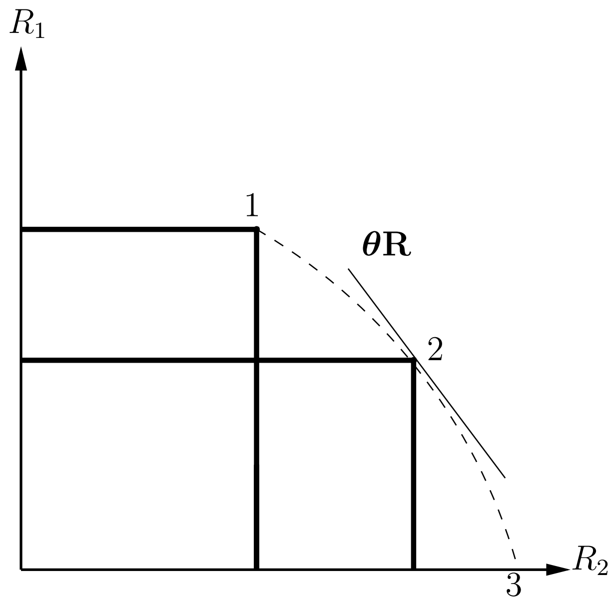
- Physical layer is a Gaussian two-way channel:

$$Y_1 = X_1 + X_2 + N_1$$

$$Y_2 = X_1 + X_2 + N_2$$

N_1, N_2 are Gaussian noises with zero mean and unit power.

Capacity Region



- Convex region, boundary is characterized by solving

$$\begin{aligned} \max_{\bar{P}_i, P_i, \delta_i} \quad & \sum_{i=1}^{T} \theta_1 \frac{1}{2} \log(1+P_i) + \theta_2 \frac{1}{2} \log(1+\bar{P}_i) \\ \text{s.t.} \quad & (\boldsymbol{\delta}, \mathbf{P}, \bar{\mathbf{P}}) \in \mathcal{F} \end{aligned}$$

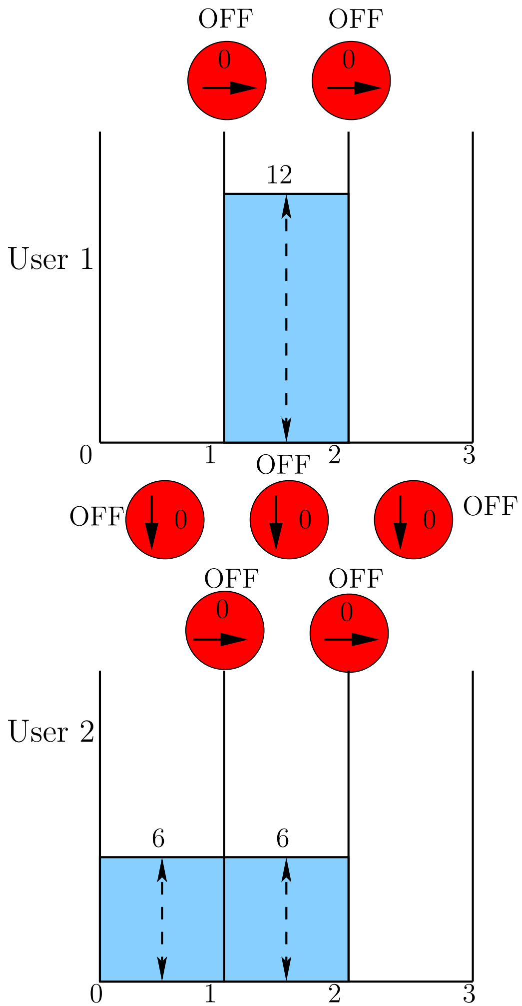
- Point 1 is achieved by $\boldsymbol{\delta} = 0$: **no energy transfer.**
- Point 3 is achieved by $\boldsymbol{\delta} = \mathbf{E}$: **full energy transfer.**

Water-filling Approach

- **Generalized two-dimensional directional water-filling algorithm.**
- Transfer energy from one user to another while maintaining optimal allocation in time.
- Spread the energy as much as possible in time and user dimensions.
- Now we give a numerical example for $\theta_1 = \theta_2$ and $\alpha = 1$.

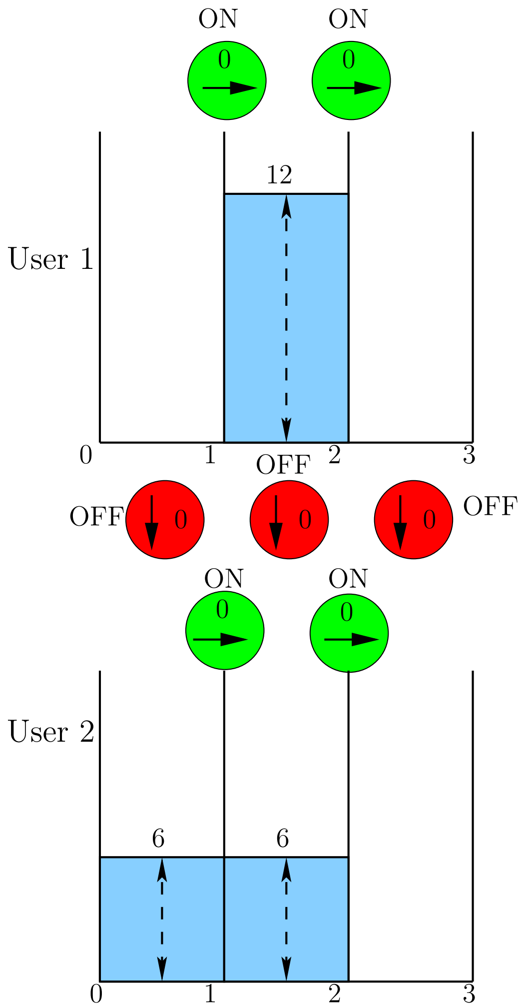
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



Numerical Example

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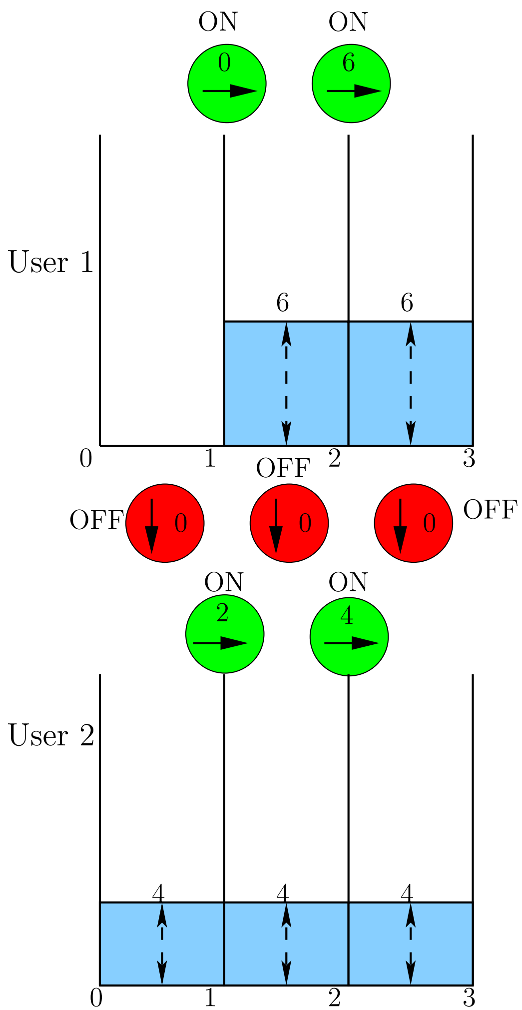
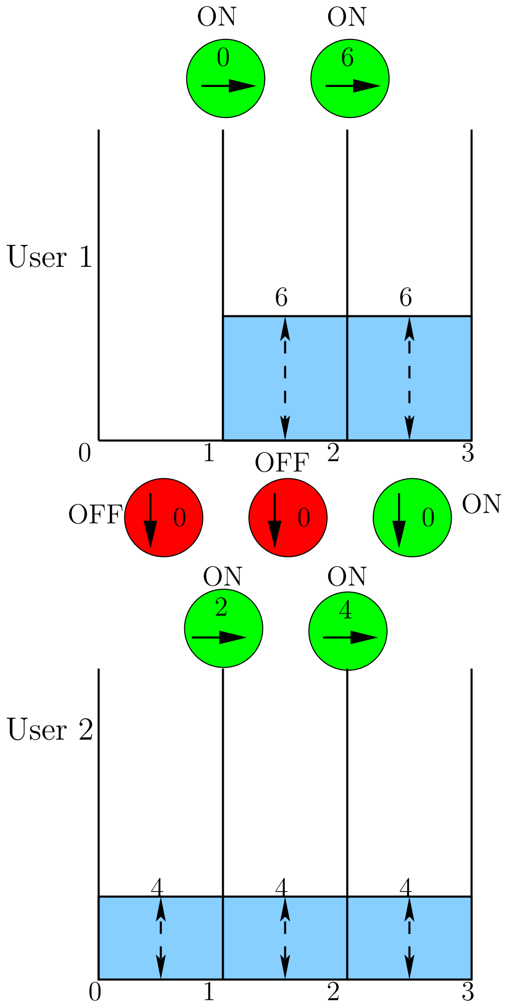


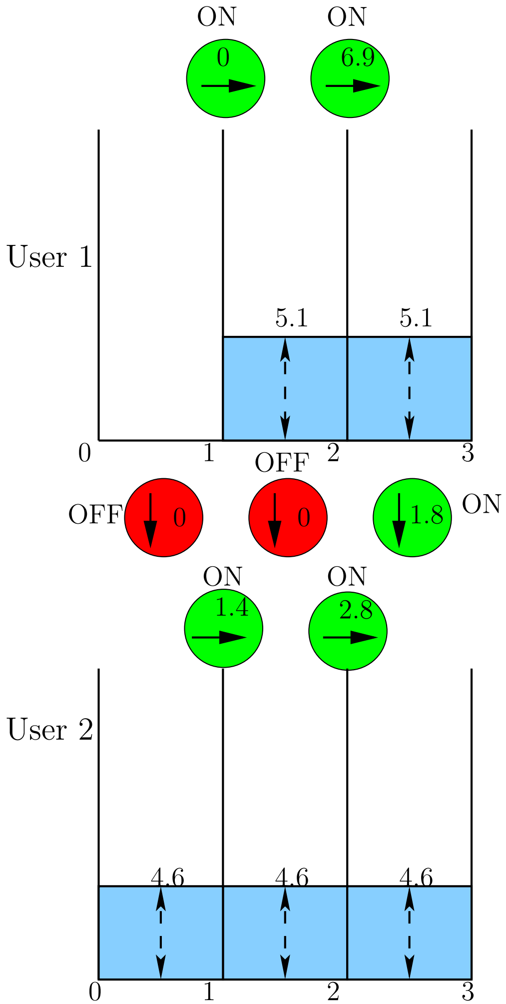
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



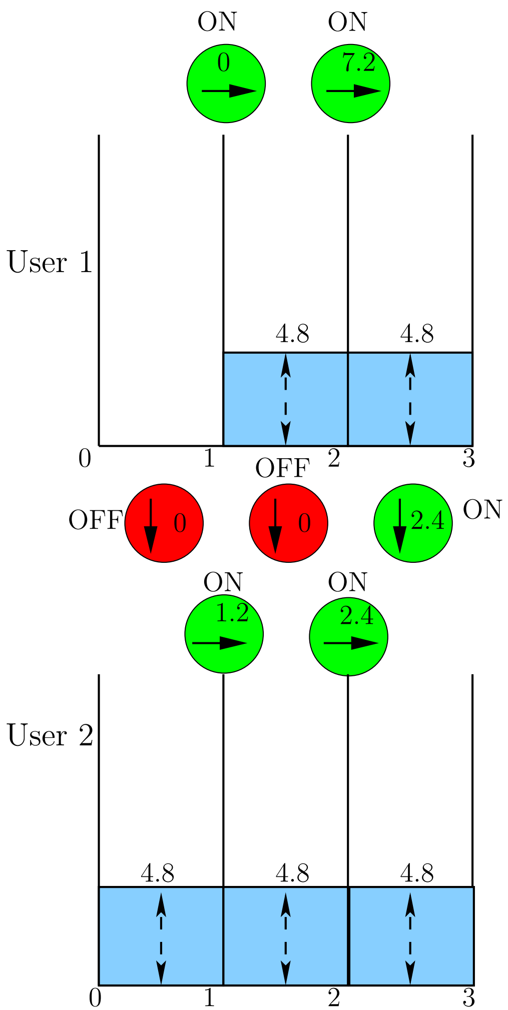
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



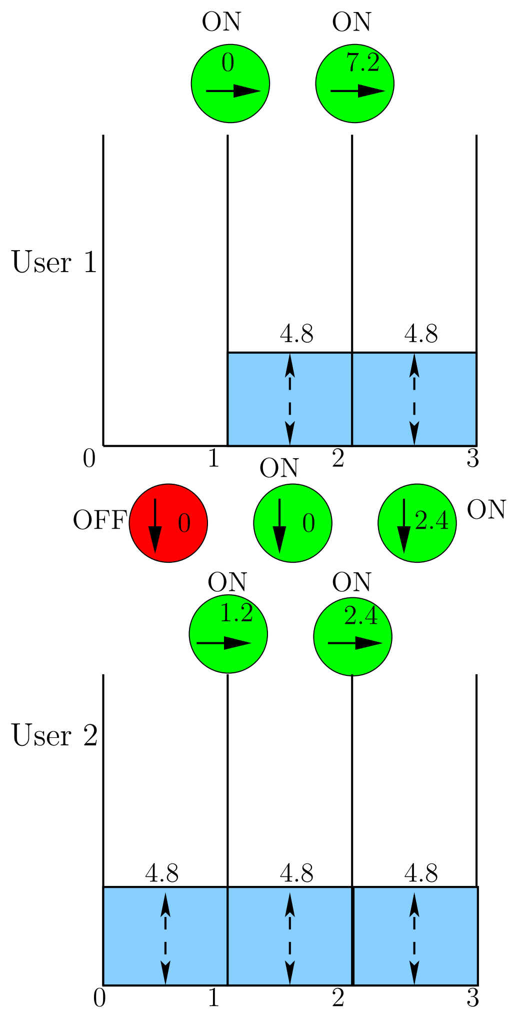
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



