Energy Harvesting and Remotely Powered Wireless Networks

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Outline of This Tutorial

- Introduction to energy harvesting (EH)
- Single-user offline power/rate optimization [Aylin]
- Single-user online power/rate optimization [Ayfer]
- Multi-user offline power optimization [Sennur]
- Multi-user online power optimization [Sennur]
- Energy cooperation (EC) and optimization [Sennur]
- Information theory of EH, infinite/zero/unit battery [Aylin]
- Information theory w/ finite battery, connections to online & offline optimization; IT of EC [Ayfer]
Prerequisites for the Tutorial

Basic command of

- Optimization
- Communication Theory

Reasonable fluency in

- Shannon Theory

Fairly self-contained otherwise
Energy Harvesting and Remotely Powered Wireless Networks- Part I

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Outline – Aylin – Part I

- Introduction to energy harvesting (EH)
- Communication theory of EH – the optimization set up
  - Short term throughput maximization for single link with finite battery
  - Transmission completion time minimization with finite battery
  - Extension to fading channels
  - Transmission policies for nodes with inefficient energy storage
Introduction

Ubiquitous Mobile / Remote Energy-limited

Wireless Communications

Energy Harvesting

Green Many sources Abundant energy

Energy Harvesting Wireless Networks
Energy Harvesting Networks

- Wireless networking with rechargeable (energy harvesting) nodes:
  - Green, self-sufficient nodes,
  - Extended network lifetime,
  - Smaller nodes with smaller batteries.
What could EH bring to communications?
Energy Harvesting Applications

- Communications satellites
- Space communications
- Deep space exploration
Wireless Energy Cooperation
Energy Harvesting Applications

Body area networks

- Heart sensor
- Motion sensor
- Personal access point
- Wearable

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Energy Harvesting Applications

MC10's biostamps for medical monitoring, powered wirelessly


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Energy Harvesting Applications

Fujitsu’s hybrid device utilizing **heat or light**.

Health tracker built at the ASSIST Center at North Carolina State University, utilizing **solar cells**

Energy Harvesting Applications

In-body (intravascular) wireless devices

Proteus Biomedical pills, powered by stomach acids

(middle) http://www.imedicalapps.com/2012/03/robotic-medical-devices-controlled-wireless-technology-nanotechnology/
(bottom) http://scitechdaily.com/smart-pills-will-track-patients-from-the-inside-out/
What is in it for us?

- New: communication theory of EH nodes
- New: information theory of EH nodes

Key new ingredient:

A set of energy feasibility constraints based on harvests govern the communication resources.
Communications

- New Wireless Network Design Challenge:
  A set of energy feasibility constraints based on harvests govern the communication resources.
- Design question:
  When and at what rate/power should a “rechargeable” (energy harvesting) node transmit?
- Optimality? Throughput; Delivery Delay
- Outcome: Optimal Transmission Schedules
Two main metrics

- **Short-Term Throughput Maximization (STTM):**
  Given a deadline, maximize the number of bits sent before the end of transmission.

- **Transmission Completion Time Minimization (TCTM):**
  Given a number of bits to send, minimize the time at which all bits have departed the transmitter.
One Energy harvesting transmitter.

Find optimal power allocation/transmission policy that departs maximum number of bits in a given duration $T$.

Energy available intermittently.

Up to a certain amount of energy can be stored by the transmitter $\Rightarrow$ BATTERY CAPACITY.
System Model

- Energy harvesting transmitter:
  - Transmitter has **backlogged data** to send by deadline $T$
  - Energy **arrives intermittently** from harvester
  - Stored in a **finite battery** of capacity $E_{max}$

---

$E_{max}$

Energy queue

Data queue

transmitter

receiver

[Diagram of system model]
System Model

- Energy arrivals of energy $E_i$ at times $s_i$

- Arrivals known non-causally by transmitter,

- Design parameter: \textbf{power} $\rightarrow$ \textbf{rate} $r(p)$.
Power-Rate Function

- Transmission with power $p$ yields a rate of $r(p)$

- Assumptions on $r(p)$:
  
  i. $r(0)=0$, $r(p) \to \infty$ as $p \to \infty$
  
  ii. increases monotonically in $p$
  
  iii. strictly concave
  
  iv. $r(p)$ continuously differentiable

Example: AWGN Channel, $r(p) = \frac{1}{2} \log \left(1 + \frac{p}{N}\right)$
Notations and Assumptions

- Power allocation function: $p(t)$
- Energy consumed: $\int_0^T p(t)\,dt$
- Short-term throughput: $\int_0^T r(p(t))\,dt$

Concave rate in power $\Rightarrow$ Given a fixed energy, a longer transmission with lower power departs more bits.
Energy Constraints

(Energy arrivals of $E_i$ at times $s_i$)

- **Energy Causality:**
  \[ \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t) dt \geq 0 \quad s_{n-1} \leq t' \leq s_n \]

- **Battery Capacity:**
  \[ \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t) dt \leq E_{\max} \quad s_{n-1} \leq t' \leq s_n \]

- **Set of energy-feasible power allocations**
  \[ \mathcal{P} = \left\{ p(t) \mid 0 \leq \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t) dt \leq E_{\max}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\} \]
Energy "Tunnel"

Energy Causality

Feasible Policy

Battery Capacity

$E_c$

$E_0$

$E_1$

$E_2$

$E_{\text{max}}$

$s_1$

$s_2$

$t$

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Optimization Problem

- Maximize total number of transmitted bits by deadline $T$

$$\max_{p(t)} \int_0^T r(p(t)) dt, \quad s.t. \quad p(t) \in \mathcal{P}$$

$$\mathcal{P} = \left\{ p(t) \mid 0 \leq \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t) dt \leq E_{max}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\}$$

- Convex constraint set, concave maximization problem
Necessary conditions for optimality of a transmission policy

- **Property 1**: Transmission power remains constant between energy arrivals.

Let the total consumed energy in epoch \([s_i, s_{i+1}]\) be \(E_{\text{total}}\) which is available at \(t = s_i\). Then the power policy

\[
p' = \frac{E_{\text{total}}}{s_{i+1} - s_i}, \quad t \in [s_i, s_{i+1}]
\]

is feasible and better than a variable power transmission; shown easily using concavity of \(r(p)\).
Property 2: Battery never overflows.

Proof:

Assume an energy of $\Delta$ overflows at time $\tau$

Define

$$p'(t) = \begin{cases} 
  p(t) + \frac{\Delta}{\delta} & [\tau - \delta, \tau] \\
  p(t) & \text{else}
\end{cases}$$

Then

$$\int_0^T r(p'(t)) dt > \int_0^T r(p(t)) dt$$

since $r(p)$ is increasing in $p$
Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

\[ \int r(p'(t))dt > \int r(p(t))dt \]

- Policy can be improved
- Policy cannot be improved
Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

\[ \int r(p'(t))dt > \int r(p(t))dt \]

**Policy can be improved**

\[ \int r(p'(t))dt < \int r(p(t))dt \]

**Policy cannot be improved**
Necessary conditions for optimality of a transmission policy

- **Property 4:** Battery is depleted at the end of transmission.

**Proof:** Assume an energy of $\Delta$ remains after $p(t)$

Define

$$p'(t) = \begin{cases} 
  p(t) + \frac{\Delta}{\delta} & [T - \delta, T] \\
  p(t) & \text{else} 
\end{cases}$$

Then

$$\int_0^T r(p'(t)) dt > \int_0^T r(p(t)) dt \quad \text{since } r(p) \text{ is increasing}$$
Implications of the properties [Tutuncuoglu-Yener'12]

- Structure of optimal policy is piece-wise linear.

\[ p(t) = \begin{cases} 
  p_n & i_{n-1} < t < i_n \\
  0 & t > T 
\end{cases}, \quad i_n \in \{s_n\}, \quad p_n \text{ constant} \]

- For power to increase or decrease, policy must meet the upper or lower boundary of the tunnel respectively.

- At termination step, battery is depleted.

- Utilizing this structure, a recursive algorithm emerges to find the unique optimum policy [Tutuncuoglu-Yener'12].
Energy “Tunnel”

Energy Causality

Optimal Policy
Feasible Policy

Battery Capacity

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Shortest Path Interpretation

- Optimal policy is identical for any concave power-rate function!
- Let \( r(p) = -\sqrt{p^2 + 1} \), then the problem solved becomes:

\[
\max_{p(t)} \int_0^T -\sqrt{p^2(t) + 1} \, dt \quad \text{s.t. } p(t) \in \mathcal{P}
\]

\[
= \min_{p(t)} \int_0^T \sqrt{p^2(t) + 1} \, dt \quad \text{s.t. } p(t) \in \mathcal{P}
\]

**length** of policy path in energy tunnel

\[\Rightarrow \text{The throughput maximizing policy yields the shortest path through the energy tunnel for any concave power-rate function.}\]
Shortest Path Interpretation

- **Property 1**: Constant power is better than any other alternative
- **Shortest path** between two points is a line (constant slope)
Alternative Solution (Using Property 1)

- Transmission power is constant within each epoch:

\[ p(t) = \{p_i, t \in \text{epoch } i, \ i = 1, \ldots, N \} \]

\[ \max_{p_i} \sum_{i=1}^{N} L_i.r(p_i) \]

\[ \text{s.t. } 0 \leq \sum_{i=1}^{n} E_i - L_i p_i \leq E_{\text{max}} \quad n = 1, \ldots, N \]

- KKT conditions \( \rightarrow \) optimum power policy.
Solution

- Complementary Slackness

\[ \lambda_n \left( \sum_{i=1}^{n} L_i p_i - E_i \right) = 0 \quad \forall n \]

\[ \mu_n \left( \sum_{i=1}^{n} E_i - L_i p_i - E_{max} \right) = 0 \quad \forall n \]

\( \lambda_n \)'s are positive only when battery is empty \( \left( \sum_{i=1}^{n} L_i p_i - E_i \right) = 0 \)

\( \mu_n \)'s only positive only when battery is full \( \left( \sum_{i=1}^{n} E_i - L_i p_i - E_{max} \right) = 0 \)

\[ p_n^* = \left[ \frac{1}{\sum_{j=n}^{N} (\lambda_j - \mu_j)} - 1 \right]^+ \]

This increases with positive \( \lambda_n \)

This decreases with positive \( \mu_n \)
Directional Water-Filling

- [Ozel, Tutuncuoglu, Ulukus, Yener’11]
- Harvested energies filled into epochs individually

Water levels ($v_i$)
Directional Water-Filling

- Harvested energies filled into epochs individually

- Constraints:
  - **Energy Causality:** water-flow only forward in time

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Water levels ($v_i$)
Directional Water-Filling

- Harvested energies filled into epochs individually

- Constraints:
  - **Energy Causality**: water-flow only forward in time
  - **Battery Capacity**: water-flow limited to $E_{\text{max}}$ by taps

\[
E_0 \quad E_1 \quad E_2
\]

Water levels ($v_i$)
Example

\[ E_0 = 2 \]

\[ E_1 = 5 \]
\[ E_2 = 1 \]
\[ E_3 = 9 \]
\[ E_4 = 7 \]

\[ E_{\text{max}} = 10 \]
Directional Water-Filling

- Energy tunnel and directional water-filling approaches yield the same policy.
Directional Water-Filling

- Energy tunnel and directional water-filling approaches yield the same policy

![Graph showing directional water-filling](image-url)
Simulation Results

- Improvement of optimal algorithm over an on-off transmitter in a simulation with truncated Gaussian arrivals.

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Fading Channels
[Ozel-Tutuncuoglu-Ulukus-Yener'11]

- **AWGN Channel with fading** $h$: $r(p, h) = \frac{1}{2} \log(1 + hp)$
- Each “epoch” defined as the interval between two “events”.

$$h = h_1 \quad h_2 = h_3 = h_4 \quad h_5 \quad h_6 = h_7 \quad h_8$$
Directional Water-Filling for Fading Channels

- Same directional water filling with base levels adjusted according to channel quality.
  - Directional water flow (Energy causality)
  - Limited water flow (Battery capacity)
Transmission Completion Time Minimization (TCTM) [Yang-Ulukus'12]

- Given the total number of bits to send as $B$, complete transmission in the shortest time possible.

\[
\min_{p(t)} T \quad \text{s.t.} \quad B - \int_0^T r(p(t)) \, dt \leq 0, \quad p(t) \in \mathcal{P}
\]

\[
\mathcal{P} = \left\{ p(t) \mid 0 \leq \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t) \, dt \leq E_{\text{max}}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\}
\]
Relationship of STTM and TCTM

- Lagrangian dual of TCTM problem becomes:

\[
\max_{u \geq 0} \left( \min_{p(t) \in \mathcal{P}, T} T + u \left( B - \int_0^T r(p(t)) \, dt \right) \right)
\]

\[
= \max_{u \geq 0} \left( \min_{T} \left( T + uB - u \max_{p(t) \in \mathcal{P}} \int_0^T r(p(t)) \, dt \right) \right)
\]

STTM problem for deadline \( T \)
Relationship of STTM and TCTM

- Optimal allocations are identical:

\[
\text{STTM’s solution for deadline } T \quad \equiv \quad \text{TCTM’s solution for departing } B \text{ bits in time } T
\]

- STTM solution can be used to solve the TCTM problem
Maximum Service Curve

\[ s(T) = \max_{p(t)} \int_0^T r(p(t)) dt, \quad \text{s.t.} \quad p(t) \in \mathcal{P} \]

- Maximum number of bits that can be sent in time \( T \).
- Each point calculated by solving the corresponding STTM problem.
Maximum Service Curve

- Continuous, monotone increasing, invertible

Maximum Departure (B)

Deadline (T)

- Optimal allocation for TCTM with $B_1$ bits

Optimal allocation for STTM with deadline $T_1$
Maximum Service Curve: Fading

- Continuous, non-decreasing
  (flat regions when fading is severe)

- Inverse can be considered as the smallest $T$ that achieves $B_1$
Transmission Policies with Inefficient Energy Storage

- Energy stored in a battery, supercapacitor, ...

