Scheduling in Energy Harvesting Networks
and
Energy Cooperation in Energy Harvesting Networks
and
Information Theory for Energy Harvesting Communications

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ICC, June 2013

So Far, We Learned...

- **Single-user** communication with an energy harvesting transmitter.

- Energy arrives (is harvested) **during the communication session**.

- A non-trivial shift from the conventional battery powered systems.

- Transmission policy is **adapted to energy arrivals**.

- Energy causality constraint and battery capacity limit.

- Objective: Maximize average throughput
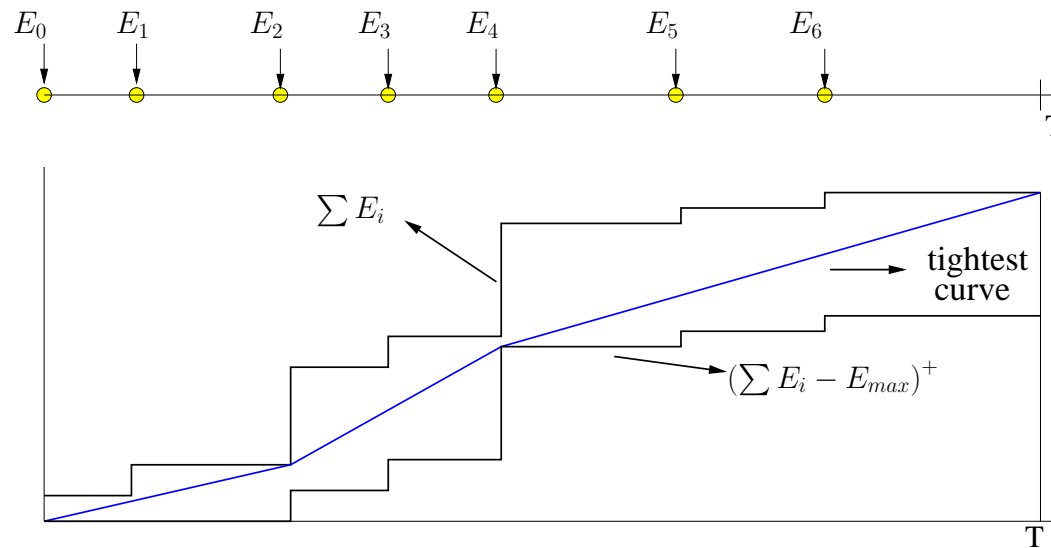
E