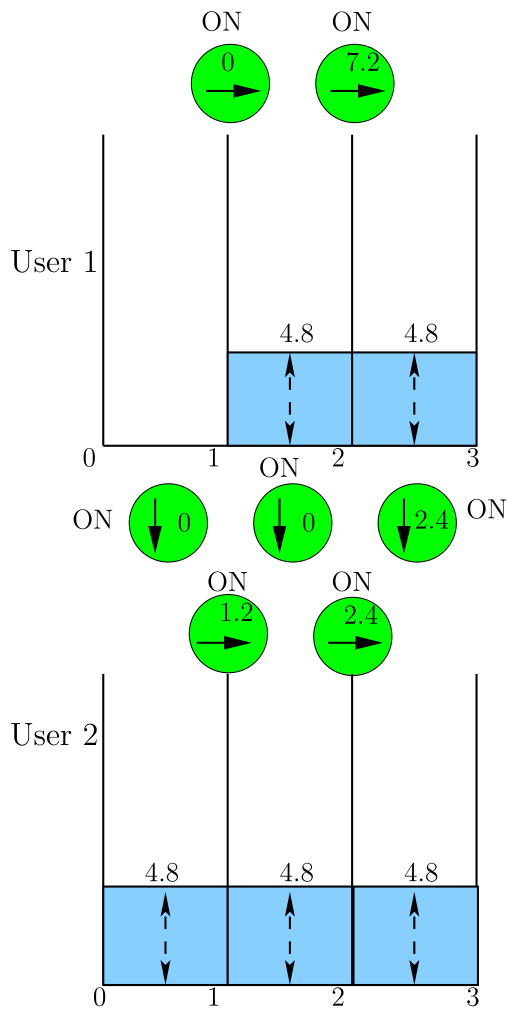
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



Conclusions for Offline Energy Cooperation Scenarios

- Energy harvesting users with infinite capacity batteries.
- Energy transfer capability in an orthogonal channel in one way.
- Energy transfer provides a new degree of freedom to smooth out the energy profiles.
- Optimal policies identified for Gaussian two-hop relay and two-way channels.
- End-to-end throughput maximization for the two-hop relay channel.
- Capacity regions for two-way channels.

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- Slides: Abdulrahman Baknina, Ahmed Arafa, Berk Gurakan, and Omur Ozel

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Energy Harvesting and Remotely Powered Wireless Networks- Part II

PennState

Wireless Communications & Networking Laboratory
WCAN@PSU

Aylin Yener

yener@ee.psu.edu

PennState

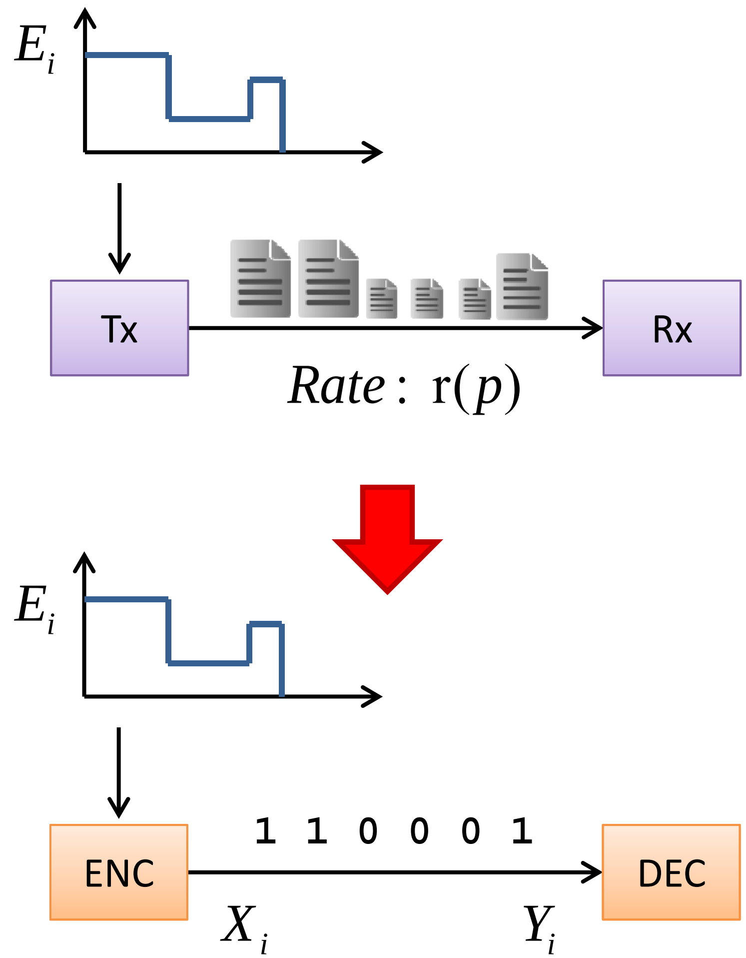
Outline – Aylin- Part II

- Information theory of energy harvesting transmitters
- Energy harvesting AWGN channel with infinite battery
- Energy harvesting AWGN channel with no battery
- Binary noiseless energy harvesting channel
- State amplification and state masking

Information Theory of EH Transmitters

- So far, we have assumed **sufficiently long time slots** and utilized the known rate expressions.

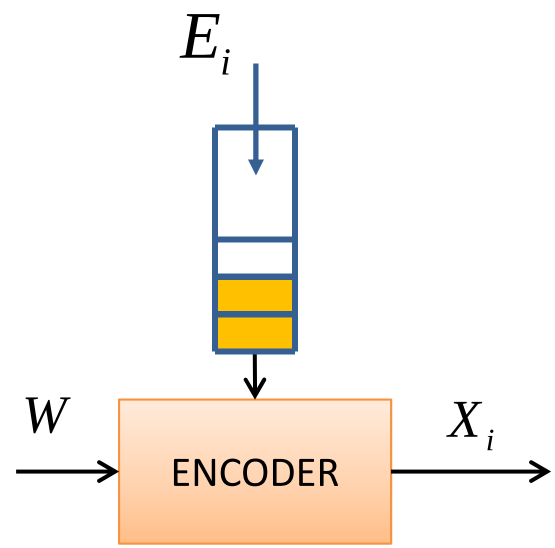
- What if **energy harvesting is at the symbol level**, i.e., each input symbol is individually limited by EH constraints?



Energy Harvesting (EH) Channel

[Tutuncuoglu-Ozel-Ulukus-Yener'13]

- The channel input is restricted by an external energy harvesting process.
- State: available energy
  - Has memory (due to energy storage)
  - Depends on channel input
  - Causally known to Tx (causal CSIT)



Energy Harvesting (EH) Channel

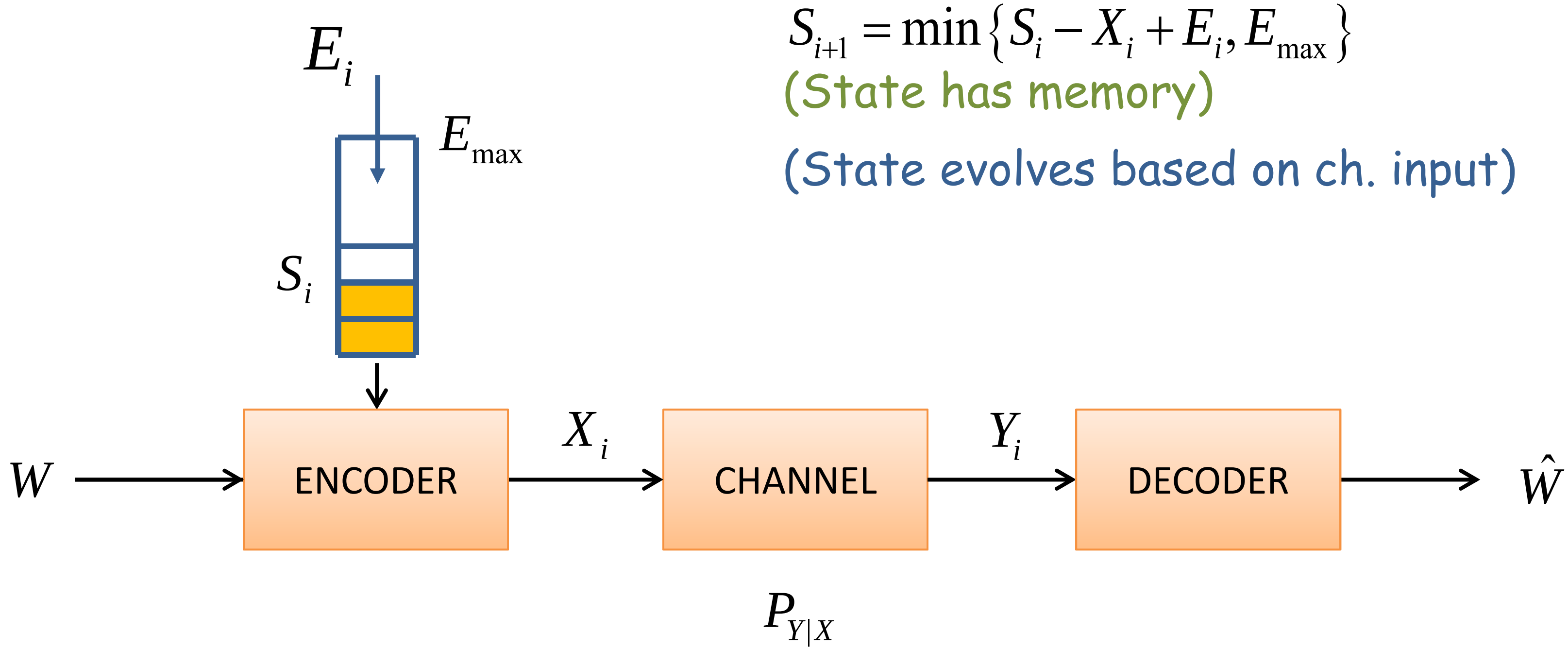
$$X_i \le S_i$$

(Ch. input constrained by state)

$$S_{i+1} = \min\{S_i - X_i + E_i, E_{\max}\}$$

(State has memory)

(State evolves based on ch. input)



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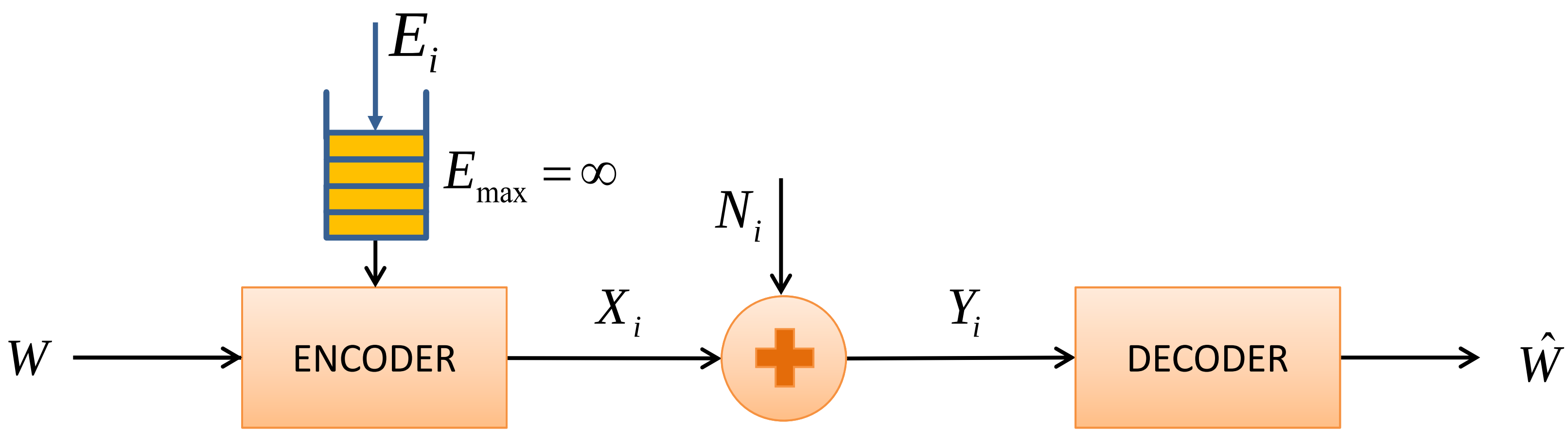
Energy Harvesting AWGN Channel



[Ozel-Ulukus '12]

- Battery capacity $E_{\max}$ is infinite.
- Average recharge rate: $P = E[E_i]$
- Capacity without energy harvesting: $C = \frac{1}{2}\log(1+P)$

Energy Harvesting AWGN Channel

- Code symbols are constrained by the energy in the battery at each channel use, i.e.,

$$\sum_{i=1}^{k} X_i^2 \leq \sum_{i=1}^{k} E_i, \quad k = 1,2,\ldots,n.$$

- Conversely, the average power constraint for a non-EH AWGN channel would be a single constraint:

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq \frac{1}{n}\sum_{i=1}^{n} E_i \rightarrow P.$$

➢ $C = \frac{1}{2}\log(1+P)$ is an upper bound on the capacity of the energy harvesting AWGN channel.

Achievability

- This upper bound is achievable.

- Two sources of error:

1. Decoding error,
2. Energy shortage.

- Idea: Design the codebook as if the channel is non-EH and show that energy shortages are insignificant.

- Two achievable schemes:

1. Save-and-Transmit,
2. Best-Effort-Transmit.

Save-and-Transmit

- Suppose $h(n) \in o(n)$, i.e., $h(n)/n \to 0.$

- Save energy for the first $h(n)$ channel uses, do not transmit.

- Transmit i.i.d. Gaussian signals in the remaining $n - h(n)$ channel uses.

- The energy saved during the first $h(n)$ channel uses is sufficient to guarantee no energy shortages occur in the remaining $n - h(n)$ channel uses.