- “Real life” issues:
  - [Devillers-Gunduz '12]: Leakage and Degradation
  - [Tutuncuoglu-Yener-Ulukus '15]: Storage and Retrieval Losses
Battery Degradation

- [Devillers-Gunduz '12]

**Optimal Policy:** Shortest path within *narrowing tunnel*

![Graph showing battery degradation over time](image)

$E$ vs. $t$
Battery Leakage

- [Devillers-Gunduz '12]
- **Optimal Policy:** When total energy in an epoch is low, deplete energy earlier to reduce leakage.
Storage/Recovery Losses

- [Tutuncuoglu-Yener-Ulukus ’15]

Main Tension:

- Concavity of $r(p)$: Use battery to maintain a constant power transmission
- Battery inefficiency: Storing energy in battery causes energy loss
Time slotted model

Time slots of duration $\tau = 1 \text{s}$

Energy harvests: Size $E_i$ at the beginning of time slot $i$

All arrivals known by transmitter beforehand.
System Model

Energy storage (ESD)

- $h_i$: Harvested power
- $s_i$: Stored power
- $u_i$: Retrieved (used) power
- $p_i$: Transmit power

ESD has finite capacity $E_{\text{max}}$ and storage efficiency $\eta$.

Energy Causality:

$$\sum_{n=1}^{i} \eta s_n - u_n \geq 0, \quad i = 1, \ldots, N$$

Storage Capacity:

$$\sum_{n=1}^{i} \eta s_n - u_n \leq E_{\text{max}}, \quad i = 1, \ldots, N$$
Find optimal energy storage policy that maximizes the average throughput of an energy harvesting transmitter within a deadline of $N$ time slots.

\[
\max_{\{s_i, r_i\}} \sum_{i=1}^{N} r(E_i - s_i + u_i)
\]

s.t. \( 0 \leq E_0 + \sum_{n=1}^{i} (\eta s_i - u_i) \leq E_{\text{max}}, \quad i = 1, \ldots, N, \)

\( E_i - s_i + u_i \geq 0, \quad s_i \geq 0, \quad u_i \geq 0, \quad i = 1, \ldots, N. \)
Throughput Maximization

Old problem:

\[
\max_{\{p_i\}} \sum_{i=1}^{N} r(p_i)
\]

s.t. \[
0 \leq \sum_{n=1}^{i} (E_i - p_i) \leq E_{\text{max}}, \quad i = 1, \ldots, N, \\
p_i \geq 0, \quad i = 1, \ldots, N.
\]

New problem:

\[
\max_{\{s,r\}} \sum_{i=1}^{N} r(E_i - s_i + u_i)
\]

s.t. \[
0 \leq \sum_{n=1}^{i} (\eta s_i - u_i) \leq E_{\text{max}}, \quad i = 1, \ldots, N, \\
E_i - s_i + u_i \geq 0, \quad s_i \geq 0, \quad u_i \geq 0, \quad i = 1, \ldots, N.
\]
Optimal Power Policy

Structure of optimal policy:

\[ p_i = \begin{cases} 
  \left[p_{s,i}\right]^+ & E_i \geq p_{s,i} \\
  E_i & p_{u,i} \leq E_i \leq p_{s,i} \\
  p_{u,i} & E_i \leq p_{u,i} 
\end{cases} \]

"Double Threshold Policy"
Optimal Power Policy

\[ p_i \]

\[ E_i \]

\[ p_{s,i}^* \]

\[ p_i^* \]

\[ p_{u,i}^* \]

\[ 0 \]

\[ i = 1 \]

\[ i = 2 \]

\[ i = 3 \]

\[ i = 4 \]

\[ i = 5 \]
Optimal Power Policy
(Fading channel)

\[ p_i \]

\[ E_i \]

\[ \frac{1}{h_i} \]

\[ i = 1 \]

\[ i = 2 \]

\[ i = 3 \]

\[ i = 4 \]

\[ i = 5 \]
Simulations

$N = 10^4$ time slots
$
\tau = 10 \text{ ms}
$
$E_{\text{max}} = 1 \text{ mJ}

E_0 = 0

E_i \sim \text{i.i.d. } U[0,200] \mu J

h = -100 \text{ dB}

B = 1 \text{ MHz}

N_0 = 10^{-19} \text{ W/Hz}$
References—Part I


[Devillers-Gunduz ’12]: Bertrand Devillers and Deniz Gunduz, A general framework for the optimization of energy harvesting communication systems with battery imperfections, Journal of Communications and Networks, 14(2):130-139, April 2012.
Online Power Control for Energy Harvesting Nodes

Ayfer Ö zgür

Tutorial on Energy Harvesting and Remotely Powered Communication

ISIT 2016, Barcelona, Spain
$E_t$: i.i.d. energy harvesting process, can be continuous or discrete, its realization is known ahead of time.

Power Control Problem:

$$T = \sup_{g} \liminf_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{t=1}^{n} \frac{1}{2} \log(1 + \gamma g_t) \right],$$

where $g_t : \mathcal{E}^n \to \mathbb{R}_+$, $t = 1, \ldots, n$ is a power control policy that satisfies:

- $0 \leq g_t \leq b_t$
- $b_{t+1} = \min(b_t - g_t + e_{t+1}, E_{max})$
Offline Setting

Optimal Solution:

- Ensure battery never overflows.
- Allocate energy as equally as possible over time.
Online Setting

Energy arrivals are known causally:

\[ g_t : E^t \rightarrow \mathbb{R}_+, \quad t = 1, \ldots, n \]

Easy to observe that this a Markov Decision Process:

- **state** \( b_t \)  
  - state space \([0, E_{\text{max}}]\)
- **action** \( g_t \)  
  - action space \([0, b_t]\)
- **disturbance** \( E_t \)  
  - disturbance distribution \( p(e) \) or \( f(e) \)
- **state evolution** \( b_{t+1} = \min(b_t - g_t + e_{t+1}, E_{\text{max}}) \)
- **stage reward** \( \frac{1}{2} \log(1 + \gamma g_t) \)
Markov Decision Processes

\[ s_{t+1} = f(s_t, u_t, w_t) \]

- state \( s_t \)
- state space \( S \)
- action \( u_t \)
- action space \( U(s_t) \)
- disturbance \( w_t \)
- disturbance distribution \( p(w|s, u) \)

history \( h_t = (s_1, w_1, w_2, \ldots, w_{t-1}) \)

- policy \( \pi = \{\mu_1, \mu_2, \ldots\} \)
- \( u_t = \mu_t(h_t) \)
- reward \( g(s_t, u_t) \)

Goal: maximize average reward

\[ J = \sup_{\pi} \liminf_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \mathbb{E}[g(S_t, \mu_t(H_t))] \]
A dynamic programming approach

Bellman Equation

If there exists a scalar $\lambda \in \mathbb{R}_+$ and a bounded function $h : [0, E_{\text{max}}] \rightarrow \mathbb{R}_+$ that satisfy

$$
\lambda + h(b) = \sup_{0 \leq g \leq b} \left\{ \frac{1}{2} \log(1 + \gamma g) + \mathbb{E}[h(\min\{b - g + E_t, E_{\text{max}}\})] \right\}
$$

for all $0 \leq b \leq E_{\text{max}}$, then the optimal policy is given by

$$
g_t^*(E^t) = g^*(b_t(E^t)).
$$

Limitations:

- can be computationally demanding;
- solution depends on the exact statistical model of energy arrivals;
- no insight on the structure of the optimal policy and the qualitative behavior of the resultant throughput;
Heuristic Online Policies

- Either no or only asymptotic guarantees on performance.
- Two natural heuristics widely considered: greedy policy and constant policy.

Greedy policy:
- instantenously uses all the incoming energy;
- ensures no battery overflow;
- becomes optimal when SNR → 0:

\[
\frac{1}{n} \sum_{t=1}^{n} \frac{1}{2} \log(1 + \gamma g_t) \approx \frac{\gamma}{2} \frac{1}{n} \sum_{t=1}^{n} g_t
\]
Constant Policy

- keep power allocation as constant as possible over time;

\[ g_t = \begin{cases} 
\mu = E[E_t] & \text{if } b_t \geq \mu \\
 b_t & \text{if } b_t < \mu.
\end{cases} \]

- becomes optimal when \( E_{max} \rightarrow \infty \):

\[ T = \frac{1}{2} \log(1 + \gamma \mu). \]
For finite parameter values

- these schemes can be arbitrarily away from optimality.
- asymptotic results provide no insights about the gap to optimality.
- which of the previous two policies is a better choice for a given problem?
A constant gap approach

Look for policies that are provably close to optimal across all parameter regimes and any distribution of the energy arrivals.

Universal near-optimal policies:

- have minimal dependence on the distribution of the energy arrivals, e.g. depend only on the mean.
- achieve the optimal throughput simultaneously within a constant additive and multiplicative gap for all parameter values and distributions of energy arrivals.
Degrees of Freedom

⇓

Generalized Degrees of Freedom

⇓

Constant Gap Approximations
Starting Point: Bernoulli Arrivals

First, we focus on i.i.d. Bernoulli energy arrival process:

\[ E_t = \begin{cases} 
E_{\text{max}} & \text{w.p. } p_0 \\
E_{\text{max}} & \text{w.p. } 1 - p_0 
\end{cases} \]
First, we focus on i.i.d. Bernoulli energy arrival process:

$$E_t = \begin{cases} E_{max} & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p, \end{cases}$$
Bernoulli Battery Recharges

Law of large numbers for regenerative processes:

\[
\sup_{g} \liminf_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{t=1}^{n} \frac{1}{2} \log(1 + g(t)) \right] = \sup_{g} \frac{1}{\mathbb{E}L} \mathbb{E} \left[ \sum_{t=1}^{L} \frac{1}{2} \log(1 + \gamma g(b_t)) \right]
\]

\[
= \max_{\{\mathcal{E}_i\}_{i=1}^{\infty}} \sum_{i=1}^{\infty} p(1 - p)^{i-1} \frac{1}{2} \log(1 + \mathcal{E}_i)
\]

The optimal power control policy:

\[
\mathcal{E}_i = \begin{cases} 
\frac{(N+E_{\text{max}})}{1-(1-p)^N} p(1 - p)^{i-1} - 1 & , i = 1, \ldots, N \\
0 & , i > N
\end{cases}
\]

where \( N \) is the smallest positive integer satisfying

\[
1 > (1 - p)^N [1 + p(E_{\text{max}} + N)].
\]
Bernoulli Battery Recharges

Law of large numbers for regenerative processes:

\[
\sup_{g} \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{t=1}^{n} \frac{1}{2} \log(1 + g(t)) \right] = \sup_{g} \frac{1}{L} \mathbb{E} \left[ \sum_{t=1}^{L} \frac{1}{2} \log(1 + \gamma g(b_t)) \right]
\]

\[
= \max \{ \mathcal{E}_i \}_{i=1}^{\infty} : \mathcal{E}_i \geq 0 \quad \forall i
\]
\[
\sum_{i=1}^{\infty} \mathcal{E}_i \leq E_{\text{max}}
\]

The optimal power control policy:

\[
\mathcal{E}_i = \begin{cases} 
\frac{(N+E_{\text{max}})}{1-(1-p)^N} p(1 - p)^{i-1} - 1, & i = 1, \ldots, N \\
0, & i > N
\end{cases}
\]

where \( N \) is the smallest positive integer satisfying

\[
1 > (1 - p)^N [1 + p(E_{\text{max}} + N)].
\]
Because rate is a concave function of energy/power, allocate the energy as equally as possible across time.

Use $p$ fraction of the available energy at each time slot:

$$g_t = pB_t$$
Exponentially decreasing power allocations

Because rate is a concave function of energy/power, allocate the energy as equally as possible across time.

Use $p$ fraction of the available energy at each time slot:

$$g_t = pB_t$$

$$g_t = p(1 - p)^j E_{\text{max}}$$

where $j = t - \max\{t' \leq t : E_{t'} = E_{\text{max}}\}$. 
Simplified policy for Bernoulli Arrivals

Fixed Fraction Policy:

\[ g_t = pB_t \]

Theorem

Let \( E_t \) be i.i.d Bernoulli\((p, E_{max})\) as before. The throughput \( T_{FF} \) achieved by the constant fraction policy satisfies

\[ T_{FF} \geq \frac{1}{2} \log(1 + \gamma pE_{max}) - 0.72, \]

and

\[ T_{FF} \geq \frac{1}{2} \frac{1}{2} \log(1 + \gamma pE_{max}). \]
Simulation

$E_t$ is Bernoulli($0, E_{max}$) with $p = 0.1$. 

