Energy Harvesting Wireless Communications-Part I

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Outline

- Transmission Completion Time Minimization for single link
- Short Term Throughput Maximization for single link with finite battery
- Transmission Completion Time Minimization for single link w/ finite battery
- Extension to fading channels
- Transmission policies for nodes with inefficient energy storage
- Energy harvesting receivers
- Energy harvesting multiuser networks
- Energy cooperation in energy harvesting networks
- Information theory of energy harvesting communications (introductory)

- Part2

Motivation

- Energy efficient communication means something different than it did a decade ago!

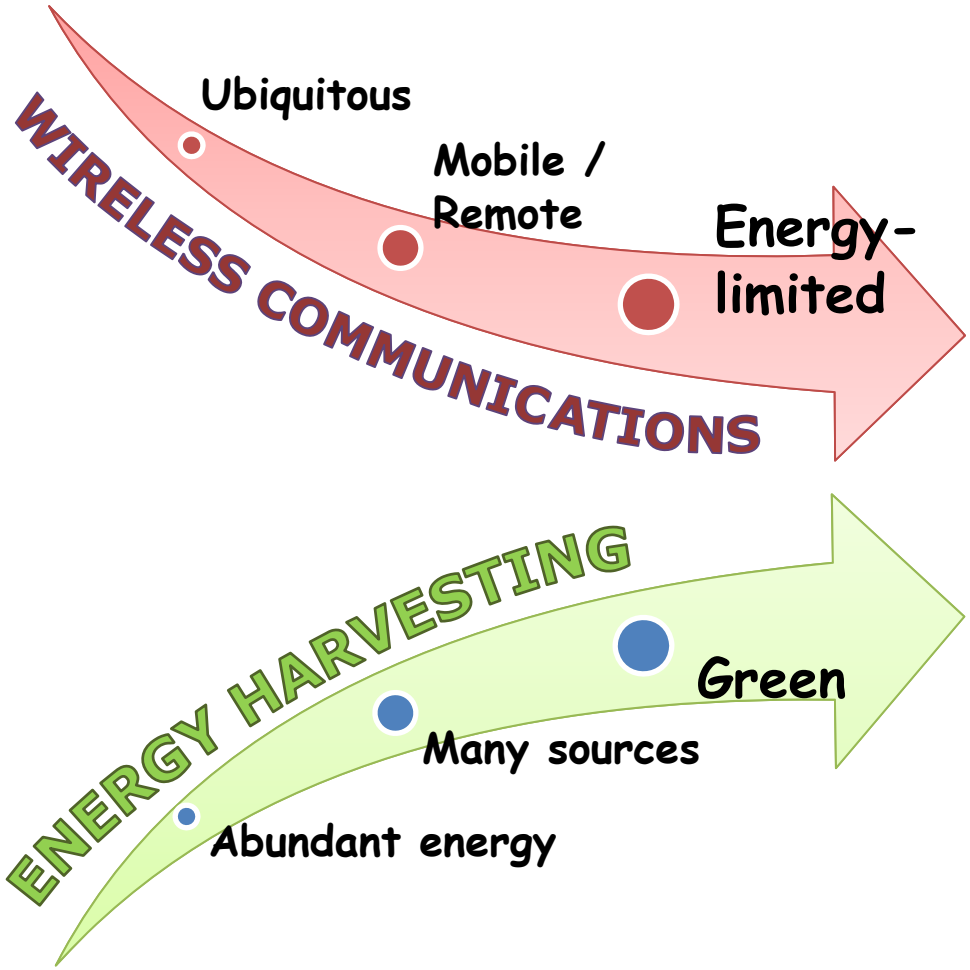
- From a communication network design perspective do rechargeable/energy harvesting networks bring?

- Communication with energy harvesting nodes:
  - green, self-sufficient nodes with extended network lifetime
  - relatively new field with increasing interest

Prerequisites for the Tutorial

- Optimization (Basic)
- Communication Theory (Basic)
- Fairly self-contained otherwise

Introduction



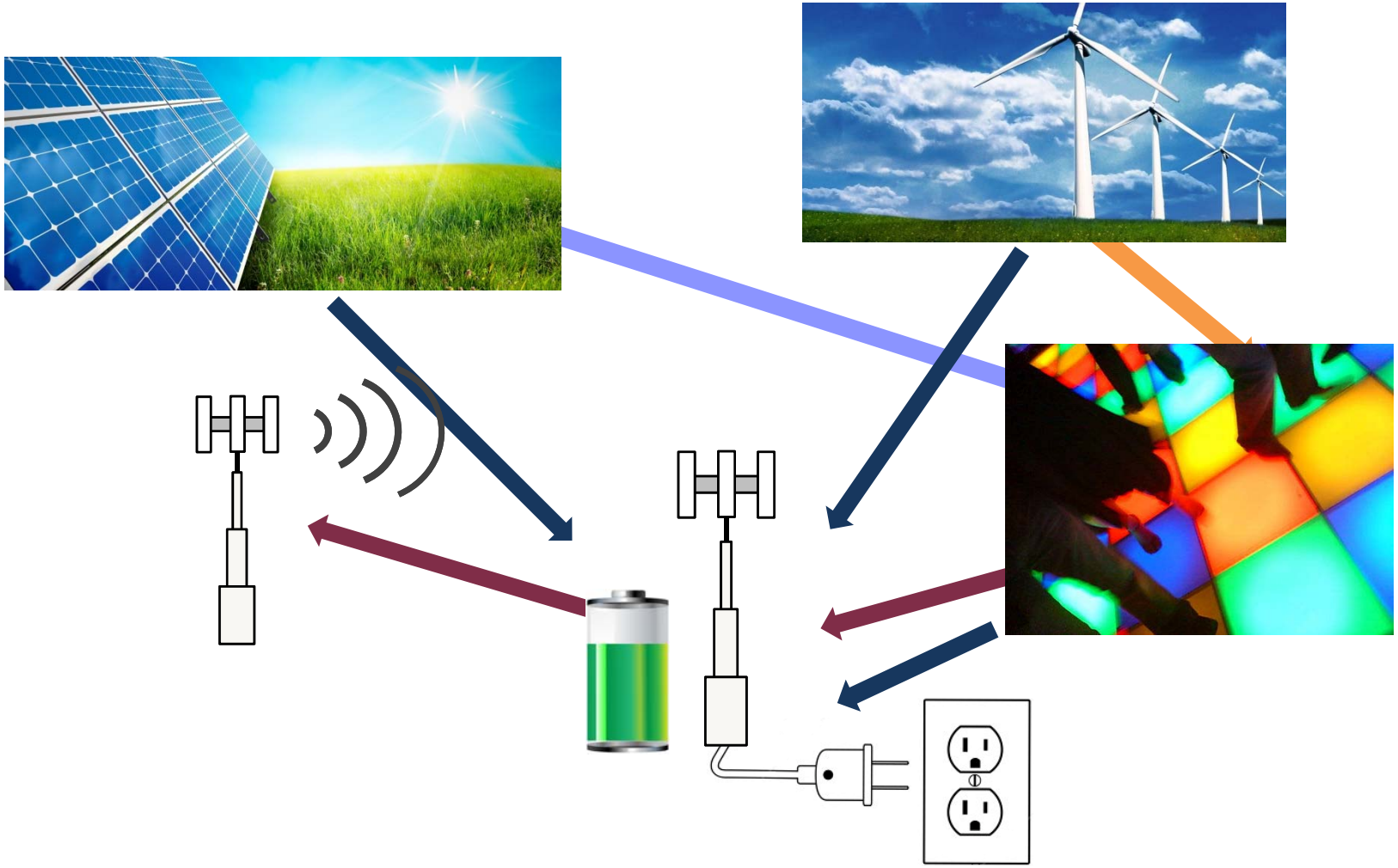
Energy Harvesting Wireless Networks

Energy Harvesting Networks

- **Wireless networking with rechargeable (energy harvesting) nodes:**
  - Green, self-sufficient nodes,
  - Extended network lifetime,
  - Smaller nodes with smaller batteries.

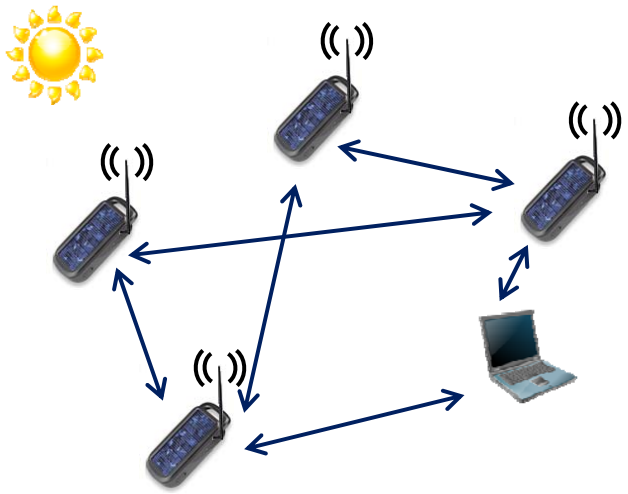
A relatively new field with increasing interest.

Introduction



Energy Harvesting Applications

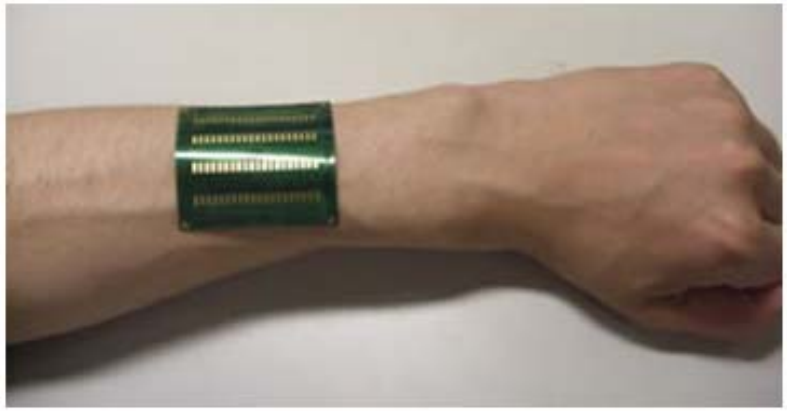
Wireless sensor networks

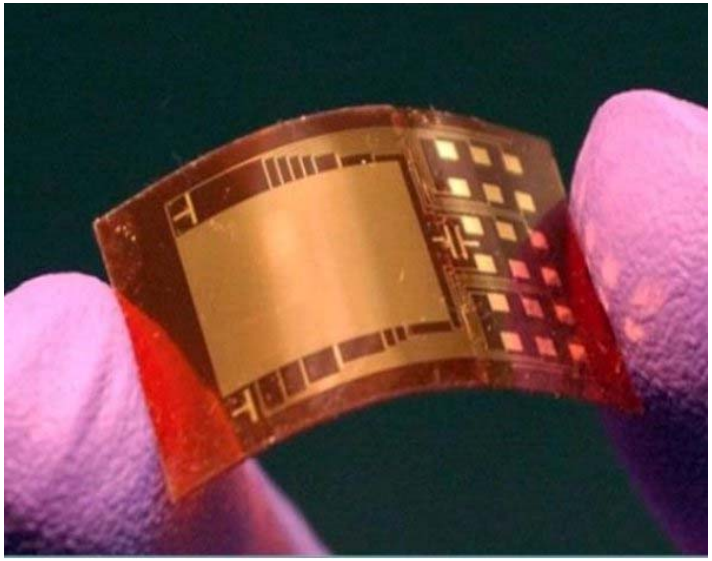


Green communications

Energy Harvesting Applications

Fujitsu's hybrid device utilizing **heat or light.**

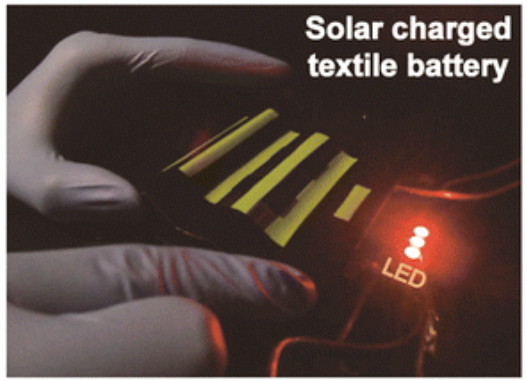


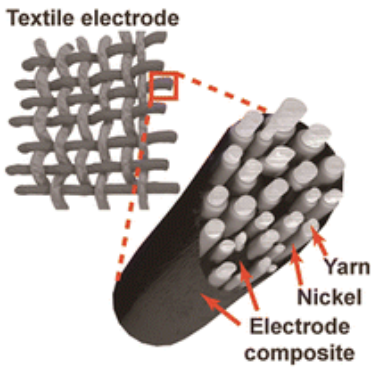


Nanogenerators built at Georgia Tech, utilizing **strain**

Image Credits: (top) http://www.fujitsu.com/global/news/pr/archives/month/2010/20101209-01.html
(bottom) http://www.zeitnews.org/nanotechnology/squeeze-power-first-practical-nanogenerator-developed.html

Energy Harvesting Applications





KAIST's Solar charged textile battery

MC10's **biostamps** for medical monitoring, powered **wirelessly**

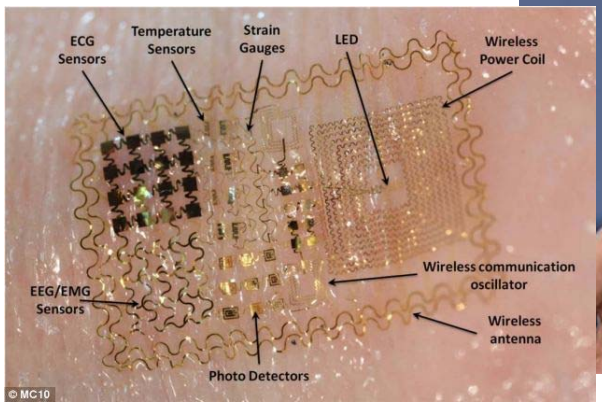
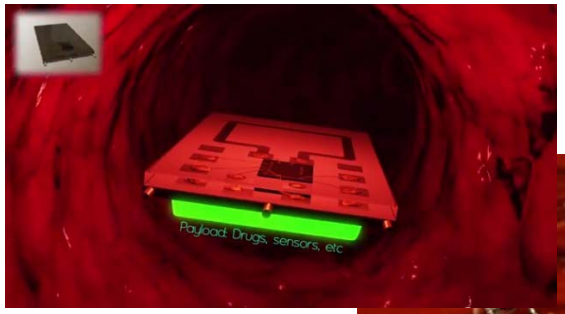


Image Credits: (top) http://pubs.acs.org/doi/abs/10.1021/nl403860k#aff1 (bottom))
http://www.dailymail.co.uk/sciencetech/article-2333203/Moto-X-Motorola-reveals-plans-ink-pills-replace-ALL-passwords.html

Energy Harvesting Applications



In-body (intravascular) wireless devices

Proteus Biomedical pills, powered by **stomach acids**

Image Credits: (top) http://www.extremetech.com/extreme/119477-stanford-creates-wireless-implantable-innerspace-medical-device (middle) http://www.imedicalapps.com/2012/03/robotic-medical-devices-controlled-wireless-technology-nanotechnology/ (bottom) http://scitechdaily.com/smart-pills-will-track-patients-from-the-inside-out/

Motivation

- **New Wireless Network Design Challenge:**

  A **set of energy feasibility constraints** based on harvests govern the communication resources.

- **Design question:**

  When and at what rate/power should a "rechargeable" (energy harvesting) node transmit?

- **Optimality? Throughput; Delivery Delay**
- **Outcome: Optimal Transmission Schedules**

Two Main Goals of Transmission Scheduling

- **Transmission Completion Time Minimization (TCTM):**

  Given a number of bits to send, minimize the time at which all bits have departed the transmitter.

- **Short-Term Throughput Maximization (STTM):**

  Given a deadline, maximize the number of bits sent before the end of transmission.

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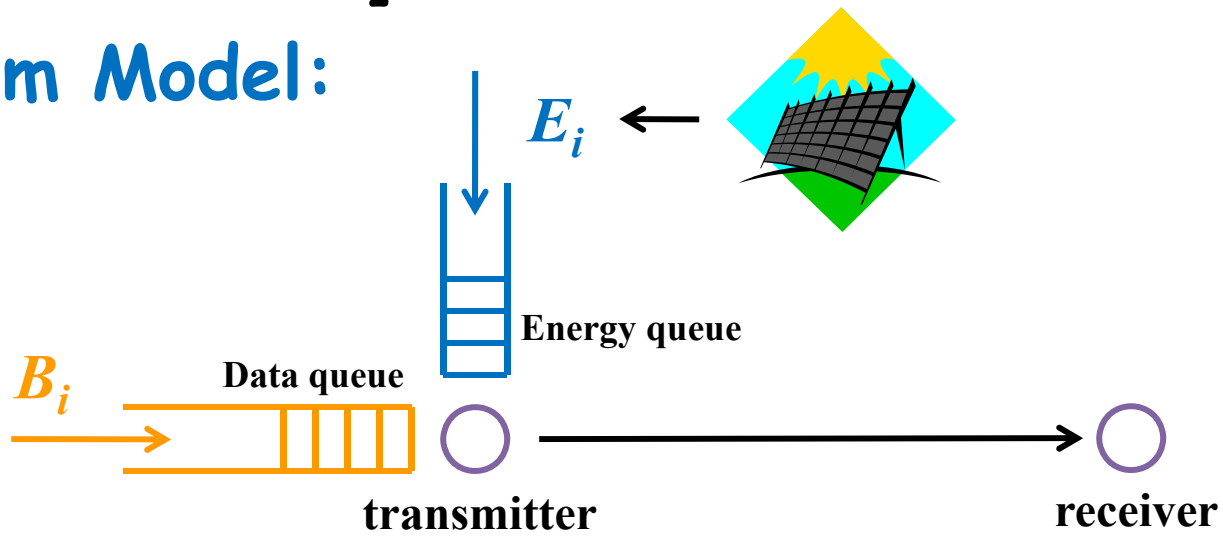
TCTM for Single Link

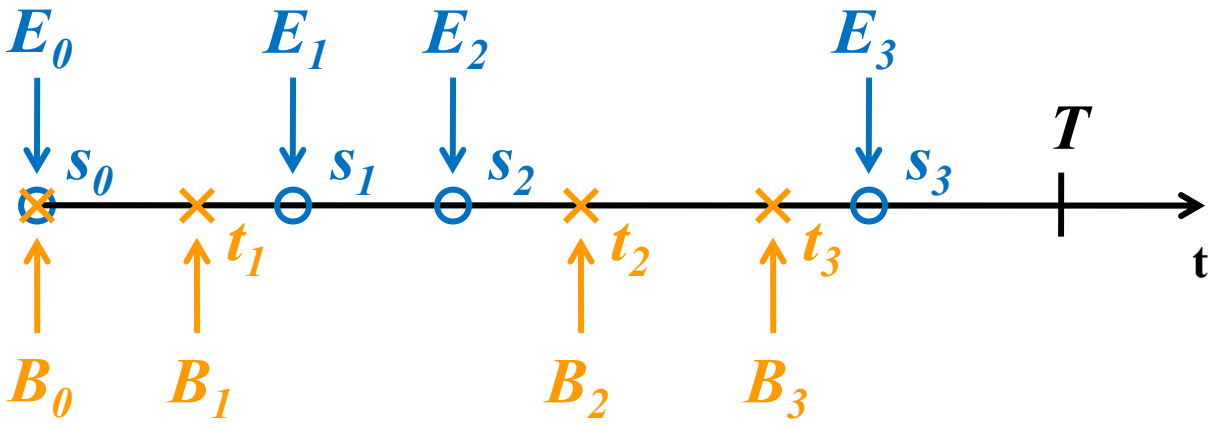
- [Yang-Ulukus '12]
- System Model:



- Energy harvesting transmitter
- Energy and data arrivals to transmitter
- Transmitting with power p achieves rate $r(p)$

TCTM for Single Link

System Model:



- **Energy harvests:** Size E_i at time t_i
- **Data packet arrivals:** Size B_i at time s_i

All arrivals known by transmitter beforehand.

TCTM for Single Link

- **Problem:**

  Find optimal transmission power/rate policy that minimizes **transmission time** for a known amount of arriving packets.

- **Constraints:**

  Cannot use energy not harvested yet.

  Cannot transmit packets not received yet.