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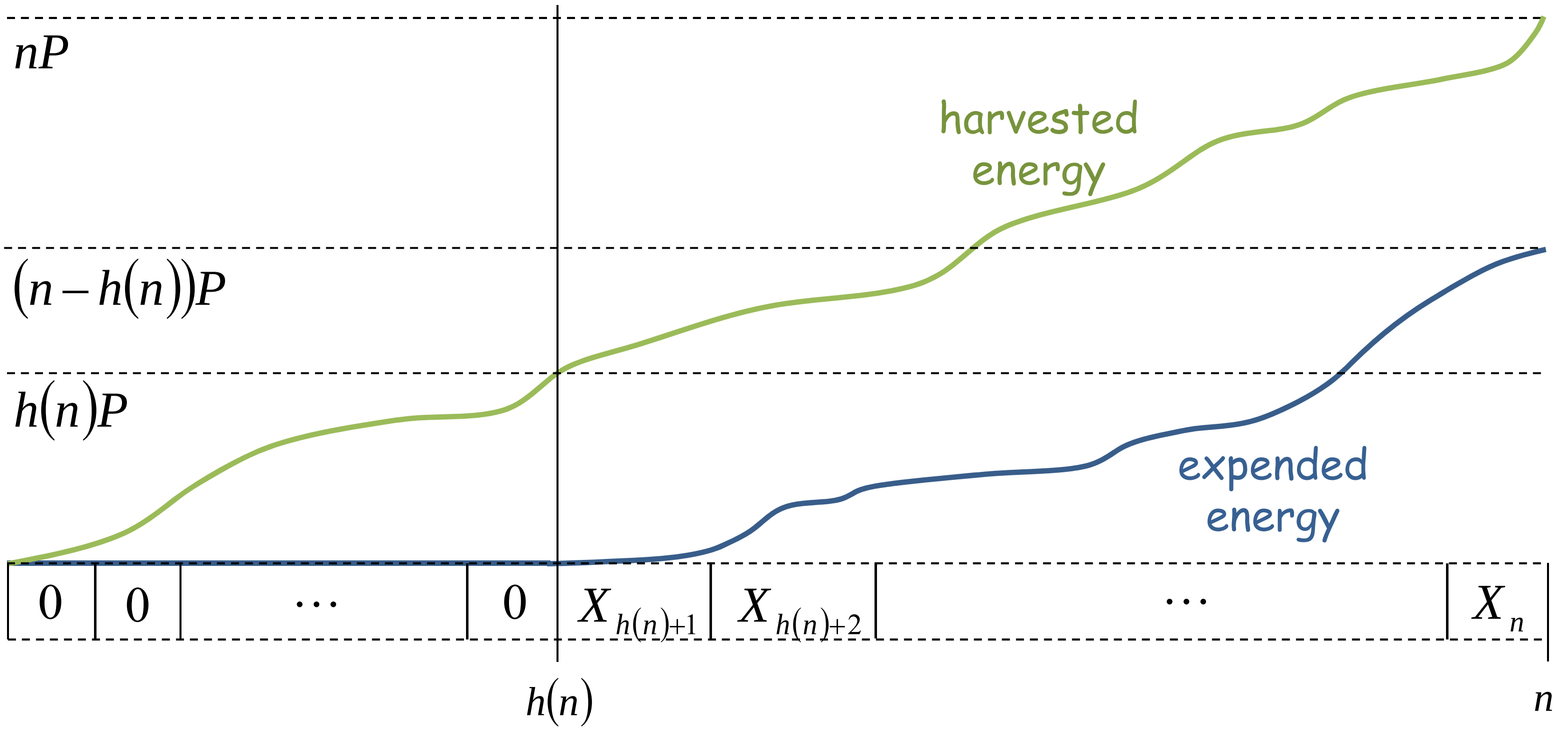
Save-and-Transmit



- Since $h(n)/n \to 0$, there is no loss in rate.
- Rates $< \frac{1}{2}\log(1+P)$ are achievable.

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Best-Effort-Transmit

- Codewords are i.i.d. Gaussian with variance $P-\varepsilon$.
- S_i : the energy in the battery, i.e., the battery state in the ith channel use.
- If $S_i \geq X_i^2$, i.e., there is enough energy in the battery, send X_i. Otherwise, send nothing.
- The battery state updates according to

$$S_{i+1} = S_i + E_i - X_i^2 1\left(S_i \geq X_i^2\right).$$

Best-Effort-Transmit

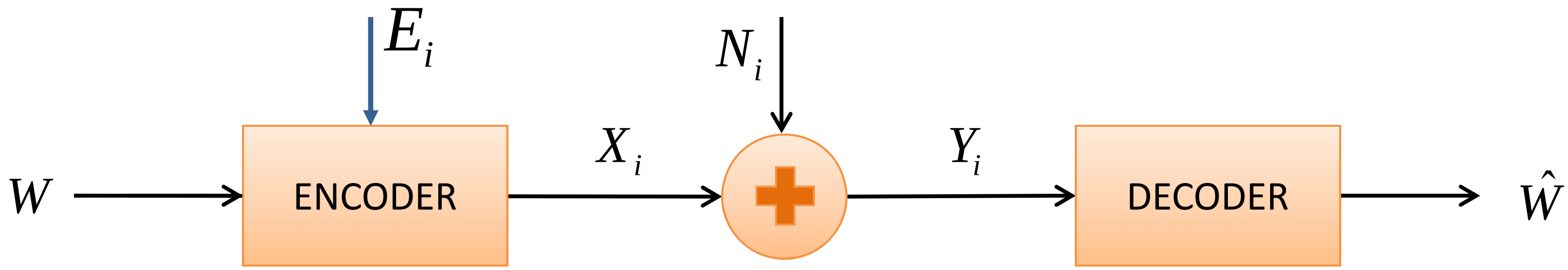
- With $E\left[X_i^2\right] = P - \varepsilon$ and $E\left[E_i\right] = P$, it is shown by SLLN that, finitely many energy shortages occur.

- Finitely many symbols are infeasible, i.e., the transmitter puts 0 to the channel instead of the desired code symbol finitely many times.

- Finitely many mismatches are insignificant for joint typical decoding.

- Rates $< \frac{1}{2}\log\left(1+P\right)$ are achievable.

EH AWGN Channel with No Battery



[Ozel-Ulukus '11]

- There is no battery at the transmitter, i.e., $E_{\max} = 0$.
- The code symbols are amplitude constrained:

$$X_i^2 \le E_i, \quad i = 1, 2, \ldots, n.$$

EH AWGN Channel with No Battery

- The transmitter has causal information of energy arrivals. The receiver does not know the energy arrivals.

- The harvested energy amount is one of finitely many possibilities. For simplicity, assume binary $\{E_1, E_2\}$.

- Background:

1. Static amplitude constrained AWGN channel [Smith'71]
2. State dependent channel with causal state information at the transmitter [Shannon'58]

Static Amplitude Constrained AWGN Channel [Smith'71]

- At each channel use, the code symbol is amplitude constrained by A.

- The channel capacity under this constraint is

$$C_{\mathrm{Sm}}(A) = \max_{|X| \leq A} I(X;Y)$$

which is a convex program.

- The capacity achieving distribution was shown to have finitely many mass points.

State Dependent Channel with Causal State Information at the Tx [Shannon'58]

- Channel model: $p(y \mid x, s)$
- State $s \in S$ is causally available at the transmitter only.
- The channel capacity is

$$C_{\mathrm{Sh}} = \max_{p_T(t)} I(T;Y).$$

- $T = [T_1, T_2, \ldots, T_{|S|}]$ is an extended channel input satisfying

$$p_{Y|T}(y \mid t) = \sum_{i=1}^{|S|} P(s = s_i) \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(y-t_i)^2}{2}}.$$

Capacity of the EH AWGN Channel with No Battery

- Suppose the harvested energy is $\begin{cases} E_1, & \text{w.p. } p_1 \\ E_2, & \text{w.p. } p_2 = 1 - p_1 \end{cases}.$

- Apply Shannon's result with $T = [T_1, T_2]$ and

$$p(y \mid t_1, t_2) = \underbrace{\frac{p_1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(y-t_1)^2}{2}}}_{t_1^2 \le E_1} + \underbrace{\frac{p_2}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(y-t_2)^2}{2}}}_{t_2^2 \le E_2}.$$

- The capacity achieving distribution is observed to have finitely many mass points.

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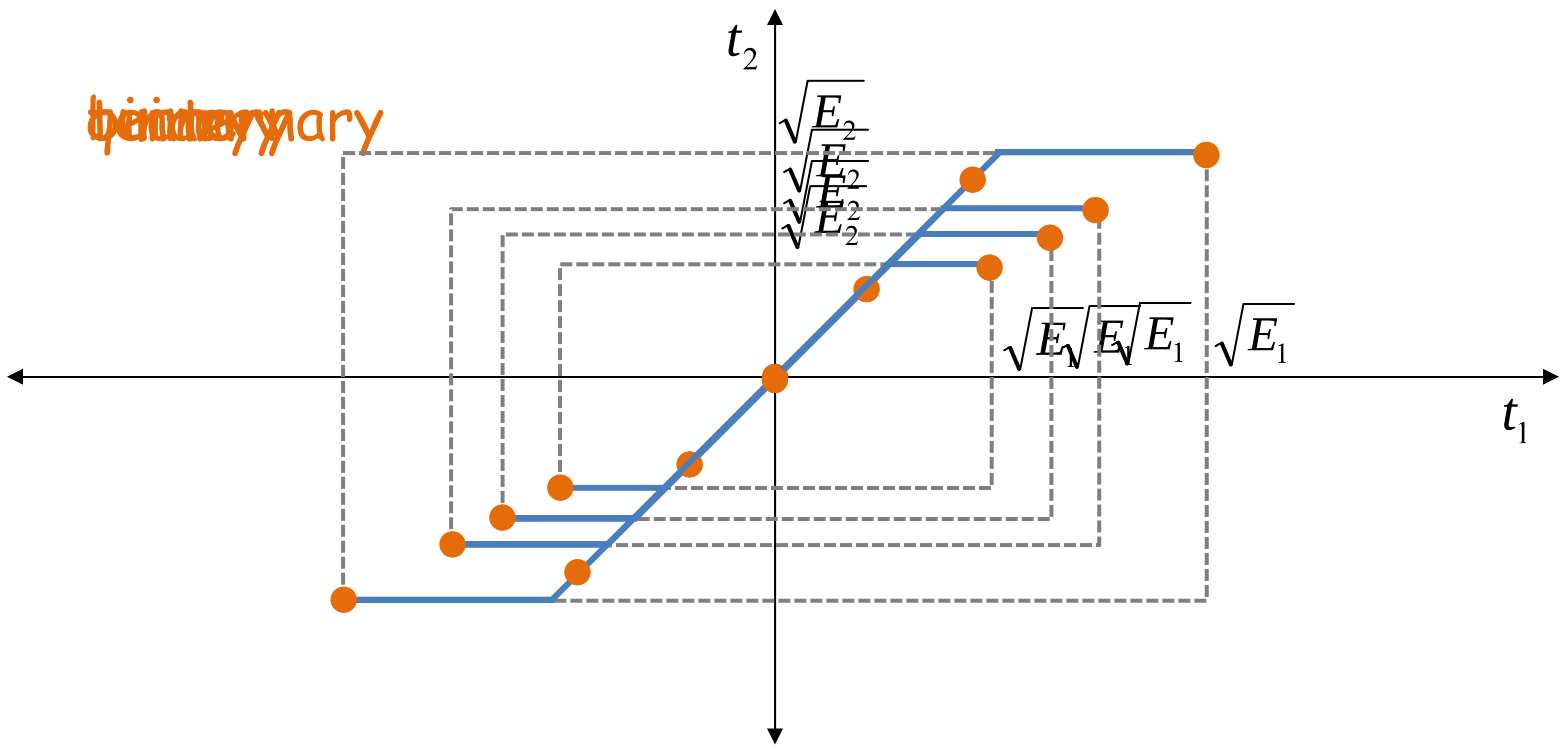
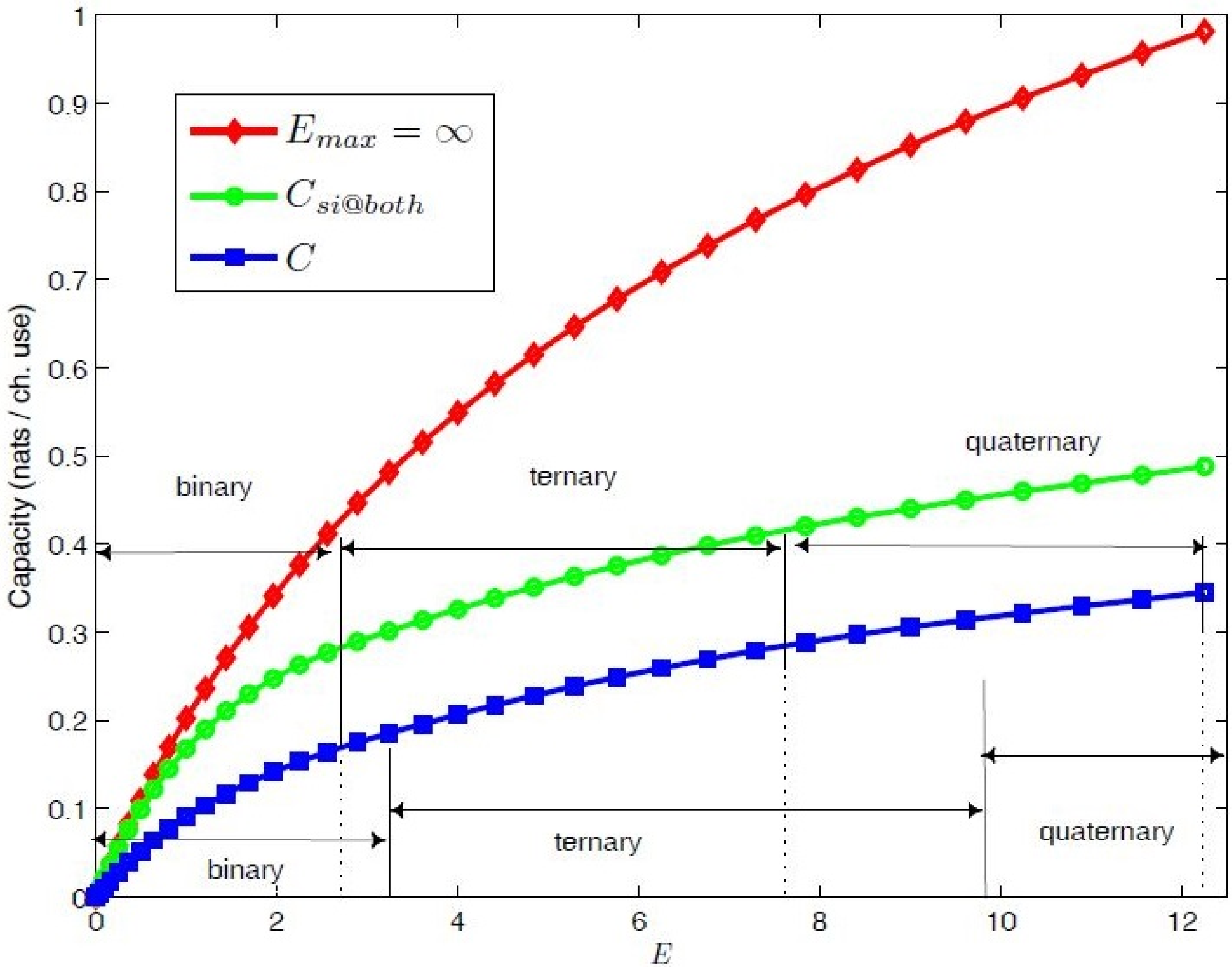
Mass Points