Throughput

Gap
Simulation

Throughput

Upper Bound $\frac{1}{2} \log(1 + \gamma \mu)$

Fixed Fraction $g_t = q b_t$

Greedy $g_t = b_t$

Constant $g_t = \mu \cdot 1\{b_t \geq \mu\}$

Optimal $\Theta$

$E_t$ is Bernoulli$(0, E_{max})$ with $p = 0.9$. 

Gap

Fixed Fraction $g_t = q b_t$

Greedy $g_t = b_t$

Constant $g_t = \mu \cdot 1\{b_t \geq \mu\}$
**General i.i.d. energy arrivals**

**Fixed Fraction Policy:**

\[ g_t = qB_t, \quad \text{where} \quad q = \frac{\mu}{E_{\text{max}}} \]

**Theorem**

*The throughput \( T_{FF} \) achieved by the constant fraction policy satisfies*

\[ T_{FF} \geq \frac{1}{2} \log(1 + \gamma \mu) - 0.72, \]

*and*

\[ T_{FF} \geq \frac{1}{2} \frac{1}{2} \log(1 + \gamma \mu). \]
Proof idea

**Theorem**

\[ \text{Bernoulli}(0, E_{\text{max}}) \text{ is the worst case among all distributions with the same mean.} \]

Previous heuristics: the greedy policy and the constant policy

- build on the insights from the best case scenario: \( E_t = \mu \) deterministically.

The fixed fraction policy

- builds on the insights from the worst case scenario: Bernoulli arrivals.
Simulation

$E_t$ is Exponential($1/0.1 \ E_{\max}$).
Open Questions and Directions

- Is Bernoulli the worst case for the optimal policy?
- Non i.i.d. energy arrivals.
- Fading Channels.
- Battery Imperfections.
- Multi-user Settings (to be discussed in the next part).

Ayfer Özgür

Online Power Control

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References


Offline Multi-user Energy Harvesting Settings
Online Multi-user Energy Harvesting Settings
Energy Cooperation in Energy Harvesting Networks

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So Far, We Learned...

- Wireless nodes **harvesting energy** from nature.
- **Single-user** communication with an energy harvesting transmitter.
- Energy arrives (is harvested) **during the communication session**.
- A non-trivial shift from the conventional battery powered systems.
- Transmission policy is **adapted to energy arrivals**.
- Objective: maximize throughput, minimize delay.
Single-User Optimal Policy for $E_{max} = \infty$

- Find the **tightest curve** under the cumulative energy arrival staircase.
Single-User Optimal Policy for $E_{\text{max}} < \infty$

- Find the tightest curve in the energy feasibility tunnel.
Equivalence of Feasibility Tunnel and Directional Water-filling
Equivalence of Feasibility Tunnel and Directional Water-filling

\[ E_0, E_1, E_2, E_3, E_4, E_5, E_6 \]

\[ T \]

tightest curve

causality no overflow
Optimal Packet Scheduling: Broadcast Channel

- Energy arrives (is harvested) during the communication session
- Assume battery has infinite storage capacity: $E_{max} = \infty$
- Broadcasting data to two users by adapting to energy arrivals
- Objective: maximize the data departure region
Broadcast Channel Model: $E_{max} = \infty$

- AWGN broadcast channel:

  $$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

  where $N_1 \sim \mathcal{N}(0, \sigma_1^2), N_2 \sim \mathcal{N}(0, \sigma_2^2)$

- $\sigma_2^2 > \sigma_1^2$: 2nd user is degraded; we call 1st user stronger and 2nd user weaker
• Broadcast capacity region:

\[ r_1 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\alpha P}{\sigma_1^2} \right), \quad r_2 \leq \frac{1}{2} \log_2 \left( 1 + \frac{(1 - \alpha)P}{\alpha P + \sigma_2^2} \right) \]

• We work in the \((r_1, r_2)\) domain:

\[ P = \sigma_1^2 2^{2(r_1 + r_2)} + (\sigma_2^2 - \sigma_1^2)2^{2r_2} - \sigma_2^2 \triangleq g(r_1, r_2) \]

• \(g(r_1, r_2)\) is the minimum power required to send at rates \((r_1, r_2)\)
Finding the Maximum Departure Region

- The maximum departure region $D(T)$: union of $(B_1, B_2)$ pairs achievable by some rate allocation policy that satisfies the energy causality constraint.

- Transmission rates, and power, remain constant between energy harvests.

- The energy causality constraint reduces to constraints on $(r_{1i}, r_{2i})$:

$$\sum_{i=1}^{k} g(r_{1i}, r_{2i}) \ell_i \leq \sum_{i=0}^{k-1} E_i, \quad k = 1, \ldots, N+1$$
Finding the Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.
- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\max_{r_1, r_2} \mu_1 \sum_{i=1}^{N+1} r_{1i} \ell_i + \mu_2 \sum_{i=1}^{N+1} r_{2i} \ell_i$$

s.t. $\sum_{i=1}^{k} g(r_{1i}, r_{2i}) \ell_i \leq \sum_{i=0}^{k-1} E_i, \quad k = 1, \ldots, N + 1$
**Structure of the Optimal Policy**

- Total transmit power is the same as the single-user case.
- The power shares follow a cut-off structure.
- **Cut-off level** $P_c$

$$P_c = \left( \frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+$$

- If total power is below $P_c$, then, **only transmit to the stronger user**.
- Otherwise, **stronger user’s power share is** $P_c$.
- $P_c$ (share of the stronger user) decreases with $\mu_2$, the priority of the weaker user.
The Structure of the Optimal Policy for $E_{max} = \infty$

$$E_0 \quad E_1 \quad E_2 \quad E_3 \quad E_4 \quad E_5 \quad E_6$$

$$\sum_{j=0}^{i} E_j$$

$$P$$

$$P_c$$
The Structure of the Optimal Policy for $E_{\text{max}} < \infty$
Conclusions for the Offline Broadcasting Scenario

- Energy harvesting transmitter with infinite and finite capacity battery
- Maximize the departure region.
- Obtain the structure of the solution, such as:
  - the monotonicity of the transmit power
  - the cut-off power property
• AWGN MAC channel $Y = X_1 + X_2 + Z$, $Z \sim N(0, 1)$.

• The capacity region is a pentagon denoted as $C(P_1, P_2)$:

$$R_1 \leq f(P_1), \quad R_2 \leq f(P_2), \quad R_1 + R_2 \leq f(P_1 + P_2)$$

where $f(p) = \frac{1}{2} \log(1 + p)$. 

$E_{1i}$

$E_{2i}$

data queue

$Tx_1$

$Tx_2$

$Rx$

$C_1$

$C_2$

$C_s$

$R_1$

$R_2$
Problem Formulation

- Maximize the departure region $\mathcal{D}(T)$ by time $T$.

- Each feasible policy gives a pentagon.

- Union of all feasible policies gives $\mathcal{D}(T)$. 

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Characterizing $\mathcal{D}(T)$

- Transmission rate remains constant between energy harvests.
- For any feasible transmit power sequences $p_1, p_2$, the departure region is a pentagon

$$
B_1 \leq \sum_{n=1}^{N} f(p_{1n}) l_n
$$

$$
B_2 \leq \sum_{n=1}^{N} f(p_{2n}) l_n
$$

$$
B_1 + B_2 \leq \sum_{n=1}^{N} f(p_{1n} + p_{2n}) l_n
$$

- $\mathcal{D}(T)$ is a union of $(B_1, B_2)$ and convex.
- The boundary points maximize $\mu_1 B_1 + \mu_2 B_2$ for some $\mu_1, \mu_2 \geq 0$. 
**Point a**

- **Single-user** power allocation.

$$\max_{\mathbf{p}_1} \sum_{n} f(p_{1n})l_n$$

s.t. $$\sum_{n=1}^{j} p_{1n}l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \leq N$$
**Point b**

- User 2 power is **fixed** to its **single-user** power allocation.

\[
\begin{align*}
\max_{p_1} & \quad \sum_{n} f(p_{1n} + p_{2n}) l_n \\
\text{s.t.} & \quad \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \leq N
\end{align*}
\]
Sum-rate: Points between $c$ and $d$

- Maximize the sum-rate of the users.

\[
\begin{align*}
\max_{p_1, p_2} & \quad \sum_n f(p_{1n} + p_{2n})l_n \\
\text{s.t.} & \quad \sum_{n=1}^{j} p_{1n}l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j: 0 < j \leq N \\
& \quad \sum_{n=1}^{j} p_{2n}l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad \forall j: 0 < j \leq N
\end{align*}
\]
Sum-Rate: Points between $c$ and $d$, Equivalent Problem

- **Equivalent** problem:

  \[
  \max_{p_1+p_2} \quad \sum_{n} f(p_{1n} + p_{2n})l_n \\
  \text{s.t.} \quad \sum_{n=1}^{j} p_{1n}l_n + p_{2n}l_n \leq \sum_{n=0}^{j-1} E_{1n} + E_{2n}, \quad \forall j : 0 < j \leq N
  \]

- Power can be divided back to $p_1, p_2$ in infinite number of ways.
Points between $b$ and $c$: Arbitrary $\mu_1, \mu_2$

- Each boundary point corresponds to a corner point of some pentagon.
- $\mu_2 > \mu_1 \Rightarrow$ achieving points between point $b$ and point $c$:

\[
\begin{align*}
\max_{\mathbf{p}_1, \mathbf{p}_2} & \quad (\mu_2 - \mu_1) \sum_n f(p_{2n}) l_n + \mu_1 \sum_n f(p_{1n} + p_{2n}) l_n \\
\text{s.t.} & \quad \sum_{n=1}^j p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j: 0 < j \leq N \\
& \quad \sum_{n=1}^j p_{2n} l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad \forall j: 0 < j \leq N
\end{align*}
\]
Generalized Iterative Backward Waterfilling

- Solve the problem via generalized iterative backward waterfilling:

- Given $p_1^*$, solve for $p_2$:

$$\max_{p_2} (\mu_2 - \mu_1) \sum_{n=1}^{N} f(p_{2n})l_n + \mu_1 \sum_{n=1}^{N} f(p_{1n}^* + p_{2n})l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{2n}l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad 0 < j \leq N$$

- Once $p_2^*$ is obtained, we do a backward waterfilling for the second user.

- We perform the optimization for both users in an alternating way.

- The iterative algorithm converges to the global optimal solution.
Conclusions for the Offline Multiple Access Scenario

- Energy harvesting transmitters sending messages to a single access point.
- The problem: maximization of the departure region.
- Obtain the structure using generalized iterative waterfilling.
So far, we learned...

\[ E_0, E_1, E_2, E_3 \]

Only causally known

Cumulative energy arrivals

Cumulative energy expenditure

- So far, mostly: dynamic programming, learning algorithms, heuristics.
A Unique Approach: Online Power Scheduling

- Feel the Bernoulli.
- Steps of the approach:
  - Study Bernoulli energy arrivals with \{0, B\} support.
  - Propose a simple sub-optimal policy for Bernoulli arrivals.
  - Bound its performance.
  - Extend this sub-optimal policy for general energy arrivals.
  - Bernoulli is the worst energy arrival for the proposed algorithm.
  - Obtain a near-optimal online power policy.

![Diagram showing rate vs. B with bounds and optimal policies]
Online Policy for the Single-User Channel

- Bernoulli energy arrivals:

  \[ \mathbb{P}[E_i = B] = 1 - \mathbb{P}[E_i = 0] = p \]

- When an energy arrives, a renewal occurs.
Long-Term Average Throughput Using Renewal Theory

- Long-term average throughput, under Bernoulli energy arrivals:

\[
\lim_{n \to \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2} \log (1 + P_i) \right] = \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^{L} \frac{1}{2} \log (1 + P_i) \right]
\]

\[
= p \sum_{k=1}^{\infty} p (1 - p)^{k-1} \sum_{i=1}^{k} \frac{1}{2} \log (1 + P_i)
\]

\[
= \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} p^2 (1 - p)^{k-1} \frac{1}{2} \log (1 + P_i)
\]

\[
= \sum_{i=1}^{\infty} p (1 - p)^{i-1} \frac{1}{2} \log (1 + P_i)
\]

- \(L\) is inter-energy arrival time, geometric with \(\mathbb{E}[L] = \frac{1}{p}\).
To characterize \( \{P_i\} \) which achieves the maximum, we solve

\[
\max_{\{P_i\}} \sum_{i=1}^{\infty} p(1 - p)^{i-1} \frac{1}{2} \log (1 + P_i)
\]

s.t. \( \sum_{i=1}^{\infty} P_i \leq B \), \( P_i \geq 0, \ \forall i \)

Solution:

\[
P_i = \frac{p(1 - p)^{i-1}}{\lambda} - 1, \quad i = 1, \ldots, \tilde{N}
\]

Decreasing power for a finite duration \( \tilde{N} \) that depends on \( B \).
Online Policy for the Single-User Channel

- Bernoulli energy arrivals:
  - **Optimal** power allocation with $\tilde{N} = 4$:

- Sub-optimal **fractional** power allocation, $P_i = B p(1 - p)^{i-1}$:
Bounds on the Online Policies

- **Upper bound** from offline policy:
  \[ r \leq \frac{1}{2} \log (1 + \mu) \]