Power-Rate Function

- Transmission with power ***p*** yields a rate of ***r(p)***

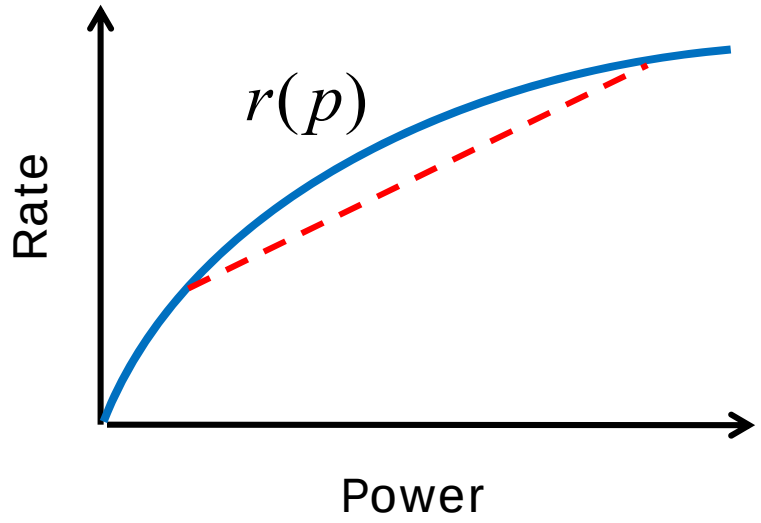
- **Assumptions on r(p):**

  i. $r(0)=0,\ r(p) \to \infty \ as\ p \to \infty$

  ii. increases monotonically in p

  iii. **strictly concave**

  iv. $r(p)$ continuously differentiable

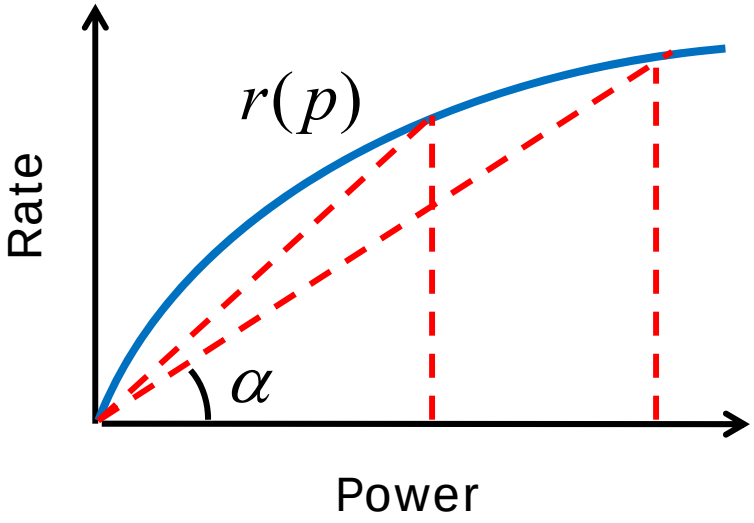


Example: AWGN Channel, $r(P) = \frac{1}{2}\log(1+\frac{P}{N})$

Power-Rate Function

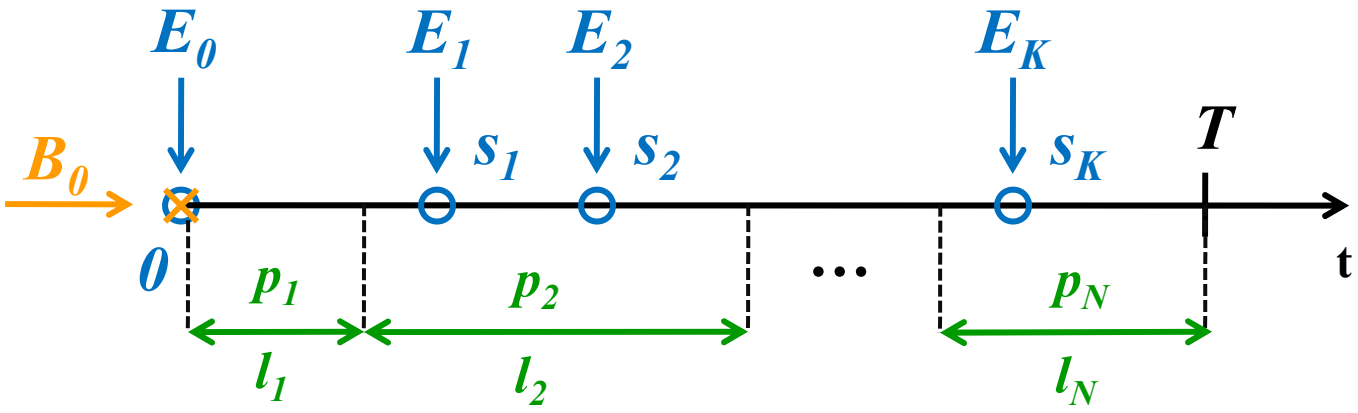
- $r(p)$ strictly concave, increasing, $r(0)=0$ implies

$$\tan(\alpha) = \frac{r(p)}{p} \text{ is monotonically decreasing in } p$$



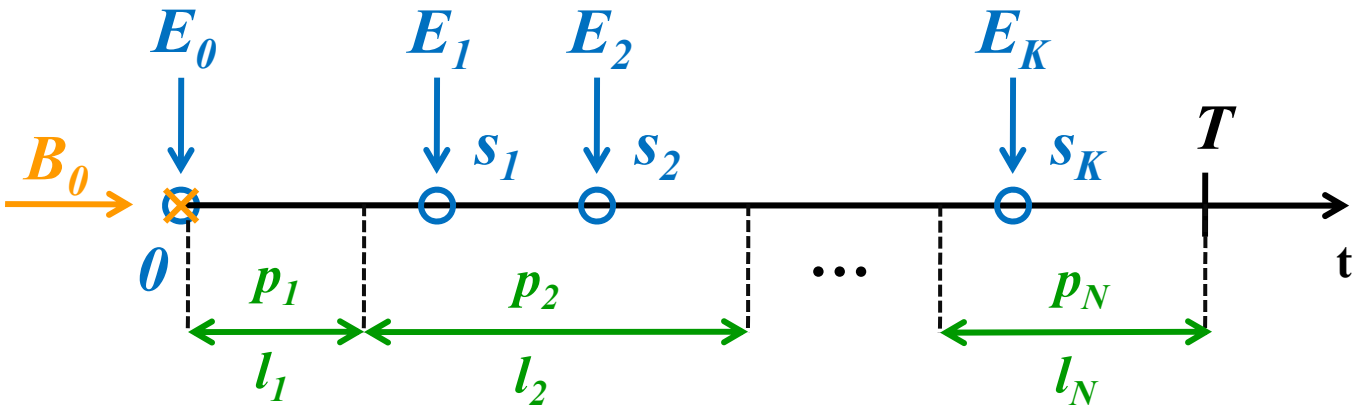
- Given a fixed energy, a longer transmission with lower power departs more bits (Lazy Scheduling).
- Also, $r^{-1}(p)$ exists and is strictly convex

Scenario I: Packets Ready before Transmission



- **Transmission structure:** Power p_i for duration l_i
- **Expended Energy:** $$E(t)=\sum_{i=1}^{\bar{i}} p_i l_i + p_{i+1}\left(t-\sum_{i=1}^{\bar{i}} l_i\right), \quad \bar{i}=\max\left\{ i : \sum_{j=1}^{i} l_j \le t \right\}$$
- **Departed bits:** $$B(t)=\sum_{i=1}^{\bar{i}} r(p_i) l_i + r(p_{i+1})\left(t-\sum_{i=1}^{\bar{i}} l_i\right)$$

Scenario I: Packets Ready before Transmission



- **Problem Definition:** $\min T$

$$\text{s.t.} \quad E(t) \le \sum_{i:s_i<t} E_i \qquad 0 \le t \le T$$

$$B(T) = B_0$$

Necessary conditions for optimality

- **Lemma 1:** Transmit powers increase monotonically,

  i.e., $p_1 < p_2 < \ldots < p_N$

  Proof: (by contradiction) assume not, i.e., $p_i > p_{i+1}$ for some i

  Energy consumed in l_i and l_{i+1} is $p_i l_i + p_{i+1} l_{i+1}$

  Consider the following constant power policy:

$$p_i' = p_{i+1}' = \frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}$$

  which does not violate energy constraint since $p_i' < p_i$

Necessary conditions for optimality

- **Lemma 1:** Transmit powers increase monotonically,

  i.e., $p_1 < p_2 < \ldots < p_N$

  Proof(cont'd): Transmitted bits then become

$$r_i' \cdot l_i + r_{i+1}' l_{i+1} = r\left(\frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}\right)(l_i + l_{i+1})$$

$$> r(p_i)\frac{l_i}{l_i + l_{i+1}}(l_i + l_{i+1}) + r(p_{i+1})\frac{l_{i+1}}{l_i + l_{i+1}}(l_i + l_{i+1})$$

$$= r(p_i)l_i + r(p_{i+1})l_{i+1}$$

  where inequality is due to **strict concavity of** ***r(p)***

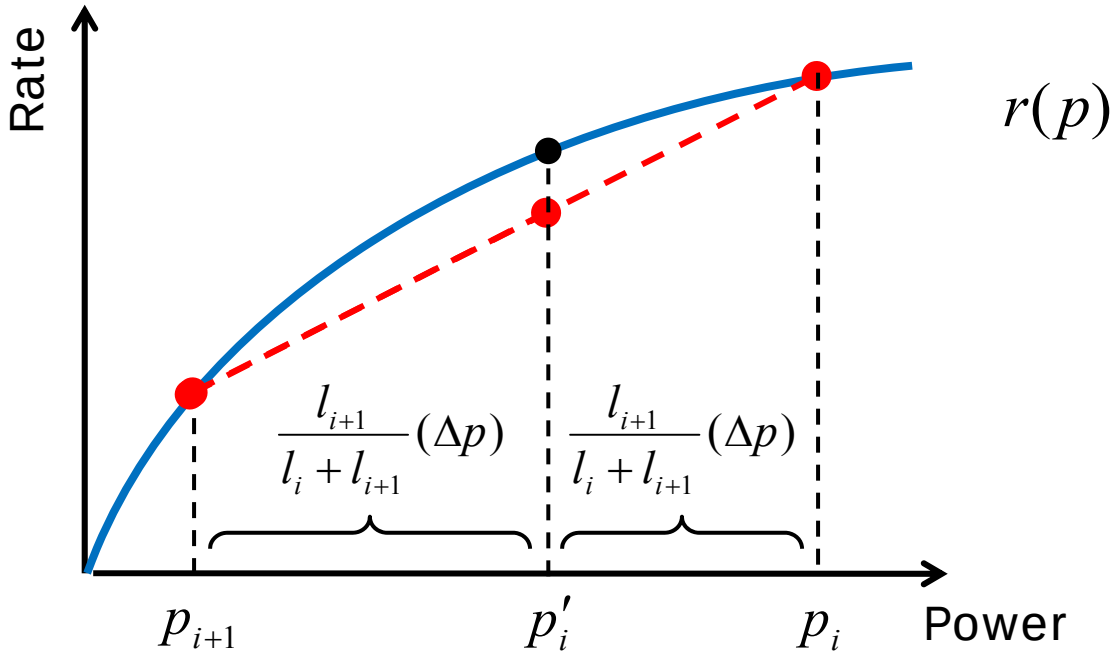
  Therefore $p_i > p_{i+1}$ cannot be optimal

Necessary conditions for optimality

- **Lemma 1:** The transmit powers increase monotonically, i.e., $p_1 < p_2 < \ldots < p_N$

Proof(cont'd):

Time-sharing between any two points is strictly suboptimal for concave r(p)



Necessary conditions for optimality

- **Lemma 2:** The transmission power remains constant between energy harvests.

Proof: (by contradiction) assume not

Let the total consumed energy in epoch $[s_i, s_{i+1}]$ be E_{total}, which is available in energy queue at $t = s_i$

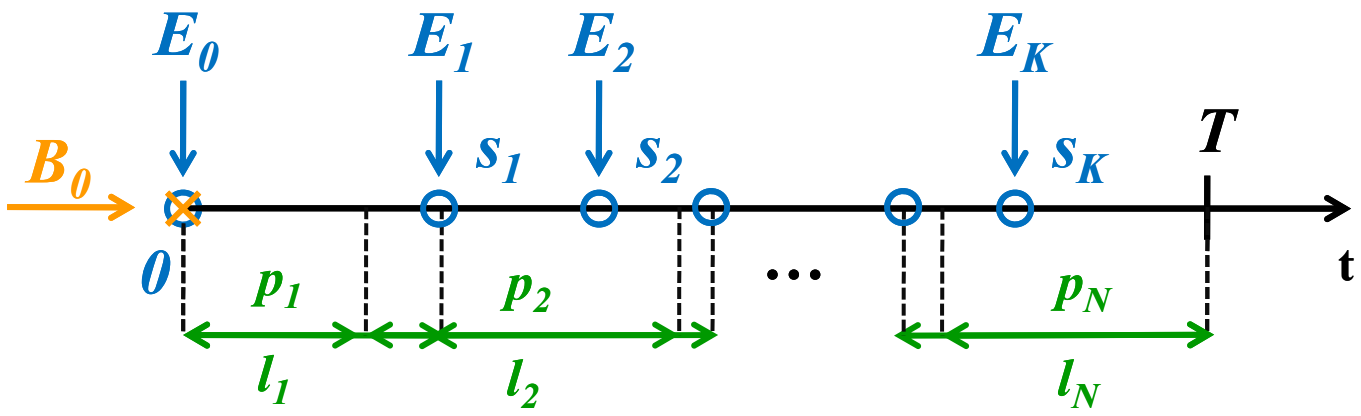
Then a constant power transmission

$$p' = \frac{E_{total}}{s_{i+1} - s_i}, \qquad t \in [s_i, s_{i+1}]$$

is feasible and strictly better than a non-constant transmission.

Necessary conditions for optimality

- **Lemma 2:** The transmission power remains constant between energy harvests.



Transmission power only changes at s_i

Necessary conditions for optimality

- **Lemma 3:** Whenever transmission rate changes, the battery is empty.

Proof: (by contradiction) assume not, i.e., $p_i < p_{i+1}$ for some i and energy buffer has Δ energy remaining at time of change.

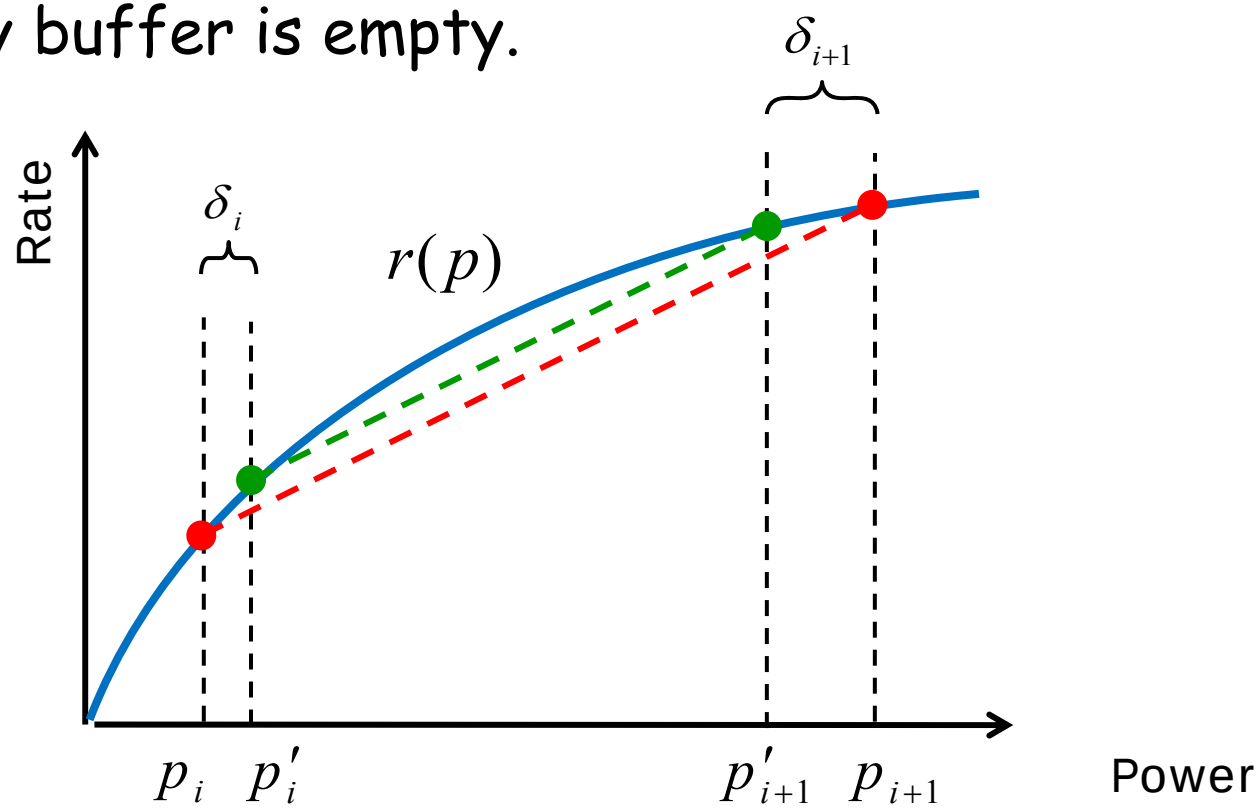
Choose δ_i and δ_{i+1} such that $\delta_i l_i = \delta_{i+1} l_{i+1} \leq \Delta$ and let

$$p_i' = p_i + \delta_i, \qquad p_{i+1}' = p_{i+1} - \delta_{i+1}$$

Since Δ amount of energy has moved from $i+1$ to i, and this was available at the buffer, this policy is feasible.

Necessary conditions for optimality

- **Lemma 3:** Whenever transmission rate changes, energy buffer is empty.



Necessary conditions for optimality

Summary:

- **L1:** Power only increases
- **L2:** Power constant between arrivals
- **L3:** At time of power change, energy buffer is empty

Conclusion:

For optimal policy, compare and sort (L1) power levels that deplete battery (L3) at arrival instances (L2).

Optimal Policy for Scenario I

For a given B_0 the optimal policy satisfies:

$$\sum_{n=1}^{N} r(p_n) l_n = B_0$$

and has the form

for $n = 1,2,\ldots, N$

$$i_n = \arg \min_{\substack{i: s_i \le T \\ s_i > s_{i_{n-1}}}} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\}$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}}, \qquad l_n = s_{i_n} - s_{i_{n-1}}$$

Algorithm for Scenario I

1. Find minimum number of energy arrivals required $i_{\min}$

   by comparing: $A_i = r^{-1}\left(\frac{B_0}{s_i}\right) s_i \le \sum_{j=0}^{i-1} E_j$

2. Find $s_{i_{\min}-1} < T_1 < s_{i_{\min}}$ satisfying $B_0 = r\underbrace{\left(\frac{\sum_{j=0}^{i-1} E_j}{T_1}\right)}_{\tilde{p}_1} s_i \cdot T_1$