B_1

Tx

Rx
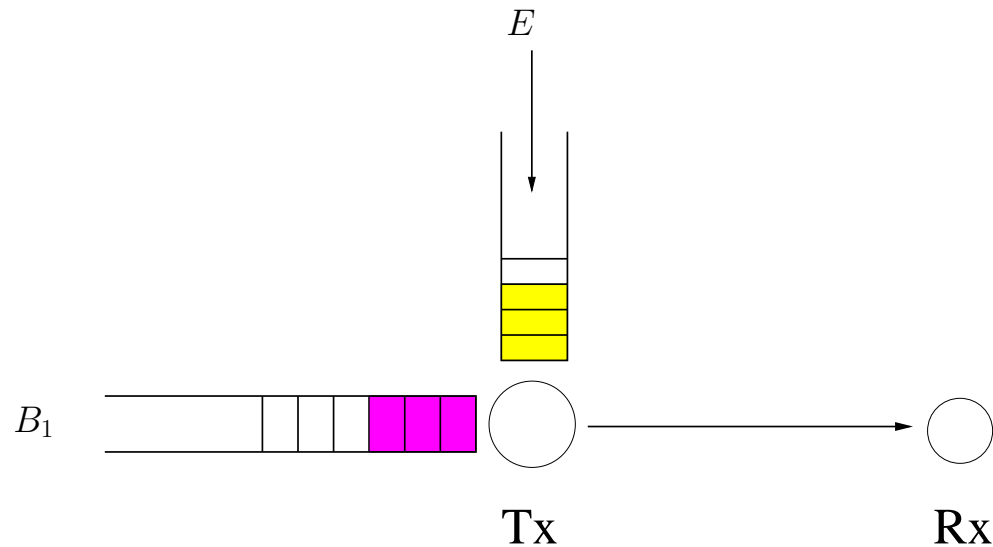
The Optimal Policy for $E_{max} = \infty$
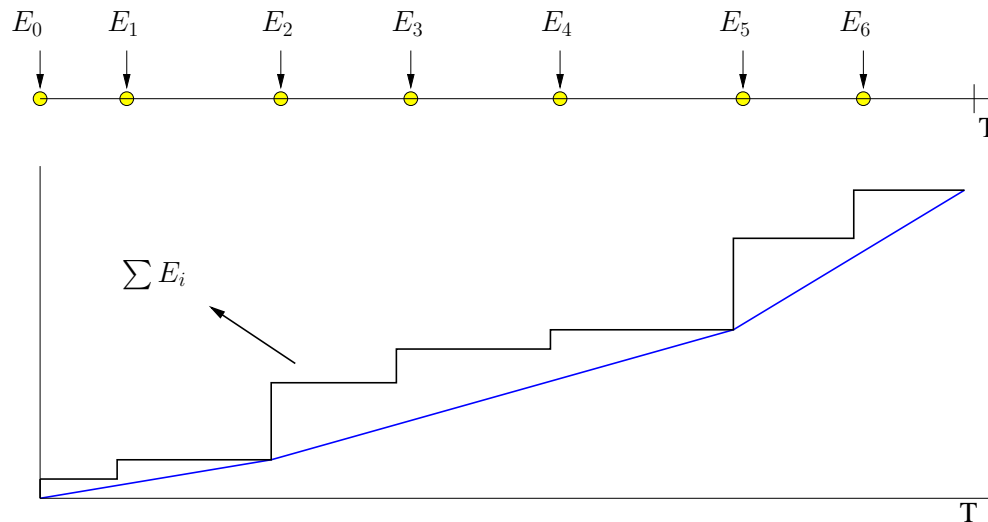
- Upper staircase is the cumulative energy arrivals

- Feasible energy consumption lies below the staircase

- Transmit power remains constant in each epoch

- The tightest curve under the cumulative energy arrival staircase

The Optimal Policy for $E_{max} < \infty$

- Upper staircase: energy arrivals

- Lower staircase: finite battery constraint (no overflows)

- Any feasible energy consumption curve must lie **in between**

- Power remains constant in each epoch

- The tightest curve in the feasibility tunnel

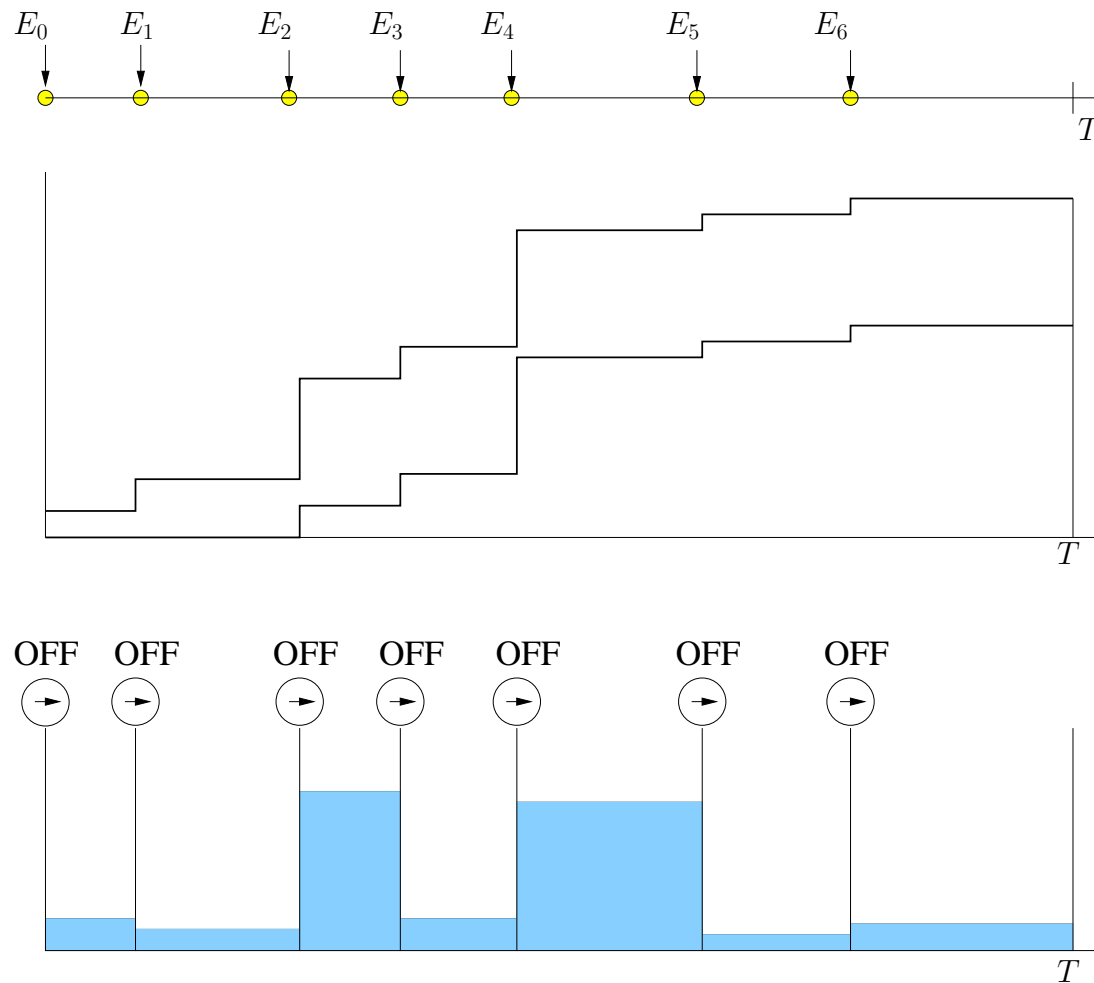# Single-User Optimal Policy for Fading Channel