- **Lower bound** algebraically for Bernoulli arrivals:
  \[ r \geq \frac{1}{2} \log (1 + \mu) - 0.72 \]

**Sketch of the proof:**

\[
\begin{align*}
  r &= \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^{L} \frac{1}{2} \log \left( 1 + Bp(1 - p)^{i-1} \right) \right] \\
  &\geq \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^{L} \frac{1}{2} \log (1 + Bp) + \frac{1}{2} \log \left( (1 - p)^{i-1} \right) \right] \\
  &\geq \frac{1}{2} \log (1 + \mu) - 0.72
\end{align*}
\]

- Bernoulli is the worst energy arrival for the fractional policy:

\[
T_{upper} - 0.72 \leq T_{Bern} \leq T_{any} \leq T_{upper}
\]
Online Policies for the Broadcast Channel

- Bernoulli energy arrivals:

\[ \mathbb{P}[E_i = B] = 1 - \mathbb{P}[E_i = 0] = p \]

- When an energy arrives, a renewal occurs.
• Long-term weighted average throughput, under Bernoulli energy arrivals:

\[
\lim_{n \to \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^{n} (\mu_1 r_{1i} + \mu_2 r_{2i}) \right] = \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^{L} (\mu_1 r_{1i} + \mu_2 r_{2i}) \right]
\]

\[
= p \sum_{k=1}^{\infty} p(1-p)^{k-1} \sum_{i=1}^{k} (\mu_1 r_{1i} + \mu_2 r_{2i})
\]

\[
= \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} p^2 (1-p)^{k-1} (\mu_1 r_{1i} + \mu_2 r_{2i})
\]

\[
= \sum_{i=1}^{\infty} p(1-p)^{i-1} (\mu_1 r_{1i} + \mu_2 r_{2i})
\]
Resulting Optimization Problem for the Broadcast Channel

- Problem becomes

$$\max_{\{r_{1i}, r_{2i}\}} \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i} + \mu_2 r_{2i})$$

s.t. $$\sum_{i=1}^{\infty} g(r_{1i}, r_{2i}) \leq B$$
$$r_{1i}, r_{2i} \geq 0, \ \forall i$$

where

$$P_i = \sigma_1^2 e^{2(r_{1i}+r_{2i})} + (\sigma_2^2 - \sigma_1^2)e^{2r_{2i}} - \sigma_2^2 \triangleq g(r_{1i}, r_{2i})$$

- Modified offline problem:
  - **One energy** arrival.
  - **Generalized fading** due to $$p(1-p)^{i-1}$$
• User 1 is served for a time no shorter than user 2.

• Both users’ powers are decreasing.

• Cut-off level $P_c$:

$$P_c = \left( \frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+$$
Proposed Sub-optimal Policy for Bernoulli Energy Arrivals

- Sub-optimal fractional total power policy:
  - Total power per slot:
    \[ P_i = P_{1i} + P_{2i} = pb_i = Bp(1 - p)^{i-1} \]
  - Optimally divided power according to cut-off:
    \[ P_{1i} = \min\{p_c, Bp(1 - p)^{i-1}\} \]
    \[ P_{2i} = Bp(1 - p)^{i-1} - P_{1i} \]
Proposed Sub-optimal Policy for General Energy Arrivals

- Defining $q = \mu/B$.
- Total power per slot:

$$P_i = qb_i$$

- Optimally divided power according to cut-off:

$$P_{1i} = \min\{P_c, qb_i\}$$

$$P_{2i} = qb_i - P_{1i}$$
Bounds on the Online Policies

- Bernoulli energy arrivals gives a lower bound for general energy arrivals.

- **Lower** bound:

\[
\begin{align*}
    r_1 & \geq \frac{1}{2} \log \left( 1 + \frac{\alpha \mu}{\sigma_1^2} \right) - 0.72 \\
    r_2 & \geq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) - 0.99
\end{align*}
\]

- **Upper** bound:

\[
\begin{align*}
    r_1 & \leq \frac{1}{2} \log \left( 1 + \frac{\alpha \mu}{\sigma_1^2} \right) \\
    r_2 & \leq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right)
\end{align*}
\]

for some \( \alpha \in [0, 1] \), where \( \mu = \mathbb{E}[E_i] \) is the average recharge rate.
Distance between any two points with the same $\alpha$ on the upper and lower bounds is equal to:

$$\sqrt{0.72^2 + 0.99^2} = 1.22$$
Conclusions for the Online Broadcasting Scenario

- Energy harvesting transmitter with finite capacity battery
- Maximize the departure region.
- Obtain the structure of the solution, such as:
  - the monotonicity of the transmit power
  - the cut-off power property
- Near-optimal policy.
Multiple Access Channel with Common Source

- Bernoulli energy arrivals:

\[ P[E_i = B] = 1 - P[E_i = 0] = p, \text{ where } B \geq \max\{B_1, B_2\}. \]

- Average admitted energies at the two users are not the same.

- When an energy arrives, a renewal occurs.
Long-Term Weighted Average Throughput

• Long-term weighted average throughput, under Bernoulli energy arrivals:

\[
\lim_{n \to \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^{n} (\mu_1 r_{1i} + \mu_2 r_{2i}) \right] = \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^{L} (\mu_1 r_{1i} + \mu_2 r_{2i}) \right]
\]

\[
= p \sum_{k=1}^{\infty} p(1-p)^{k-1} \sum_{i=1}^{k} (\mu_1 r_{1i} + \mu_2 r_{2i})
\]

\[
= \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} p^2 (1-p)^{k-1} (\mu_1 r_{1i} + \mu_2 r_{2i})
\]

\[
= \sum_{i=1}^{\infty} p(1-p)^{i-1} (\mu_1 r_{1i} + \mu_2 r_{2i})
\]
Online Policies for the Multiple Access Channel with Common Source

- For Bernoulli energy arrivals:

\[
\max_{\{P_{1i}, P_{2i}\}} \sum_{i=1}^{\infty} p(1 - p)^{i-1} (\mu_1 r_{1i} + \mu_2 r_{2i}) \\
\text{s.t. } (r_{1i}, r_{2i}) \in C(P_{1i}, P_{2i}) \\
\sum_{i=1}^{\infty} P_{1i} \leq B_1, \sum_{i=1}^{\infty} P_{2i} \leq B_2
\]

where \(C(P_{1i}, P_{2i})\) of this channel in slot \(i\) is:

\[
r_{1i} \leq \frac{1}{2} \log \left(1 + \frac{P_{1i}}{\sigma^2}\right)
\]

\[
r_{2i} \leq \frac{1}{2} \log \left(1 + \frac{P_{2i}}{\sigma^2}\right)
\]

\[
r_{1i} + r_{2i} \leq \frac{1}{2} \log \left(1 + \frac{P_{1i} + P_{2i}}{\sigma^2}\right)
\]

- Modified offline problem:
  - One energy arrival.
  - Generalized fading due to \(p(1 - p)^{i-1}\)
Online Policies for the Multiple Access Channel with Common Source

- Achievable rate region

Each feasible policy achieves a pentagon

Rate region is the union of all such pentagons

Points $a$ and $f$ are single-user rates
Online Policies for the Multiple Access Channel with Common Source

- Achievable rate region

Each feasible policy achieves a pentagon

- Rate region is the union of all such pentagons

- Points $a$ and $f$ are single-user rates
Point \( b \)

- User 2 power is fixed to:

\[
P^*_{2i} = \frac{p(1 - p)^{i-1}}{\lambda_2} - \sigma^2, \quad i = 1, \ldots, \tilde{N}_2
\]

- Optimization problem becomes:

\[
\max_{\{P_{1i}\}} \sum_{i=1}^{\infty} p(1 - p)^{i-1} r_{1i}
\]

s.t. \( r_{1i} \in C(P_{1i}, P^*_{2i}) \), \( \sum_{i=1}^{\infty} P_{1i} \leq B_1 \)

- The optimal power:

\[
P_{1i} = \frac{p(1 - p)^{i-1}}{\lambda_1 - \nu_{1i}} - \sigma^2 - P^*_{2i}
\]

- At point \( b \), user 1 transmits for a duration no shorter than user 2.

- Power of both users are monotonically decreasing.
Sum-Rate
• $\mu_1 = \mu_2 = 1$

• The optimization problem becomes:

$$\max_{\{P_{1i}, P_{2i}\}} \frac{1}{2} \sum_{i=1}^{\infty} p(1-p)^{i-1} \log \left( 1 + \frac{P_{1i} + P_{2i}}{\sigma^2} \right)$$

s.t. \ $\sum_{i=1}^{\infty} P_{1i} \leq B_1$, \ $\sum_{i=1}^{\infty} P_{2i} \leq B_2$

• A relaxed problem:

$$\max_{\{P_{1i}, P_{2i}\}} \frac{1}{2} \sum_{i=1}^{\infty} p(1-p)^{i-1} \log \left( 1 + \frac{P_{1i} + P_{2i}}{\sigma^2} \right)$$

s.t. \ $\sum_{i=1}^{\infty} P_{1i} + P_{2i} \leq B_1 + B_2$

• Equivalent problems.
  – Use $P_{1i} = (P_{1i}^* + P_{2i}^*) \frac{B_1}{B_1 + B_2}$

• Hence, solve a single-user problem for $(P_{1i} + P_{2i})$. 
Sum-Rate

- \((P_{1i} + P_{2i})^*\) is positive for a duration \(\tilde{N}_s \geq \max\{\tilde{N}_1, \tilde{N}_2\}\)
- It is sufficient to show that:

\[
(P_{1i} + P_{2i})^* - P_{2i}^* \geq 0
\]

- Implies that the single-user power allocation is feasible
Online Policies for the Multiple Access Channel with Common Source

- Optimal capacity region with Bernoulli arrivals is a single pentagon

![Diagram of pentagon]

- Distributed sub-optimal policy, let $q_k \triangleq \frac{\bar{p}_k}{B_k}$:
  - For Bernoulli energy arrivals:
    \[
    P_{1i} = B_1 p (1 - p)^{i-1} \\
    P_{2i} = B_2 p (1 - p)^{i-1}
    \]
  - For general energy arrivals:
    \[
    P_{1i} = q_1 b_{1i} \\
    P_{2i} = q_2 b_{2i}
    \]
Bounds for the Multiple Access Channel with Common Source

- Bernoulli energy arrivals gives a lower bound for general energy arrivals.
- Lower bound:

\[
\begin{align*}
    r_1 & \geq \frac{1}{2} \log \left( 1 + \frac{\bar{P}_1}{\sigma^2} \right) - 0.72 \\
    r_2 & \geq \frac{1}{2} \log \left( 1 + \frac{\bar{P}_2}{\sigma^2} \right) - 0.72 \\
    r_1 + r_2 & \geq \frac{1}{2} \log \left( 1 + \frac{\bar{P}_1 + \bar{P}_2}{\sigma^2} \right) - 0.72
\end{align*}
\]

- Upper bound for any energy arrival:

\[
\begin{align*}
    r_1 & \leq \frac{1}{2} \log \left( 1 + \frac{\bar{P}_1}{\sigma^2} \right) \\
    r_2 & \leq \frac{1}{2} \log \left( 1 + \frac{\bar{P}_2}{\sigma^2} \right) \\
    r_1 + r_2 & \leq \frac{1}{2} \log \left( 1 + \frac{\bar{P}_1 + \bar{P}_2}{\sigma^2} \right)
\end{align*}
\]
Multiple Access Channel with General (Arbitrarily Correlated) Arrivals

- Bounds are the same for any arbitrary energy arrivals.

Using \( q_k = \frac{\bar{p}_k}{B_k} \), the lower bound is:

\[
\begin{align*}
    r_1 &\geq \frac{1}{2} \log \left( 1 + \frac{\bar{p}_1}{\sigma^2} \right) - 0.72 \\
    r_2 &\geq \frac{1}{2} \log \left( 1 + \frac{\bar{p}_2}{\sigma^2} \right) - 0.72 \\
    r_1 + r_2 &\geq \frac{1}{2} \log \left( 1 + \frac{\bar{p}_1 + \bar{p}_2}{\sigma^2} \right) - 0.72
\end{align*}
\]
Multiple Access Channel with Arbitrary Number of Users

- Bounds are the same for any arbitrary number of users.

- Using $q_k = \frac{\bar{P}_k}{\bar{B}_k}$, the lower bound is:

$$\sum_{i \in S} r_i \geq \frac{1}{2} \log \left( 1 + \frac{\sum_{i \in S} \bar{P}_i}{\sigma^2} \right) - 0.72, \quad \forall S \subset \{1, \ldots, K\}$$

- Upper bound:

$$\sum_{i \in S} r_i \leq \frac{1}{2} \log \left( 1 + \frac{\sum_{i \in S} \bar{P}_i}{\sigma^2} \right), \quad \forall S \subset \{1, \ldots, K\}$$
Multiple Access Channel with Large Number of Users

- **Sum-rate** approaches the **capacity** for very large number of users.