3. Set $p_1 = \min\left\{\tilde{p}_1, \left\{\frac{\sum_{j=0}^{i-1} E_j}{s_i}, i = 1 \ldots i_{\min}\right\}\right\}$,

   $l_1 = s_{i_1}$ where i_1 is the minimizer of p_1

4. Repeat starting from s_{i_1}

Illustration – Step 1

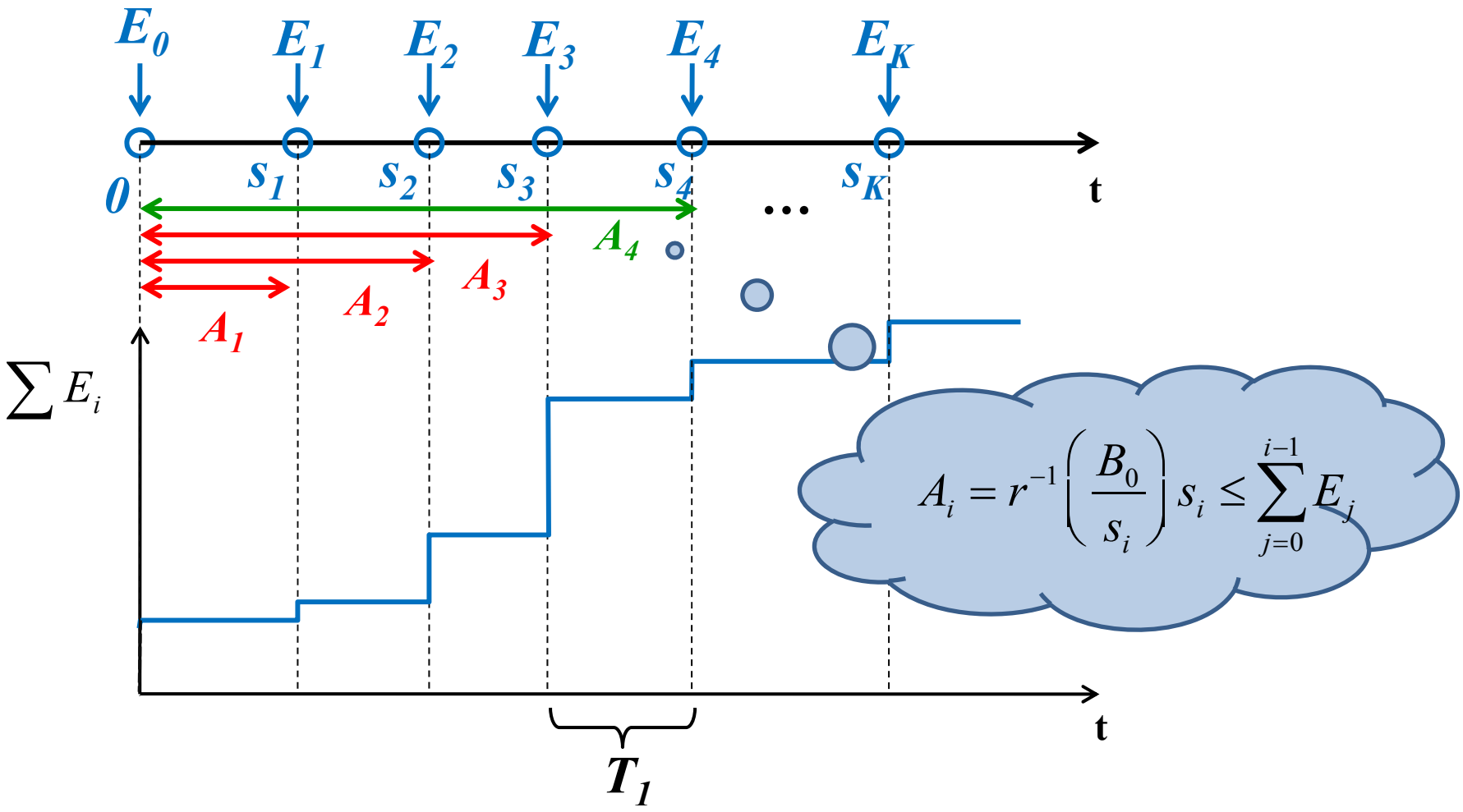


Illustration – Step 2

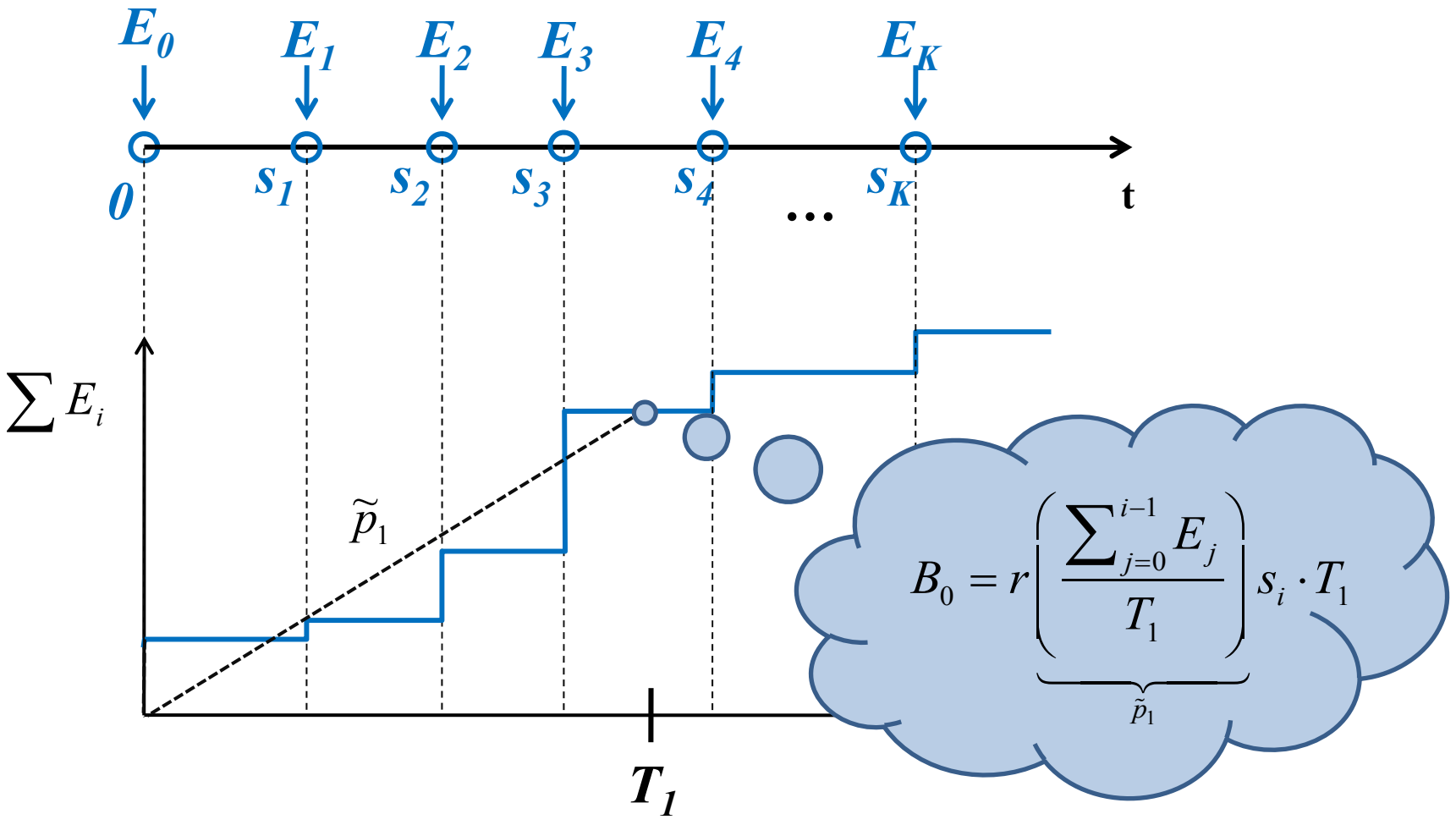


Illustration – Step 3

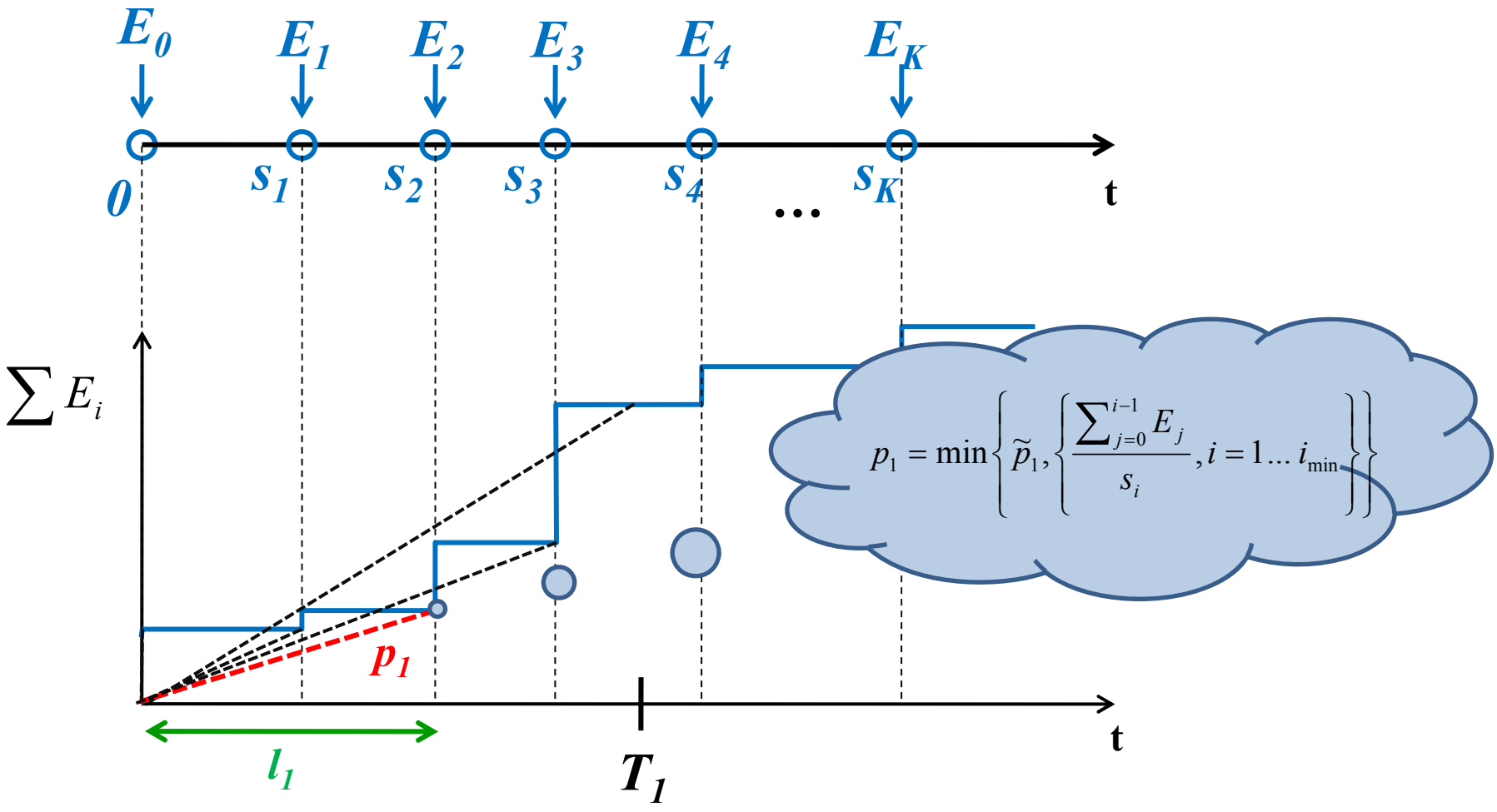
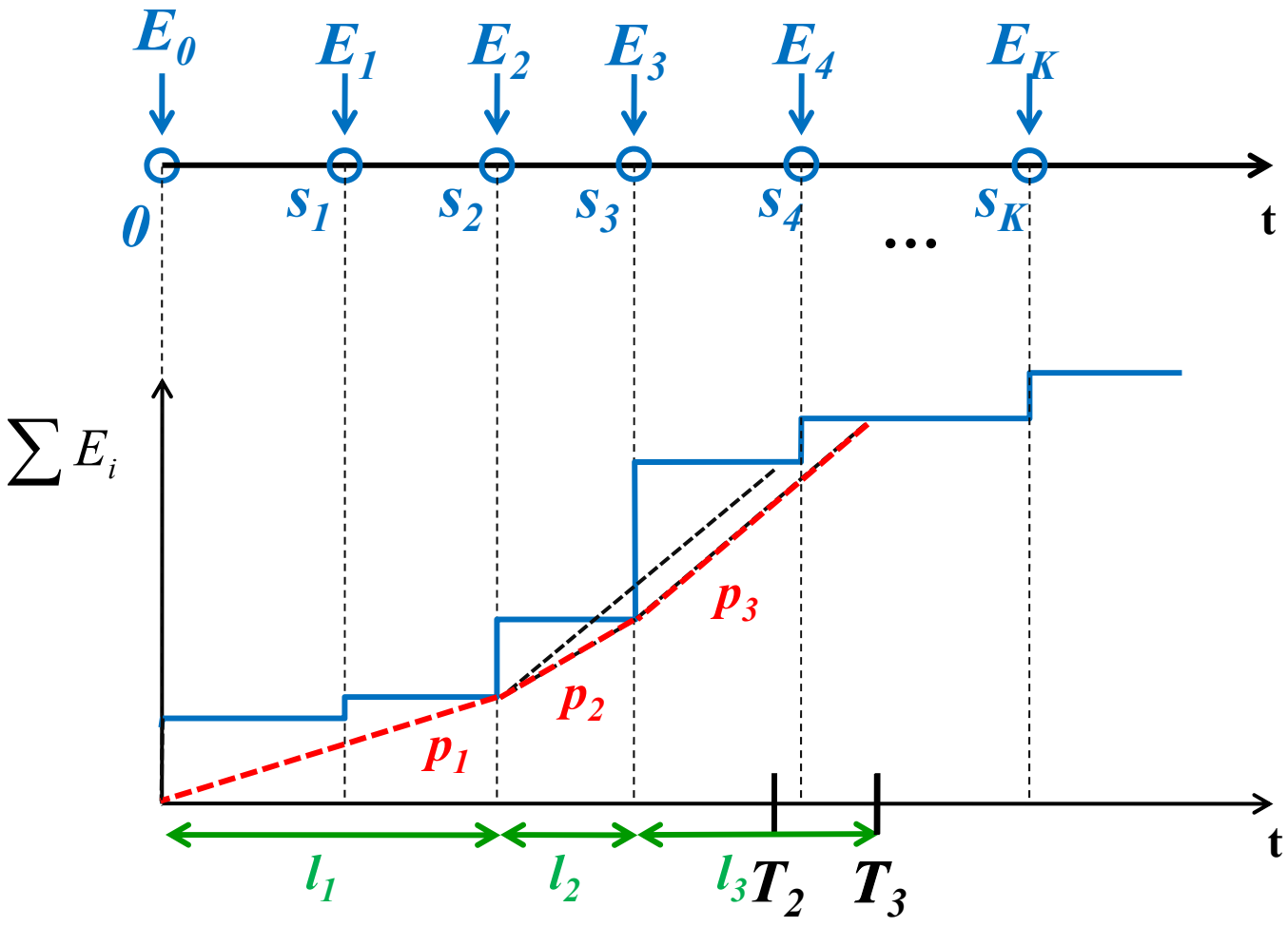
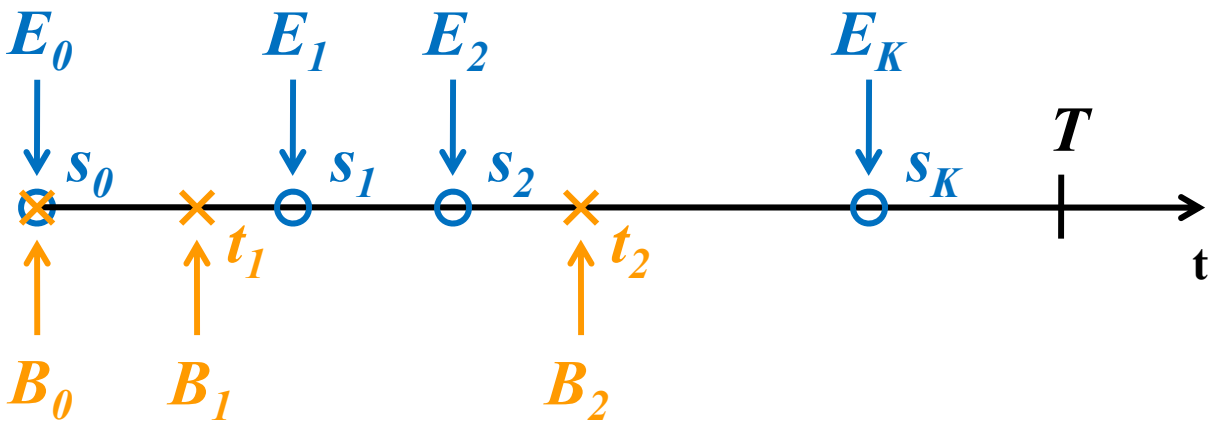


Illustration – Step 4



Scenario II: Packets Arrive During Transmission



- Transmitter cannot depart packets not received yet!
- Additional packet constraints apply

Scenario II: Packets Arrive During Transmission

Expended Energy: $E(t) = \sum_{i=1}^{\bar{i}} p_i l_i + p_{i+1}\left(t - \sum_{i=1}^{\bar{i}} l_i\right), \quad \bar{i} = \max\left\{ i : \sum_{j=1}^{i} l_j \le t \right\}$

Departed bits: $B(t) = \sum_{i=1}^{\bar{i}} r(p_i) l_i + r(p_{i+1})\left(t - \sum_{i=1}^{\bar{i}} l_i\right)$

Problem Definition:

$$\min \ T$$

$$\text{s.t.} \quad E(t) \le \sum_{i:s_i<t} E_i \qquad 0 \le t \le T$$

Energy Causality

$$B(t) \le \sum_{i:t_i<t} B_i \qquad 0 \le t \le T$$

Packet Causality

$$B(T) = \sum_{i=0}^{M} B_i$$

Necessary conditions for optimality

- **Lemma 4:** Power only increases.
- **Lemma 5:** Power constant between 2 arrivals of any kind.
- **Lemma 6:** At time of power change:

  if $t = s_i$ (energy arrival), energy buffer is empty;

  if $t = t_i$ (packet arrival), packet buffer is empty.

(Proofs are similar to Lemmas 1-3)

Optimal Policy for Scenario II

The optimal policy satisfies $\sum_{n=1}^{N} r(p_n) l_n = \sum_{n=1}^{M} B_i$

and has the form

$$r(p_1) = \min_{i:u_i \le T} \left\{ r\left(\frac{\sum_{j:s_j < u_i} E_j}{u_i} \right), \frac{\sum_{j:t_j < u_i} B_j}{u_i} \right\}$$

where $\{u_i\}$ is the ordered combination of $\{s_i\}$ and $\{t_i\}$ and subsequent rates are found iteratively

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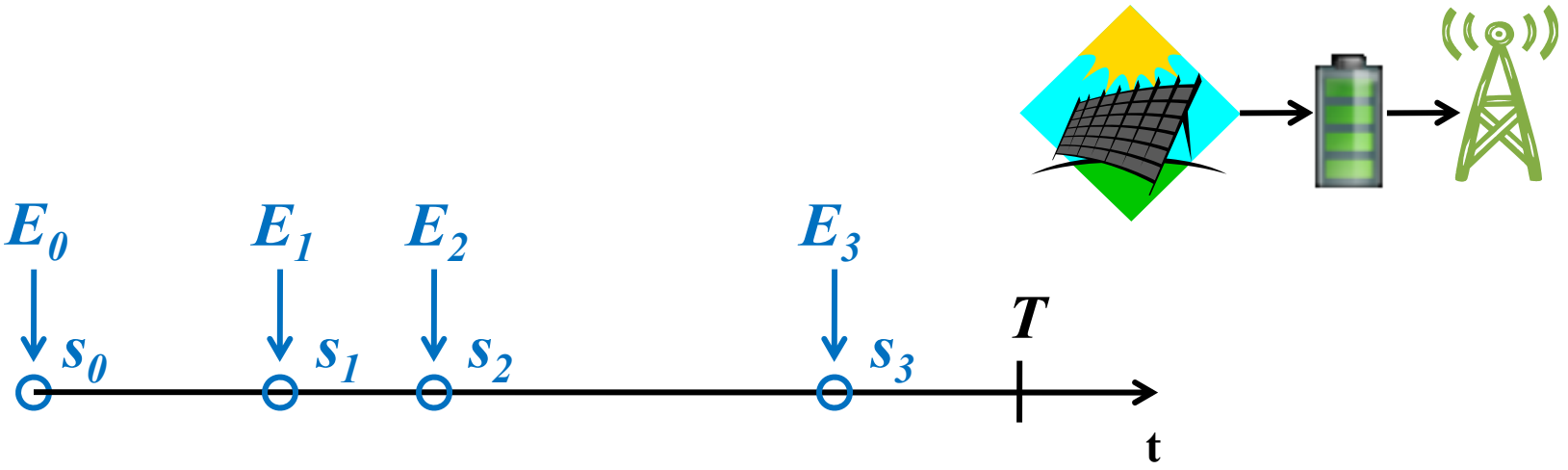
STTM for single link with Finite Battery

- [Tutuncuoglu-Yener '12a]
- Maximize the throughput of an energy harvesting transmitter by **deadline T.**
- Find optimal power allocation/transmission policy that departs maximum number of bits in a given duration.

- Up to a certain amount of energy can be stored by the transmitter ➔ **BATTERY CAPACITY**

System Model

- Energy arrivals of energy E_i at times s_i



- Arrivals known **non-causally** by transmitter,
- Stored in a **finite battery** of capacity $E_{\max}$,
- Design parameter: **power** $\rightarrow$ **rate** $r(p)$.

Notation

- Power allocation function: $p(t)$
- Energy consumed: $\int_0^T p(t)dt$
- Short-term throughput: $\int_0^T r(p(t))dt$

- Power-rate function $r(p)$: Strictly concave in p
- **Overflowing energy is lost (not optimal)**

Energy Constraints

(Energy arrivals of E_i at times s_i)

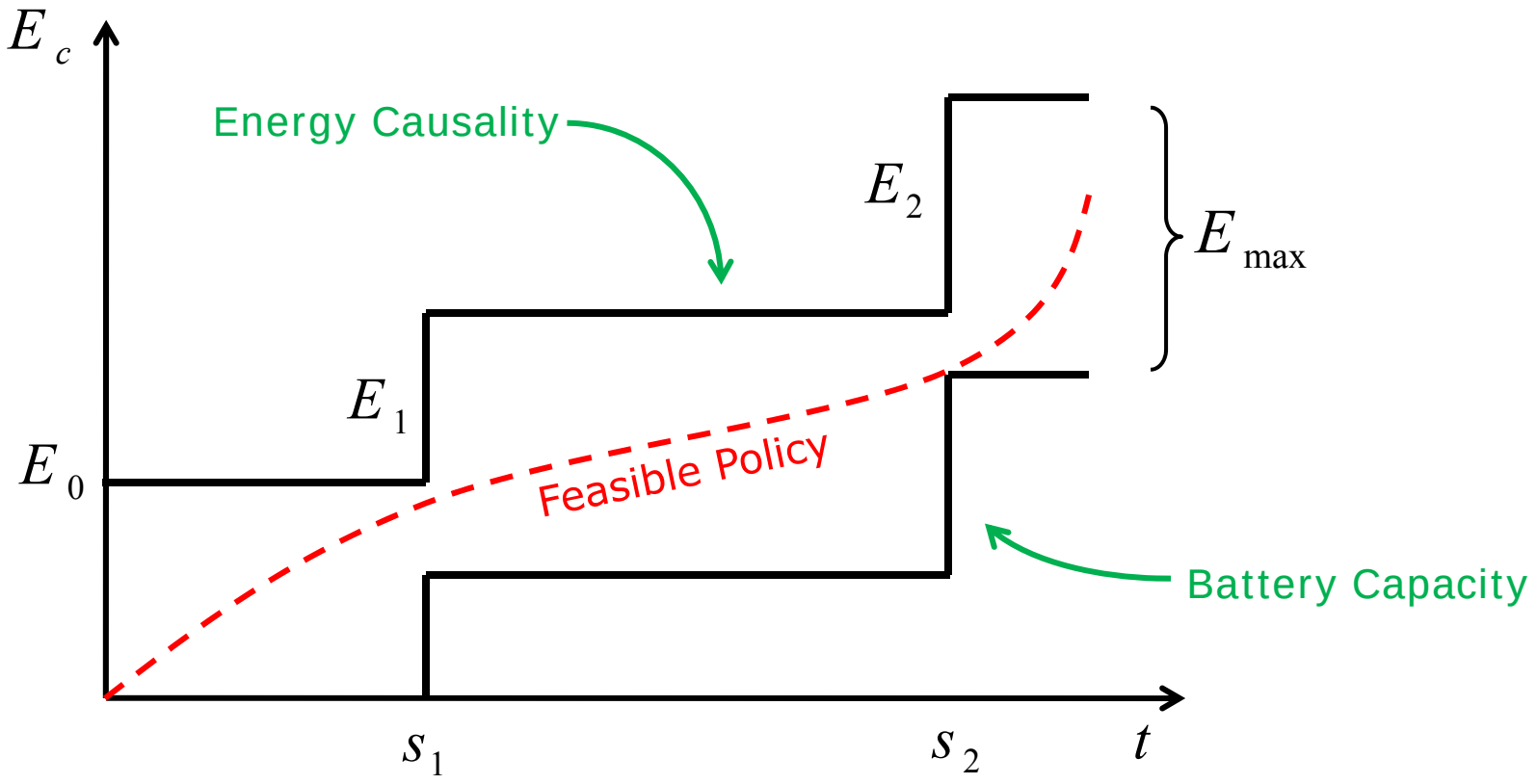
- **Energy Causality:** $\sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \geq 0 \qquad s_{n-1} \leq t' \leq s_n$

- **Battery Capacity:** $\sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{\max} \qquad s_{n-1} \leq t' \leq s_n$

- **Set of energy-feasible power allocations**

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \leq \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{\max}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\}$$

Energy "Tunnel"



STTM Problem for Single Link

- Maximize total number of transmitted bits by deadline **T**

$$\max_{p(t)} \int_0^T r(p(t))dt, \quad s.t. \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

- Convex constraint set, concave maximization problem

Necessary conditions for optimality of a transmission policy

- **Property 1:** Transmission power remains constant between arrivals.
- **Property 2:** Battery never overflows.

Proof: Assume an energy of Δ overflows at time τ

Define $$p'(t) = \begin{cases} p(t) + \dfrac{\Delta}{\delta} & [\tau - \delta, \tau] \\ p(t) & else \end{cases}$$

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\,dt$ since $r(p)$ is increasing in p

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

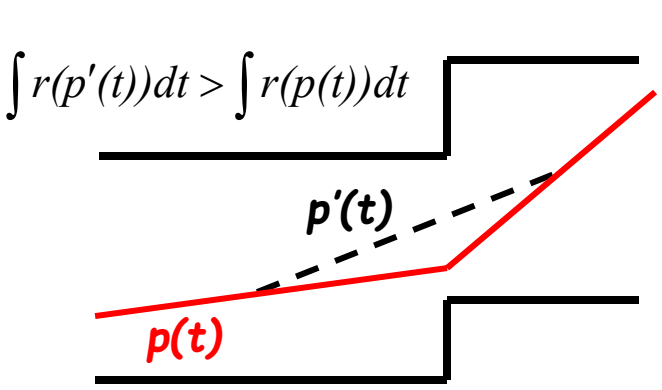
  Proof: Let $p(\tau^-) < p(\tau^+)$

  Define $p'(t) = \begin{cases} p(t) - \varepsilon & [\tau, \tau + \delta] \\ p(t) + \varepsilon & [\tau - \delta, \tau] \\ p(t) & else \end{cases}$ (Feasible unless battery is depleted)

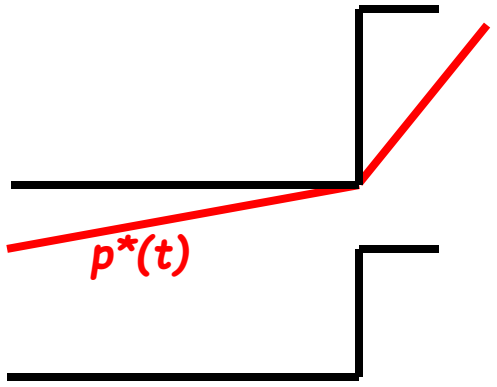
  Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\,dt$ due to strict concavity of $r(p)$

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.



Policy can be improved



Policy cannot be improved

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

Proof: Let $p(\tau^-) > p(\tau^+)$

Define $p'(t) = \begin{cases} p(t)+\varepsilon & [\tau, \tau+\delta] \\ p(t)-\varepsilon & [\tau-\delta, \tau] \\ p(t) & else \end{cases}$ (Feasible unless battery is full)

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\,dt$ due to strict concavity of $r(p)$

Necessary conditions for optimality of a transmission policy

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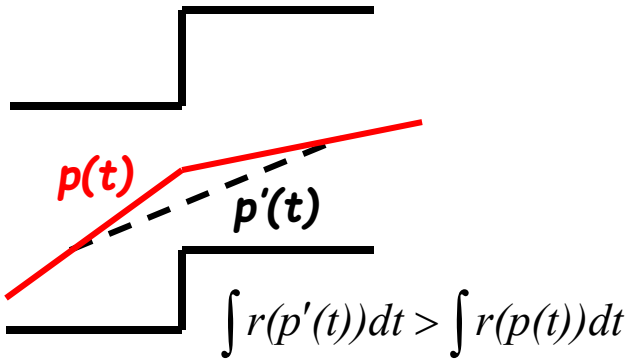
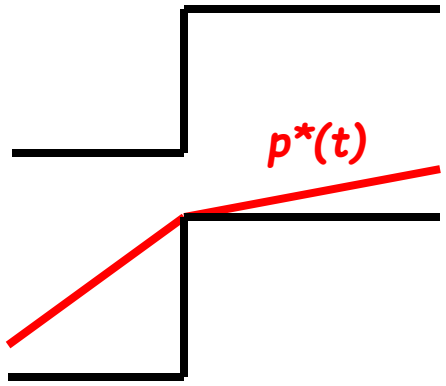


Policy can be improved



Policy cannot be improved

Necessary conditions for optimality of a transmission policy

- **Property 4:** Battery is depleted at the end of transmission.