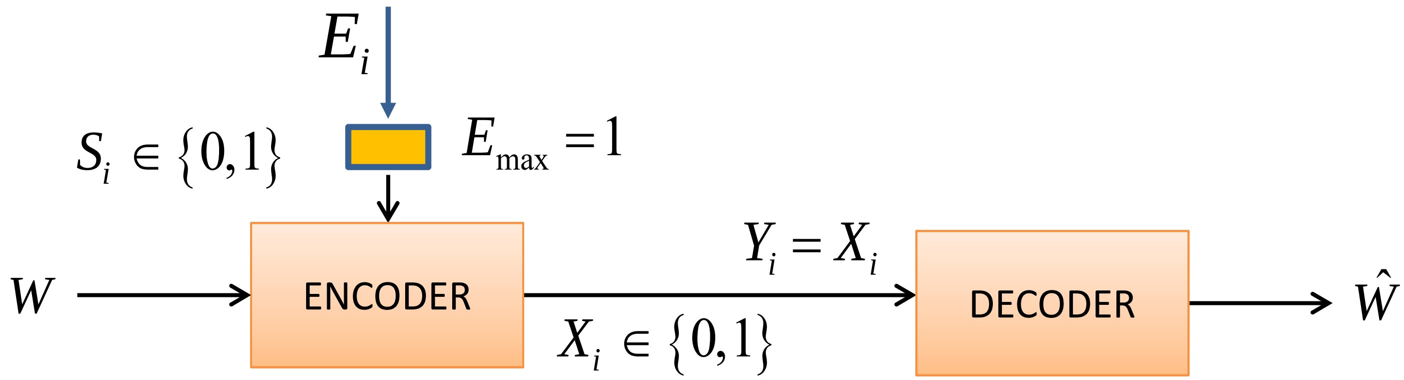
- Symmetric about the origin
- Constrained to the blue line



Numerical Results



Binary Noiseless EH Channel

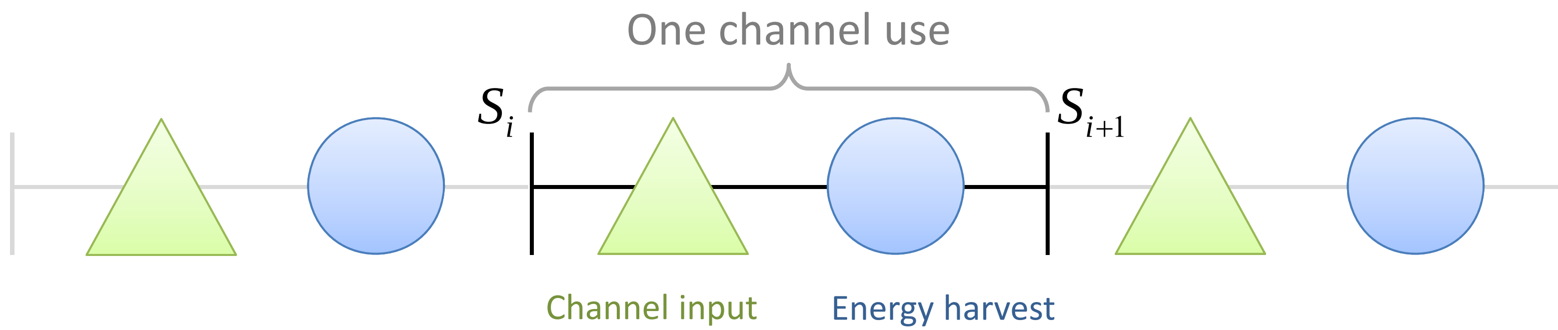


[Tutuncuoglu-Ozel-Ulukus-Yener'13]

- Transmitting $X_i \in \{0,1\}$ requires X_i units of energy
- Unit battery, $E_{\max} = 1$
- Binary noiseless channel, $Y_i = X_i$

Energy Model

- In channel use i, the transmitter **first puts input symbol X_i to the channel**, and **then harvests energy E_i**:



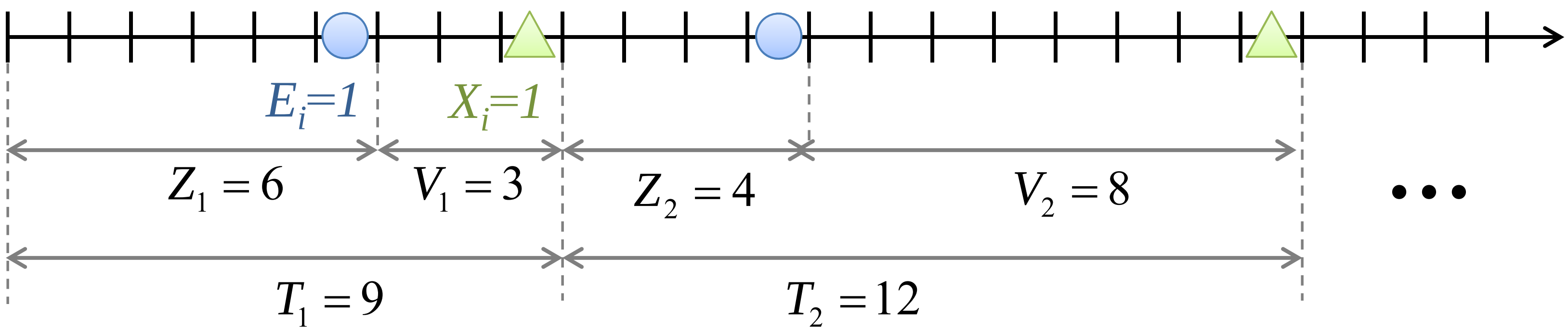
- At the **beginning** of channel use i, battery state is S_i.
- **State evolution:** $S_{i+1} = \min\{S_i - X_i + E_i, 1\}$ **(next state depends on input)**
- **Energy harvest:** E_i are i.i.d. Bernoulli with $\Pr[E_i = 1] = q_h$

Timing Channel

- A representation that simplifies the problem.

X_i 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 ...

E_i 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 ...



- $Z_i \in \{0,1,\ldots\}$: # of channel uses spent waiting for energy, $\sim$ ***Geometric*** (q_h), ***i.i.d.***
- $V_i \in \{1,2,\ldots\}$: # of channel uses the energy is kept in storage
- $T_i \in \{1,2,\ldots\}$: # of channel uses between *1*s at the receiver side

$$\boxed{T_i = V_i + Z_i}$$

Memoryless!

Timing Channel

- The two sets of variables, (V^m, Z^m, T^m) and (X^n, E^n, Y^n), are **alternative representations of the same sequences.**

$$X^n = \{0,0,0,1,0,0,1,0,1,0,0,0,0,0,1,0\} \qquad V^m = \{1,2,2,2\}$$
$$Y^n = \{0,0,0,1,0,0,1,0,1,0,0,0,0,0,1,0\} \quad \Leftrightarrow \quad T^m = \{4,3,2,6\}$$
$$E^n = \{0,0,1,0,1,0,1,0,0,0,0,0,1,0,1,0\} \qquad Z^m = \{3,1,0,4\}$$

- **Lemma:** The **timing channel capacity** with additive causally known state C_T and the originally formulated **binary EH channel capacity** C are equal, i.e., $\boldsymbol{C = C_T}$.

Capacity of the Timing Channel

- **[Shannon 1958]**

  Capacity of a memoryless channel with causal CSIT:

$$C_{CSIT} = \max_{p(u),v(u,z)} I(U;T)$$

- **[Anantharam-Verdu 1996]**

  Capacity of the timing channel: $C_T = \max_{p(x)} \frac{I(X;T)}{\mathrm{E}[T]}$

- Capacity of the **timing channel with causal CSIT**

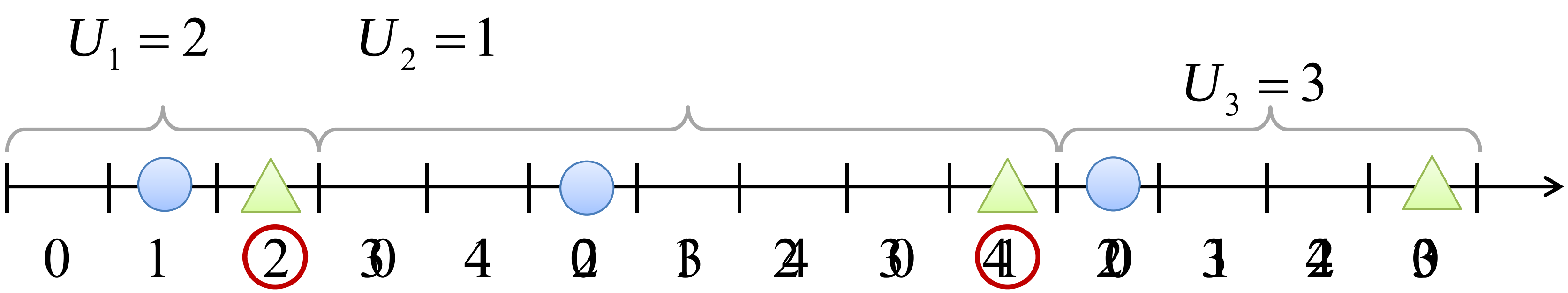
$$C_T = \max_{p(u),v(u,z)} \frac{I(U;T)}{\mathrm{E}[T]} = C_{BEHC}$$

- **Main challenge:** selection of auxiliary variable U

$$|Z|,|V| \to \infty \;\Rightarrow\; |U| \to \infty, \qquad v:(U,Z) \to V$$

Modulo Encoding

- $U \in \{0,1,\ldots,N-1\}, \quad U \sim p_U(u), \quad V = (U - Z \bmod N) + 1$
- **Binary encoding interpretation:** The encoder indexes channel uses in $mod\ N$, and sends U_i by transmitting a 1 at the earliest feasible channel use with index U_i.
- **Achievable rate:** $R_A^{(N)} = \max\limits_{p(u)} \dfrac{I(U;T)}{\mathrm{E}[V+Z]} = \max\limits_{p(u)} \dfrac{H(U)}{\mathrm{E}[V]+\mathrm{E}[Z]}$
- **Example:** $N = 5 \qquad U_i = \{2,1,3,\ldots\}, \quad Z_i = \{2,3,1,\ldots\}$

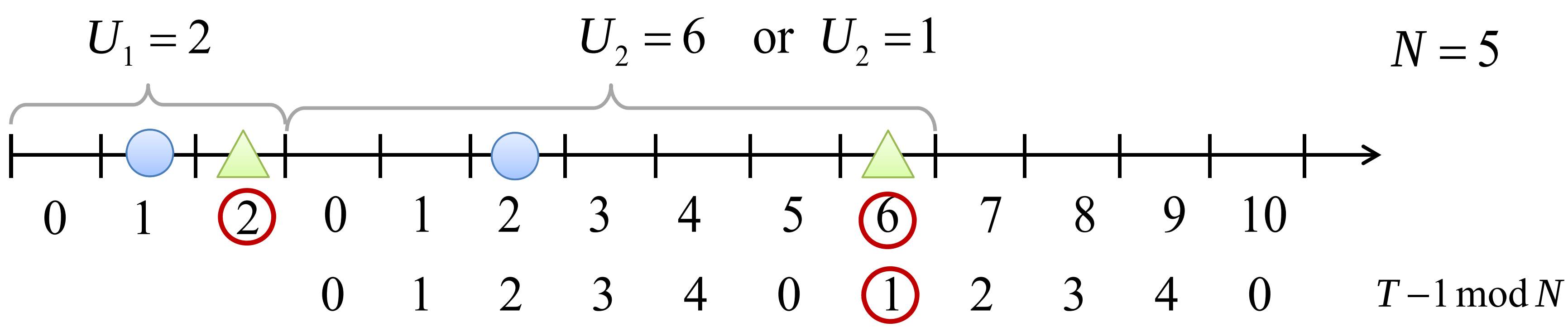


Extended Modulo Encoding

- **Choose** $V = \begin{cases} U - Z + 1 & U \geq Z \\ (U - Z \mod N) + 1 & U < Z \end{cases}$ $\quad U \in \{0,1,\ldots\}$

- **Decoder:** $T' = T - 1 \mod N = U \mod N$ (U mod N decoded without error)

- **Achievable Rate:** $R_A^{ext} = \max_N \max_{p(u)} \dfrac{I(U;T)}{\mathrm{E}[V+Z]}$



Genie Upper Bound

- Provide channel state Z_i as side information at the decoder.