![Diagram showing data queues and transmission symbols](image)

- Using $q_k = \frac{\bar{P}_k}{B_k}$, the **lower bound** is:

$$\sum_{i=1}^{K} r_i \geq \frac{1}{2} \log \left( 1 + \frac{\sum_{i=1}^{K} \bar{P}_i}{\sigma^2} \right) - 0.72$$

- **Upper bound**:

$$\sum_{i=1}^{K} r_i \leq \frac{1}{2} \log \left( 1 + \frac{\sum_{i=1}^{K} \bar{P}_i}{\sigma^2} \right)$$
• Energy harvesting transmitters sending messages to a single access point.
• The problem: maximization of the departure region.
• Obtain the structure of the solution, such as:
  – Monotonicity of the power.
  – Synchronous multiple access capacity region is a pentagon.
• Near-optimal policy.
Wireless Energy Transfer

- Newly emerging technologies have enabled us to perform wireless energy transfer efficiently.
- Inductive coupling can be used to wirelessly transfer energy.
Energy Cooperation in Multi-user Energy Harvesting Communications

- Wireless energy transfer is a **new cooperation paradigm**.

- **Energy cooperation**: Nodes share their energy as well as their information.
**Gaussian Two-Hop Relay Channel with Energy Cooperation**

- Energy harvesting source and relay with deterministic energy arrivals $E_i, \bar{E}_i$.
- **Wireless energy transfer** unit that allows the source to transfer some of its energy to the relay (with $0 \leq \alpha \leq 1$ efficiency).
- Unlimited data and energy buffers at the source and the relay.
- New energy arrivals at every slot $i$, $1 \leq i \leq T$.
- The source transfers $\delta_i$ energy to the relay at slot $i$.
- Relay receives $\alpha \delta_i$ of this transferred energy at the next slot.
Optimal source/relay profile is a separable policy.

Source performs single-user throughput maximization with respect to its own energy arrivals.

Relay forwards as many of the received bits as possible, satisfying data causality and energy causality.
Two Hop Relay Channel without Energy Cooperation

- Separable policy, source maximizes its own throughput.
Two Hop Relay Channel without Energy Cooperation

- **Separable policy**, source maximizes its own throughput.
- Relay tries to send as much as it can.
Two Hop Relay Channel without Energy Cooperation

- Separable policy, source maximizes its own throughput.
- Relay tries to send as much as it can.
- 1 bit sent to destination, 2 bits remaining at the relay.
- End-to-end throughput is 1 bit.
Two Hop Relay Channel with Energy Cooperation

- Source sends less data, but some energy to assist the relay.
Two Hop Relay Channel with Energy Cooperation

- Source sends less data, but some energy to assist the relay.
- Relay uses this extra energy to forward more data.
Two Hop Relay Channel with Energy Cooperation

- Source sends less data, but some energy to assist the relay.
- Relay uses this extra energy to forward more data.
- 2 bits sent to destination, 0 bits remaining at the relay.
- End-to-end throughput is 2 bits.
End-to-end Throughput Maximization

- Maximize end-to-end throughput

\[
\max_{i=1}^{T} \frac{1}{2} \log (1 + \bar{P}_i)
\]

s.t.

\[
\sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} (E_i - \delta_i), \quad \forall k
\]

\[
\sum_{i=1}^{k} \bar{P}_i \leq \sum_{i=1}^{k} (\bar{E}_i + \alpha \delta_i), \quad \forall k
\]

\[
\sum_{i=1}^{k} \frac{1}{2} \log (1 + \bar{P}_i) \leq \sum_{i=1}^{k} \frac{1}{2} \log (1 + P_i), \quad \forall k
\]

subject to:

- Data causality at the relay node
- Energy causality at both nodes
- (Possibly) non-zero energy transfers
Gaussian Two Way Channel with Energy Cooperation

- Energy harvesting users with deterministic energy arrivals $E_i, \tilde{E}_i$

- One-way wireless energy transfer with efficiency $0 < \alpha < 1$.

- Physical layer is a Gaussian two-way channel:
  \[
  Y_1 = X_1 + X_2 + N_1 \\
  Y_2 = X_1 + X_2 + N_2
  \]

  $N_1, N_2$ are Gaussian noises with zero mean and unit power.
• Convex region, boundary is characterized by solving

\[
\max_{\tilde{P_i}, P_i, \delta_i} \sum_{i=1}^{T} \frac{1}{2} \log(1 + P_i) + \theta_2 \frac{1}{2} \log(1 + \tilde{P}_i)
\]

\[\text{s.t.} \quad (\delta, P, \tilde{P}) \in \mathcal{F}\]

• Point 1 is achieved by \(\delta = 0\): no energy transfer.
• Point 3 is achieved by \(\delta = E\): full energy transfer.
Water-filling Approach

- Generalized two-dimensional directional water-filling algorithm.
- Transfer energy from one user to another while maintaining optimal allocation in time.
- Spread the energy as much as possible in time and user dimensions.
- Now we give a numerical example for $\theta_1 = \theta_2$ and $\alpha = 1$. 
**Numerical Example**

\[ \mathbf{E} = [0, 12, 0] \text{ mJ} \quad \tilde{\mathbf{E}} = [6, 6, 0] \text{ mJ} \]
Numerical Example

\[ E = [0, 12, 0] \text{ mJ} \quad \tilde{E} = [6, 6, 0] \text{ mJ} \]
Numerical Example

\[ E = [0, 12, 0] \text{ mJ} \quad \bar{E} = [6, 6, 0] \text{ mJ} \]
**Numerical Example**

\( E = [0, 12, 0] \text{ mJ} \quad \tilde{E} = [6, 6, 0] \text{ mJ} \)
Numerical Example

\[ E = [0, 12, 0] \text{ mJ} \quad \bar{E} = [6, 6, 0] \text{ mJ} \]
Numerical Example

\[ E = [0, 12, 0] \text{ mJ} \quad \bar{E} = [6, 6, 0] \text{ mJ} \]
Numerical Example

$$E = [0, 12, 0] \text{ mJ} \quad \bar{E} = [6, 6, 0] \text{ mJ}$$
**Numerical Example**

\[ E = [0, 12, 0] \text{ mJ} \quad \bar{E} = [6, 6, 0] \text{ mJ} \]
Conclusions for Offline Energy Cooperation Scenarios

- **Energy harvesting** users with infinite capacity batteries.
- **Energy transfer capability** in an orthogonal channel in one way.
- **Energy transfer** provides a new degree of freedom to smooth out the energy profiles.
- Optimal policies identified for Gaussian two-hop relay and two-way channels.
- End-to-end throughput maximization for the two-hop relay channel.
- Capacity regions for two-way channels.
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Energy Harvesting and Remotely Powered Wireless Networks - Part II
Outline – Aylin- Part II

- Information theory of energy harvesting transmitters
- Energy harvesting AWGN channel with infinite battery
- Energy harvesting AWGN channel with no battery
- Binary noiseless energy harvesting channel
- State amplification and state masking
Information Theory of EH Transmitters

- So far, we have assumed sufficiently long time slots and utilized the known rate expressions.

- What if energy harvesting is at the symbol level, i.e., each input symbol is individually limited by EH constraints?
Energy Harvesting (EH) Channel

[Tutuncuoglu-Ozel-Ulukus-Yener’13]

- **The channel input** is restricted by an external energy harvesting process.

- **State**: available energy
  - Has memory (due to energy storage)
  - Depends on channel input
  - Causally known to Tx (causal CSIT)
Energy Harvesting (EH) Channel

\[ X_i \leq S_i \]

(Ch. input constrained by state)

\[ S_{i+1} = \min \{ S_i - X_i + E_i, E_{\text{max}} \} \]

(State has memory)

(State evolves based on ch. input)

\[ W \rightarrow \text{ENCODER} \rightarrow X_i \rightarrow \text{CHANNEL} \rightarrow Y_i \rightarrow \text{DECODER} \rightarrow \hat{W} \]

\[ P_{Y|X} \]
Energy Harvesting AWGN Channel

- Battery capacity $E_{\text{max}}$ is infinite.
- Average recharge rate: $P = E[E_i]$
- Capacity without energy harvesting: $C = \frac{1}{2} \log(1+P)$

[Ozel-Ulukus '12]
Energy Harvesting AWGN Channel

- Code symbols are constrained by the energy in the battery at each channel use, i.e.,
  \[
  \sum_{i=1}^{k} X_i^2 \leq \sum_{i=1}^{k} E_i, \quad k = 1, 2, \ldots, n.
  \]

- Conversely, the average power constraint for a non-EH AWGN channel would be a single constraint:
  \[
  \frac{1}{n} \sum_{i=1}^{n} X_i^2 \leq \frac{1}{n} \sum_{i=1}^{n} E_i \rightarrow P.
  \]

- \( C = \frac{1}{2} \log(1+P) \) is an upper bound on the capacity of the energy harvesting AWGN channel.
Achievability

- This upper bound is achievable.

- Two sources of error:
  1. Decoding error,
  2. Energy shortage.

- Idea: Design the codebook as if the channel is non-EH and show that energy shortages are insignificant.

- Two achievable schemes:
  1. Save-and-Transmit,
  2. Best-Effort-Transmit.
Save-and-Transmit

- Suppose $h(n) \in o(n)$, i.e., $h(n)/n \to 0$.

- Save energy for the first $h(n)$ channel uses, do not transmit.

- Transmit i.i.d. Gaussian signals in the remaining $n - h(n)$ channel uses.

- The energy saved during the first $h(n)$ channel uses is sufficient to guarantee no energy shortages occur in the remaining $n - h(n)$ channel uses.
Since \( h(n)/n \to 0 \), there is no loss in rate.\[
\text{Rates } < \frac{1}{2} \log(1 + P) \text{ are achievable.}
\]
Best-Effort-Transmit

- Codewords are i.i.d. Gaussian with variance $P - \varepsilon$.

- $S_i$: the energy in the battery, i.e., the battery state in the $i$th channel use.

- If $S_i \geq X_i^2$, i.e., there is enough energy in the battery, send $X_i$. Otherwise, send nothing.

- The battery state updates according to

$$S_{i+1} = S_i + E_i - X_i^2 1(S_i \geq X_i^2).$$
Best-Effort-Transmit

- With $E[X_i^2] = P - \varepsilon$ and $E[E_i] = P$, it is shown by SLLN that, finitely many energy shortages occur.

- Finitely many symbols are infeasible, i.e., the transmitter puts 0 to the channel instead of the desired code symbol finitely many times.

- Finitely many mismatches are insignificant for joint typical decoding.

- Rates $< \frac{1}{2} \log(1 + P)$ are achievable.
There is no battery at the transmitter, i.e., $E_{\text{max}} = 0$.

The code symbols are amplitude constrained:

$$X_i^2 \leq E_i, \quad i = 1, 2, \ldots, n.$$
The transmitter has causal information of energy arrivals. The receiver does not know the energy arrivals.

The harvested energy amount is one of finitely many possibilities. For simplicity, assume binary \( \{E_1, E_2\} \).

Background:
1. Static amplitude constrained AWGN channel [Smith’71]
2. State dependent channel with causal state information at the transmitter [Shannon’58]
Static Amplitude Constrained AWGN Channel [Smith’71]

- At each channel use, the code symbol is amplitude constrained by $A$.

- The channel capacity under this constraint is

$$C_{Sm}(A) = \max_{|X| \leq A} I(X; Y)$$

which is a convex program.

- The capacity achieving distribution was shown to have finitely many mass points.
State Dependent Channel with Causal State Information at the Tx [Shannon’58]

- **Channel model:** $p(y \mid x, s)$
- **State $s \in S$ is causally available at the transmitter only.**
- **The channel capacity is**

$$C_{sh} = \max_{p_T(t)} I(T; Y).$$

- **$T = [T_1, T_2, \ldots, T_{|S|}]$** is an extended channel input satisfying

$$p_{Y|T}(y \mid t) = \sum_{i=1}^{\vert S \vert} P(s = s_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-t_i)^2}{2}}.$$
Capacity of the EH AWGN Channel with No Battery

- Suppose the harvested energy is
  \[
  \begin{cases}
  E_1, & \text{w.p. } p_1 \\
  E_2, & \text{w.p. } p_2 = 1 - p_1
  \end{cases}
  \]

- Apply Shannon’s result with \( T = [T_1, T_2] \) and
  \[
  p(y | t_1, t_2) = \frac{p_1}{\sqrt{2\pi}} e^{-\frac{(y-t_1)^2}{2}} + \frac{p_2}{\sqrt{2\pi}} e^{-\frac{(y-t_2)^2}{2}}.
  \]
  \[
  t_1^2 \leq E_1 \quad \text{and} \quad t_2^2 \leq E_2
  \]

- The capacity achieving distribution is observed to have finitely many mass points.
Mass Points

- Symmetric about the origin
- Constrained to the blue line
Numerical Results
Binary Noiseless EH Channel

- Transmitting $X_i \in \{0,1\}$ requires $X_i$ units of energy
- Unit battery, $E_{\text{max}} = 1$
- Binary noiseless channel, $Y_i = X_i$

[Tutuncuoglu-Ozel-Ulukus-Yener'13]
Energy Model

- In channel use $i$, the transmitter first puts input symbol $X_i$ to the channel, and then harvests energy $E_i$:

- At the beginning of channel use $i$, battery state is $S_i$.

- State evolution: $S_{i+1} = \min\{S_i - X_i + E_i, 1\}$ (next state depends on input)

- Energy harvest: $E_i$ are i.i.d. Bernoulli with $\Pr[E_i = 1] = q_h$
A representation that simplifies the problem.