  Proof: Assume an energy of Δ remains after p(t)

  Define $$p'(t) = \begin{cases} p(t) + \dfrac{\Delta}{\delta} & [T-\delta, T] \\ p(t) & else \end{cases}$$

  Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))\,dt$ since $r(p)$ is increasing

Necessary Conditions for Optimality

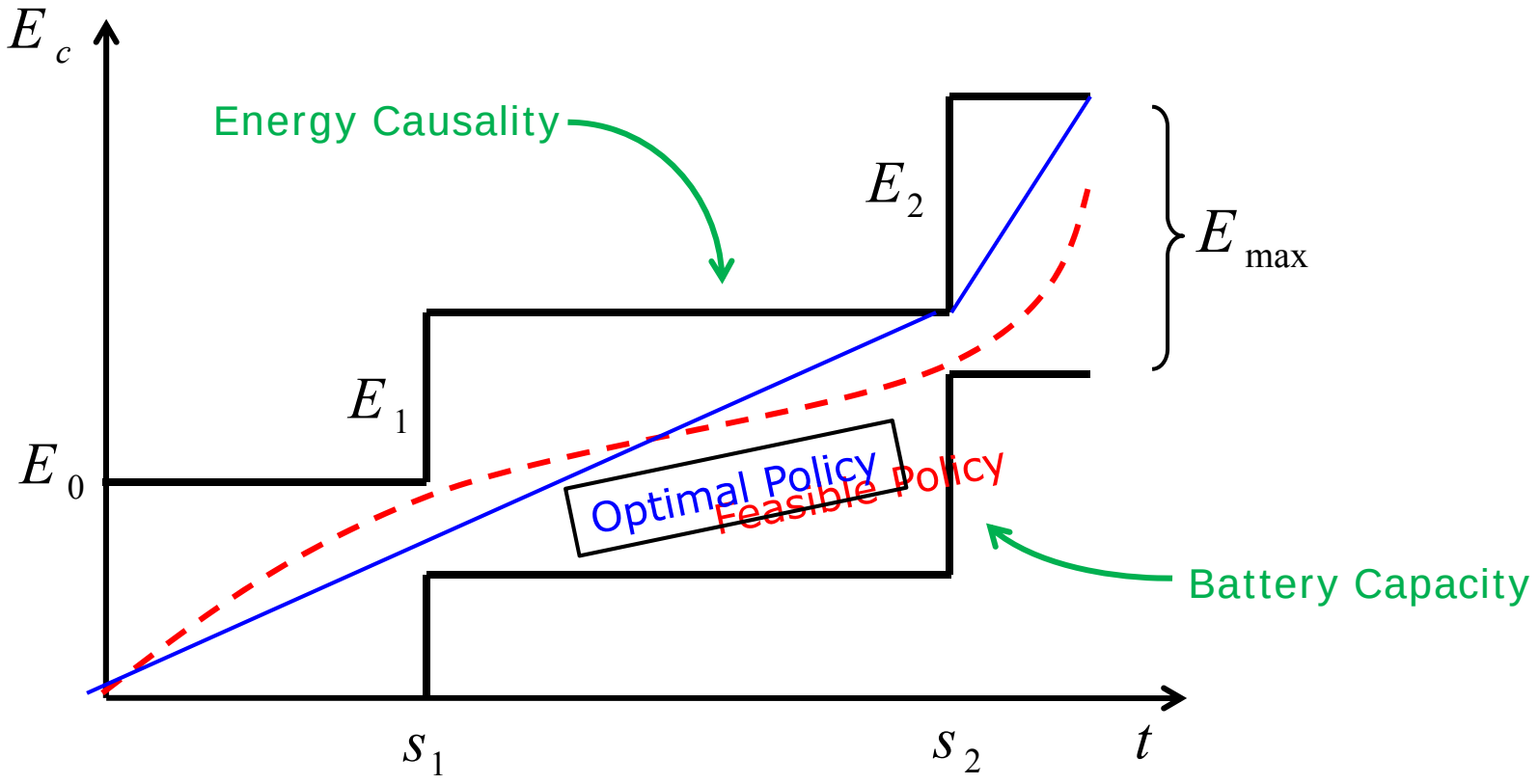
Implications of Properties 1-4:

- **Structure of optimal policy:** (Property 1)

$$p(t) = \begin{cases} p_n & i_{n-1} < t < i_n \\ 0 & t > T \end{cases}, \qquad i_n \in \{s_n\}, \quad p_n \text{ constant}$$

- For power to increase or decrease, policy must meet the upper or lower boundary of the tunnel respectively (Property 3)
- At termination step, battery is depleted (Property 4).

Energy "Tunnel"



Shortest Path Interpretation

- **Optimal policy is identical for any concave power-rate function!**
- Let $r(p) = -\sqrt{p^2+1}$, then the problem solved becomes:

$$\max_{p(t)} \int_0^T -\sqrt{p^2(t)+1}\, dt \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

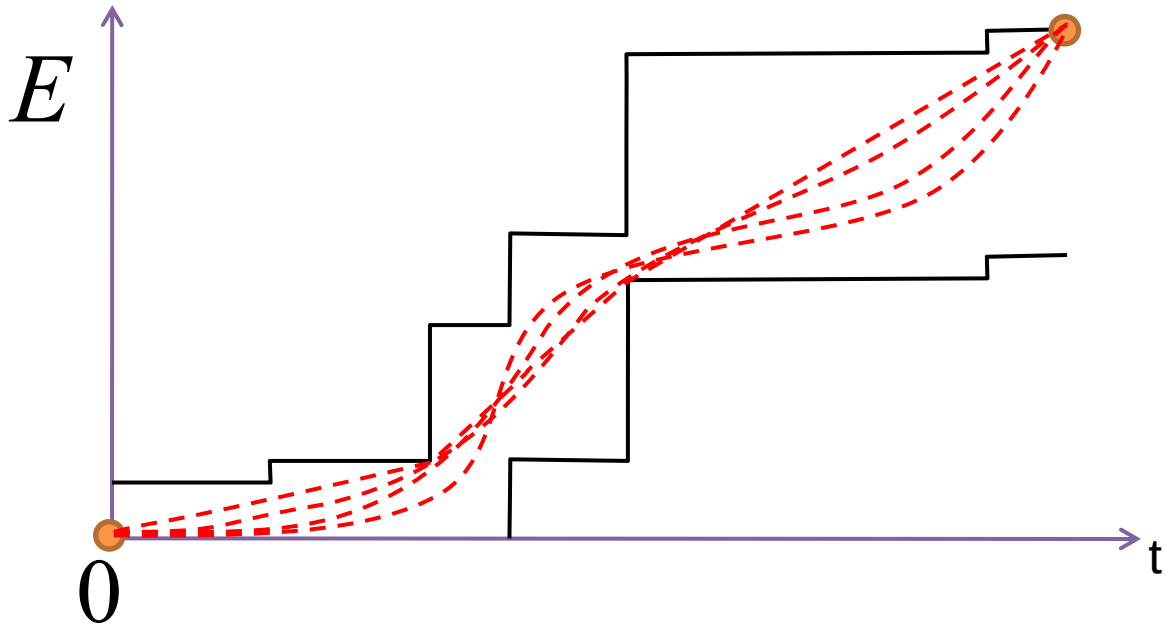
$$= \min_{p(t)} \underbrace{\int_0^T \sqrt{p^2(t)+1}\, dt}_{\textbf{length}} \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

length of policy path in energy tunnel

$\Rightarrow$ The **throughput maximizing policy** yields the **shortest path** through the energy tunnel for any concave power-rate function.

Shortest Path Interpretation

- **Property 1**: Constant power is better than any other alternative
- **Shortest path** between two points is a line (constant slope)



Throughput Maximizing Algorithm (TMA)

- Knowing the structure of the policy, we can construct an iterative algorithm to get the tightest string in the tunnel.
- Note: After a step (p_1, i_1) is determined, the rest of the policy is the solution to a ***shifted problem*** with shifted arrivals and deadline:

$$E_0' = \sum_{k=0}^{n_1} E_k - i_1 . p_1, \quad T' = T - i_1, \quad n'_{\max} = n_{\max} - n_1,$$

$$E_n' = E_{n+n_1}, \quad s_n' = s_{n+n_1} - i_1, \quad \text{for } n = 0, \ldots, n'_{\max}$$

- **Essentially, the algorithm compares and find the tightest segment that hits the upper or lower wall staying feasible all along.**

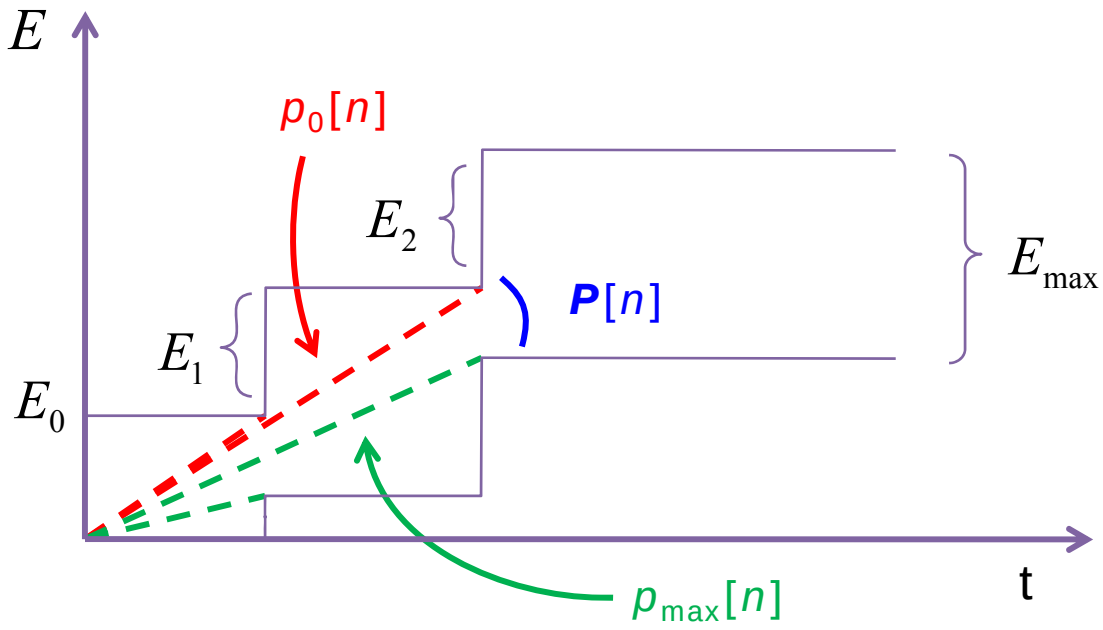
Throughput Maximizing Algorithm (TMA)

$$p_{\max}[n] = \max\left\{\frac{\sum_{k=0}^{n} E_k - E_{\max}}{s_n}, 0\right\}$$

$$p_0[n] = \frac{\sum_{k=0}^{n-1} E_k}{s_n}$$

$$\mathbf{P}[n] = [p_{\max}[n], p_0[n]]$$

$$\mathbf{P}[n_{\max}] = \{p_0[n_{\max}]\}$$



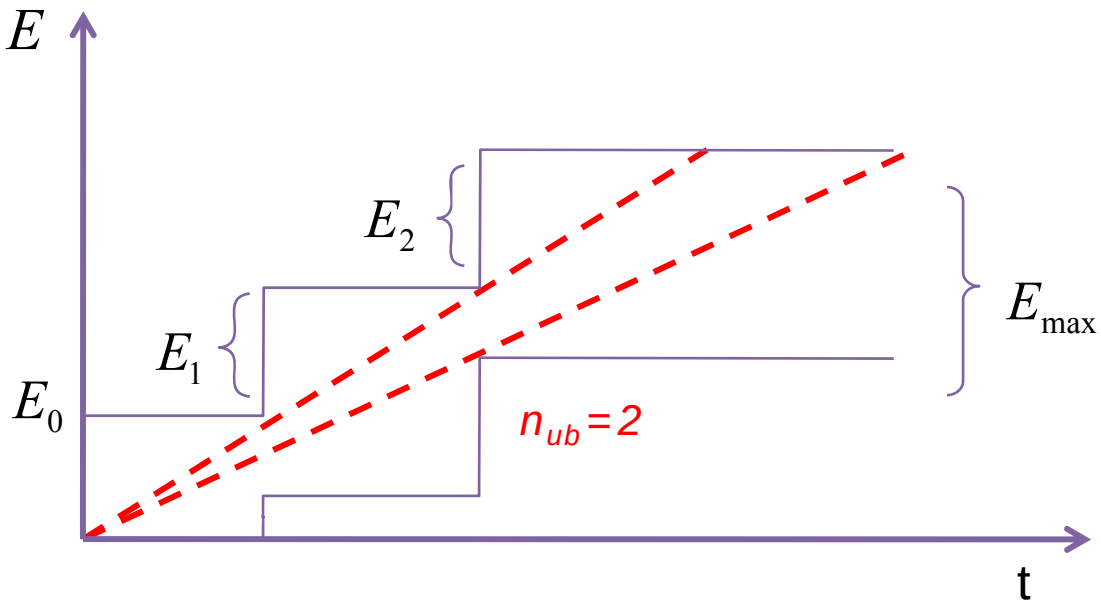
Throughput Maximizing Algorithm (TMA)

$$n_{ub} = \max\{n \mid \bigcap_{k=1}^{n} \mathbf{P}[k] \neq \varnothing, n = 1,2,\ldots,n_{\max}\}$$

Upper bound for the duration of the first step

- The **transmission power must change** before arrival n_{ub+1} to stay in the feasible tunnel

$\Rightarrow$ At or before n_{ub}, battery must be **empty or full** to allow the necessary change. (Prop. 3)



Throughput Maximizing Algorithm (TMA)

1. Find n_{ub} . If $n_{ub} = n_{\max}$ terminate with power $(\sum_{k=0}^{n_{\max}} E_k)/T$
2. Determine relation between $\mathbf{P}[n_{ub}+1]$ and $\bigcap_{k=0}^{n_{ub}} \mathbf{P}[k]$
3. Transmit based on the outcome of step 2 with:

$$n_1 = \max\{n \mid p_0[n] \in \bigcap_{k=0}^{n} \mathbf{P}[k]\}$$
$$p_1 = p_0[n_1]$$
$$i_1 = s_{n_1}$$

$$n_1 = \max\{n \mid p_{\max}[n] \in \bigcap_{k=0}^{n} \mathbf{P}[k]\}$$
$$p_1 = p_{\max}[n_1]$$
$$i_1 = s_{n_1}$$

4. Repeat for shifted problem with updated parameters:

$$E_0' = \sum_{k=0}^{n_1} E_k - i_1 . p_1, \quad T' = T - i_1, \quad n'_{\max} = n_{\max} - n_1,$$
$$E_n' = E_{n+n_1}, \quad s_n' = s_{n+n_1} - i_1, \quad \text{for } n = 0,\ldots,n'_{\max}$$

Alternative Solution

- Transmission power is constant within each epoch:

$$p(t) = \{p_i \qquad t \in \text{epoch } i,\ i = 1,..., N + M + 1\}$$

- STTM problem expressed with above notation

$$\max_{p_i} \sum_{i=1}^{M+N+1} L_i . r(p_i)$$

(L_i: length of epoch i)

$$s.t. \quad 0 \le \sum_{i=1}^{l} E_i - L_i p_i \le E_{\max} \qquad \forall l$$

Energy constraints: sufficient to check arrivals only

Water-filling approach

- Lagrangian function for STTM

$$\max_{p_i} \sum_{i=1}^{M+N+1} L_i.r(p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right)$$

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0 \quad \forall j$$

$$\mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0 \quad \forall j$$

(Complementary slackness conditions)

- KKT Stationarity Condition

$$\nabla \left(\sum_{i=1}^{M+N+1} L_i.r(p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) \right) = 0 \quad \text{at } p = p^*$$

Water-filling approach

- Gradient for kth component

$$\nabla_k \left(\sum_{i=1}^{M+N+1} L_i . r(p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) \right) = 0 \quad \forall n$$

$$= \sum_{i=1}^{M+N+1} L_i . \underbrace{\nabla_k r(p_i)}_{\frac{\delta_{(i-k)}}{1+p_i}} - \sum_{j=1}^{M+N+1} \lambda_j \underbrace{\left(\sum_{i=1}^{j} L_i (\nabla_k p_i) \right)}_{\left\{ \begin{matrix} L_k & if\ j>k \\ 0 & if\ j<k \end{matrix} \right\}} - \sum_{j=1}^{M+N+1} \mu_j \underbrace{\left(\sum_{i=1}^{j} L_i (\nabla_k p_i) \right)}_{\left\{ \begin{matrix} L_k & if\ j>k \\ 0 & if\ j<k \end{matrix} \right\}} = 0$$

$$= L_k \cdot \frac{1}{1+p_k^*} - L_k \sum_{j=k}^{M+N+1} \lambda_j - L_k \sum_{j=k}^{M+N+1} \mu_j = 0$$

$$\Rightarrow \frac{1}{1+p_k^*} = \sum_{j=k}^{M+N+1} (\lambda_j - \mu_j)$$

$$\Rightarrow p_k^* = \frac{1}{\sum_{j=k}^{M+N+1} (\lambda_j - \mu_j)} - 1 \qquad \textbf{(Water Filling)}$$

Water-filling approach

- Complementary Slackness

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0 \qquad \forall j$$

Conditions:

$$\mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0 \qquad \forall j$$

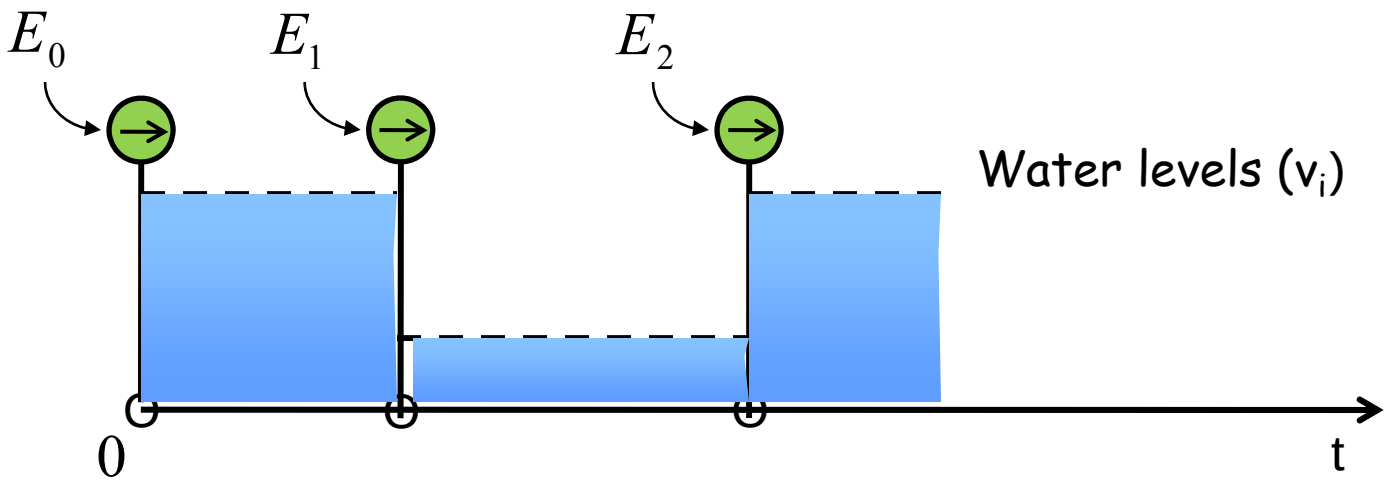
λ_j's are positive only when battery is empty $\left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0$

μ_j's only positive only when battery is full $\left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0$

$$p_k^* = \left[\frac{1}{\sum_{j=k}^{M+N+1} (\lambda_j - \mu_j)} - 1 \right]^+$$

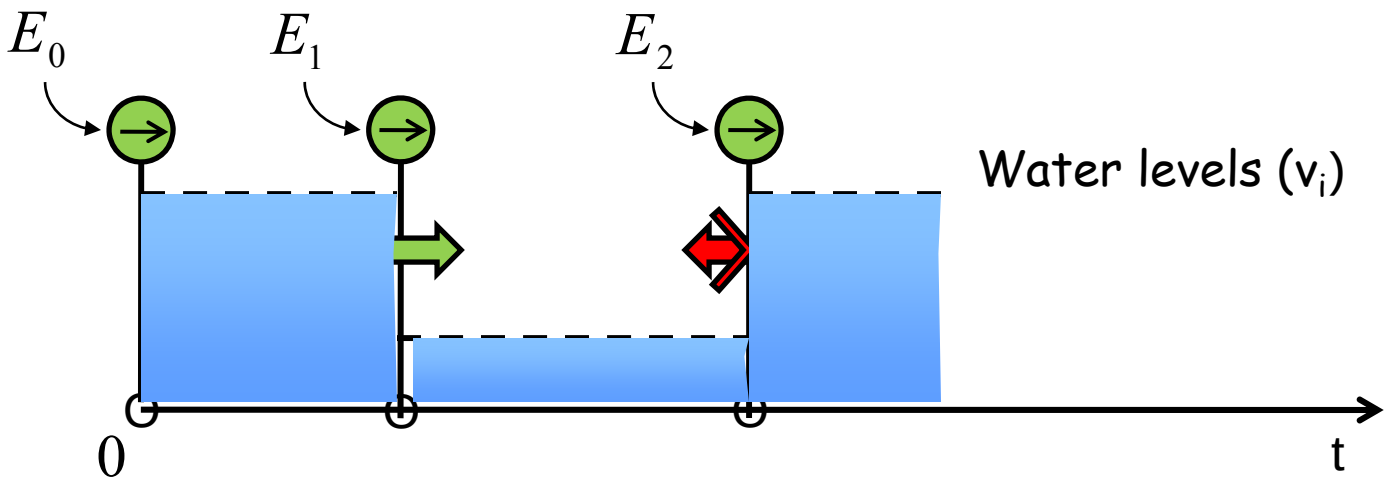
Directional Water-Filling

- Harvested energies filled into epochs individually



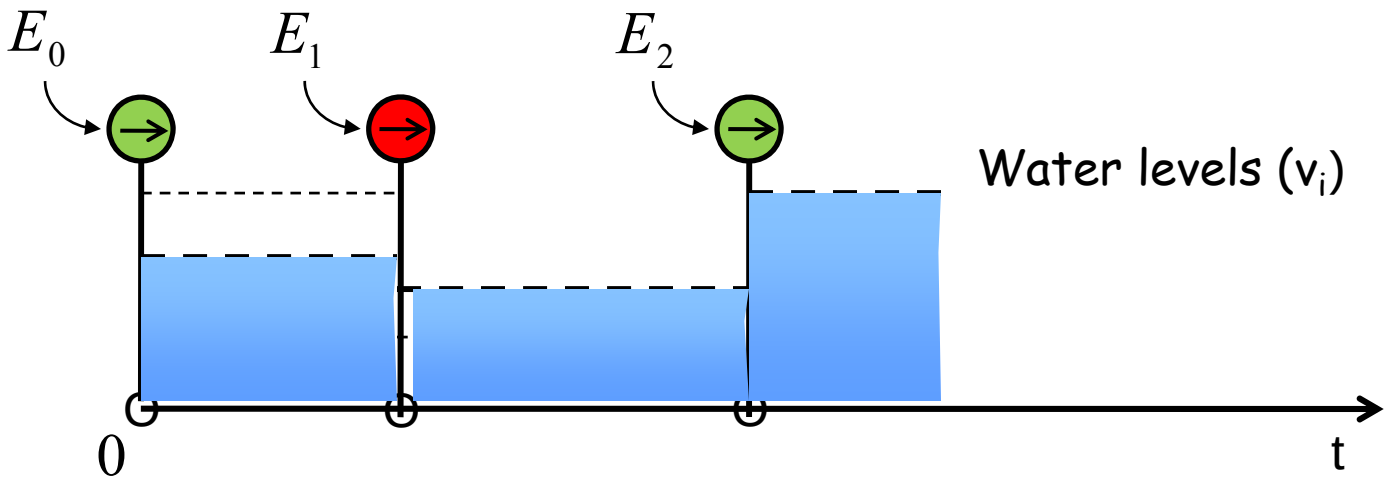
Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time ⟹