- **Directional water-filling algorithm.**

- First: fill each incoming energy till next energy arrival.
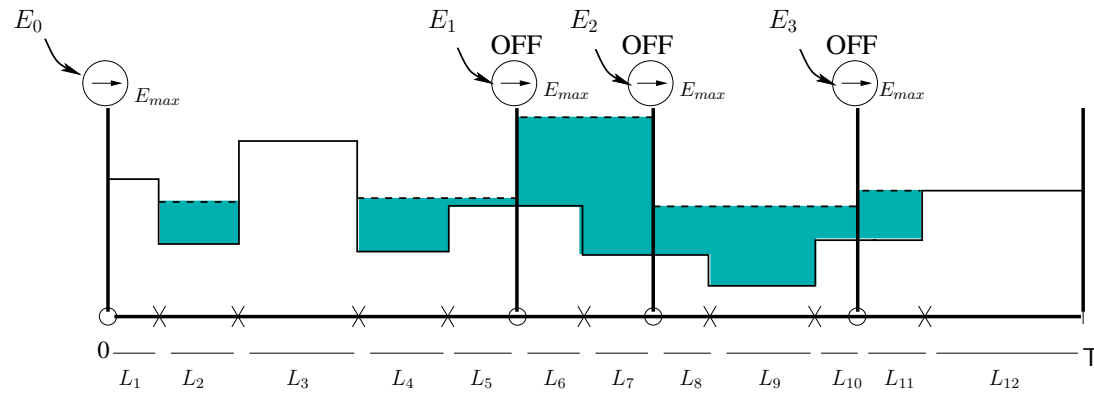

- Second: Allow transfer of energy one by one to the right only.

- Equalize the water level if the water level is higher in the left.
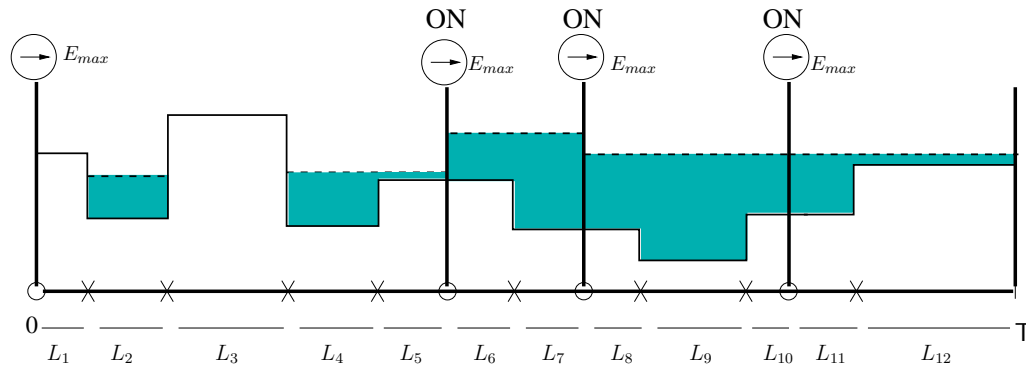
Equivalence of Feasibility Tunnel and Directional Water-filling