$$C_{UB}^{genie} = \max_{p(v)} \frac{I(V;T \mid Z)}{\mathrm{E}[V+Z]} = \max_{p(v)} \frac{H(V)}{\mathrm{E}[V]+\mathrm{E}[Z]}$$

$$= \max_{\mu \geq 0} \frac{1}{\mu + \mathrm{E}[Z]} \max_{\mathrm{E}[V] \leq \mu} H(V)$$

- The entropy maximizing distribution on $V \in \{1,2,\ldots\}$ with $\mathrm{E}[V] = \mu$ is *Geometric(1/μ).*

$$C_{UB}^{genie} = \max_{q_u \in [0,1]} \frac{q_h H(q_u)}{q_h + q_u(1-q_h)}$$

Asymptotic Optimality

- **Modulo Encoding:**

$$R_A^{\mathrm{mod}} = \max_{q_u, N} \frac{H(U)}{\mathrm{E}[V] + \mathrm{E}[Z]}$$

Genie Upper bound:

$$C_{UB}^{genie} = \max_{q_u \in [0,1]} \frac{q_h H(q_u)}{q_h + q_u(1-q_h)}$$

- Choose $N = \left\lceil \frac{1}{q_u^*} \right\rceil$, $U \sim Unif(\{0,1,\ldots,N-1\})$

$$\Rightarrow \lim_{q_h \to 0} \frac{C_{UB}^{genie}}{R_A^{\mathrm{mod}}} = 1$$

Modulo encoding is asymptotically optimal for low harvesting rates

Leakage Upper Bound [Tutuncuoglu-Ozel-Yener-Ulukus'14]

- Timing Channel Capacity: $C_T = \max_{p(u),v(u,z)} \frac{I(U;T)}{\mathrm{E}[T]}$

- $I(U;T) = H(T) - \boxed{I(Z;T \mid U)}$ (Mutual dependence of Z and T given U)

- $I(Z;T \mid U) = \sum_{t=1}^{\infty} \sum_{u} p(t,u)[H(Z) - \boxed{H(Z \mid T=t, U=u)}]$ (Entropy of Z upon observing $T=t$ and decoding $U=u$)

Lemma: $H(Z \mid T=t, U=u) \le H(Z_t)$

where $p_{Z_t}(z) = \begin{cases} \frac{q_h(1-q_h)^z}{1-(1-q_h)^t}, & \text{if } z < t \\ 0, & \text{otherwise} \end{cases}$ (Truncated geometric)

Leakage Upper Bound

$$C_T = \max_{p(u),v(u,z)} \frac{I(U;T)}{\mathrm{E}[T]}$$

$$= \max_{p(u),v(u,z)} \frac{H(T)-I(Z;T|U)}{\mathrm{E}[T]}$$

$$C_T \le \max_{p(t)} \frac{H(T)-\sum_{t=1}^{\infty} p(t)[H(Z)-H(Z_t)]}{\mathrm{E}[T]}$$

Leakage Upper Bound

- Easier to evaluate than C_T since the maximization is over $p(t)$ instead of $p(u),v(u,z)$

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Computing the Leakage Upper Bound

$$C_T \le \max_{p(t)} \frac{H(T) - \sum_{t=1}^{\infty} p(t)[H(Z) - H(Z_t)]}{\mathbf{E}[T]}$$

$$= \max_{\beta} \frac{1}{\beta} \boxed{\max_{p(t), \mathbf{E}[T] \le \beta} H(T) - \sum_{t=1}^{\infty} \Delta_t p(t)}$$

(Inner problem is convex)

- KKT optimality conditions give

$$\boxed{p(t) = A \exp\left(-\mu t - \Delta_t - \sum_{n=1}^{t} \gamma_n \right)}$$

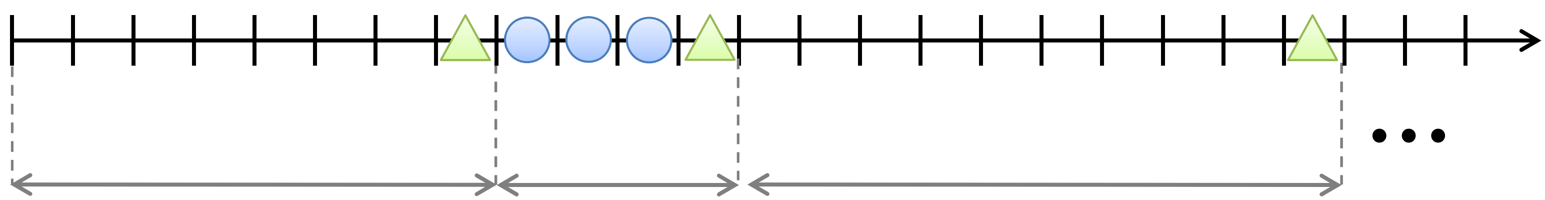
$$A = \left(\sum_{t=1}^{\infty} - \mu t - \Delta_t - \sum_{n=1}^{t} \gamma_n \right)^{-1}$$

- Calculate UB by exhaustive search over μ for each β

Interpretation of the UB

$$C_T \le \max_{p(t)} \frac{H(T) - \boxed{\sum_{t=1}^{\infty} p(t)[H(Z)-H(Z_t)]}}{\mathrm{E}[T]}$$

T_i 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 ...



$T_1 = 8$ $T_2 = 4$ $T_2 = 10$

Revealed: $Z_1 < 8$ $Z_2 < 4$ $Z_3 < 10$

We inadvertently **"waste"** part of the potential rate of the channel

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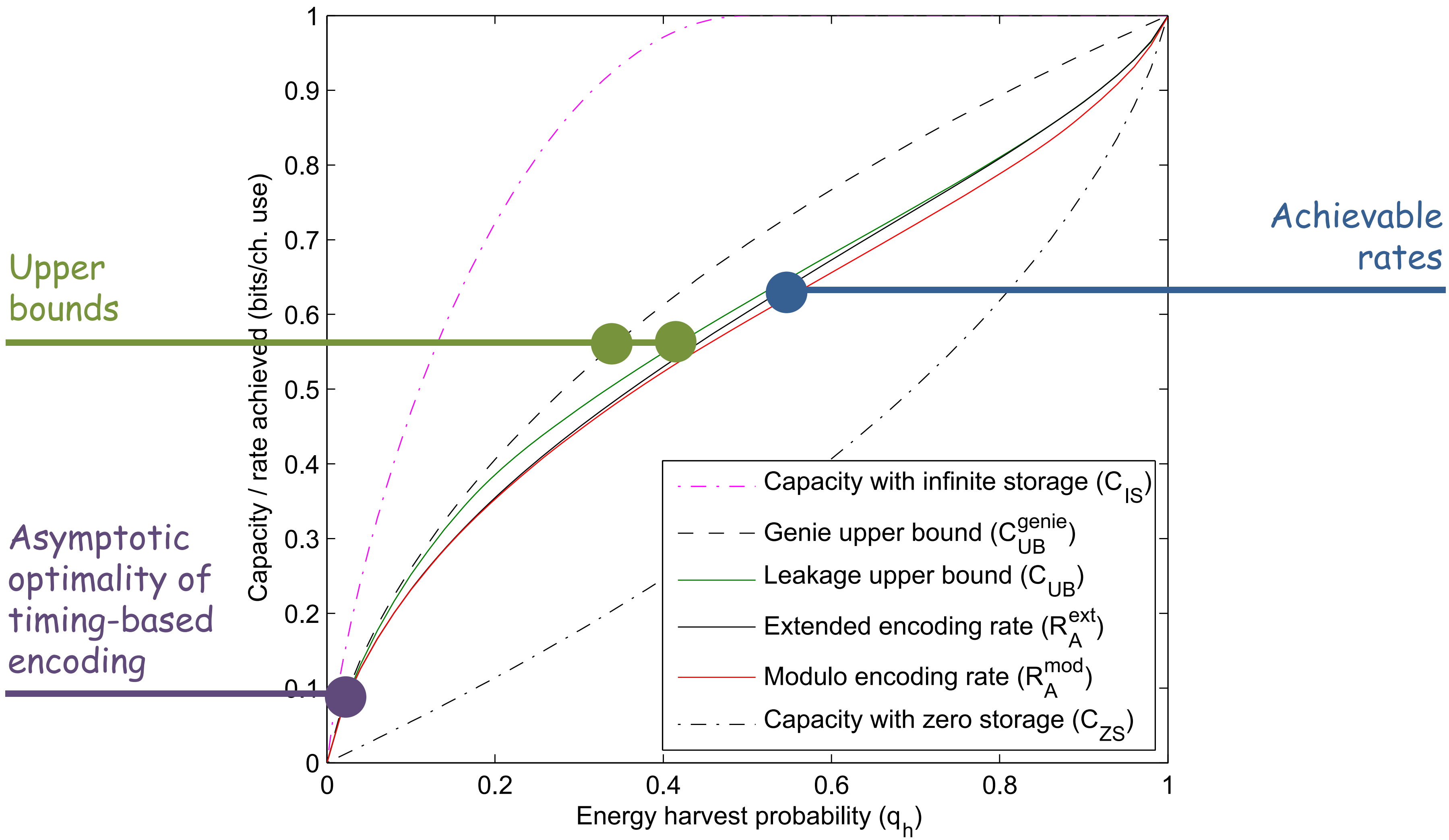
Numerical Results



Numerical Results

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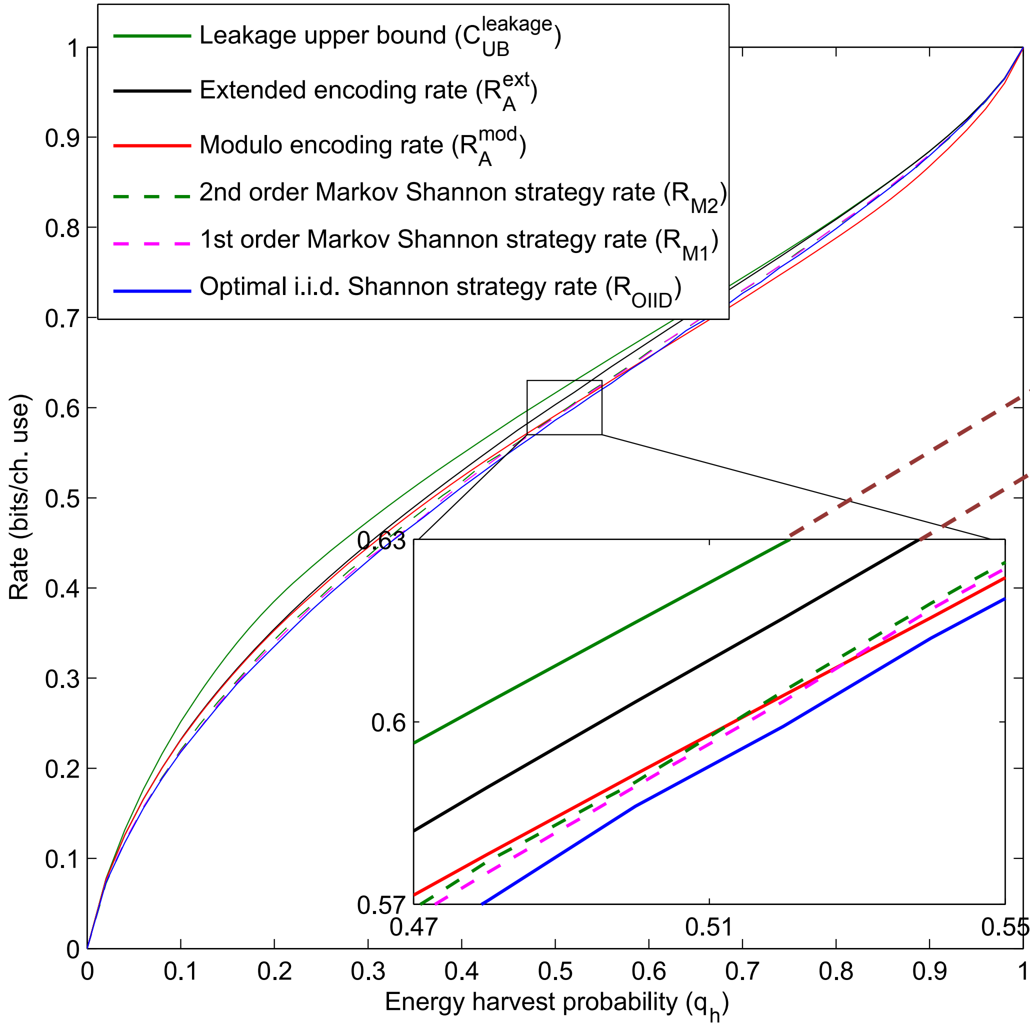
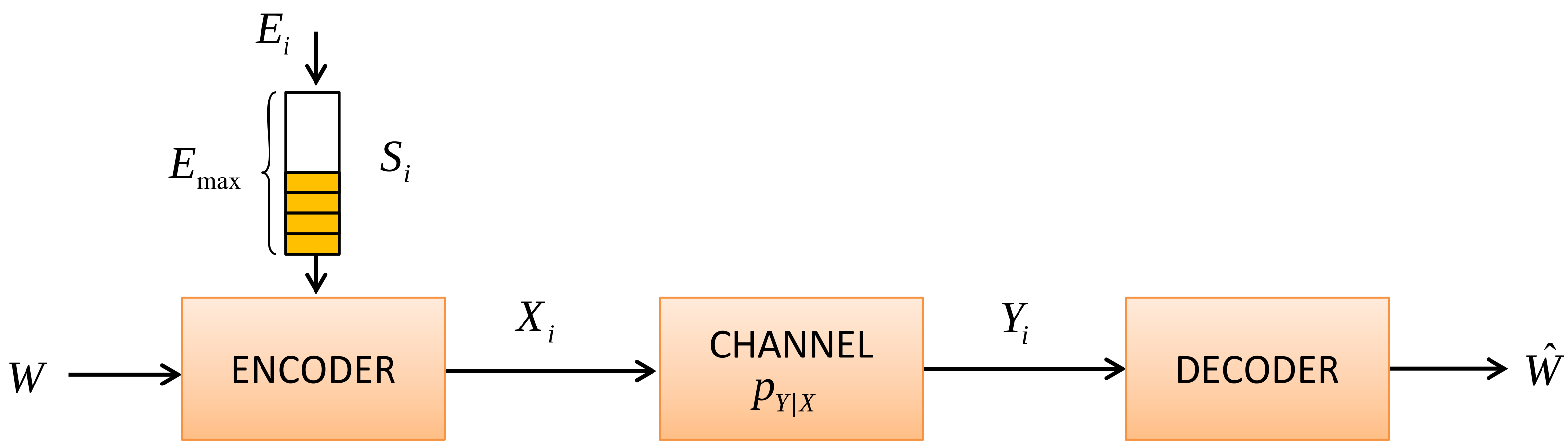