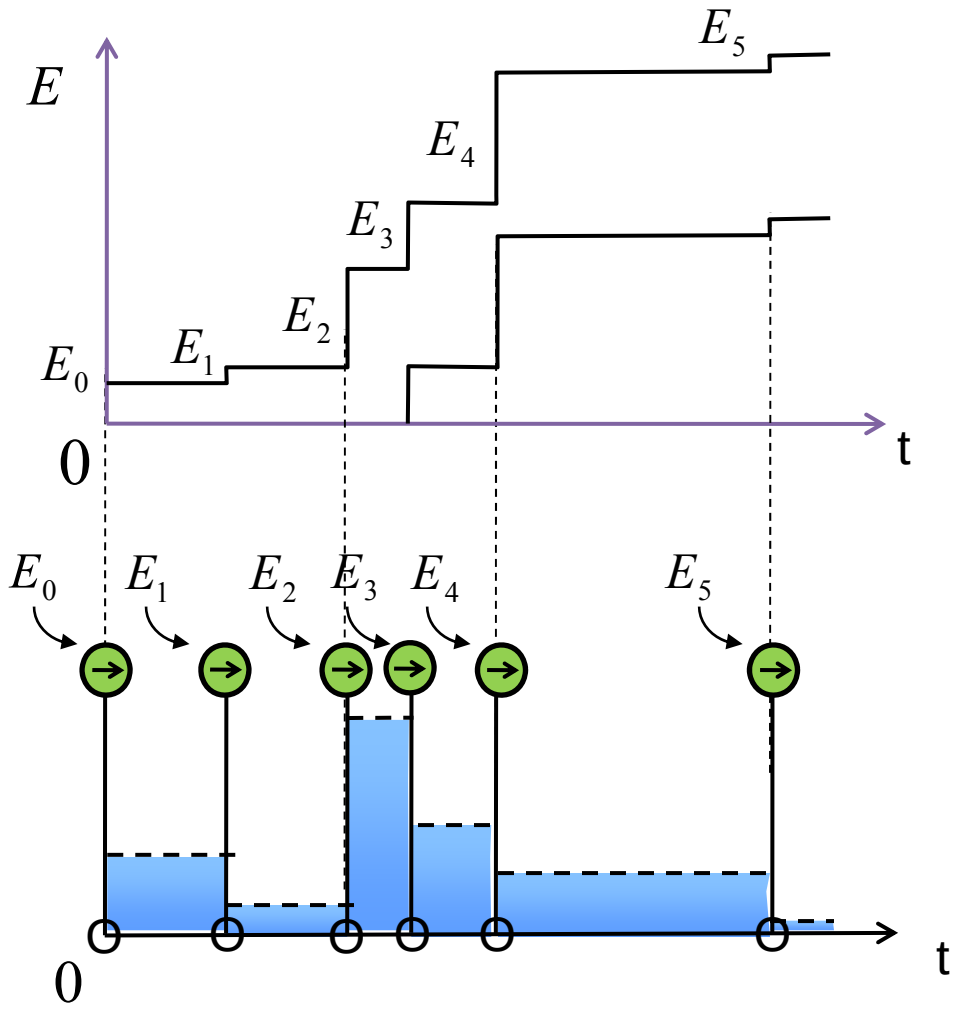


Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time
  - **Battery Capacity**: water-flow limited to E_{max} by taps

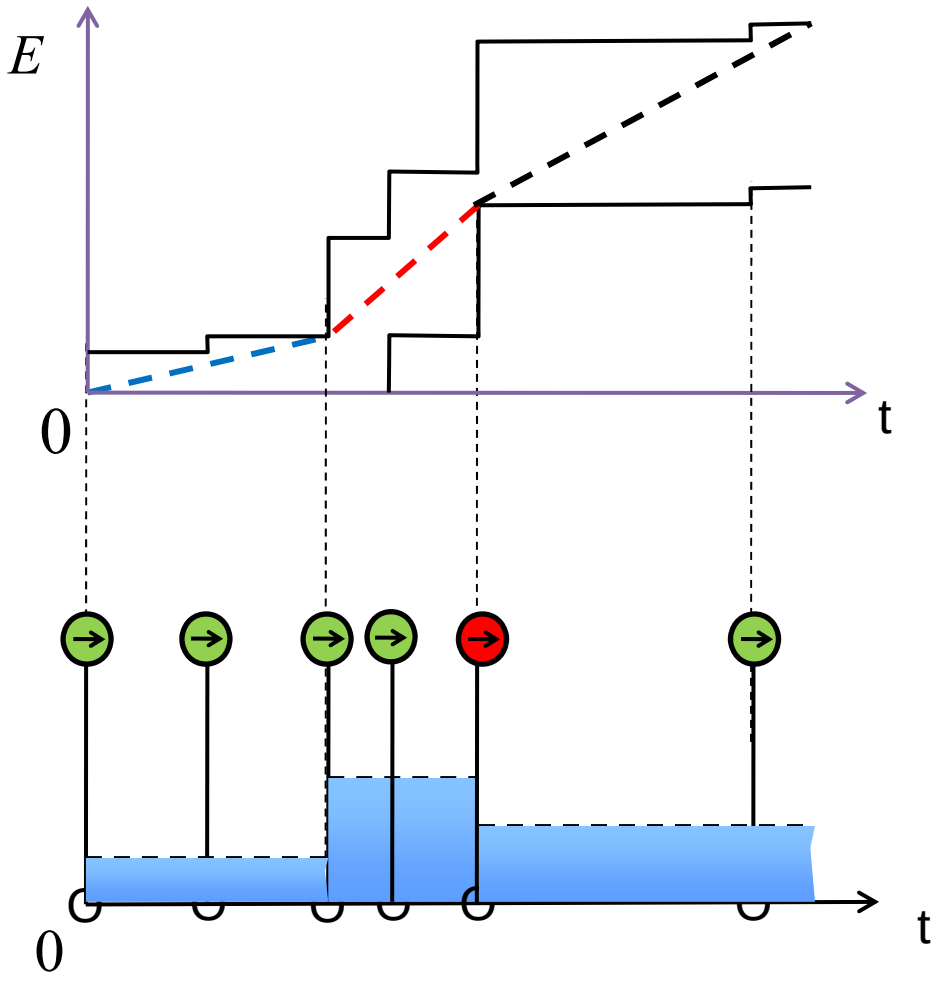


Directional Water-Filling



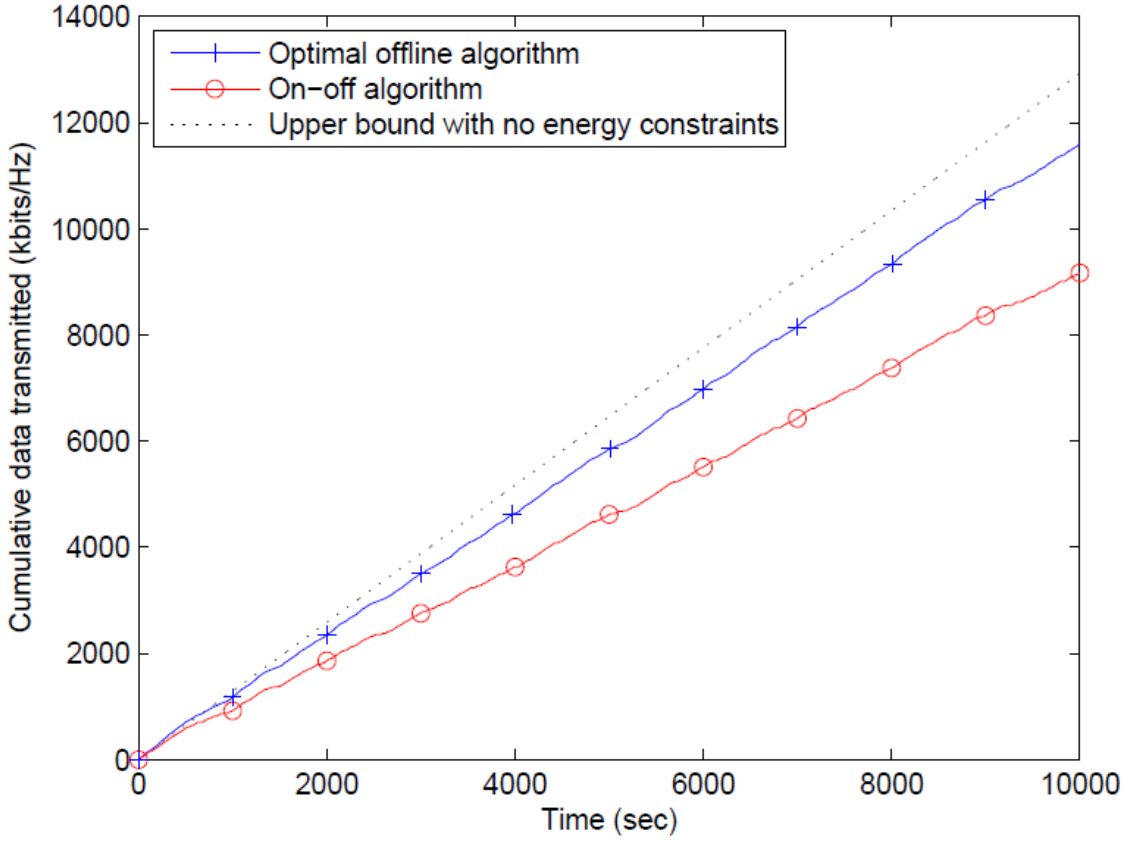
- Energy tunnel and directional water-filling approaches yield the same policy

Directional Water-Filling



- Energy tunnel and directional water-filling approaches yield the same policy

Simulation Results



- Improvement of optimal algorithm over an *on-off transmitter* in a simulation with truncated Gaussian arrivals.

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Outline

- Transmission Completion Time Minimization for single link
- Short Term Throughput Maximization for single link with finite battery
- **Transmission Completion Time Minimization for single link w/finite battery**
- Extension to fading channels
- Transmission policies for nodes with inefficient energy storage
- Energy harvesting receivers
- Energy harvesting multiuser networks
- Energy cooperation in energy harvesting networks
- Information theory of energy harvesting communications (introductory)

Transmission Completion Time Minimization (TCTM) for single link with finite battery

- Given the total number of bits to send as B, complete transmission in the shortest time possible.

$$\min_{p(t)} T \quad s.t. \quad B - \int_0^T r(p(t))dt \le 0, \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

Relationship of STTM and TCTM problems

- Lagrangian dual of TCTM problem becomes:

$$\max_{u \geq 0} \left(\min_{p(t) \in \mathfrak{P}, T} T + u\left(B - \int_0^T r(p(t))dt \right) \right)$$

$$= \max_{u \geq 0} \left(\min_{T} \left(T + uB - u.\underbrace{\max_{p(t) \in \mathfrak{P}} \int_0^T r(p(t))dt}_{\text{STTM problem for deadline } T} \right) \right)$$

Relationship of STTM and TCTM problems

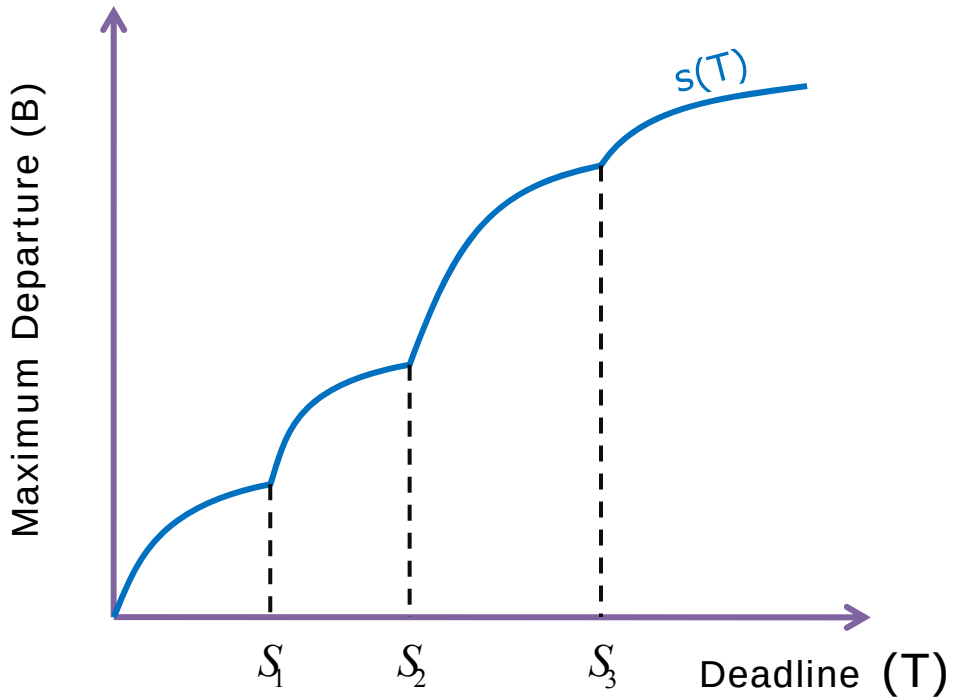
- Optimal allocations are identical:

STTM's solution for deadline T departing B bits $\equiv$ TCTM's solution for departing B bits in time T

- STTM solution can be used to solve the TCTM problem

Maximum Service Curve

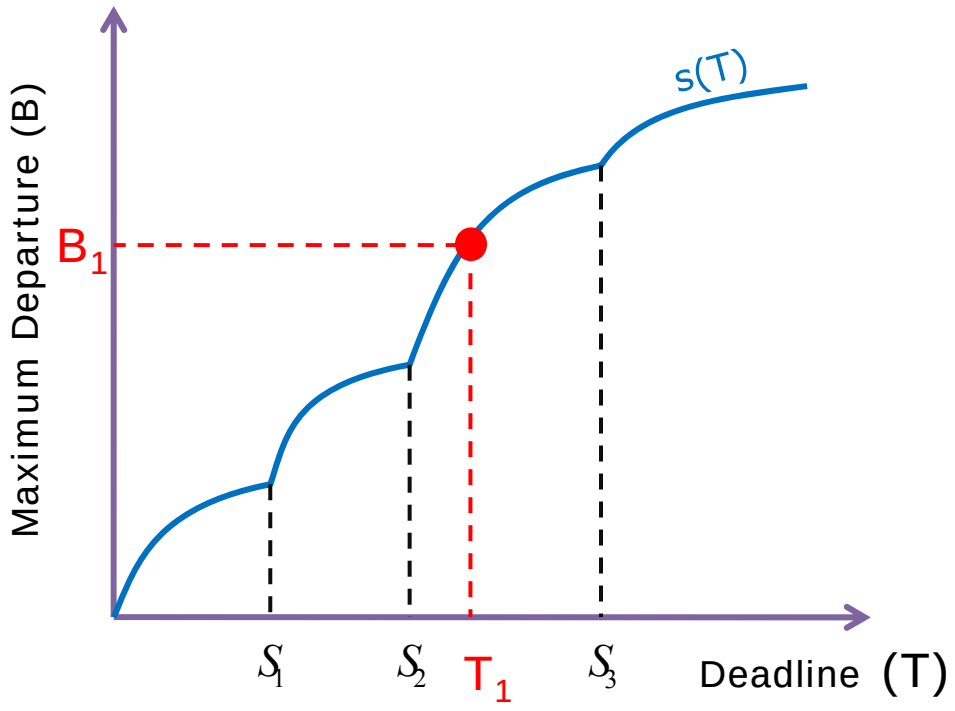
$$s(T) = \max_{p(t)} \int_0^T r(p(t))dt, \quad s.t. \quad p(t) \in \mathfrak{P}$$



- Maximum number of bits that can be sent in time T.
- Each point calculated by solving the corresponding STTM problem.

Maximum Service Curve

- Continuous, monotone increasing, **invertible**



- Optimal allocation for TCTM with B_1 bits

=

Optimal allocation for STTM with deadline T_1

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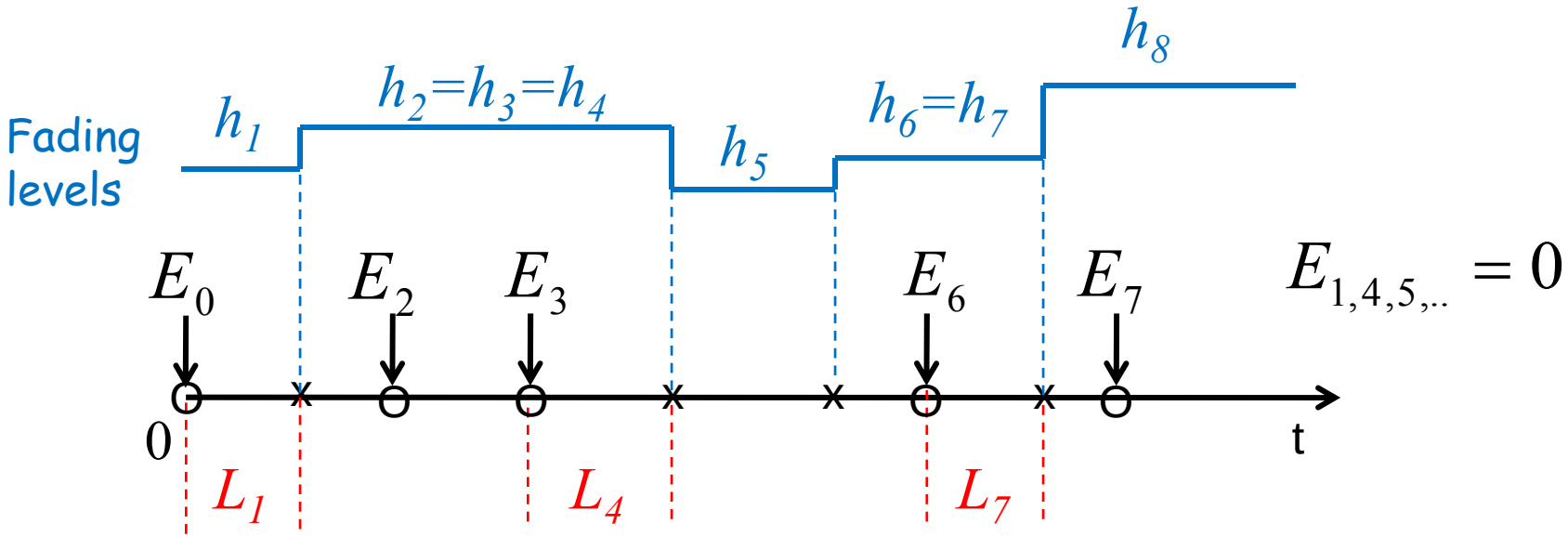
Extension to Fading Channels

- [Ozel-Tutuncuoglu-Ulukus-Yener '11]
- Find the short-term throughput maximizing and transmission completion time minimizing power allocations in a fading channel with non-causally known channel states.
- Finite battery.