E_0 E_1 E_2 E_3 E_4 E_5 E_6

T

T

OFF   OFF   OFF   OFF   OFF   OFF   OFF

T

Equivalence of Feasibility Tunnel and Directional Water-filling

E_0 E_1 E_2 E_3 E_4 E_5 E_6

T

tightest
curve

T

ON   ON   ON   ON   ON   ON   ON

T

causality   no overflow
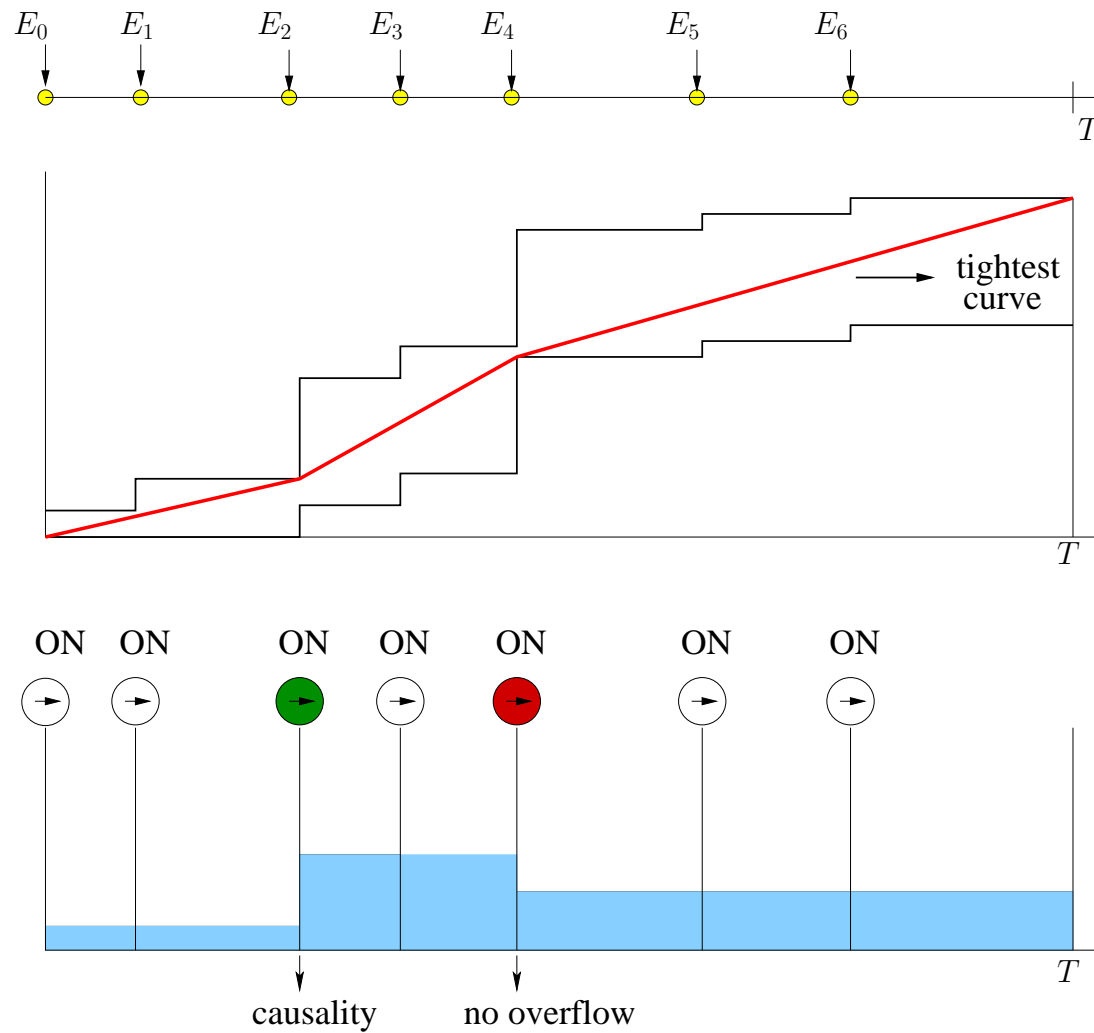
Scheduling in Multi-user Energy Harvesting Systems