Capacity within 0.03 bits/ch.use

Binary Symmetric EH Channel



- **Binary symmetric channel**: $\Pr[Y_i \neq X_i] = p_e \in \left[0, \frac{1}{2}\right], \quad X_i, Y_i \in \{0,1\}$
- The energy arrivals are **i.i.d. Bernoulli**, $E_i \sim \text{Bernoulli}(q)$.
- Two sources of errors:

1. Energy shortage: Without energy, the encoder **must send a zero.**
2. Channel errors: Any bit sent can be **flipped by the channel.**

Binary Symmetric EH Channel

- Observing Y^n, decoder **also obtains information** about E^n
- Rate of this information flow can be quantified by

$$\Delta = \frac{1}{n}\Big[\underbrace{H(E^n)}_{\text{Randomness of energy arrival process}} - \underbrace{H(E^n \mid Y^n)}_{\text{Randomness remaining after channel output is observed}} \Big] = \frac{1}{n} I(E^n; Y^n).$$

The encoder may wish to **[Tutuncuoglu-Ozel-Yener-Ulukus'14ITW]**:

- **Maximize** entropy reduction rate Δ: **State Amplification**

  (Cooperative scenario) [Kim et al. '08]

- **Minimize** entropy reduction rate Δ: **State Masking**

  (Privacy or stealth scenario) [Merhav-Shamai '07]

No Battery Case [Tutuncuoglu-Ozel-Yener-Ulukus'14ITW]

- The encoder can send $X_i = 1$ only when $E_i = 1$.
- For i.i.d. arrivals, this is a **memoryless channel with CSIT.**
- Capacity achieved using **Shannon strategies**:

$$U \in \{0,1\}^n, \quad U_i = \text{Bern}(p), \quad X_i = \begin{cases} 1 & E_i = 1, U_i = 1 \\ 0 & else \end{cases}$$

Shorthand for $U = (0,0)$ and $U = (0,1)$.

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No Battery Case

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- State Amplification

$$R \le H(pq * p_e) - pH(q * p_e) - (1-p)H(p_e)$$
$$\Delta \le H(q)$$
$$R + \Delta \le H(pq * p_e) - H(p_e)$$

- State Masking

$$R \le H(pq * p_e) - pH(q * p_e) - (1-p)H(p_e)$$
$$\Delta \ge pH(q * p_e) - pH(p_e)$$

where $p * q = p(1-q) + (1-p)q$

Infinite Battery Case [Tutuncuoglu-Ozel-Yener-Ulukus'14ITW]

- Capacity achieved via extending the **save-and-transmit scheme** [Ozel-Ulukus '12].
- Channel input constrained as $\mathbf{E}[X] \leq q$

$$C = C_{BSC} = \begin{cases} H(q * p_e) - H(p_e) & q \leq \frac{1}{2} \\ 1 - H(p_e) & q > \frac{1}{2} \end{cases}$$

State Amplification

Lemma: The (R,Δ) region is given by

$$R+\Delta \le C_{BSC}, \qquad 0 \le \Delta \le H(q)$$

- **Achievability:** Compress part of E^n and send **as a part of the message**, i.e., decoder obtains $W=(W',E^k)$
- **Converse:** Using the Markov Chain $(W,E^n)-X^n-Y^n$

$$\begin{aligned} I(X^n;Y^n) &\ge I(E^n,W;Y^n) \\ &\ge I(E^n;Y^n)+H(W)-H(\varepsilon)-\varepsilon\log(nR) \\ &= n\Delta+nR-\boxed{H(\varepsilon)-\varepsilon\log(nR) \quad \to 0 \text{ as } \varepsilon\to 0} \end{aligned}$$

State Masking

For $E_{\max} = \infty$, perfect state masking is possible, i.e.,

$(R, \Delta) = (C_{BSC}, 0)$ is achievable

- In the save-and-transmit scheme, channel input X^n is independent of harvested energy E^n
- Any rate $R < C_{BSC}$ is also achievable.
- Due to converse proof, no better rate can be achieved

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Unit-sized Battery Case

- Capacity of this channel is **open** as we have just seen.
- Some **achievable rates** proposed: [Tutuncuoglu-Ozel-Y.-Ulukus'13][Mao-Hassibi '13].
- [Mao-Hassibi '13]: Two strategies, $U_i \in \{0,1\}$
- Channel input: $X_i = \begin{cases} 1 & S_i = 1, U_i = 1 \\ 0 & else \end{cases}$
- $R_{IID} = \lim_{n\to\infty} \frac{1}{n} I(U^n; Y^n)$
- If S_i was memoryless, this **would be capacity achieving**.

Numerical Results

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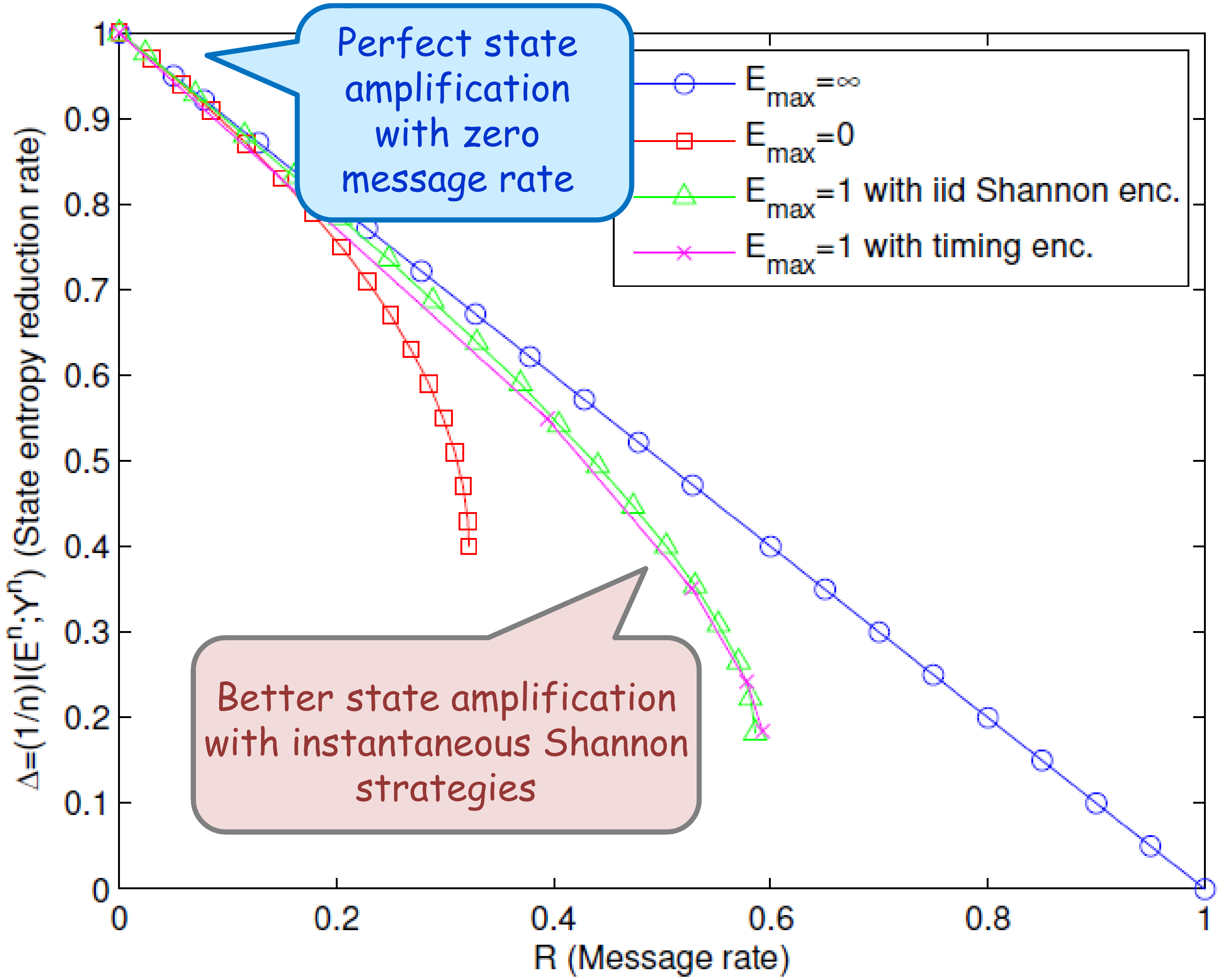
State Amplification

- Noiseless channel $(p_e = 0)$
- $q = \frac{1}{2}$



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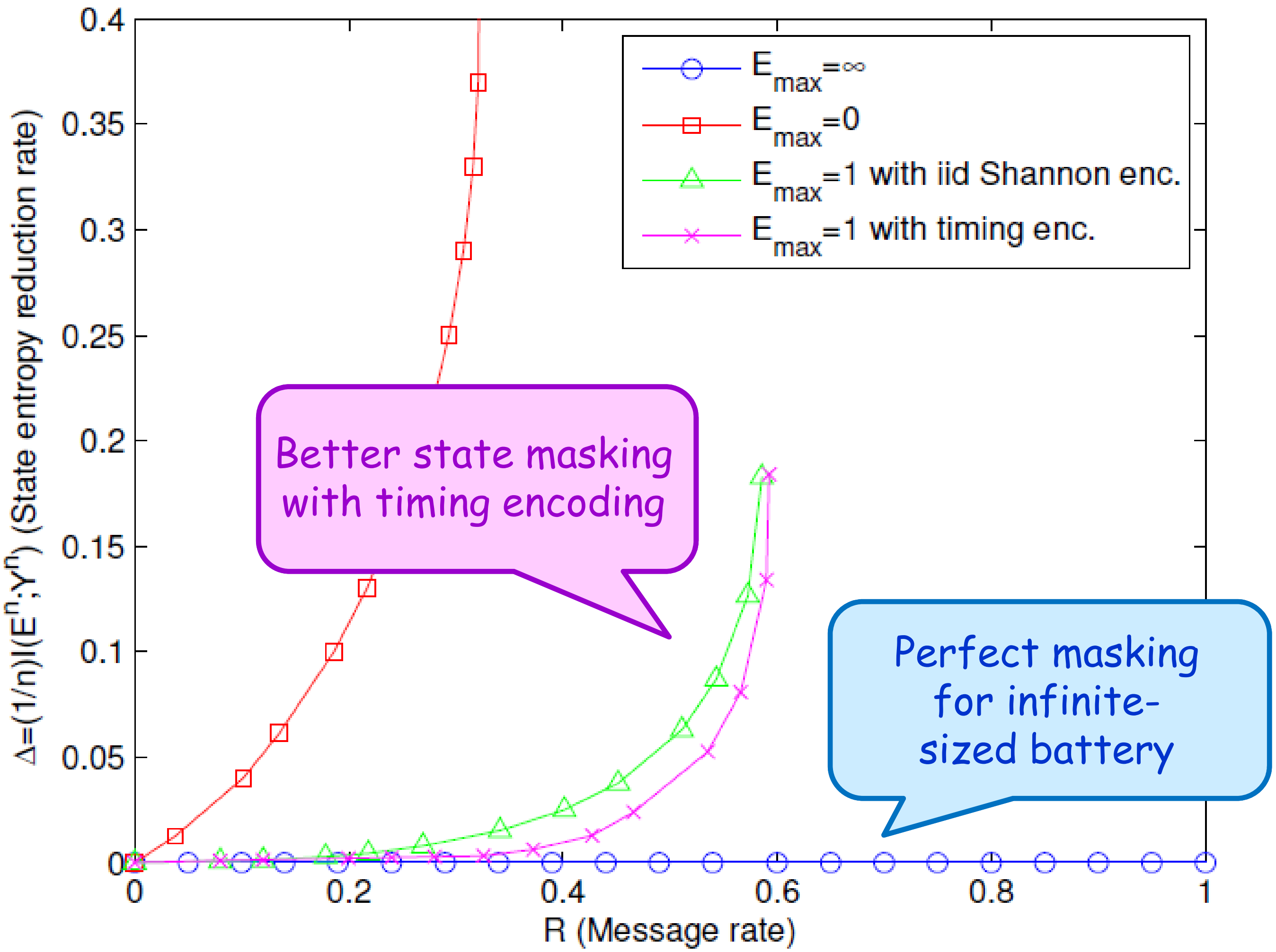
Numerical Results

State Masking

- Noiseless channel $(p_e = 0)$
- $q = \frac{1}{2}$



Conclusion

- New wireless communications paradigm: energy harvesting nodes
- New design insights arising from
  - new energy constraints
  - energy storage limitations and inefficiencies
  - interaction of multiple EH transmitters
  - energy cooperation
- New problems in the information theory domain
- Lots of open problems related to all layers of the network design: e.g. Signal processing/PHY design; MAC protocol design; channel capacity…

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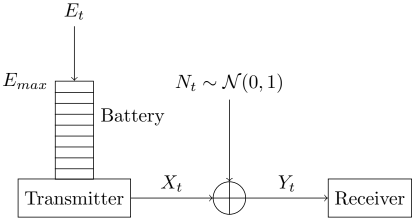
Information-Theoretic Capacity of Energy Harvesting and Remotely Powered Systems

Ayfer Özgür

Tutorial on Energy Harvesing and Remotely Powered Communication

ISIT 2016, Barcelona, Spain

Model



$$|X_t|^2 \le B_t$$
$$B_{t+1} = \min\left(B_t - |X_t|^2 + E_{t+1}, E_{max}\right).$$

E_t: i.i.d. energy harvesting process known causally at the transmitter and not at the receiver.