System Model



- AWGN Channel with fading h: $R(P,h) = \frac{1}{2}\log(1 + h.P)$
- Each "epoch" defined as the interval between two "events".
- Fading states and harvests known **non-causally**

STTM Problem with Fading

- Transmission power constant within each epoch:

$$p(t) = \{p_i, \quad t \in \text{epoch } i, \quad i = 1,\ldots,N+M+1\}$$

- Maximize total number of transmitted bits by a deadline T

$$\max_{p_i} \sum_{i=1}^{M+N+1} \frac{L_i}{2} \log(1 + h_i p_i)$$

$$s.t. \quad 0 \le \sum_{i=1}^{l} E_i - L_i p_i \le E_{\max} \quad \forall l$$

STTM Problem with Fading

- Lagrangian of the STTM problem

$$\max_{p_i} \sum_{i=1}^{M+N+1} \frac{L_i}{2}\log(1+h_i p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) + \sum_{i=1}^{M+N+1} \eta_i p_i$$

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0 \quad \forall j$$

$$\mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0 \quad \forall j$$

$$\eta_j p_j = 0 \quad \forall j$$

(Complementary slackness conditions)

- **Solution:** *directional* water-filling with fading levels:

$$p_i^* = \left[v_i - \frac{1}{h_i} \right]^+, \qquad v_i = \frac{1}{\sum_{j=i}^{M+N+1} \lambda_j - \mu_j}$$

STTM Problem with Fading

$$\nabla_k \left(\sum_{i=1}^{M+N+1} L_i . r(p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) + \sum_{i=1}^{M+N+1} \eta_i p_i \right) = 0 \quad \forall n$$

$$= \sum_{i=1}^{M+N+1} L_i . \underbrace{\nabla_k r(p_i)}_{\frac{h_k \cdot \delta_{(i-k)}}{1+h_k p_i}} - \sum_{j=1}^{M+N+1} \lambda_j \underbrace{\left(\sum_{i=1}^{j} L_i (\nabla_k p_i) \right)}_{\left\{ \begin{matrix} L_k & if\ j>k \\ 0 & if\ j<k \end{matrix} \right\}} - \sum_{j=1}^{M+N+1} \mu_j \underbrace{\left(\sum_{i=1}^{j} L_i (\nabla_k p_i) \right)}_{\left\{ \begin{matrix} L_k & if\ j>k \\ 0 & if\ j<k \end{matrix} \right\}} + \underbrace{\sum_{i=1}^{M+N+1} \eta_i (\nabla_k p_i)}_{\eta_k} = 0$$

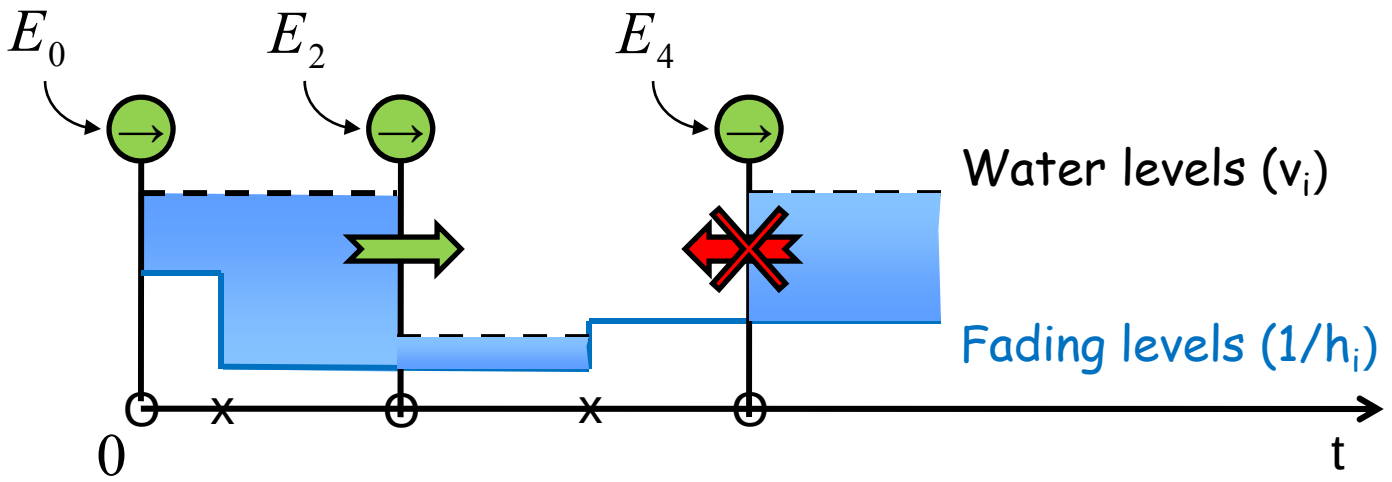
$$= L_k \cdot \frac{h_k}{1+h_k p_k^*} - L_k \sum_{j=k}^{M+N+1} \lambda_j - L_k \sum_{j=k}^{M+N+1} \mu_j + \eta_k = 0$$

$$\Rightarrow \frac{h_k}{1+h_k p_k^*} = \sum_{j=k}^{M+N+1} (\lambda_j - \mu_j) \qquad \text{(if } p_k^* > 0 \text{ is satisfied. Otherwise } p_k^* = 0 \text{ and } \eta_k > 0 \text{)}$$

$$\Rightarrow p_k^* = \left[\frac{1}{\sum_{j=k}^{M+N+1} (\lambda_j - \mu_j)} - \frac{1}{h_k} \right]^+ \quad \textbf{(Water Filling)}$$

Directional Water-Filling

- Same directional water filling model with added fading levels.
  - Directional water flow **(Energy causality)**
  - Limited water flow **(Battery capacity)**



Directional Water-Filling

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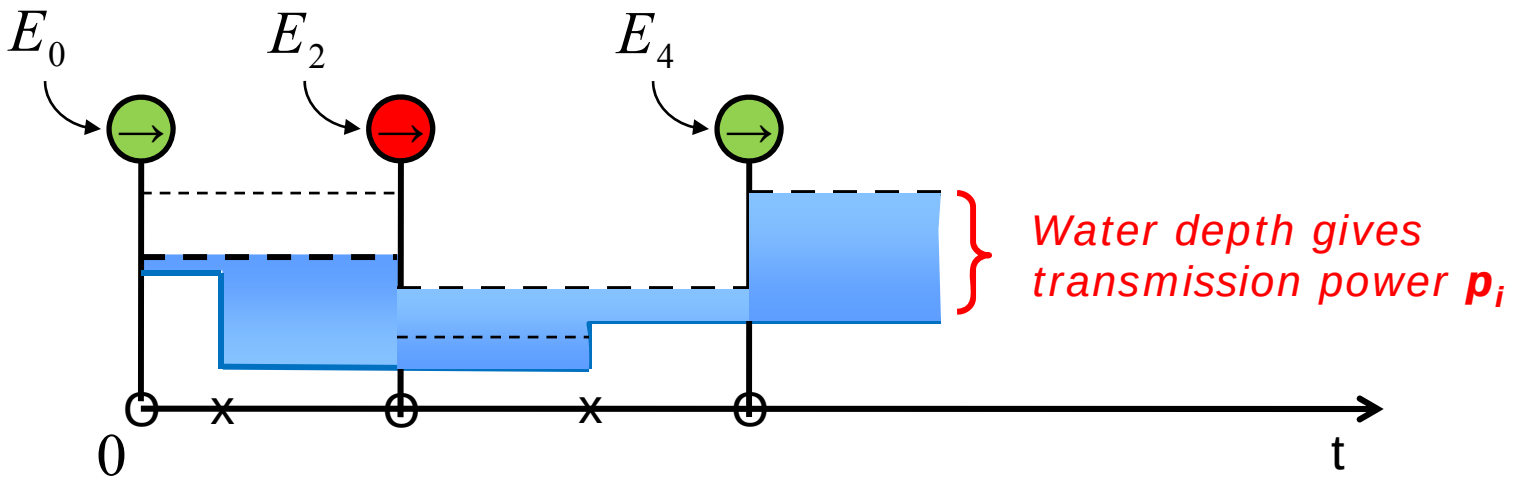
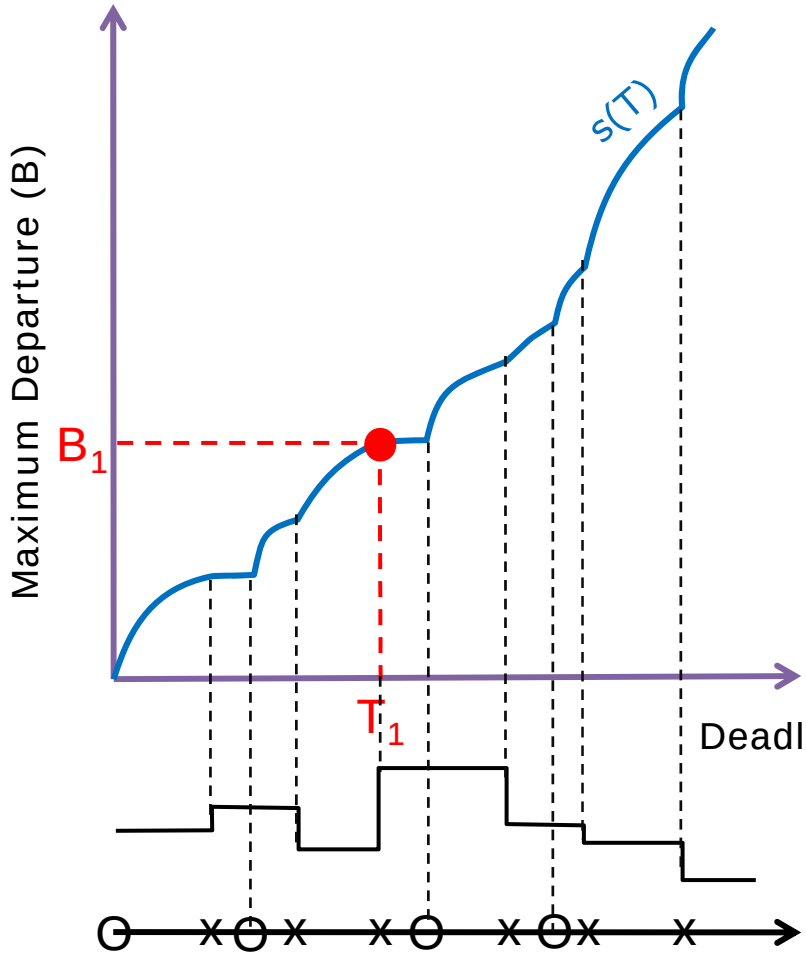
  - Directional water flow **(Energy causality)**
  - Limited water flow **(Battery capacity)**



Maximum Service Curve



- Continuous, **non-decreasing**

  (flat regions when fading is severe)

- Inverse can be considered as the **smallest** T that achieves B_1

Online Algorithms

Optimal online policy can be found using dynamic programming

- States of the system: fade level: h, battery energy: e

$$J_g(e,h,t) = E\left[\int_t^T \frac{1}{2}\log(1+h(\tau)g(e,h,\tau))d\tau\right]$$

$$J(e,h,t) = \sup_g J_g$$

- Quantizing time by δ, $g^*(e,h,k\delta)$ can be found by iteratively solving

$$\max_{g(e,h,t)}\left(\frac{\delta}{2}\log(1+h.g(e,h,t)) + J(e',h',t+\delta)\right)$$

$$\begin{pmatrix} e' = e+\delta(-g(e,h,t)+P_{avg}) \\ h' = E[h(t+\delta)\,|\,h(t)] \end{pmatrix}$$

Online Algorithms

Constant Water Level

- A cutoff fading level h_0 is determined by the **average harvested power** P_{avg} as:

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh = P_{avg}$$

$f(h)$:Fading distribution

- Transmitter uses the corresponding water-filling power if available, is silent otherwise

$$p_i = \left(\frac{1}{h_0}-\frac{1}{h_i}\right)^+$$

Online Algorithms

Time-Energy Adaptive Water-filling

- h_0 determined by **remaining energy scaled by remaining time** as

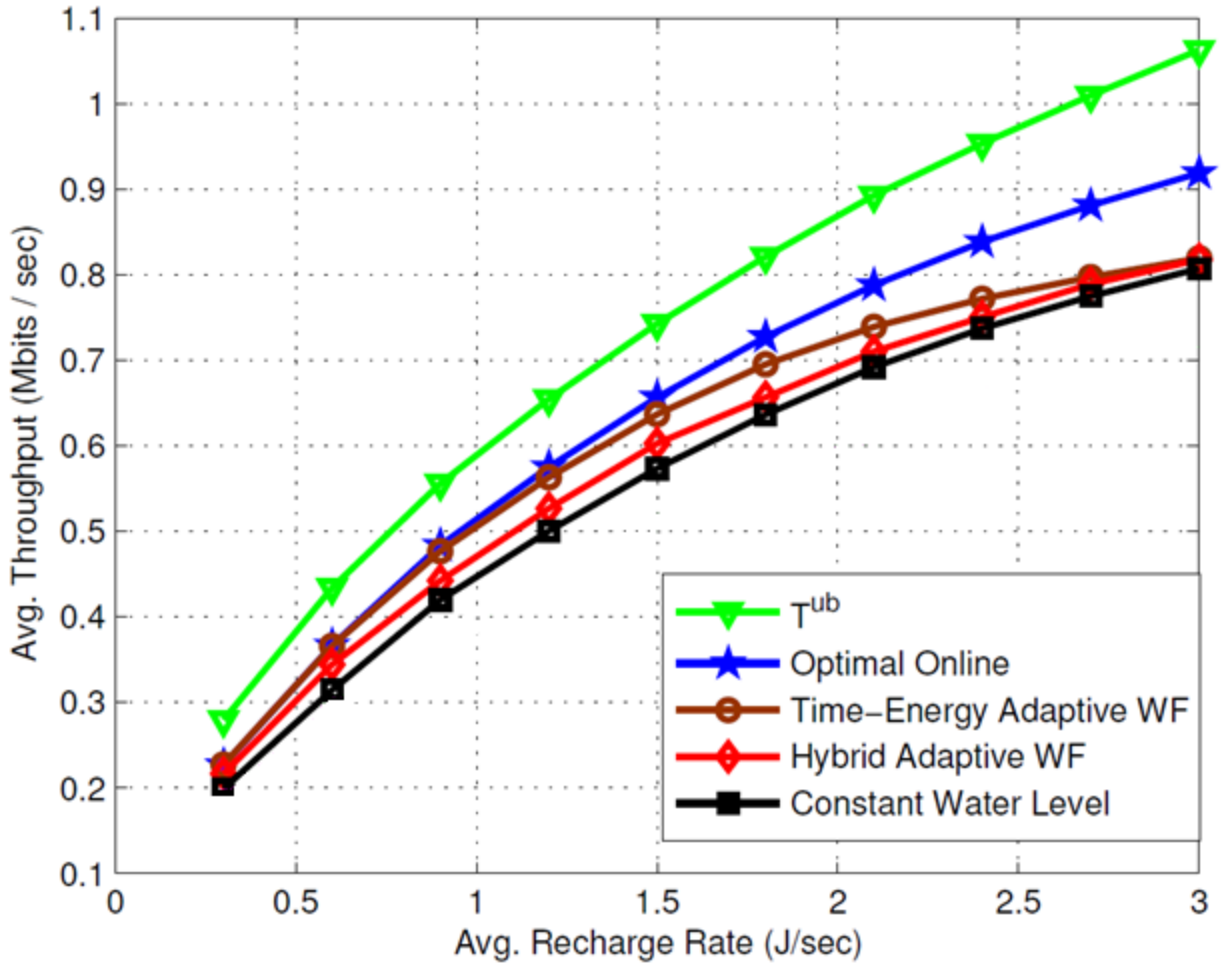
$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh=\frac{E_{current}}{T-t}$$

Hybrid Adaptive Water-filling

- h_0 determined similarly but by **adding average received power**

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh=\frac{E_{current}}{T-t}+P_{avg}$$

Simulations



Performances of the policies w.r.t. **energy arrival rates** under:

- unit mean Rayleigh fading
- T = 10 sec
- E_{max} = 10 J.

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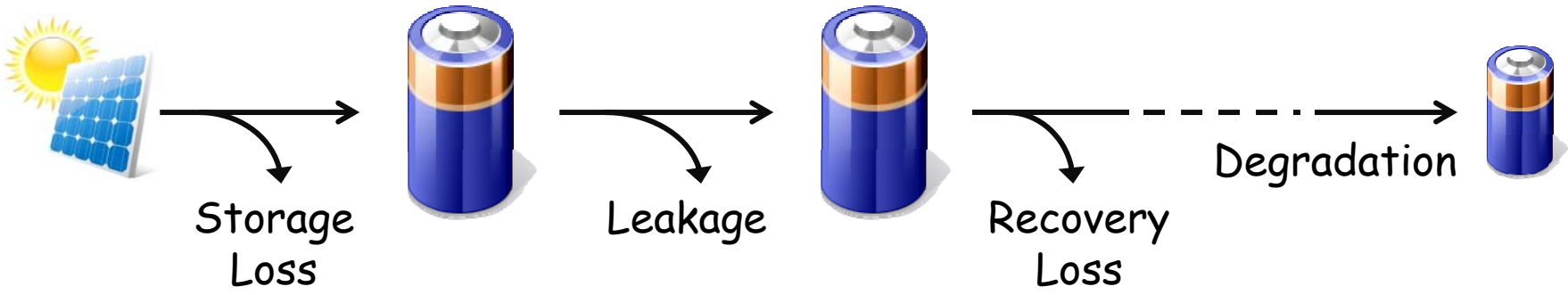
Outline

- Transmission Completion Time Minimization for single link
- Short Term Throughput Maximization for single link with finite battery
- Transmission Completion Time Minimization for single link w/ finite battery
- Extension to fading channels
- **Transmission policies for nodes with inefficient energy storage**
- Energy harvesting receivers
- Energy harvesting multiuser networks
- Energy cooperation in energy harvesting networks
- Information theory of energy harvesting communications (introductory)

Transmission Policies with Inefficient Energy Storage

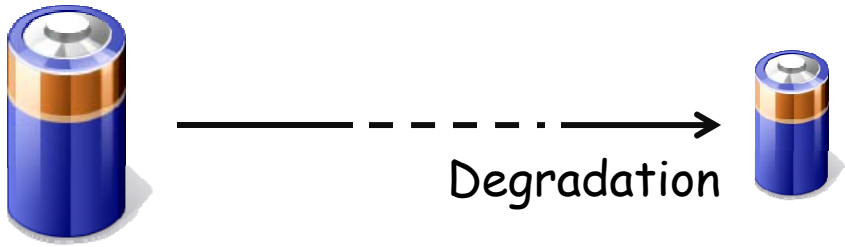
- Energy stored in a **battery, supercapacitor**, . . .
- **"Real life" issues:**



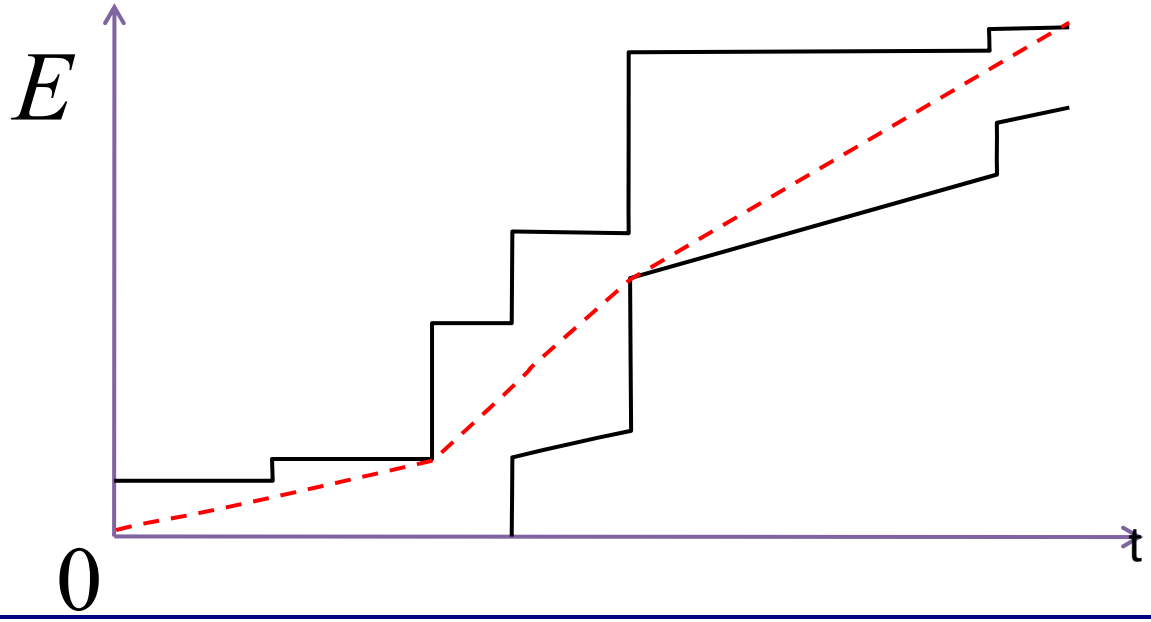
- [Devillers-Gunduz '11]: Leakage and Degradation
- [Tutuncuoglu-Yener-Ulukus'15]: Storage/Recovery Losses

Battery Degradation

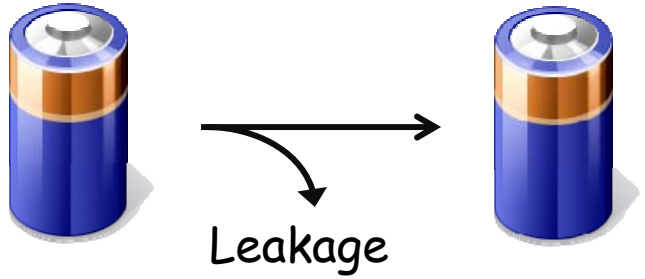
- [Devillers-Gunduz '11]



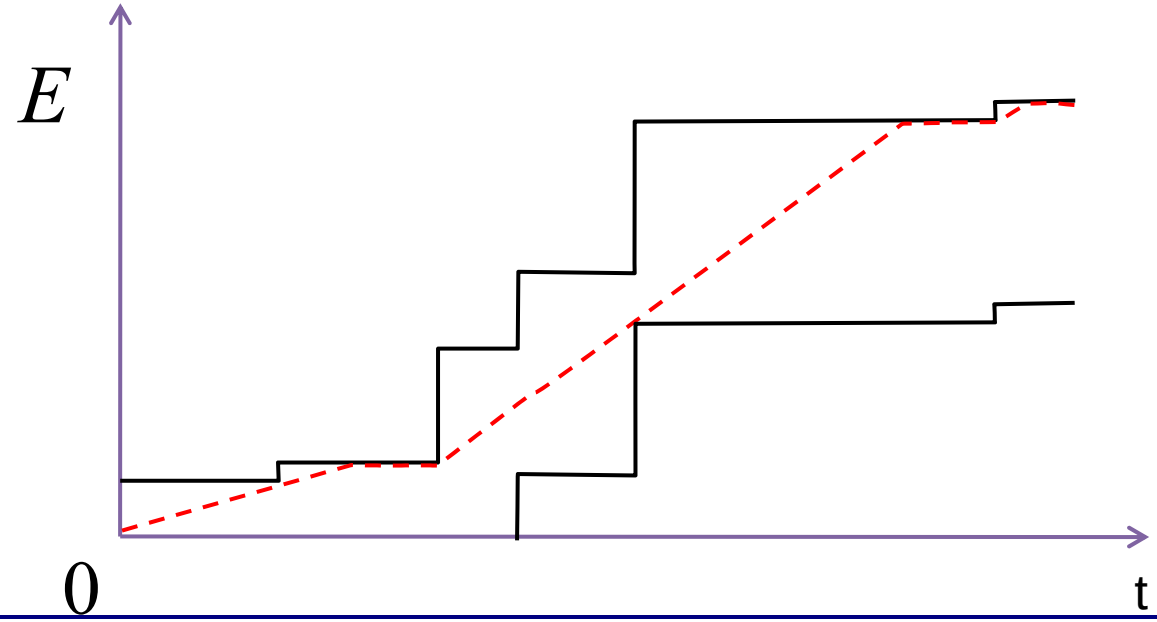
- **Optimal Policy:** Shortest path within **narrowing tunnel**



Battery Leakage

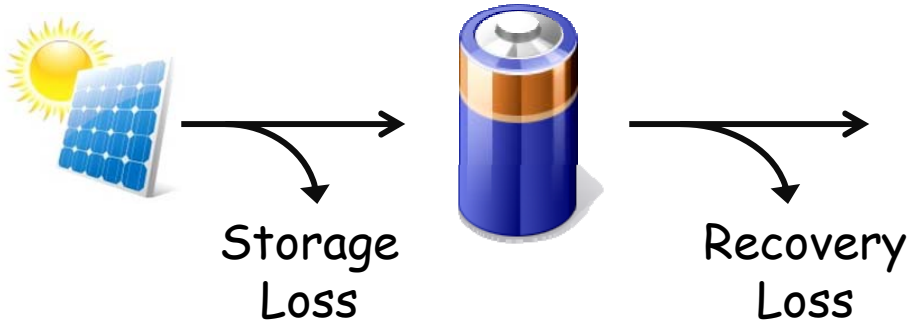


- [Devillers-Gunduz '11]
- **Optimal Policy:** When total energy in an epoch is low, deplete energy earlier to reduce leakage.