- Broadcasting with an energy harvesting transmitter

  – An energy harvesting transmitter sends messages to two users

  – E.g., a wireless access device sending different messages to users

- Multiple access with energy harvesting transmitters

  – Energy harvesting transmitters communicating with a single receiver

  – E.g., multiple sensors sending data to a center

- Interference channel with energy harvesting transmitters

  – Tx-Rx pairs communicate simultaneously where Txs are energy harvesting.

  – E.g., multiple sensors sending data to different centers.

- Two-hop communication with energy harvesting nodes

  – Source and relay nodes send messages using harvested energy.

  – E.g., end-to-end data delivery in sensor networks.

Broadcasting with an Energy Harvesting Transmitter

E

B_1

B_2

Tx

Rx1

Rx2
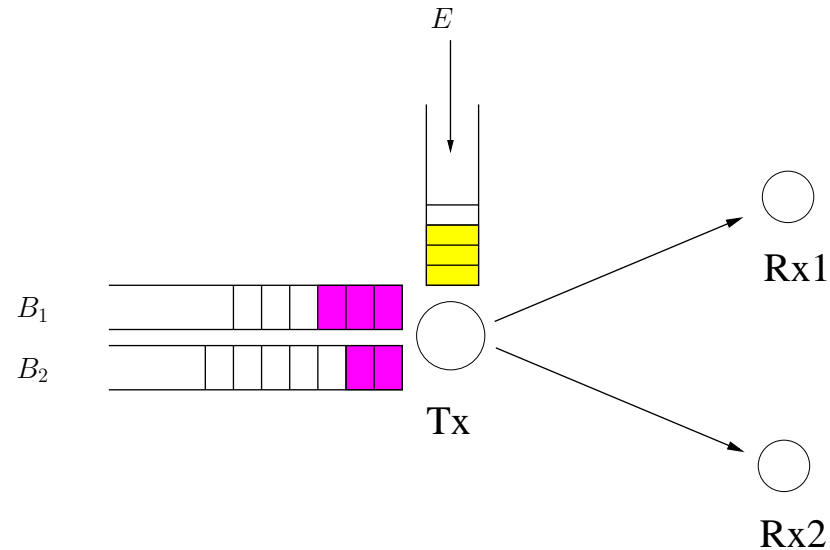
- Energy arrives (is harvested) **during the communication session**.

- Assume battery has infinite storage capacity: $E_{max} = \infty$

- Broadcasting data to two users by **adapting to energy arrivals**

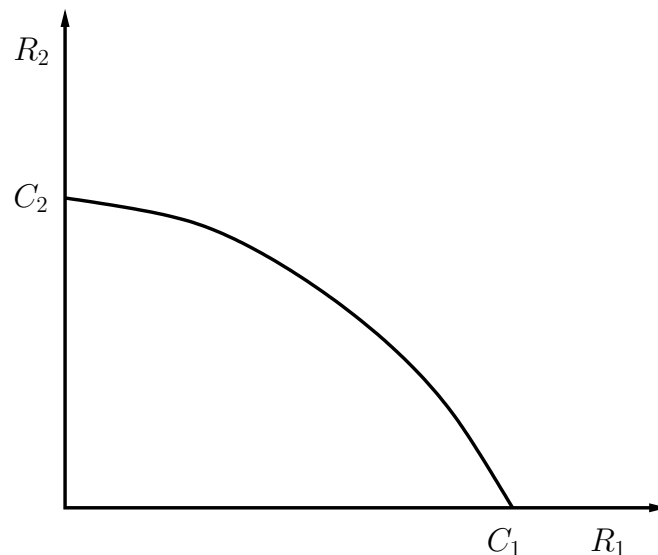- Objective: maximize the data departure region