- $X_i$: # of channel uses spent waiting for energy, $\sim \text{Geometric} (q_h)$, i.i.d.
- $E_i$: # of channel uses the energy is kept in storage
- $T_i$: # of channel uses between 1s at the receiver side

\[ T_i = V_i + Z_i \]

Timing Channel

- $Z_i \in \{0,1,...\}$: # of channel uses spent waiting for energy, $\sim \text{Geometric} (q_h)$, i.i.d.
- $V_i \in \{1,2,...\}$: # of channel uses the energy is kept in storage
- $T_i \in \{1,2,...\}$: # of channel uses between 1s at the receiver side

Memoryless!
The two sets of variables, \((V^m, Z^m, T^m)\) and \((X^n, E^n, Y^n)\), are alternative representations of the same sequences.

\[
X^n = \{0,0,0,1,0,0,1,0,0,0,0,0,1,0,0\} \quad \quad V^m = \{1,2,2,2\}
\]

\[
Y^n = \{0,0,0,1,0,0,1,0,0,0,0,0,0,1,0\} \quad \quad T^m = \{4,3,2,6\}
\]

\[
E^n = \{0,0,1,0,1,0,1,0,0,0,0,0,0,1,0,0\} \quad \quad Z^m = \{3,1,0,4\}
\]

**Lemma:** The timing channel capacity with additive causally known state \(C_T\) and the originally formulated binary EH channel capacity \(C\) are equal, i.e., \(C = C_T\).
Capacity of the Timing Channel

- **[Shannon 1958]**
  Capacity of a memoryless channel with causal CSIT:
  \[
  C_{CSIT} = \max_{p(u), v(u, z)} I(U; T)
  \]

- **[Anantharam-Verdu 1996]**
  Capacity of the timing channel:
  \[
  C_T = \max_{p(x)} \frac{I(X; T)}{E[T]}
  \]

- Capacity of the timing channel with causal CSIT
  \[
  C_T = \max_{p(u), v(u, z)} \frac{I(U; T)}{E[T]} = C_{BEHC}
  \]

- **Main challenge:** selection of auxiliary variable \( U \)
  \[
  |Z|, |V| \to \infty \Rightarrow |U| \to \infty, \quad v : (U, Z) \to V
  \]
Modulo Encoding

- $U \in \{0,1,...,N-1\}$, $U \sim p_U(u)$, $V = (U - Z \mod N) + 1$
- **Binary encoding interpretation:** The encoder indexes channel uses in mod N, and sends $U_i$ by transmitting a 1 at the earliest feasible channel use with index $U_i$.
- **Achievable rate:** $R^{(N)}_A = \max_{p(u)} \frac{I(U;T)}{\mathbb{E}[V + Z]} = \max_{p(u)} \frac{H(U)}{\mathbb{E}[V] + \mathbb{E}[Z]}$
- **Example:** $N = 5$, $U_i = \{2,1,3,...\}$, $Z_i = \{2,3,1,...\}$

\[U_1 = 2, \quad U_2 = 1, \quad U_3 = 3\]
Extended Modulo Encoding

- **Choose** \[ V = \begin{cases} U - Z + 1 & U \geq Z \\ (U - Z \mod N) + 1 & U < Z \end{cases} \] \( U \in \{0,1,\ldots\} \)

- **Decoder:** \( T' = T - 1 \mod N = U \mod N \) (\( U \mod N \) decoded without error)

- **Achievable Rate:** \( R_A^{ext} = \max_N \max_u I(U;T) / E[V + Z] \)

\[ U_1 = 2 \quad U_2 = 6 \quad \text{or} \quad U_2 = 1 \]

\[ N = 5 \]

\[ T - 1 \mod N \]

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Genie Upper Bound

- Provide channel state $Z_i$ as side information at the decoder.

$$C_{UB}^{\text{genie}} = \max_{p(v)} \frac{I(V;T | Z)}{E[V + Z]} = \max_{p(v)} \frac{H(V)}{E[V] + E[Z]}$$

$$= \max_{\mu \geq 0} \frac{1}{\mu + E[Z]} \max_{E[V] \leq \mu} H(V)$$

- The entropy maximizing distribution on $V \in \{1,2,\ldots\}$ with $E[V] = \mu$ is $\text{Geometric}(1/\mu)$.

$$C_{UB}^{\text{genie}} = \max_{q_u \in [0,1]} \frac{q_h H(q_u)}{q_h + q_u (1 - q_h)}$$
Asymptotic Optimality

- **Modulo Encoding:**
  
  $$R_A^{\text{mod}} = \max_{q_u,N} \frac{H(U)}{E[V] + E[Z]}$$

- **Choose**
  
  $$N = \left\lfloor \frac{1}{q_u^*} \right\rfloor$$

  $$U \sim \text{Unif}(\{0,1,\ldots,N-1\})$$

- **Genie Upper bound:**
  
  $$C_{UB}^{\text{genie}} = \max_{q_u \in [0,1]} \frac{q_h H(q_u)}{q_h + q_u(1-q_h)}$$

\[ \Rightarrow \lim_{q_h \to 0} \frac{C_{UB}^{\text{genie}}}{R_A^{\text{mod}}} = 1 \quad \text{Modulo encoding is asymptotically optimal for low harvesting rates} \]
Timing Channel Capacity: \[ C_T = \max_{p(u), v(u, z)} \frac{I(U; T)}{E[T]} \]

\[ I(U; T) = H(T) - I(Z; T | U) \] (Mutual dependence of \(Z\) and \(T\) given \(U\))

\[ I(Z; T | U) = \sum_{t=1}^{\infty} \sum_{u} p(t, u) [H(Z) - H(Z | T = t, U = u)] \] (Entropy of \(Z\) upon observing \(T=t\) and decoding \(U=u\))

Lemma: \[ H(Z | T = t, U = u) \leq H(Z_t) \]

where \[ p_{Z_t}(z) = \begin{cases} q_h (1-q_h)^z, & \text{if } z < t \\ \frac{q_h (1-q_h)^z}{1-(1-q_h)^t}, & \text{otherwise} \end{cases} \] (Truncated geometric)
Leakage Upper Bound

\[ C_T = \max_{p(u), v(u, z)} \frac{I(U; T)}{E[T]} \]

\[ = \max_{p(u), v(u, z)} \frac{H(T) - I(Z; T | U)}{E[T]} \]

\[ C_T \leq \max_{p(t)} \frac{H(T) - \sum_{t=1}^{\infty} p(t)[H(Z) - H(Z_t)]}{E[T]} \]

- Easier to evaluate than \( C_T \) since the maximization is over \( p(t) \) instead of \( p(u), v(u, z) \)
Computing the Leakage Upper Bound

\[
C_T \leq \max_{p(t)} \frac{H(T) - \sum_{t=1}^{\infty} p(t)[H(Z) - H(Z_t)]}{\mathbb{E}[T]}
\]

\[
= \max_{\beta} \frac{1}{\beta} \max_{p(t), \mathbb{E}[T] \leq \beta} H(T) - \sum_{t=1}^{\infty} \Delta_t p(t)
\]

(Inner problem is convex)

- KKT optimality conditions give

\[
p(t) = A \exp\left( -\mu t - \Delta_t - \sum_{n=1}^{t} \gamma_n \right)
\]

\[
A = \left( \sum_{t=1}^{\infty} -\mu t - \Delta_t - \sum_{n=1}^{t} \gamma_n \right)^{-1}
\]

- Calculate UB by exhaustive search over \(\mu\) for each \(\beta\)
Interpretation of the UB

\[ C_T \leq \max_{p(t)} \frac{H(T) - \sum_{t=1}^{\infty} p(t)[H(Z) - H(Z_t)]}{E[T]} \]

\[ Ti \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad ... \]

\[ T_1 = 8 \quad T_2 = 4 \quad T_2 = 10 \]

\textbf{Revealed:} \quad Z_1 < 8 \quad Z_2 < 4 \quad Z_3 < 10

We inadvertently “waste” part of the potential rate of the channel

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Numerical Results

Upper bounds

Asymptotic optimality of timing-based encoding

Achievable rates

Energy harvest probability ($q_h$)

- Capacity with infinite storage ($C_{IS}$)
- Genie upper bound ($C_{UB}^{\text{genie}}$)
- Leakage upper bound ($C_{UB}^{\text{UB}}$)
- Extended encoding rate ($R_{A}^{\text{ext}}$)
- Modulo encoding rate ($R_{A}^{\text{mod}}$)
- Capacity with zero storage ($C_{ZS}$)
Numerical Results

![Graph showing rates and energy harvest probability]

- Leakage upper bound ($C_{UB}^{\text{leakage}}$)
- Extended encoding rate ($R_e^{\text{ext}}$)
- Modulo encoding rate ($R_e^{\text{mod}}$)
- 2nd order Markov Shannon strategy rate ($R_M^{\text{M2}}$)
- 1st order Markov Shannon strategy rate ($R_M^{\text{M1}}$)
- Optimal i.i.d. Shannon strategy rate ($R_{\text{IID}}$)

Capacity within 0.03 bits/ch.use
Binary Symmetric EH Channel

- **Binary symmetric channel:** \( \Pr[Y_i \neq X_i] = p_e \in \left[0, \frac{1}{2}\right] \), \( X_i, Y_i \in \{0,1\} \)

- The energy arrivals are **i.i.d. Bernoulli**, \( E_i \sim \text{Bernoulli}(q) \).

- Two sources of errors:
  1. **Energy shortage:** Without energy, the encoder **must send a zero**.
  2. **Channel errors:** Any bit sent can be **flipped by the channel**.
Binary Symmetric EH Channel

- Observing \( Y^n \), decoder also obtains information about \( E^n \)

- Rate of this information flow can be quantified by

\[
\Delta = \frac{1}{n} \left[ H(E^n) - H(E^n | Y^n) \right] = \frac{1}{n} I(E^n; Y^n).
\]

Randomness of energy arrival process \hspace{1cm} \text{Randomness remaining after channel output is observed}

The encoder may wish to [Tutuncuoglu-Ozel-Yener-Ulukus'14ITW]:

- **Maximize** entropy reduction rate \( \Delta \): State Amplification
  
  (Cooperative scenario) [Kim et al. '08]

- **Minimize** entropy reduction rate \( \Delta \): State Masking
  
  (Privacy or stealth scenario) [Merhav-Shamai '07]
No Battery Case
[Tutuncuoglu-Ozel-Yener-Ulukus'14ITW]

- The encoder can send $X_i = 1$ only when $E_i = 1$.
- For i.i.d. arrivals, this is a memoryless channel with CSIT.
- Capacity achieved using Shannon strategies:

  $U \in \{0,1\}^n$, $U_i = \text{Bern}(p)$, $X_i = \begin{cases} 1 & E_i = 1, U_i = 1 \\ 0 & \text{else} \end{cases}$

  Shorthand for $U = (0,0)$ and $U = (0,1)$.
No Battery Case

- **State Amplification**

  \[ R \leq H(pq * p_e) - pH(q * p_e) - (1 - p)H(p_e) \]
  \[ \Delta \leq H(q) \]
  \[ R + \Delta \leq H(pq * p_e) - H(p_e) \]

- **State Masking**

  \[ R \leq H(pq * p_e) - pH(q * p_e) - (1 - p)H(p_e) \]
  \[ \Delta \geq pH(q * p_e) - pH(p_e) \]

where \( p * q = p(1 - q) + (1 - p)q \)
Infinite Battery Case
[Tutuncuoglu-Ozel-Yener-Ulukus’14ITW]

- Capacity achieved via extending the save-and-transmit scheme [Ozel-Ulukus ‘12].

- Channel input constrained as $\mathbb{E}[X] \leq q$

\[
C = C_{BSC} = \begin{cases} 
H(q \ast p_e) - H(p_e) & q \leq \frac{1}{2} \\
1 - H(p_e) & q > \frac{1}{2}
\end{cases}
\]
Lemma: The \((R, \Delta)\) region is given by

\[ R + \Delta \leq C_{BSC}, \quad 0 \leq \Delta \leq H(q) \]

- **Achievability:** Compress part of \(E^n\) and send as a part of the message, i.e., decoder obtains \(W = (W', E^k)\)

- **Converse:** Using the Markov Chain \((W, E^n) - X^n - Y^n\)

\[
I(X^n; Y^n) \geq I(E^n, W; Y^n) \\
\geq I(E^n; Y^n) + H(W) - H(\varepsilon) - \varepsilon \log(nR) \\
= n\Delta + nR - H(\varepsilon) - \varepsilon \log(nR) \quad \rightarrow 0 \quad \text{as} \quad \varepsilon \rightarrow 0
\]
For $E_{\text{max}} = \infty$, perfect state masking is possible, i.e.,

$$(R, \Delta) = (C_{\text{BSC}}, 0)$$
is achievable.

- In the save-and-transmit scheme, channel input $X^n$ is independent of harvested energy $E^n$.
- Any rate $R < C_{\text{BSC}}$ is also achievable.
- Due to converse proof, no better rate can be achieved.
Unit-sized Battery Case

- Capacity of this channel is open as we have just seen.
- Some achievable rates proposed: [Tutuncuoglu-Ozel-Y.-Ulukus’13][Mao-Hassibi ’13].
  