State-dependent channel:

- State process has memory and is input dependent.
- State is known causally at the transmitter but not at the receiver.

$E_{max} = \infty$: Capacity equal to that of a classical AWGN channel with $P = \mathbb{E}[E_t]$:

$$C = \log\left(1 + \mathbb{E}[E_t]\right)$$

First-order questions:

- How does the capacity of the energy harvesting AWGN channel depend on system parameters such as E_{max} and E_t?

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- What are the properties of E_t most relevant to capacity? What are more favorable and less favorable energy profiles?

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- What are the properties of E_t most relevant to capacity? What are more favorable and less favorable energy profiles?
- Are there different operating regimes where the dependence to E_{max} and E_t is qualitatively different?
- For a given E_t, how can we "optimally" choose E_{max}?

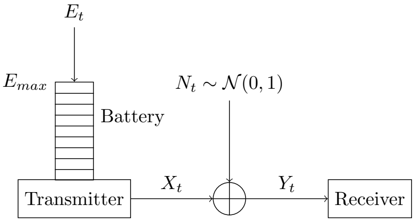
$E_{max} = \infty$: Capacity equal to that of a classical AWGN channel with $P = \mathbb{E}[E_t]$:

$$C = \log(1 + \mathbb{E}[E_t])$$

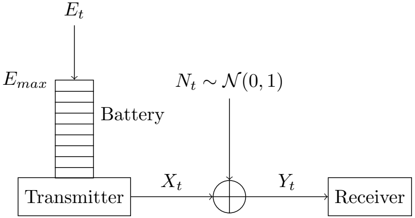
First-order questions:

- How does the capacity of the energy harvesting AWGN channel depend on system parameters such as E_{max} and E_t?
- What are the properties of E_t most relevant to capacity? What are more favorable and less favorable energy profiles?
- Are there different operating regimes where the dependence to E_{max} and E_t is qualitatively different?
- For a given E_t, how can we "optimally" choose E_{max}?

A channel with random battery recharges

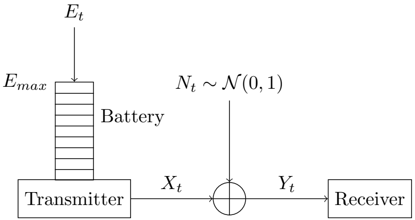


A channel with random battery recharges



$$|X_t|^2 \leq B_t$$
$$B_{t+1} = \min\left(B_t + E_{t+1} - |X_t|^2, E_{max}\right)$$

A channel with random battery recharges

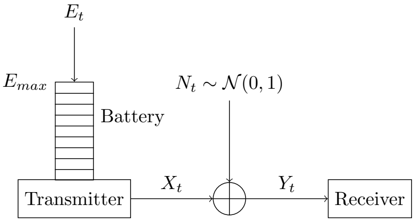


$$|X_t|^2 \leq B_t$$
$$B_{t+1} = \min\left(B_t + E_{t+1} - |X_t|^2, E_{max}\right)$$

We focus on i.i.d. Bernoulli energy arrival process:

$$E_t = \begin{cases} E_{max} & \text{w.p. } p \\ 0 & \text{w.p. } 1-p, \end{cases}$$

A channel with random battery recharges



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We focus on i.i.d. Bernoulli energy arrival process:

$$E_t = \begin{cases} E_{max} & \text{w.p. } p \\ 0 & \text{w.p. } 1-p, \end{cases}$$

- The energy arrival process $\{E_t\}$ is causally known both at the transmitter and the receiver.

Results for this model

- n-letter expression for capacity:

$$C = \lim_{N \to \infty} \max_{\substack{p(x^N): \\ \|X^N\|^2 \leq E_{max}}} \sum_{k=1}^{N} p^2 (1-p)^{k-1} I(X^k; X^k + Z^k)$$

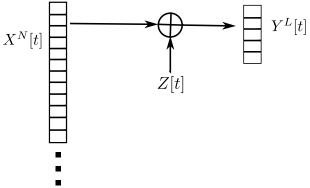
- Connection to online power control:

$$T - 1.05 \leq C \leq T.$$

- Bounded gap to AWGN capacity:

$$\log(1 + pE_{max}) - 1.77 \leq C \leq \log(1 + pE_{max}).$$

Clipping Channel

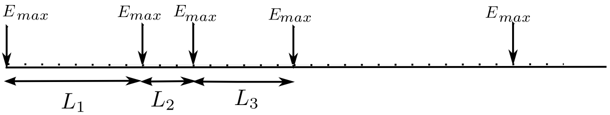


$$Y_j[t] = \begin{cases} X_j[t] + Z_j[t] & ,j \leq L[t] \\ 0 & ,j > L[t] \end{cases}$$

where $L[t]$ are i.i.d. Geometric(p) and $||X[t]||^2 \leq E_{max}$.

Theorem

$$C_{EH} = p \cdot C_{clp}$$



Capacity of the Clipping Channel

$$C_{\mathrm{clp}} = \lim_{N \to \infty} \max_{\substack{p(x^N): \\ \|X^N\|^2 \le E_{max}}} I(X^N; Y^L | L)$$

$$= \lim_{N \to \infty} \max_{\substack{p(x^N): \\ \|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p(1-p)^{k-1} I(X^k; X^k + Z^k)$$

Bounding C_{EH}

$$C_{\text{EH}} = \lim_{N\to\infty} \max_{\substack{p(x^N): \\ \|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p^2(1-p)^{k-1} I(X^k; X^k + Z^k)$$

Bounding C_{EH}

$$
\begin{aligned}
C_{\text{EH}} &= \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p^2(1-p)^{k-1} I(X^k; X^k + Z^k) \\
&\le \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \mathbb{E}\|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p^2(1-p)^{k-1} \sum_{i=1}^{k} I(X_i; X_i + Z_i)
\end{aligned}
$$

Bounding C_{EH}

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\begin{aligned}
C_{\mathrm{EH}} &= \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p^2(1-p)^{k-1} I(X^k; X^k + Z^k) \\
&\le \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \mathbb{E}\|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p^2(1-p)^{k-1} \sum_{i=1}^{k} I(X_i; X_i + Z_i) \\
&= \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \mathbb{E}\|X^N\|^2 \le E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} I(X_i; X_i + Z_i)
\end{aligned}
$$

Bounding C_{EH}

$$C_{\text{EH}} = \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p^2(1-p)^{k-1} I(X^k; X^k + Z^k)$$

$$\le \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \mathbb{E}\|X^N\|^2 \le E_{max}}} \sum_{k=1}^{N} p^2(1-p)^{k-1} \sum_{i=1}^{k} I(X_i; X_i + Z_i)$$

$$= \lim_{N\to\infty} \max_{\substack{p(x^N):\\ \mathbb{E}\|X^N\|^2 \le E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} I(X_i; X_i + Z_i)$$

$$= \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N:\\ \mathcal{E}_i \ge 0\ \forall i\\ \sum_{i=1}^N \mathcal{E}_i \le E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \frac{1}{2} \log(1 + \mathcal{E}_i)$$

Bounding C_{EH}

$$C_{\text{EH}} \le \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \ge 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \le E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \frac{1}{2}\log(1+\mathcal{E}_i)$$

$$C_{\text{EH}} \ge \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \ge 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \le E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \max_{|X_i| \le \sqrt{\mathcal{E}_i}} I(X_i; X_i + Z_i)$$

Bounding C_{EH}

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We can show

$$\max_{|X_i| \leq \sqrt{\mathcal{E}}} I(X_i; X_i + Z_i) \geq \frac{1}{2} \log(1+\mathcal{E}) - 1.05.$$

Bounding C_{EH}

$$C_{\mathsf{EH}} \leq \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \geq 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \leq E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \frac{1}{2} \log(1+\mathcal{E}_i)$$

$$C_{\mathsf{EH}} \geq \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \geq 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \leq E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \max_{|X_i| \leq \sqrt{\mathcal{E}_i}} I(X_i; X_i + Z_i)$$

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$$C_{\mathsf{EH}} \geq \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \geq 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \leq E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \frac{1}{2} \log(1+\mathcal{E}_i) - 1.05.$$

Bounding C_{EH}

$$C_{\text{EH}} \leq \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \geq 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \leq E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \frac{1}{2} \log(1+\mathcal{E}_i)$$

$$C_{\text{EH}} \geq \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \geq 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \leq E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \max_{|X_i| \leq \sqrt{\mathcal{E}_i}} I(X_i; X_i + Z_i)$$

We can show

$$\max_{|X_i| \leq \sqrt{\mathcal{E}}} I(X_i; X_i + Z_i) \geq \frac{1}{2} \log(1+\mathcal{E}) - 1.05.$$

$$C_{\text{EH}} \geq \lim_{N\to\infty} \max_{\substack{\{\mathcal{E}_i\}_{i=1}^N: \\ \mathcal{E}_i \geq 0 \ \forall i \\ \sum_{i=1}^N \mathcal{E}_i \leq E_{max}}} \sum_{i=1}^{N} p(1-p)^{i-1} \frac{1}{2} \log(1+\mathcal{E}_i) - 1.05.$$

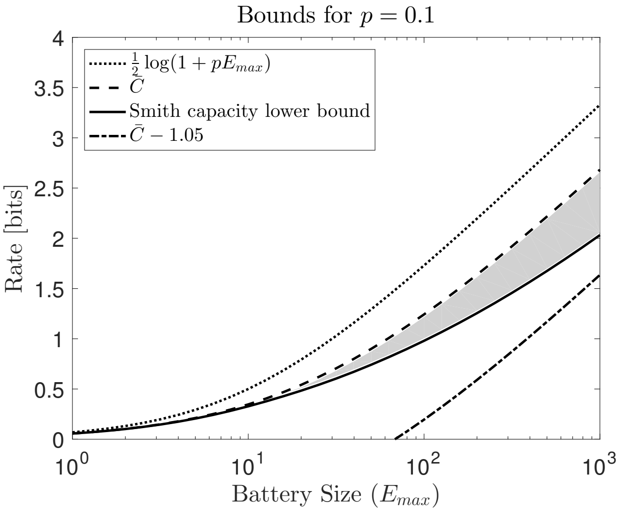
- Connection to the online throughput:

$$T - 1.05 \leq C \leq T.$$

- Bounded gap to AWGN capacity:

$$\log(1 + pE_{max}) - 1.77 \leq C_{TxRx} \leq \log(1 + pE_{max}).$$

Bounding C_{EH}



Capacity Improvement due to CSIR

Proposition

In a general channel (not necessarily stationary memoryless), capacity improvement due to receiver side information is bounded by the entropy rate of the side information itself.

For the Bernoulli case, capacity improvement is bounded by $H(p) \leq 1$.

Capacity with no receiver energy arrival information:

$$\log(1 + pE_{max}) - 2.77 \leq C_{Tx} \leq \log(1 + pE_{max}).$$

Capacity

Theorem

The capacity of the energy harvesting channel with i.i.d. energy arrivals is given by

$$C_{Tx}^{causal} = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{U^n} \in \mathcal{P}_n(b)} I(U^n; Y^n), \tag{1}$$

$$C_{TxRx}^{causal} = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{U^n} \in \mathcal{P}_n(b)} I(U^n; Y^n | E^n), \tag{2}$$

$$C_{TxRx}^{noncausal} = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{X^n|E^n} \in \mathcal{F}_n(b)} I(X^n; Y^n | E^n), \tag{3}$$

where

$$\mathcal{F}_n(b) = \Big\{ P_{X^n|E^n} \ s.t.\ \forall e^n \in \mathcal{E}^n,\ a.s.\ for\ t = 1, \ldots, n : \\ X_t^2 \leq B_t,\ B_0 = b,\ B_t = \min\{B_{t-1} - |X_{t-1}|^2 + e_t, E_{max}\} \Big\}.$$

Capacity

Theorem

The capacity of the energy harvesting channel with i.i.d. energy arrivals is given by

$$C_{TxRx}^{causal} = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{U^n} \in \mathcal{P}_n(b)} I(U^n; Y^n), \tag{1}$$

$$C_{TxRx}^{causal} = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{U^n} \in \mathcal{P}_n(b)} I(U^n; Y^n | E^n), \tag{2}$$

$$C_{TxRx}^{noncausal} = \lim_{n\to\infty} \frac{1}{n} \sup_{P_{X^n|E^n} \in \mathcal{F}_n(b)} I(X^n; Y^n | E^n), \tag{3}$$

where

$$\mathcal{P}_n(b) = \Big\{ P_{U^n} \text{ s.t. } U_t : e^t \to \mathcal{X} \text{ for } t = 1, \ldots, n \text{ and } \forall e^n \in \mathcal{E}^n \text{a.s.} : \\ |U_t(e^t)|^2 \leq B_t,\ B_0 = b,\ B_t = \min\{B_{t-1} - |U_{t-1}(e^{t-1})|^2 + e_t, E_{max}\} \Big\}.$$

Connection to the Energy Allocation Problem

Theorem

The capacities of the energy harvesting channel with various levels of energy arrival information can be bounded by

$$\begin{aligned} T^{online} - 1.05 - H(g_t(E_t)) \leq C_{Tx}^{causal} &\leq T^{online}, \\ T^{online} - 1.05 \leq C_{TxRx}^{causal} &\leq T^{online}, \\ T^{offline} - 1.05 \leq C_{TxRx}^{noncausal} &\leq T^{offline} \end{aligned}$$

where $H(g_t(E_t))$ is the entropy rate of the power control process.