Storage/Recovery Losses

- [Tutuncuoglu-Yener-Ulukus'15]

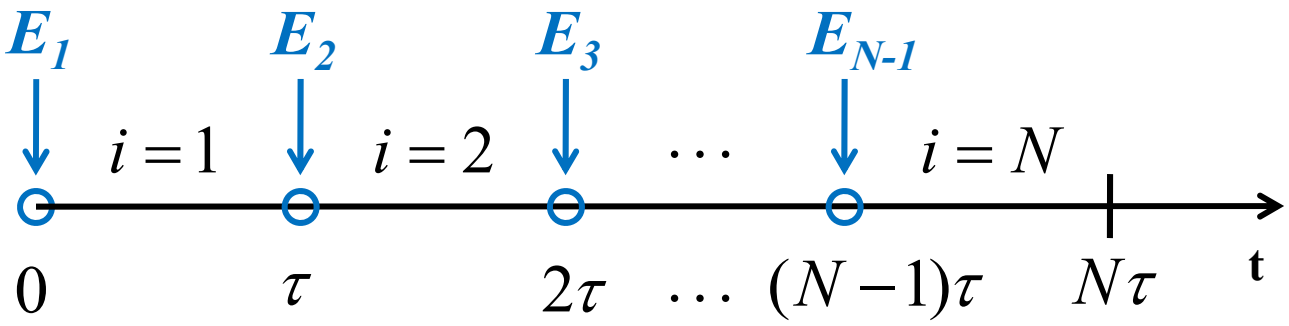


- **Main Tension:**

Concavity of r(p): Use battery to maintain a constant power transmission

Battery inefficiency: Storing energy in battery causes energy loss

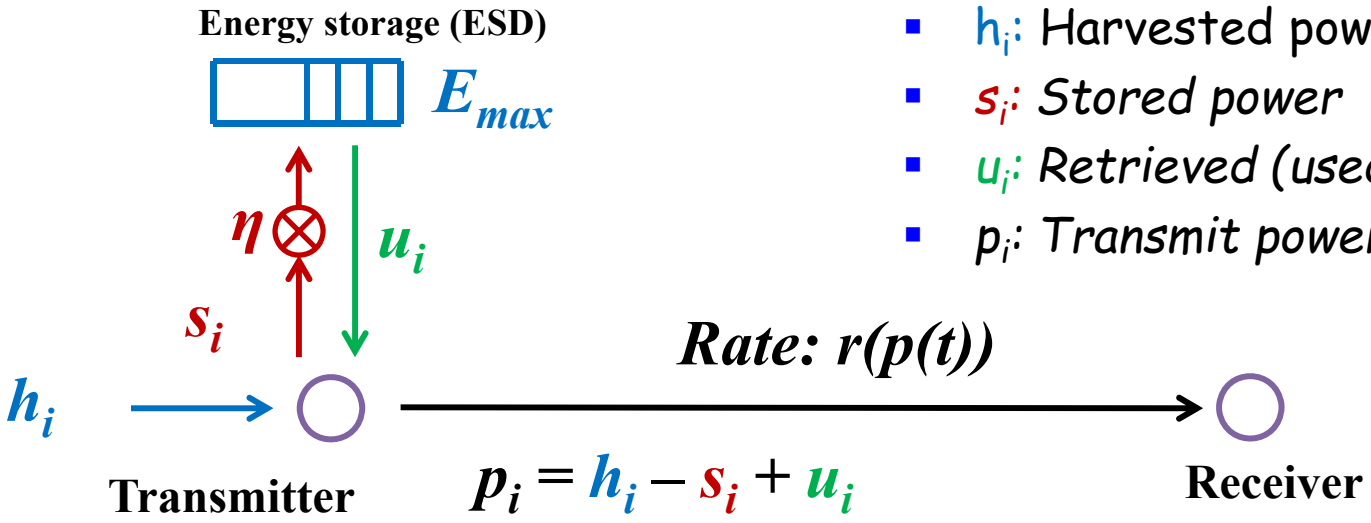
Time slotted model



- **Time slots** of duration $\tau = 1\ s$
- **Energy harvests:** Size E_i at the beginning of time slot i

Energy arrivals known offline first.

System Model



- h_i: Harvested power
- s_i: Stored power
- u_i: Retrieved (used) power
- p_i: Transmit power

- ESD has finite capacity E_{max} and storage efficiency η.
- **Energy Causality:** $\sum_{n=1}^{i} \eta s_n - u_n \geq 0, \qquad i = 1,\ldots,N$
- **Storage Capacity:** $\sum_{n=1}^{i} \eta s_n - u_n \leq E_{\max}, \qquad i = 1,\ldots,N$

Throughput Maximization

- Find optimal energy storage policy that maximizes the average throughput of an energy harvesting transmitter within a deadline of N time slots.

$$\max_{\{s_i, r_i\}} \sum_{i=1}^{N} r(E_i - s_i + u_i)$$

$$s.t. \quad 0 \le E_0 + \sum_{n=1}^{i} (ns_i - u_i) \le E_{\max}, \qquad i = 1, \ldots, N,$$

$$E_i - s_i + u_i \ge 0, \; s_i \ge 0, \; u_i \ge 0, \qquad i = 1, \ldots, N.$$

Throughput Maximization

- Find optimal energy storage policy (γ_i, ρ_i) that maximizes the average transmission rate of an energy harvesting transmitter within a deadline D.

Old problem:

$$\max_{\{p_i\}} \sum_{i=1}^{N} r(p_i)$$

$$s.t. \quad 0 \le \sum_{n=1}^{i} (E_i - p_i) \le E_{\max}, \quad i = 1, \ldots, N,$$

$$p_i \ge 0, \qquad i = 1, \ldots, N.$$

$$\max_{\{s_i, r_i\}} \sum_{i=1}^{N} r(E_i - s_i + u_i)$$

$$s.t. \quad 0 \le E_0 + \sum_{n=1}^{i} (ns_i - u_i) \le E_{\max}, \qquad i = 1, \ldots, N,$$

$$E_i - s_i + u_i \ge 0, \quad s_i \ge 0, \quad u_i \ge 0, \qquad i = 1, \ldots, N.$$

Optimal Power Policy

i) No storage capacity constraint

KKT Stationarity

$$\frac{h}{1+hp_i} - \eta \sum_{n=i}^{N} \lambda_n + \mu_i - \varphi_i = 0, \quad i = 1, \ldots, N,$$

$$\frac{h}{1+hp_i} - \sum_{n=i}^{N} \lambda_n + \mu_i + \psi_i = 0, \quad i = 1, \ldots, N,$$

Complementary Slackness

$$\lambda_i E_{bat,i} = 0, \quad \mu_i (E_i - s_i + u_i) = 0, \quad i = 1, \ldots, N,$$

$$\varphi_i s_i = 0, \quad \psi_i u_i = 0, \quad i = 1, \ldots, N.$$

$$p_i = \frac{1}{\eta \sum_{n=i}^{N} \lambda_n - \mu_i + \varphi_i} - \frac{1}{h} = \frac{1}{\sum_{n=i}^{N} \lambda_n - \mu_i + \psi_i} - \frac{1}{h}, \quad i = 1, \ldots, N,$$

Optimal Power Policy

i) No storage capacity constraint

$$p_i = \frac{1}{\eta \sum_{n=i}^{N} \lambda_n - \mu_i + \varphi_i} - \frac{1}{h} = \frac{1}{\sum_{n=i}^{N} \lambda_n - \mu_i + \psi_i} - \frac{1}{h}, \qquad i = 1, \ldots, N,$$

- When battery is charging ($s_i > 0$)

$$p_i = \frac{1}{\eta \sum_{n=i}^{N} \lambda_n} - \frac{1}{h} = p_{s,i}$$

- When battery is discharging ($u_i > 0$)

$$p_i = \frac{1}{\sum_{n=i}^{N} \lambda_n} - \frac{1}{h} = p_{u,i}$$

$$\frac{1 + h p_{u,i}}{1 + h p_{s,i}} = \eta$$

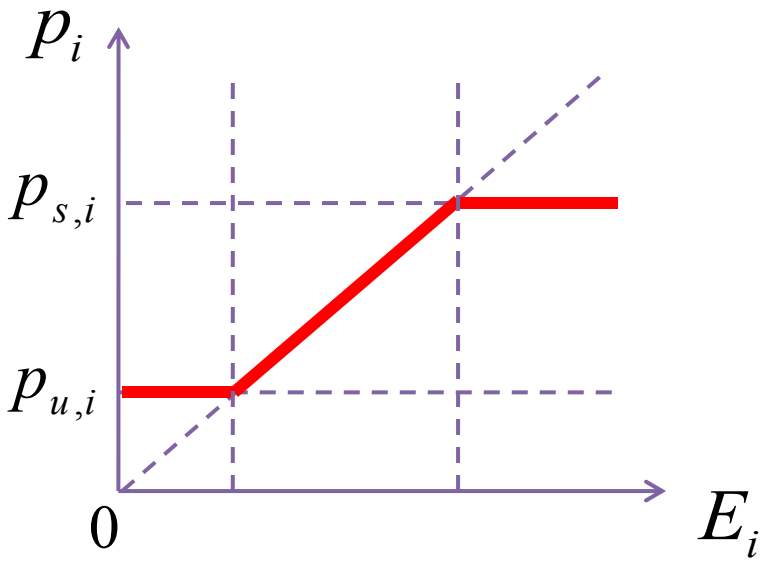
Optimal Power Policy

i) No storage capacity constraint

- **Structure of optimal policy:**

$$p_i = \begin{cases} [p_{s,i}]^+, & E_i \geq p_{s,i}, \\ E_i, & p_{u,i} \leq E_i \leq p_{s,i}, \\ p_{u,i}, & E_i \leq p_{u,i}. \end{cases}$$

"Double Threshold Policy"



Optimal Power Policy

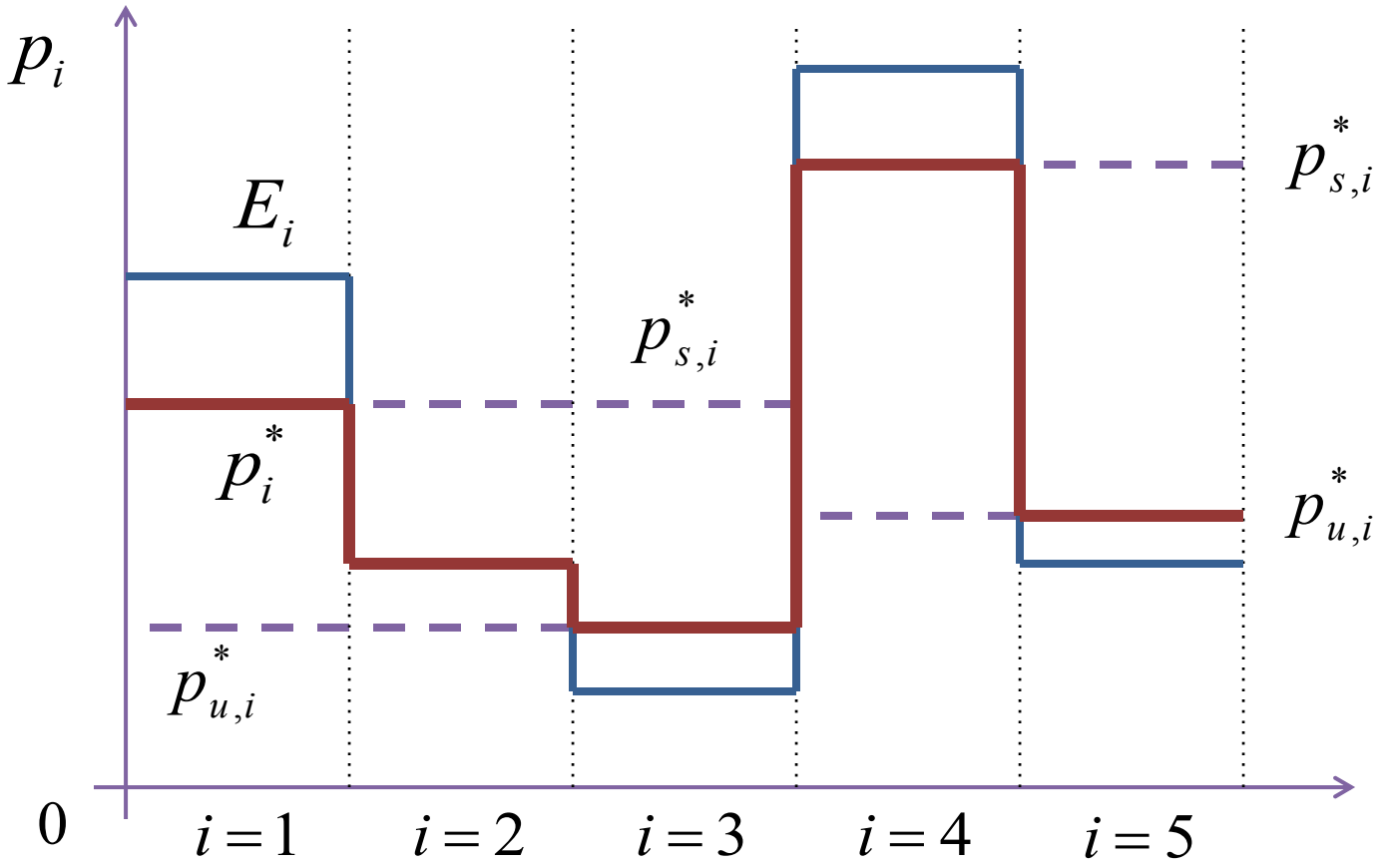
i) No storage capacity constraint

- **How to find $p_{s,i}$ and $p_{u,i}$?**
  - Only change when $E_{bat,i} = 0$ (battery empty)
  - Both increasing in time since $\lambda_i \geq 0$
  - $$\frac{1+hp_{u,i}}{1+hp_{s,i}} = \eta$$

Iteratively find the smallest feasible $p_{s,i}$ and $p_{u,i}$ that depletes the battery in the future.

Optimal Power Policy

i) No storage capacity constraint



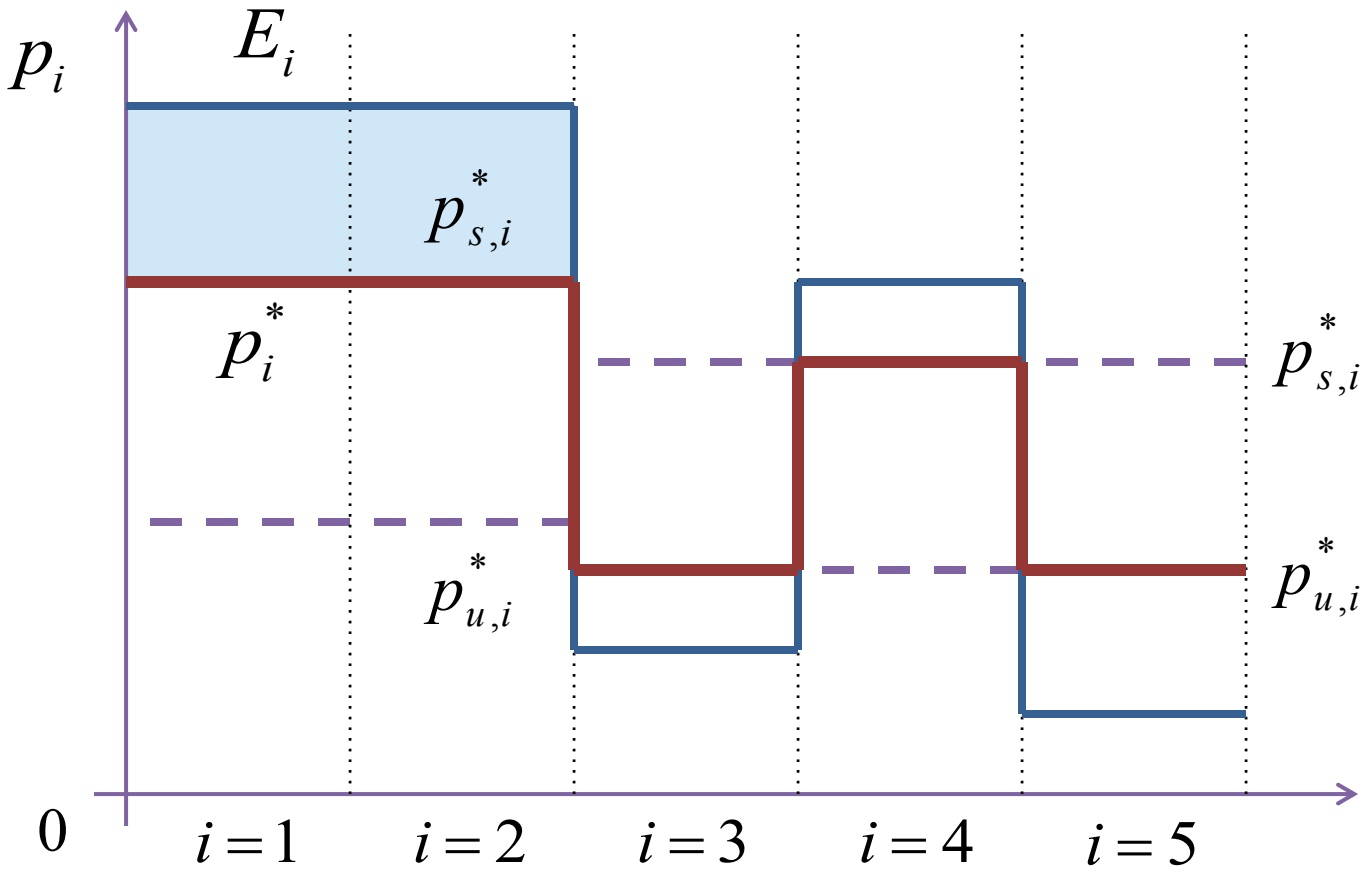
Optimal Power Policy
ii) Finite-sized storage

- **How to find $p_{s,i}$ and $p_{u,i}$?**
  - Only change when $E_{bat,i} = 0$ or $E_{bat,i} = E_{\max}$
  - Increasing if $E_{bat,i} = 0$ and decreasing if $E_{bat,i} = E_{\max}$
  - $$\frac{1+hp_{u,i}}{1+hp_{s,i}} = \eta$$

Find feasible pairs $(p_{s,i}, p_{u,i})$ that fill or deplete the battery, choose the one based on the second property.