  [Mao-Hassibi ’13]: Two strategies, $U_i \in \{0,1\}$

- Channel input: $X_i = \begin{cases} 1 & S_i = 1, U_i = 1 \\ 0 & \text{else} \end{cases}$

- $R_{IID} = \lim_{n \to \infty} \frac{1}{n} I(U^n; Y^n)$

- If $S_i$ was memoryless, this would be capacity achieving.
Numerical Results

State Amplification

- Noiseless channel ($p_e = 0$)
- $q = \frac{1}{2}$
Numerical Results

State Masking

- Noiseless channel \((p_e = 0)\)
  - \(q = \frac{1}{2}\)

Better state masking with timing encoding

Perfect masking for infinite-sized battery
Conclusion

- New wireless communications paradigm: energy harvesting nodes
- New design insights arising from
  - new energy constraints
  - energy storage limitations and inefficiencies
  - interaction of multiple EH transmitters
  - energy cooperation
- New problems in the information theory domain
- Lots of open problems related to all layers of the network design: e.g. Signal processing/PHY design; MAC protocol design; channel capacity...


Information-Theoretic Capacity of Energy Harvesting and Remotely Powered Systems

Ayfer Özgür

Tutorial on Energy Harvesting and Remotely Powered Communication

ISIT 2016, Barcelona, Spain
**Model**

$E_t$ : i.i.d. energy harvesting process known causally at the transmitter and not at the receiver.

State-dependent channel:
- State process has memory and is input dependent.
- State is known causally at the transmitter but not at the receiver.

\[
|X_t|^2 \leq B_t \\
B_{t+1} = \min \left( B_t - |X_t|^2 + E_{t+1}, E_{\text{max}} \right).
\]
$E_{\text{max}} = \infty$: Capacity equal to that of a classical AWGN channel with $P = \mathbb{E}[E_t]$:

$$C = \log (1 + \mathbb{E}[E_t])$$

First-order questions:

- How does the capacity of the energy harvesting AWGN channel depend on system parameters such as $E_{\text{max}}$ and $E_t$?
$E_{\text{max}} = \infty$: Capacity equal to that of a classical AWGN channel with $P = \mathbb{E}[E_t]$:

$$C = \log (1 + \mathbb{E}[E_t])$$

First-order questions:

- How does the capacity of the energy harvesting AWGN channel depend on system parameters such as $E_{\text{max}}$ and $E_t$?
- What are the properties of $E_t$ most relevant to capacity? What are more favorable and less favorable energy profiles?
$E_{\text{max}} = \infty$: Capacity equal to that of a classical AWGN channel with $P = \mathbb{E}[E_t]$: 

\[ C = \log (1 + \mathbb{E}[E_t]) \]

First-order questions:

- How does the capacity of the energy harvesting AWGN channel depend on system parameters such as $E_{\text{max}}$ and $E_t$?
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- Are there different operating regimes where the dependence to $E_{\text{max}}$ and $E_t$ is qualitatively different?
\( E_{\text{max}} = \infty \): Capacity equal to that of a classical AWGN channel with 
\[ P = \mathbb{E}[E_t]: \]

\[
C = \log (1 + \mathbb{E}[E_t])
\]

First-order questions:

- How does the capacity of the energy harvesting AWGN channel depend on system parameters such as \( E_{\text{max}} \) and \( E_t \)?
- What are the properties of \( E_t \) most relevant to capacity? What are more favorable and less favorable energy profiles?
- Are there different operating regimes where the dependence to \( E_{\text{max}} \) and \( E_t \) is qualitatively different?
- For a given \( E_t \), how can we “optimally” choose \( E_{\text{max}} \)?
$E_{\text{max}} = \infty$: Capacity equal to that of a classical AWGN channel with $P = \mathbb{E}[E_t]$:

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First-order questions:

- How does the capacity of the energy harvesting AWGN channel depend on system parameters such as $E_{\text{max}}$ and $E_t$?
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- Are there different operating regimes where the dependence to $E_{\text{max}}$ and $E_t$ is qualitatively different?
- For a given $E_t$, how can we “optimally” choose $E_{\text{max}}$?
A channel with random battery recharges

\[ E_t \]

\[ E_{max} \]

Battery

Transmitter

\[ N_t \sim \mathcal{N}(0, 1) \]

Receiver

\[ X_t \]

\[ Y_t \]

\[ |X_t|^2 \leq B_t \]

\[ B_{t+1} = \min(B_t + E_t + 1 - |X_t|^2, E_{max}) \]

We focus on i.i.d. Bernoulli energy arrival process:

\[ E_t = \begin{cases} E_{max} \\
0 \end{cases} \quad \text{w.p. } p_0 \\
1 - p_0 \]

The energy arrival process \( \{E_t\} \) is causally known both at the transmitter and the receiver.
A channel with random battery recharges

\[ |X_t|^2 \leq B_t \]

\[ B_{t+1} = \min \left( B_t + E_{t+1} - |X_t|^2, E_{\text{max}} \right) \]
A channel with random battery recharges

\[ |X_t|^2 \leq B_t \]
\[ B_{t+1} = \min \left( B_t + E_{t+1} - |X_t|^2, E_{max} \right) \]

We focus on i.i.d. Bernoulli energy arrival process:

\[ E_t = \begin{cases} E_{max} & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p, \end{cases} \]
A channel with random battery recharges

We focus on i.i.d. Bernoulli energy arrival process:

\[
E_t = \begin{cases} 
E_{\text{max}} \quad \text{w.p. } p \\
0 \quad \text{w.p. } 1 - p,
\end{cases}
\]

The energy arrival process \( \{E_t\} \) is causally known both at the transmitter and the receiver.
Results for this model

- $n$-letter expression for capacity:

$$C = \lim_{N \to \infty} \max_{p(x^N) \colon \|X^N\|_2^2 \leq E_{\text{max}}} \sum_{k=1}^{N} p^2(1 - p)^{k-1} I(X^k; X^k + Z^k)$$

- Connection to online power control:

$$T - 1.05 \leq C \leq T.$$ 

- Bounded gap to AWGN capacity:

$$\log(1 + pE_{\text{max}}) - 1.77 \leq C \leq \log(1 + pE_{\text{max}}).$$
Clipping Channel

\[ Y_j[t] = \begin{cases} 
X_j[t] + Z_j[t] , & j \leq L[t] \\
0 , & j > L[t] 
\end{cases} \]

where \( L[t] \) are i.i.d. \( \text{Geometric}(p) \)
and \( \|X[t]\|^2 \leq E_{\text{max}} \).

**Theorem**

\[ C_{EH} = p \cdot C_{clp} \]
Capacity of the Clipping Channel

\[ C_{\text{clp}} = \lim_{N \to \infty} \max_{p(x^N): \|X^N\|^2 \leq E_{\text{max}}} I(X^N; Y^L|L) \]

\[ = \lim_{N \to \infty} \max_{p(x^N): \|X^N\|^2 \leq E_{\text{max}}} \sum_{k=1}^{N} p(1-p)^{k-1} I(X^k; X^k + Z^k) \]
Bounding $C_{EH}$

\[
C_{EH} = \lim_{N \to \infty} \max_{p(x^N): \|X^N\|^2 \leq E_{\text{max}}} \sum_{k=1}^{N} p^2 (1 - p)^{k-1} I(X^k; X^k + Z^k)
\]
Bounding $C_{EH}$

$$C_{EH} = \lim_{N \to \infty} \max_{p(x^N):} \sum_{k=1}^{N} p^2 (1 - p)^{k-1} I(X^k; X^k + Z^k)$$

$$\leq \lim_{N \to \infty} \max_{p(x^N):} \sum_{k=1}^{N} p^2 (1 - p)^{k-1} \sum_{i=1}^{k} I(X_i; X_i + Z_i)$$

$$\leq \lim_{N \to \infty} \max_{p(x^N):} \sum_{k=1}^{N} p^2 (1 - p)^{k-1} \sum_{i=1}^{k} I(X_i; X_i + Z_i)$$
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$$C_{EH} = \lim_{N \to \infty} \max_{p(x^N)} \sum_{k=1}^{N} p^2 (1 - p)^{k-1} I(X^k; X^k + Z^k)$$

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$$= \lim_{N \to \infty} \max_{p(x^N)} \sum_{i=1}^{N} p(1 - p)^{i-1} I(X_i; X_i + Z_i)$$
Bounding $C_{EH}$

\[
C_{EH} = \lim_{N \to \infty} \max_{p(x^N)} \quad \sum_{k=1}^{N} p^2(1 - p)^{k-1} I(X^k; X^k + Z^k)
\]
\[
\leq \lim_{N \to \infty} \max_{p(x^N)} \quad \sum_{k=1}^{N} p^2(1 - p)^{k-1} \sum_{i=1}^{k} I(X_i; X_i + Z_i)
\]
\[
= \lim_{N \to \infty} \max_{p(x^N)} \quad \sum_{i=1}^{N} p(1 - p)^{i-1} I(X_i; X_i + Z_i)
\]
\[
= \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^{N}: \mathcal{E}_i \geq 0 \ \forall i} \quad \sum_{i=1}^{N} p(1 - p)^{i-1} \frac{1}{2} \log(1 + \mathcal{E}_i)
\]
Bounding $C_{EH}$

\[ C_{EH} \leq \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^N: \mathcal{E}_i \geq 0 \ \forall i} \sum_{i=1}^N p(1 - p)^{i-1} \frac{1}{2} \log(1 + \mathcal{E}_i) \]

\[ \sum_{i=1}^N \mathcal{E}_i \leq E_{max} \]

\[ C_{EH} \geq \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^N: \mathcal{E}_i \geq 0 \ \forall i} \sum_{i=1}^N p(1 - p)^{i-1} \max_{|X_i| \leq \sqrt{\mathcal{E}_i}} I(X_i; X_i + Z_i) \]

\[ \sum_{i=1}^N \mathcal{E}_i \leq E_{max} \]
Bounding $C_{EH}$

\[
C_{EH} \leq \lim_{N \to \infty} \max \{ \{ \varepsilon_i \}_{i=1}^N : \varepsilon_i \geq 0 \ \forall i \} \sum_{i=1}^N p(1 - p)^{i-1} \frac{1}{2} \log(1 + \varepsilon_i)
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C_{EH} \geq \lim_{N \to \infty} \max \{ \{ \varepsilon_i \}_{i=1}^N : \varepsilon_i \geq 0 \ \forall i \} \sum_{i=1}^N p(1 - p)^{i-1} \max \frac{1}{2} \log(1 + \varepsilon_i)
\]

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Bounding $C_{EH}$

\[
C_{EH} \leq \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^N: \mathcal{E}_i \geq 0 \ \forall i, \ \sum_{i=1}^N \mathcal{E}_i \leq E_{\text{max}}} \sum_{i=1}^N p(1 - p)^{i-1} \frac{1}{2} \log(1 + \mathcal{E}_i)
\]

\[
C_{EH} \geq \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^N: \mathcal{E}_i \geq 0 \ \forall i, \ \sum_{i=1}^N \mathcal{E}_i \leq E_{\text{max}}} \sum_{i=1}^N p(1 - p)^{i-1} \max_{|X_i| \leq \sqrt{\mathcal{E}_i}} I(X_i; X_i + Z_i)
\]

We can show

\[
\max_{|X_i| \leq \sqrt{\mathcal{E}}} I(X_i; X_i + Z_i) \geq \frac{1}{2} \log(1 + \mathcal{E}) - 1.05.
\]
Bounding $C_{EH}$

$$C_{EH} \leq \lim_{N \to \infty} \max_{\{\varepsilon_i\}^N_{i=1}: \varepsilon_i \geq 0 \ \forall \ i} \sum_{i=1}^{N} p(1 - p)^{i-1} \frac{1}{2} \log(1 + \varepsilon_i)$$

$$\sum_{i=1}^{N} \varepsilon_i \leq E_{\text{max}}$$

$$C_{EH} \geq \lim_{N \to \infty} \max_{\{\varepsilon_i\}^N_{i=1}: \varepsilon_i \geq 0 \ \forall \ i} \sum_{i=1}^{N} p(1 - p)^{i-1} \max_{|X_i| \leq \sqrt{\varepsilon_i}} I(X_i; X_i + Z_i)$$

$$\sum_{i=1}^{N} \varepsilon_i \leq E_{\text{max}}$$

We can show

$$\max_{|X_i| \leq \sqrt{\varepsilon}} I(X_i; X_i + Z_i) \geq \frac{1}{2} \log(1 + \varepsilon) - 1.05.$$
Bounding $C_{EH}$

$$C_{EH} \leq \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^N} \sum_{i=1}^N p(1 - p)^{i-1} \frac{1}{2} \log(1 + \mathcal{E}_i)$$

$$\sum_{i=1}^N \mathcal{E}_i \leq E_{max}$$

$$C_{EH} \geq \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^N} \sum_{i=1}^N p(1 - p)^{i-1} \max_{|X_i| \leq \sqrt{\mathcal{E}_i}} I(X_i; X_i + Z_i)$$

$$\sum_{i=1}^N \mathcal{E}_i \leq E_{max}$$

We can show

$$\max_{|X_i| \leq \sqrt{\mathcal{E}}} \log(1 + \mathcal{E}) - 1.05.$$

$$C_{EH} \geq \lim_{N \to \infty} \max_{\{\mathcal{E}_i\}_{i=1}^N} \sum_{i=1}^N p(1 - p)^{i-1} \frac{1}{2} \log(1 + \mathcal{E}_i) - 1.05.$$
Connection to the online throughput:

\[ T - 1.05 \leq C \leq T. \]

Bounded gap to AWGN capacity:

\[ \log(1 + pE_{max}) - 1.77 \leq C_{TxRx} \leq \log(1 + pE_{max}). \]
Bounding $C_{EH}$

Bounds for $p = 0.1$

- $\frac{1}{2} \log(1 + pE_{max})$
- $\bar{C}$
- Smith capacity lower bound
- $\bar{C} - 1.05$

Rate [bits] vs. Battery Size ($E_{max}$)
Capacity Improvement due to CSIR

**Proposition**

In a general channel (not necessarily stationary memoryless), capacity improvement due to receiver side information is bounded by the entropy rate of the side information itself.