Connection to the Energy Allocation Problem

Theorem

The capacities of the energy harvesting channel with various levels of energy arrival information can be bounded by

$$
\begin{aligned}
T^{online} - 1.05 - H(g_t(E_t)) \leq C^{causal}_{Tx} &\leq T^{online}, \\
T^{online} - 1.05 \leq C^{causal}_{TxRx} &\leq T^{online}, \\
T^{offline} - 1.05 \leq C^{noncausal}_{TxRx} &\leq T^{offline}
\end{aligned}
$$

where $H(g_t(E_t))$ is the entropy rate of the power control process. Also for $\eta \geq 0.7473$,

$$
\begin{aligned}
\eta\, T^{online} - H(g_t(E_t)) \leq C^{causal}_{Tx} &\leq T^{online}, \\
\eta\, T^{online} \leq C^{causal}_{TxRx} &\leq T^{online}, \\
\eta\, T^{offline} \leq C^{noncausal}_{TxRx} &\leq T^{offline}.
\end{aligned}
$$

Approximate Capacity for General i.i.d. Processes

Theorem:

The capacity of the energy harvesting channel can be approximated as

$$\begin{aligned}
\frac{1}{2}\log(1+\mu) - 3.85 \leq C_{\mathsf{Tx}}^{\mathsf{causal}} &\leq \frac{1}{2}\log(1+\mu), \\
\frac{1}{2}\log(1+\mu) - 1.77 \leq C_{\mathsf{TxRx}}^{\mathsf{causal}} &\leq \frac{1}{2}\log(1+\mu), \\
\frac{1}{2}\log(1+\mu) - 1.77 \leq C_{\mathsf{TxRx}}^{\mathsf{noncausal}} &\leq \frac{1}{2}\log(1+\mu).
\end{aligned}$$

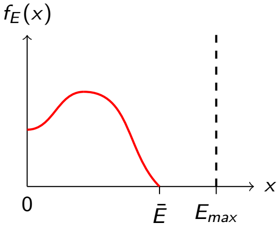
Proof: For the case where the the receiver does not have side information devise a new online power control policy which is universally near-optimal and at the same time has low entropy rate:

$$g_t = q(1-q)^j E_{max},$$

where $j = t - \max\{t' \leq t : B_t = E_{max}\}$ and $q = \mu / E_{max}$.

Insights

$$C \approx \frac{1}{2}\log(1 + \mathbb{E}[\min\{E_t, E_{max}\}])$$

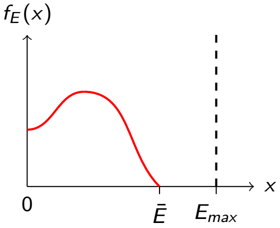


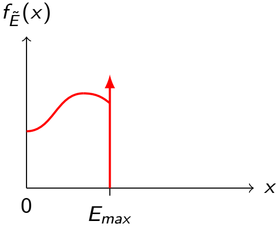
$$E_{max} > \bar{E}$$

$$C \approx \frac{1}{2}\log(1 + \mathbb{E}[E_t])$$

Insights

$$C \approx \frac{1}{2} \log(1 + \mathbb{E}[\min\{E_t, E_{max}\}])$$





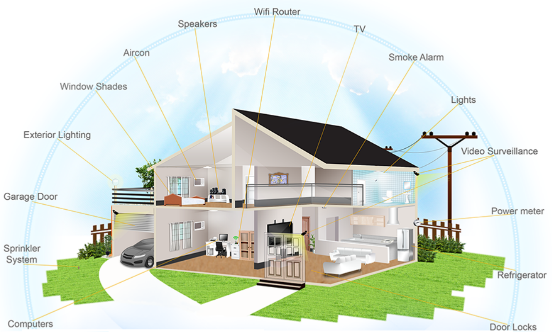
$$E_{max} > \bar{E}$$

$$E_{max} < \bar{E}$$

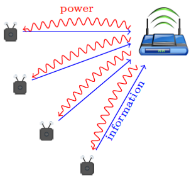
$$C \approx \frac{1}{2} \log(1 + \mathbb{E}[E_t])$$

$$C \approx \frac{1}{2} \log(1 + \mathbb{E}[\tilde{E}_t])$$

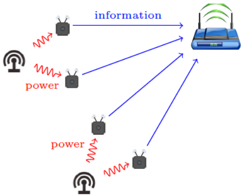
Home IoT



Wifi Router
Speakers
TV
Aircon
Smoke Alarm
Window Shades
Lights
Exterior Lighting
Video Surveillance
Garage Door
Power meter
Sprinkler System
Refrigerator
Computers
Door Locks

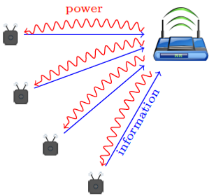


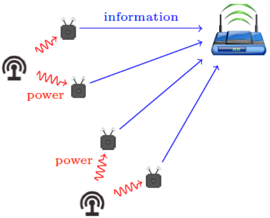
power
information



information
power
power

Two topologies for home IoT

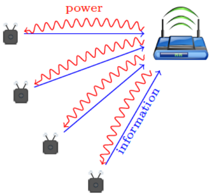


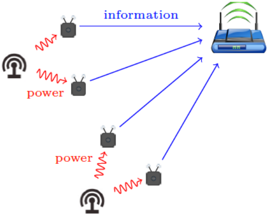


Current practice:

- Transfer energy at a constant rate.
- Periodically charge transmitter's battery.

Exploit Side Information

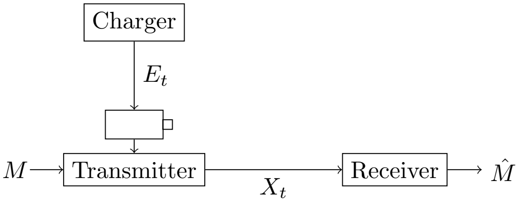




Charger observes the output of the channel.

Charger observes the input to the channel.

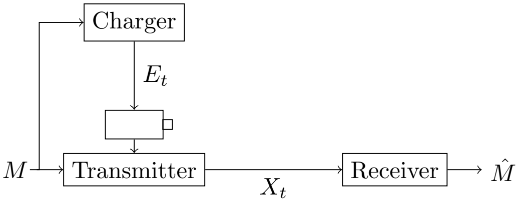
Binary Example



$$Y_t = X_t \qquad X_t \in \{0, 1\} \qquad E_t \in \{0, 1\}$$

- Charger has no side information:
  - $E_t = 1, \forall t$: $C_\emptyset = 1$ bits/channel use, $\Gamma = 1$ unit/channel use.

Binary Example



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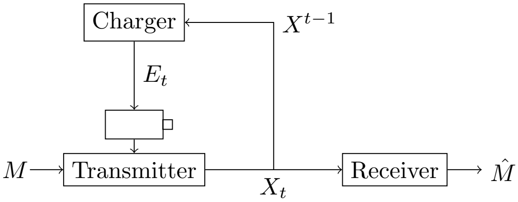
- Charger knows the message:
  - Charge when the transmitter intends to send a 1:
    $C_M = 1$ bits/channel use, $\Gamma = 1/2$ units/channel use.

Binary Example



$$Y_t = X_t \qquad X_t \in \{0,1\} \qquad E_t \in \{0,1\}$$

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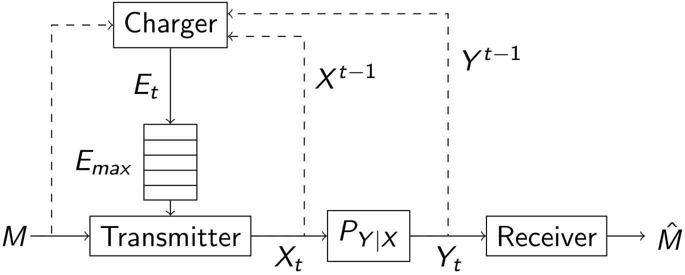
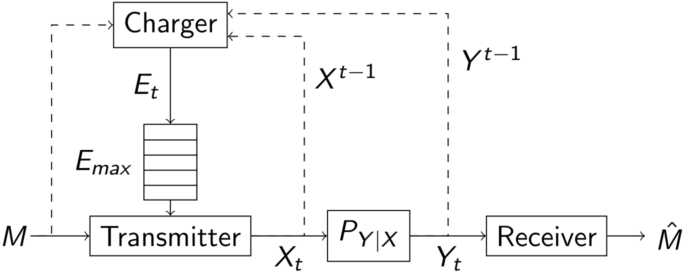
- Charger knows the message:
  - Charge when the transmitter intends to send a 1:
    $C_M = 1$ bits/channel use, $\Gamma = 1/2$ units/channel use.
- Charger can observe the transmitted signal X^{t-1}:
  - Charge when battery is empty:
    $C_X = 1$ bits/channel use, $\Gamma = 1/2$ units/channel use.

Charger Side Information



- $C_\emptyset$: Generic Charger; $f_t^{\mathrm{C}} : \emptyset \to \mathcal{E}$
- C_M: Charger and Tx connected through a backhaul link; $f_t^{\mathrm{C}} : \mathcal{M} \to \mathcal{E}$
- C_X: Charger observes the transmitted signal; $f_t^{\mathrm{C}} : \mathcal{X}^{t-1} \to \mathcal{E}$.
- C_Y: Receiver charges the transmitter; $f_t^{\mathrm{C}} : \mathcal{Y}^{t-1} \to \mathcal{E}$

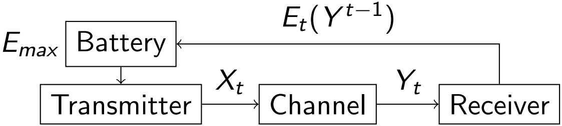
Remotely Powered Communications



- Charger: Dynamically decide how much energy to transfer to the receiver based on its side information regarding the transmission (subject to an average power constraint Γ).
- Transmitter: Dynamically adapt its transmission scheme based on its instantaneous battery level.

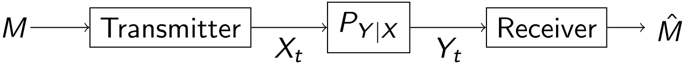
Exploiting side information at the charger can enable performance close to the centralized case.

Receiver powering transmitter



Receiver can convey both feedback information and energy with its charging actions.

Simultaneous Information and Energy Transfer



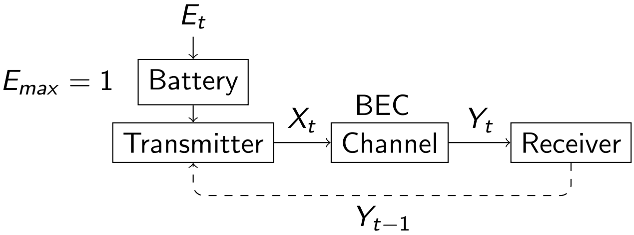
Maximize information rate under a minimum received power constraint.

$$C(P) = \max_{p(X):\mathbb{E}[b(Y)]\geq P} I(X;Y).$$

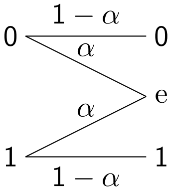
For a BSC(α),

$$C(P) = \begin{cases} 1 - h_2(\alpha), & 0 \leq P \leq 1/2 \\ h_2(P) - h_2(p), & 1/2 \leq P \leq 1-\alpha. \end{cases}$$

Can feedback increase capacity?



$\mathcal{X} = \{0, 1\} \quad \mathcal{Y} = \{0, 1, \mathrm{e}\}$



$$E_t = \begin{cases} 1 & , t \text{ odd} \\ 0 & , t \text{ even} \end{cases}$$

$$B_t = \begin{cases} 1 & , t \text{ odd} \\ 1 - X_{t-1} & , t \text{ even} \end{cases}$$

A claim by Shannon

THE ZERO ERROR CAPACITY OF A NOISY CHANNEL

Claude E. Shannon
Bell Telephone Laboratories, Murray Hill, New Jersey
Massachusetts Institute of Technology, Cambridge, Mass.

<u>Theorem 6</u>: In a memoryless discrete channel with feedback, the forward capacity is equal to the ordinary capacity C (without feedback). The average change in mutual information I_{vm} between received sequence v and message m for a letter of text is not greater than C.

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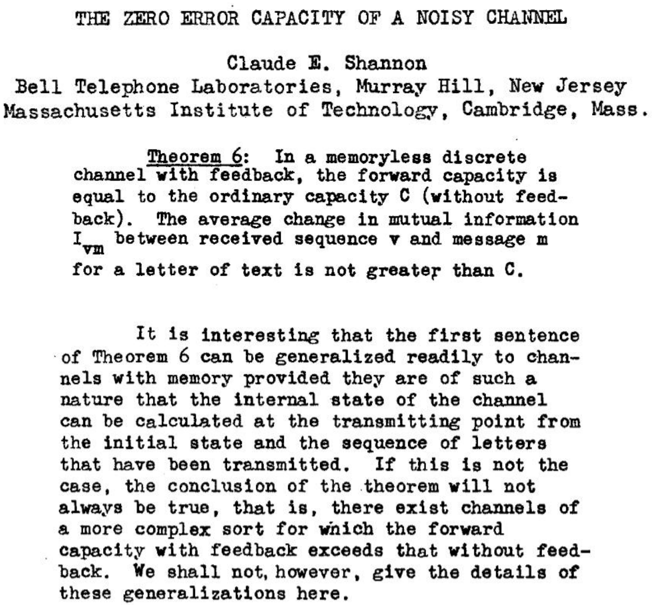
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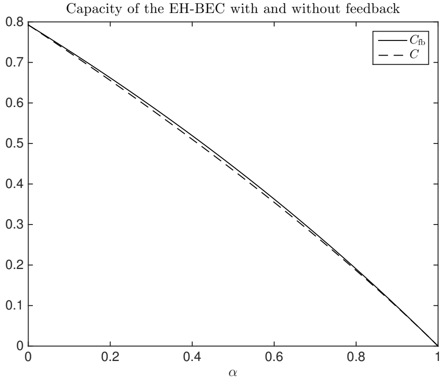
It is interesting that the first sentence of Theorem 6 can be generalized readily to channels with memory provided they are of such a nature that the internal state of the channel can be calculated at the transmitting point from the initial state and the sequence of letters that have been transmitted. If this is not the case, the conclusion of the theorem will not always be true, that is, there exist channels of a more complex sort for which the forward capacity with feedback exceeds that without feedback. We shall not, however, give the details of these generalizations here.

Feedback increases capacity



Open Questions and Directions

- Extension to more realistic energy harvesting and battery models.

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- Networking and multi-user systems.

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- Slides: Dor Shaviv.

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