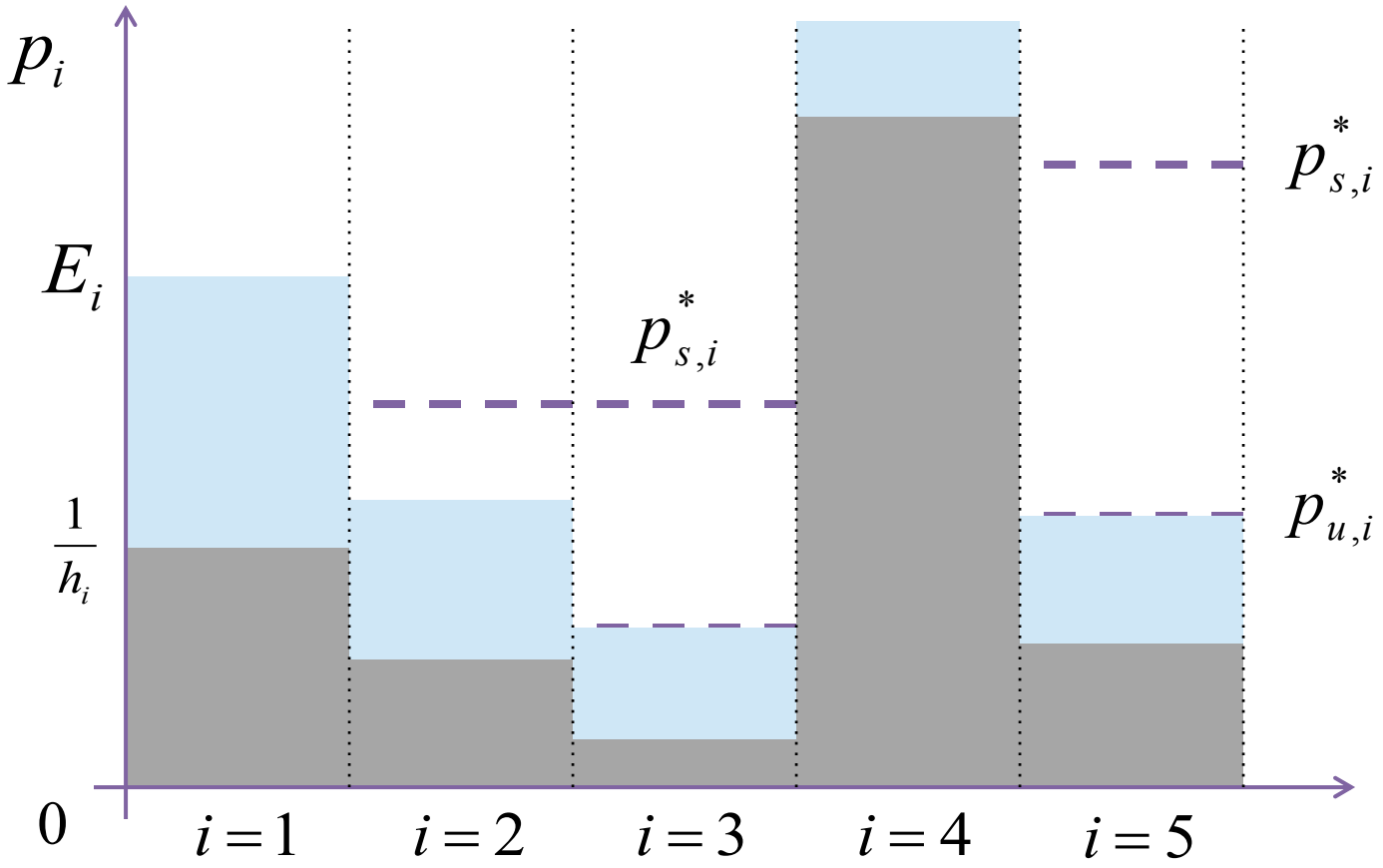
Optimal Power Policy

ii) Finite-sized storage



Optimal Power Policy

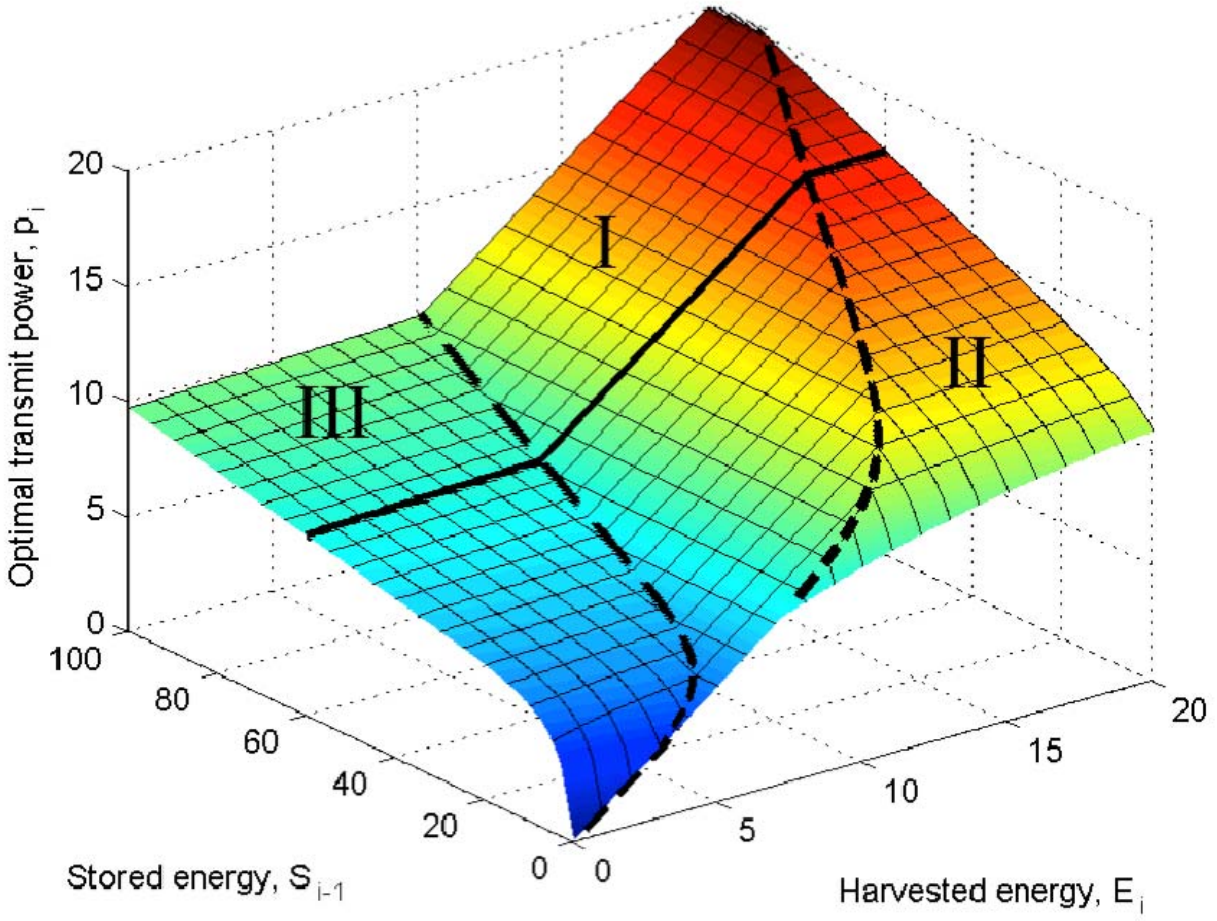
iii) Fading channel



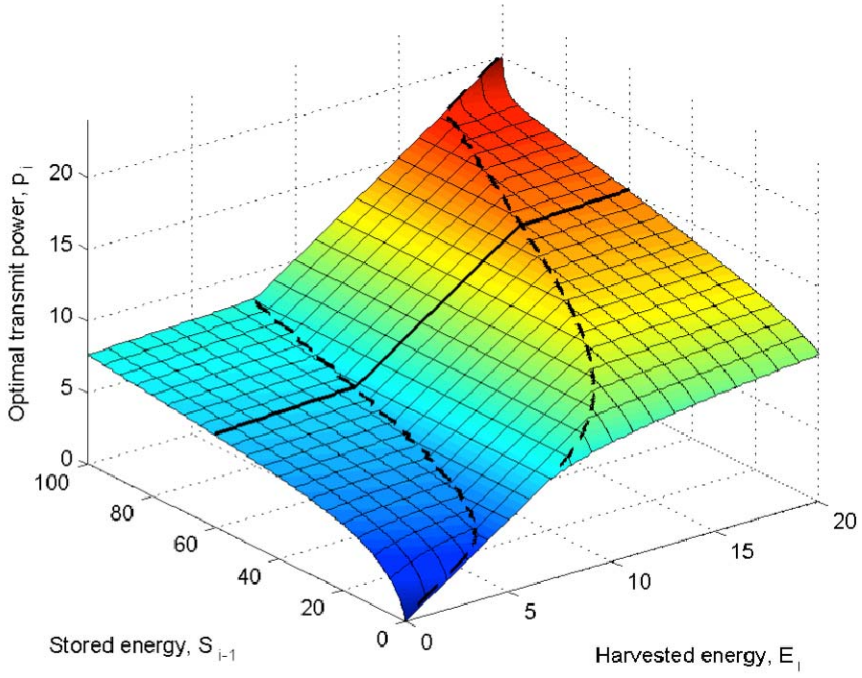
Optimal Online Policy

- So far, we have discussed **offline** policies.
- Energy harvesting scenario may not be predictable, or may not be available prior to transmission
- **Markov Decision Process (MDP)** formulation:
  - Action: $p_i = g_i(E^i, h^i)$
  - Value: $$J_i(E^i, h^i) = \max_{\pi_i} r\left(g_i(E^i, h^i), h_i\right) + \mathbf{E}\left[\sum_{n=i+1}^{N} r\left(g_i(E^i, h^i), h_i\right)\right]$$
$$= \max_{\pi_i} r\left(g_i(E^i, h^i), h_i\right) + \mathbf{E}\left[J_{i+1}(E^{i+1}, h^{i+1})\right]$$

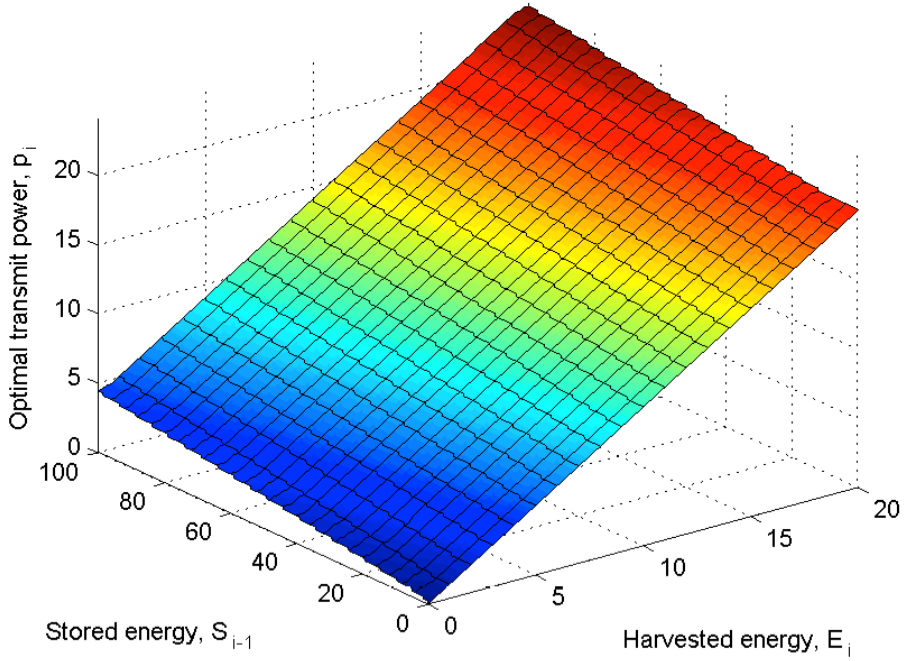
Optimal Online Policy



Optimal Online Policy



Bursty energy harvesting



Random walk energy harvesting

Proposed Online Policy

- Both offline and online policies point to **thresholds**
- Choose **fixed thresholds** throughout transmission

$$p_i = \begin{cases} \max\{p_s, E_i + S_i - E^{\max}\} & E_i \geq p_s \\ E_i & p_u \leq E_i \leq p_s \\ \min\{p_u, E_i + S_i\} & E_i \leq p_u \end{cases}$$

to satisfy

$$\eta \int_{p_s}^{\infty} (e - p_s) p_E(e) de - \int_0^{p_u} (p_u - e) p_E(e) de = 0$$

Simulations

$N = 10^4$ *time slots*

$\tau = 10$ *ms*

$E_{\max} = 1$ *mJ*

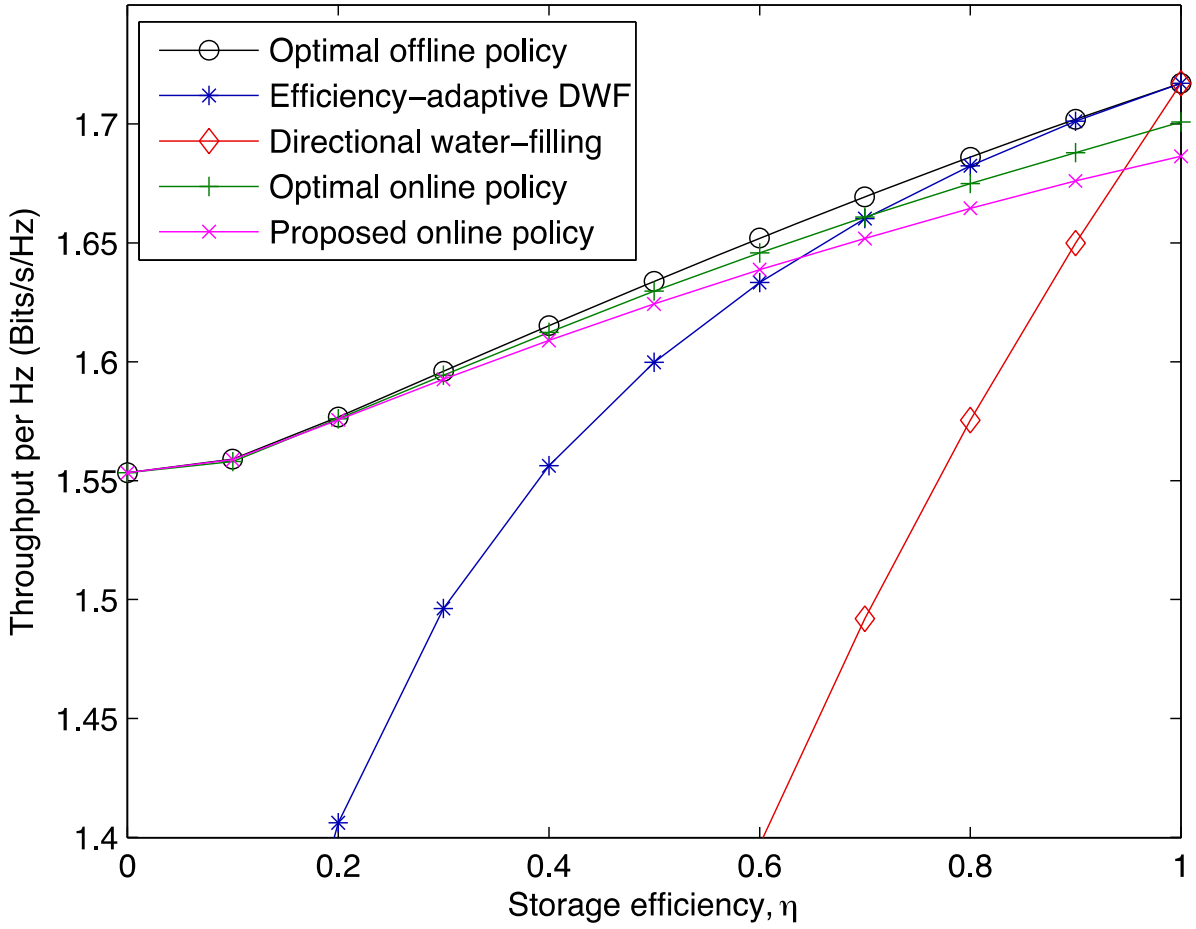
$E_0 = 0$

$E_i \sim$ *i.i.d.* $U[0,200]\,\mu J$

$h = -100$ *dB*

$B = 1$ *MHz*

$N_0 = 10^{-19}$ *W/Hz*



Simulations

$N = 10^4$ *time slots*

$\tau = 10$ *ms*

$E_{\max} = 1$ *mJ*

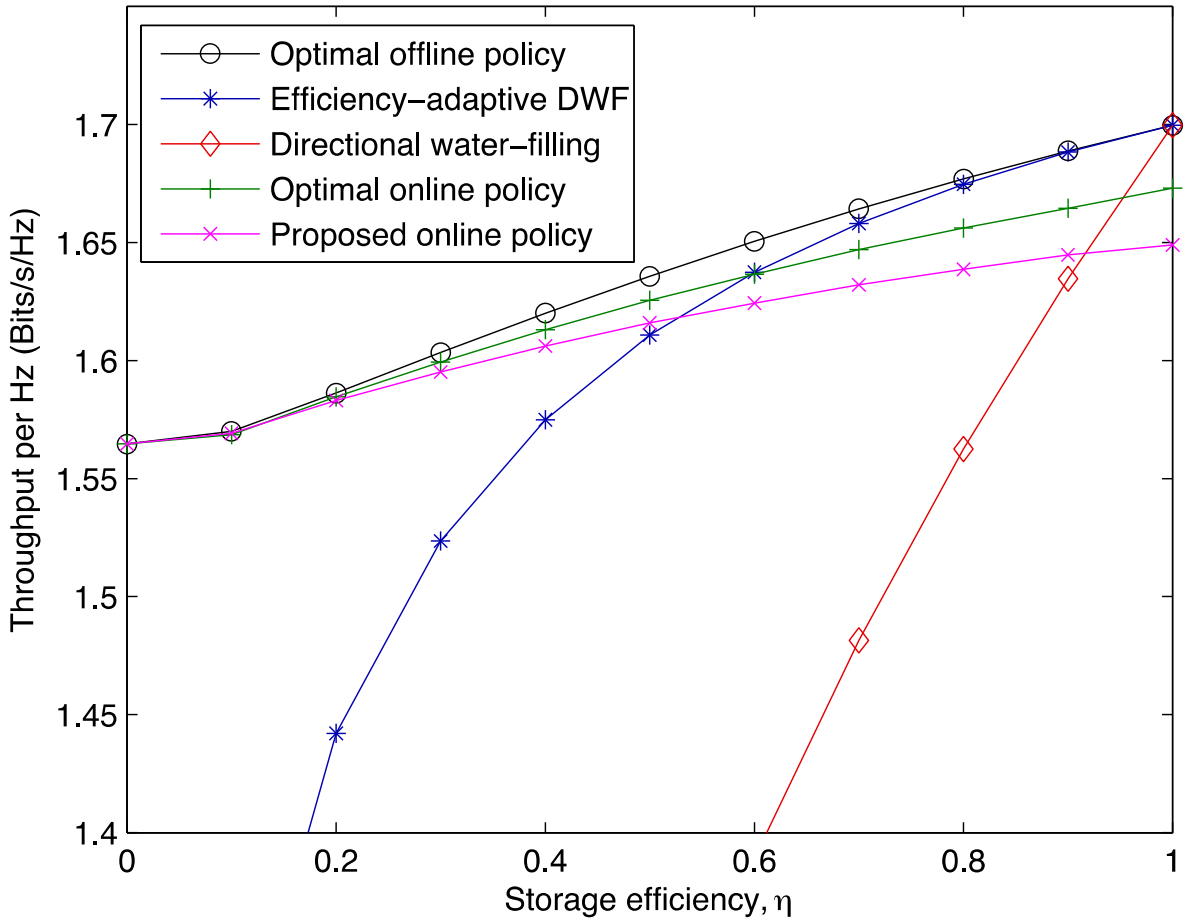
$E_0 = 0$

$E_i \sim Bursty(100\mu J)$

$h = -100$ *dB*

$B = 1$ *MHz*

$N_0 = 10^{-19}$ *W/Hz*



Simulations

$N = 10^4$ *time slots*

$\tau = 10$ *ms*

$E_{\max} = 1$ *mJ*

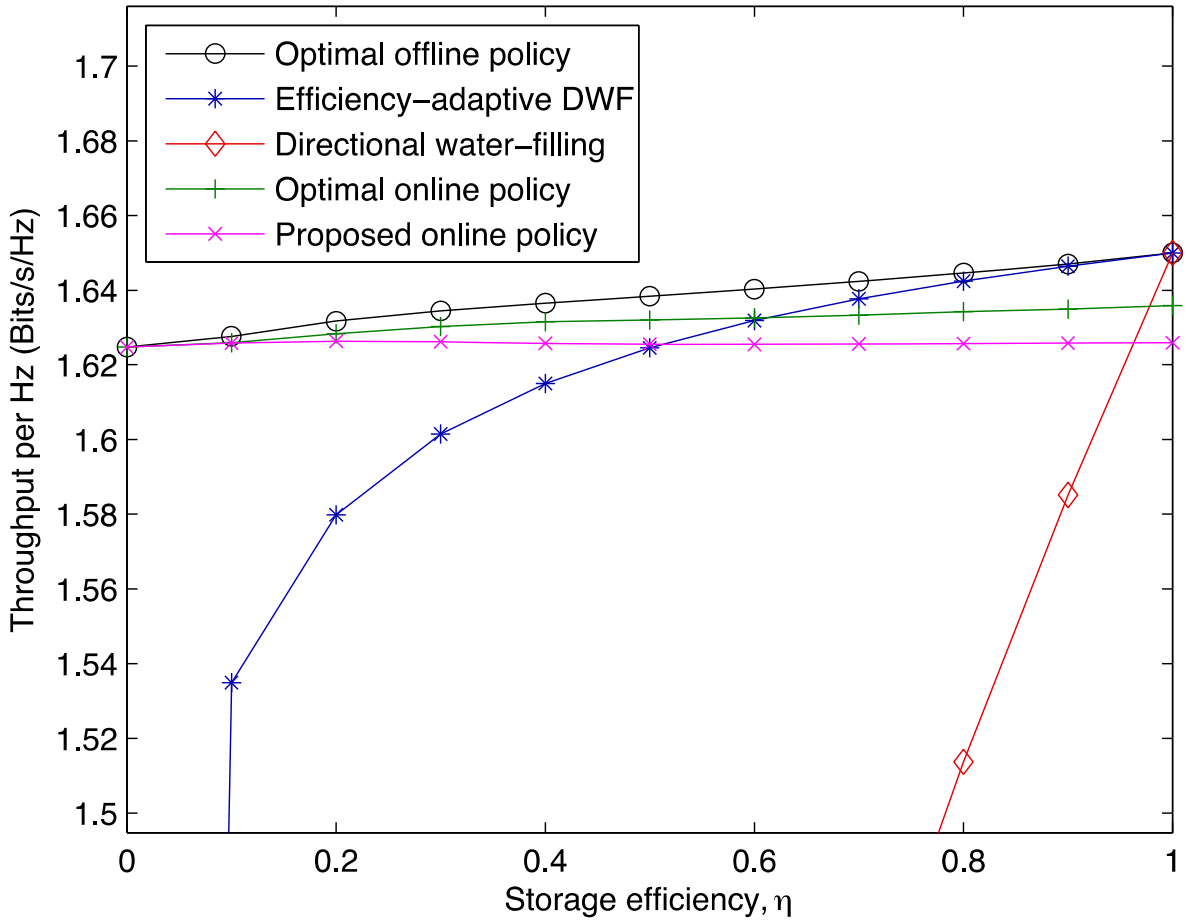
$E_0 = 0$

$E_i \sim RW(100\mu J)$

$h = -100$ *dB*

$B = 1$ *MHz*

$N_0 = 10^{-19}$ *W/Hz*



Simulations

$N = 10^4$ *time slots*

$\tau = 10$ *ms*

$\eta = 0.66$

$E^{\max} = 1$ *mJ*

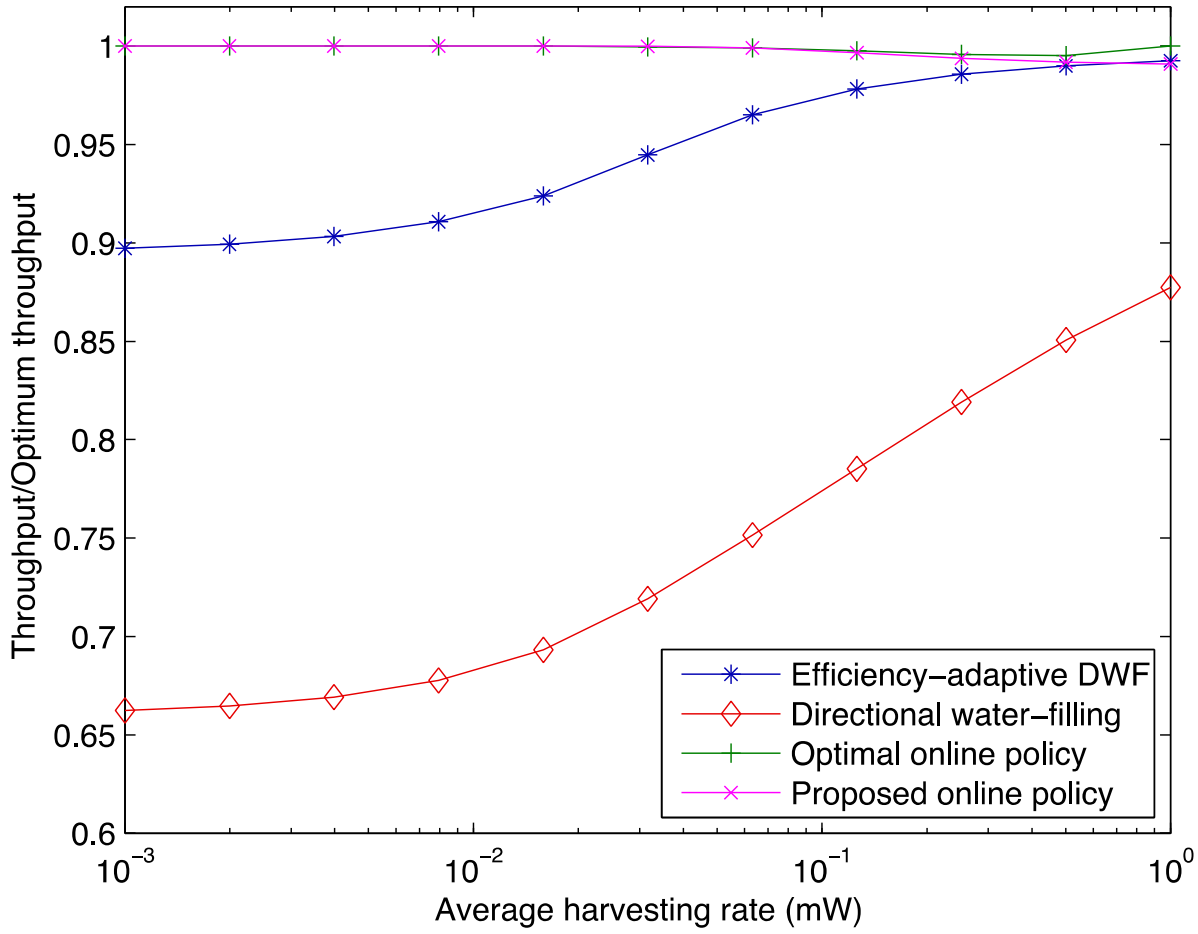
$E_0 = 0$

E_i~*i.i.d. Unif.*

$h = -100$ *dB*

$B = 1$ *MHz*

$N_0 = 10 - 19$ *W/Hz*



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- Energy cooperation in energy harvesting networks
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Conclusion

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- In this tutorial, we covered energy efficient design (optimal scheduling policies) for one energy harvesting (rechargeable) transmitter.
- New networking paradigm: energy harvesting nodes
- New design insights arising from
  - new energy constraints
  - energy storage limitations and inefficiencies
  - interaction of multiple EH transmitters
  - energy cooperation
- New problems in the information theory domain
- Lots of open problems related to all layers of the network design: e.g. signal processing/PHY design; MAC protocol design; channel capacity…

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