For the Bernoulli case, capacity improvement is bounded by \( H(p) \leq 1 \).

Capacity with no receiver energy arrival information:

\[
\log(1 + pE_{\text{max}}) - 2.77 \leq C_{\text{Tx}} \leq \log(1 + pE_{\text{max}}).
\]
The capacity of the energy harvesting channel with i.i.d. energy arrivals is given by

\[ C_{\text{causal}}^{\text{Tx}} = \lim_{n \to \infty} \frac{1}{n} \sup_{P_{U^n} \in \mathcal{P}_n(b)} I(U^n; Y^n), \quad (1) \]

\[ C_{\text{causal}}^{\text{TxRx}} = \lim_{n \to \infty} \frac{1}{n} \sup_{P_{U^n} \in \mathcal{P}_n(b)} I(U^n; Y^n|E^n), \quad (2) \]

\[ C_{\text{noncausal}}^{\text{TxRx}} = \lim_{n \to \infty} \frac{1}{n} \sup_{P_{X^n|E^n} \in \mathcal{F}_n(b)} I(X^n; Y^n|E^n), \quad (3) \]

where

\[ \mathcal{F}_n(b) = \left\{ P_{X^n|E^n} \text{ s.t. } \forall e^n \in \mathcal{E}^n, \text{ a.s. for } t = 1, \ldots, n : \right. \]

\[ X_t^2 \leq B_t, \quad B_0 = b, \quad B_t = \min\{B_{t-1} - |X_{t-1}|^2 + e_t, E_{\text{max}}\}. \]
The capacity of the energy harvesting channel with i.i.d. energy arrivals is given by

\[
C_{\text{causal}}^{Tx} = \lim_{{n \to \infty}} \frac{1}{n} \sup_{{P_{U^n} \in \mathcal{P}_n(b)}} I(U^n; Y^n),
\]

\[
C_{\text{causal}}^{TxRx} = \lim_{{n \to \infty}} \frac{1}{n} \sup_{{P_{U^n} \in \mathcal{P}_n(b)}} I(U^n; Y^n|E^n),
\]

\[
C_{\text{noncausal}}^{TxRx} = \lim_{{n \to \infty}} \frac{1}{n} \sup_{{P_{X^n|E^n} \in \mathcal{F}_n(b)}} I(X^n; Y^n|E^n),
\]

where

\[
\mathcal{P}_n(b) = \left\{ P_{U^n} \text{ s.t. } U_t : e^t \to X \text{ for } t = 1, \ldots, n \text{ and } \forall e^n \in \mathcal{E}^n \text{ a.s. : } \right. \\
\left. |U_t(e^t)|^2 \leq B_t, \ B_0 = b, \ B_t = \min\{B_{t-1} - |U_{t-1}(e^{t-1})|^2 + e_t, E_{\text{max}}\} \right\}.
\]
Connection to the Energy Allocation Problem

Theorem

The capacities of the energy harvesting channel with various levels of energy arrival information can be bounded by

\[ T_{\text{online}} - 1.05 - H(g_t(E_t)) \leq C_{T_x}^{\text{causal}} \leq T_{\text{online}}, \]
\[ T_{\text{online}} - 1.05 \leq C_{T_{xR_x}}^{\text{causal}} \leq T_{\text{online}}, \]
\[ T_{\text{offline}} - 1.05 \leq C_{T_{xR_x}}^{\text{noncausal}} \leq T_{\text{offline}} \]

where \( H(g_t(E_t)) \) is the entropy rate of the power control process.
The capacities of the energy harvesting channel with various levels of energy arrival information can be bounded by

\[ T_{\text{online}} - 1.05 - H(g_t(E_t)) \leq C_{T_x}^{\text{causal}} \leq T_{\text{online}}, \]
\[ T_{\text{online}} - 1.05 \leq C_{T_xR_x}^{\text{causal}} \leq T_{\text{online}}, \]
\[ T_{\text{offline}} - 1.05 \leq C_{T_xR_x}^{\text{noncausal}} \leq T_{\text{offline}} \]

where \( H(g_t(E_t)) \) is the entropy rate of the power control process. Also for \( \eta \geq 0.7473 \),

\[ \eta T_{\text{online}} - H(g_t(E_t)) \leq C_{T_x}^{\text{causal}} \leq T_{\text{online}}, \]
\[ \eta T_{\text{online}} \leq C_{T_xR_x}^{\text{causal}} \leq T_{\text{online}}, \]
\[ \eta T_{\text{offline}} \leq C_{T_xR_x}^{\text{noncausal}} \leq T_{\text{offline}}. \]
Approximate Capacity for General i.i.d. Processes

**Theorem:**

The capacity of the energy harvesting channel can be approximated as

\[
\frac{1}{2} \log(1 + \mu) - 3.85 \leq C_{T_x}^{\text{causal}} \leq \frac{1}{2} \log(1 + \mu),
\]

\[
\frac{1}{2} \log(1 + \mu) - 1.77 \leq C_{T_{xR}}^{\text{causal}} \leq \frac{1}{2} \log(1 + \mu),
\]

\[
\frac{1}{2} \log(1 + \mu) - 1.77 \leq C_{T_{xR}}^{\text{noncausal}} \leq \frac{1}{2} \log(1 + \mu).
\]

**Proof:** For the case where the receiver does not have side information, devise a new online power control policy which is universally near-optimal and at the same time has low entropy rate:

\[
g_t = q(1 - q)^j E_{\max},
\]

where \( j = t - \max\{t' \leq t : B_t = E_{\max}\} \) and \( q = \mu / E_{\max} \).
Insights

\[ C \approx \frac{1}{2} \log(1 + \mathbb{E}[\min\{E_t, E_{\text{max}}\}]) \]

\[ f_E(x) \]

\[ 0 \quad \bar{E} \quad E_{\text{max}} \quad x \]

\[ E_{\text{max}} > \bar{E} \]

\[ C \approx \frac{1}{2} \log(1 + \mathbb{E}[E_t]) \]
Insights

\[ C \approx \frac{1}{2} \log(1 + \mathbb{E}[\min\{E_t, E_{\text{max}}\}]) \]

\[ f_E(x) \]

\[ f_{\tilde{E}}(x) \]

\[ E_{\text{max}} > \bar{E} \]

\[ E_{\text{max}} < \bar{E} \]

\[ C \approx \frac{1}{2} \log(1 + \mathbb{E}[E_t]) \]

\[ C \approx \frac{1}{2} \log(1 + \mathbb{E}[\tilde{E}_t]) \]
Home IoT

[Diagram of a home with various devices connected via Wi-Fi]

- Speakers
- Wifi Router
- TV
- Aircon
- Smoke Alarm
- Lights
- Window Shades
- Exterior Lighting
- Garage Door
- Sprinkler System
- Computers
- Power meter
- Refrigerator
- Door Locks

[Power and information flows shown with arrows]

[Information-Theoretic Capacity]
Two topologies for home IoT

Current practice:
- Transfer energy at a constant rate.
- Periodically charge transmitter’s battery.
Charger observes the output of the channel.

Charger observes the input to the channel.
Binary Example

\[ Y_t = X_t \quad X_t \in \{0, 1\} \quad E_t \in \{0, 1\} \]

- **Charger has no side information:**
  - \( E_t = 1, \forall t: \ C_{\emptyset} = 1 \text{ bits/channel use}, \Gamma = 1 \text{ unit/channel use}. \)
Binary Example

Y_t = X_t \quad X_t \in \{0, 1\} \quad E_t \in \{0, 1\}

- Charger has no side information:
  - \( E_t = 1, \forall t: C_\emptyset = 1 \text{ bits/channel use}, \Gamma = 1 \text{ unit/channel use.} \)
- Charger knows the message:
  - Charge when the transmitter intends to send a 1:
    - \( C_M = 1 \text{ bits/channel use}, \Gamma = 1/2 \text{ units/channel use.} \)
Binary Example

\[ Y_t = X_t \quad X_t \in \{0, 1\} \quad E_t \in \{0, 1\} \]

- Charger has no side information:
  - \( E_t = 1, \forall t: \ C_\emptyset = 1 \ \text{bits/channel use, } \Gamma = 1 \ \text{unit/channel use.} \)

- Charger knows the message:
  - Charge when the transmitter intends to send a 1:
    \( C_M = 1 \ \text{bits/channel use, } \Gamma = 1/2 \ \text{units/channel use.} \)

- Charger can observe the transmitted signal \( X^{t-1} \):
  - Charge when battery is empty:
    \( C_X = 1 \ \text{bits/channel use, } \Gamma = 1/2 \ \text{units/channel use.} \)
Charger Side Information

- $C_\emptyset$: Generic Charger; $f_t^C : \emptyset \rightarrow \mathcal{E}$
- $C_M$: Charger and Tx connected through a backhaul link; $f_t^C : \mathcal{M} \rightarrow \mathcal{E}$
- $C_X$: Charger observes the transmitted signal; $f_t^C : \mathcal{X}^{t-1} \rightarrow \mathcal{E}$
- $C_Y$: Receiver charges the transmitter; $f_t^C : \mathcal{Y}^{t-1} \rightarrow \mathcal{E}$

\[ M \rightarrow \text{Transmitter} \xrightarrow{X_t} \text{Receiver} \xrightarrow{\hat{M}} \]

\[ \text{Charger} \xrightarrow{E_t} E_{max} \]

\[ Y_t \xrightarrow{Y_t^{t-1}} \]

\[ P_{Y|X} \]

\[ X_t \xrightarrow{X_t^{t-1}} \]

\[ E \xrightarrow{E_{max}} \]
Charger: Dynamically decide how much energy to transfer to the receiver based on its side information regarding the transmission (subject to an average power constraint $\Gamma$).

Transmitter: Dynamically adapt its transmission scheme based on its instantaneous battery level.

Exploiting side information at the charger can enable performance close to the centralized case.
Receiver powering transmitter

Receiver can convey both feedback information and energy with its charging actions.
Simultaneous Information and Energy Transfer

Maximize information rate under a minimum received power constraint.

\[ C(P) = \max_{p(X) : \mathbb{E}[b(Y)] \geq P} I(X; Y). \]

For a BSC(\(\alpha\)),

\[ C(P) = \begin{cases} 
1 - h_2(\alpha), & 0 \leq P \leq 1/2 \\
h_2(P) - h_2(p), & 1/2 \leq P \leq 1 - \alpha.
\end{cases} \]
Can feedback increase capacity?

\[ E_{max} = 1 \]

Battery

Transmitter

BEC

Channel

Receiver

\[ X_t \]

\[ Y_t \]

\[ Y_{t-1} \]

\[ \mathcal{X} = \{0, 1\} \quad \mathcal{Y} = \{0, 1, e\} \]

\[ E_t = \begin{cases} 1 & , t \text{ odd} \\ 0 & , t \text{ even} \end{cases} \]

\[ B_t = \begin{cases} 1 & , t \text{ odd} \\ 1 - X_{t-1} & , t \text{ even} \end{cases} \]
THE ZERO ERROR CAPACITY OF A NOISY CHANNEL

Claude E. Shannon
Bell Telephone Laboratories, Murray Hill, New Jersey
Massachusetts Institute of Technology, Cambridge, Mass.

Theorem 6: In a memoryless discrete channel with feedback, the forward capacity is equal to the ordinary capacity $C$ (without feedback). The average change in mutual information $I_{vm}$ between received sequence $v$ and message $m$ for a letter of text is not greater than $C$. 
A claim by Shannon

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Theorem 6: In a memoryless discrete channel with feedback, the forward capacity is equal to the ordinary capacity \( C \) (without feedback). The average change in mutual information between received sequence \( v \) and message \( m \) for a letter of text is not greater than \( C \).

It is interesting that the first sentence of Theorem 6 can be generalized readily to channels with memory provided they are of such a nature that the internal state of the channel can be calculated at the transmitting point from the initial state and the sequence of letters that have been transmitted. If this is not the case, the conclusion of the theorem will not always be true, that is, there exist channels of a more complex sort for which the forward capacity with feedback exceeds that without feedback. We shall not, however, give the details of these generalizations here.
Feedback increases capacity

Capacity of the EH-BEC with and without feedback

$C_{\alpha}$

$C$

$\alpha$
Open Questions and Directions

- Extension to more realistic energy harvesting and battery models.
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- Coding and modulation techniques.
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- Extension to more realistic energy harvesting and battery models.
- Coding and modulation techniques.
- Networking and multi-user systems.
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