Broadcast Channel Model
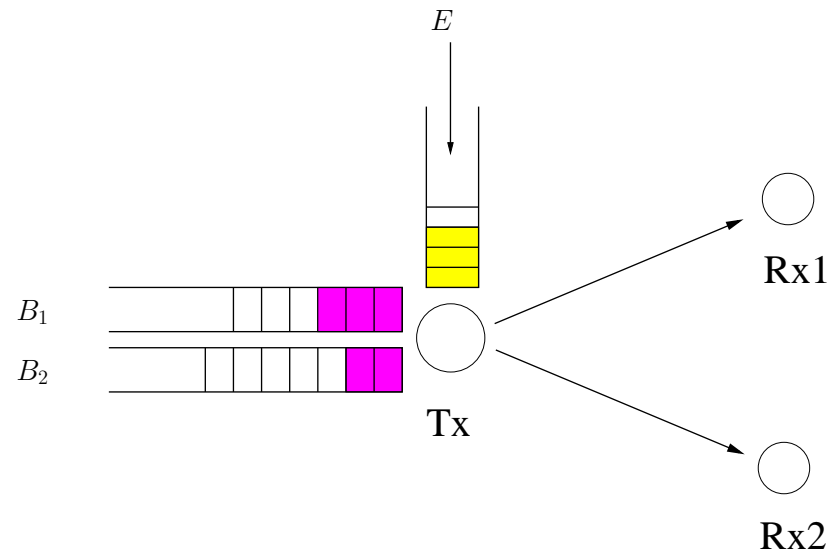
- AWGN broadcast channel:

$$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

where $N_1 \sim \mathcal{N}(0,1)$, $N_2 \sim \mathcal{N}(0,\sigma^2)$

- $\sigma^2 > 1$: 2nd user is degraded

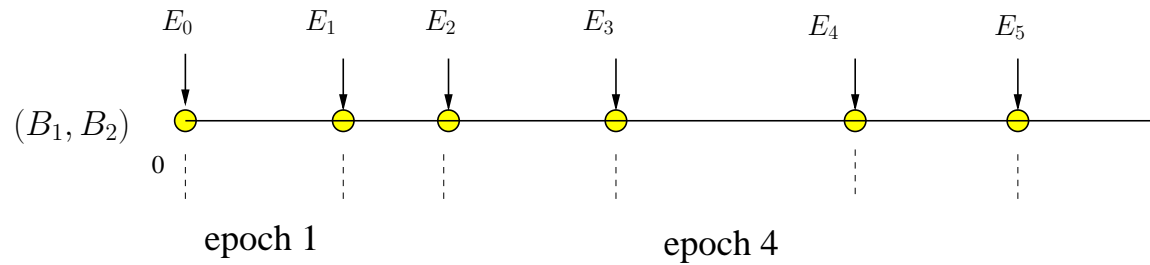- We call 1st user stronger and 2nd user weaker

Broadcast Channel Model

$$r_1 \leq \frac{1}{2} \log_2 \left(1 + \alpha P \right)$$

$$r_2 \leq \frac{1}{2} \log_2 \left(1 + \frac{(1-\alpha)P}{\alpha P + \sigma^2} \right)$$

- We work in the (r_1, r_2) domain:

$$P = 2^{2(r_1 + r_2)} + (\sigma^2 - 1)2^{2r_2} - \sigma^2 \triangleq g(r_1, r_2)$$

- $g(r_1, r_2)$ is the minimum power required to send at rates (r_1, r_2)

Energy Model

Energy $E_0, E_1, E_2, E_3, E_4, E_5$ with (B_1, B_2) at time 0, epoch 1 ... epoch 4
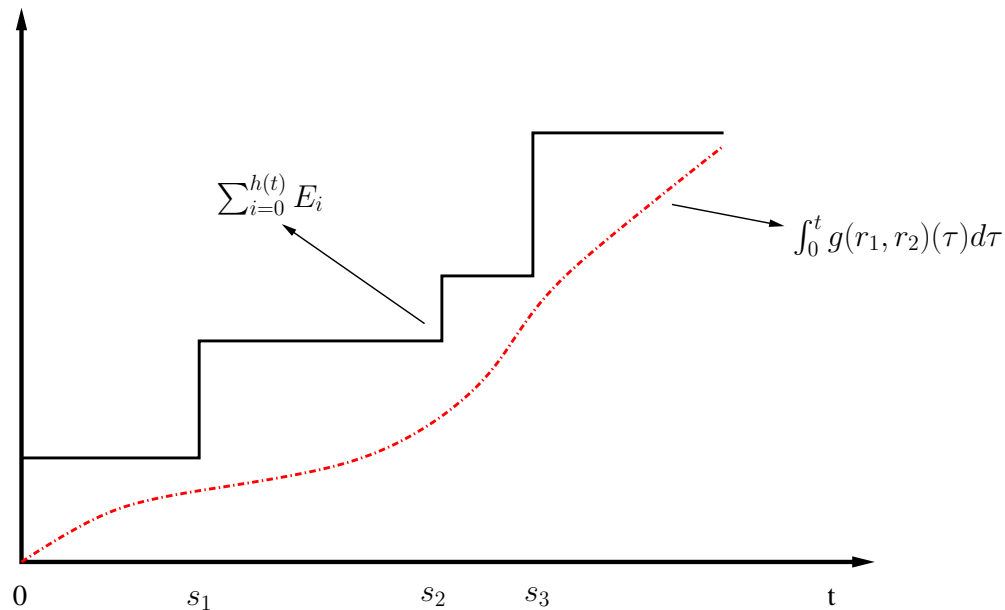
- Energy is *harvested* **during the course of communication**.

- We will consider offline policies.

- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} g(r_1, r_2)(\tau) d\tau \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

Constraints on the Power Policy

- Energy arrivals known deterministically a priori

- Upper staircase: energy arrivals

- Any feasible energy consumption curve must lie **below the upper staircase**

Find the Maximum Departure Region

- The maximum departure region $\mathcal{D}(T)$: union of (B_1, B_2) pairs achievable by some rate allocation policy that satisfies the energy causality constraint.
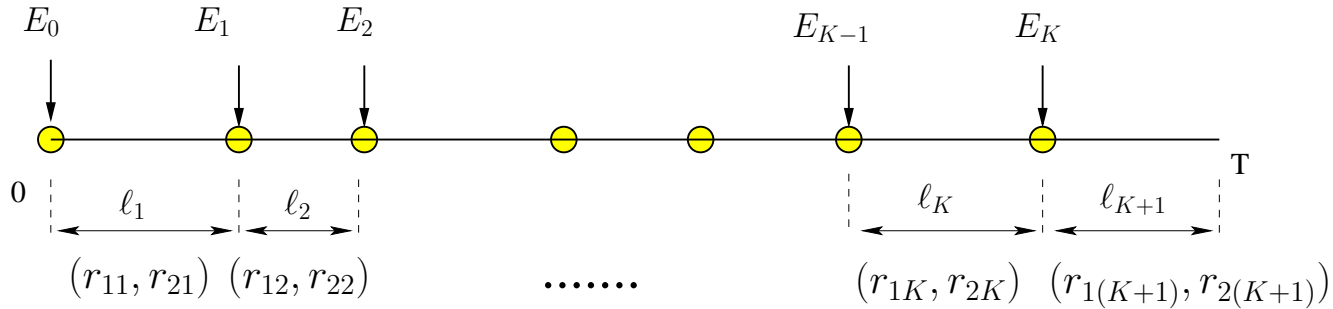

- Transmission rates, and power, remain constant between energy harvests.

- Denote the rates that go to users as (r_{1i}, r_{2i}) over epoch i.

- The **power** at epoch i: $g(r_{1i}, r_{2i})$

- The **energy spent** during epoch i: $g(r_{1i}, r_{2i})\ell_i$

- The energy causality constraint reduces to constraints on (r_{1i}, r_{2i}):

$$\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$

14

Finding the Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.

- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\max_{\mathbf{r}_1, \mathbf{r}_2} \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i$$

$$\text{s.t.} \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$

Finding the Maximum Departure Region
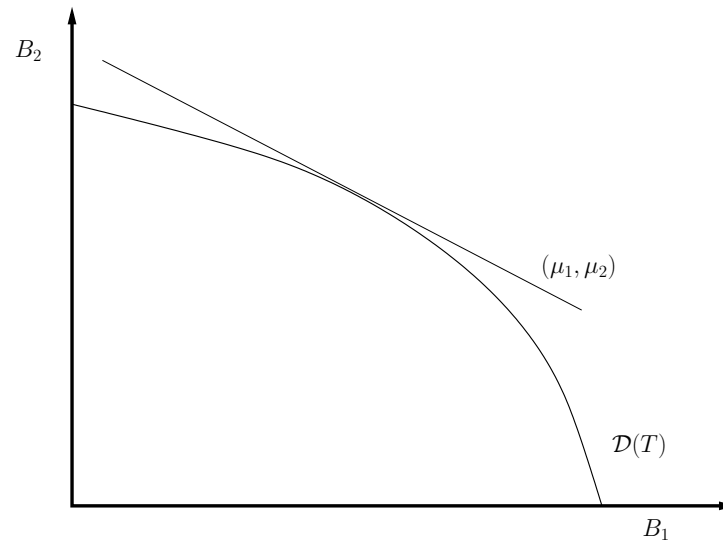
- The Lagrangian function

$$\mathcal{L} = \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i - \sum_{k=1}^{K+1} \lambda_k \left(\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i - \sum_{i=0}^{k-1} E_i \right)$$

$$+ \sum_{i=1}^{K+1} \gamma_{1i} r_{1i} + \sum_{i=1}^{K+1} \gamma_{2i} r_{2i}$$

- Total power in terms of Lagrange multipliers

$$P_i = \max \left\{ \frac{\mu_1 + \gamma_{1i}}{\sum_{k=i}^{K+1} \lambda_k} - 1, \frac{\mu_2 + \gamma_{2i}}{\sum_{k=i}^{K+1} \lambda_k} - \sigma^2 \right\}$$

- Structural properties of the optimal policy:

  - Optimal total transmit power, $\{g(r_{1i}^*, r_{2i}^*)\}_{i=1}^{K+1}$, is independent of μ_1, μ_2.

  - In particular, it is the same as the optimal single-user transmit power.

Single User Optimal Policy

- Single user optimal policy is found by calculating the tightest curve below the energy arrival curve:
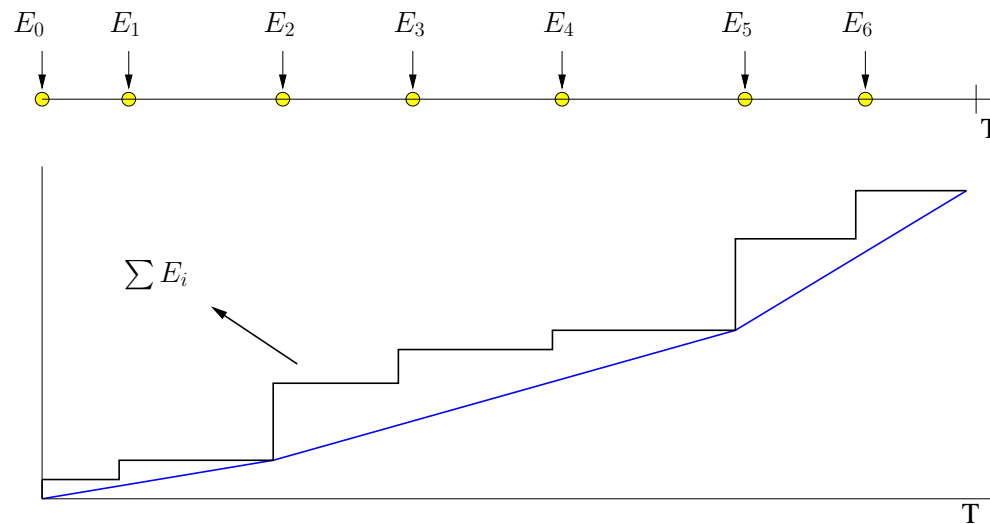
- Slope of the curve is the allocated power

- Power is monotonically increasing

Structure of the Optimal Policy

- Total transmit power is the same as the single-user case.

- The power shares follow a cut-off structure:

- Cut-off level P_c

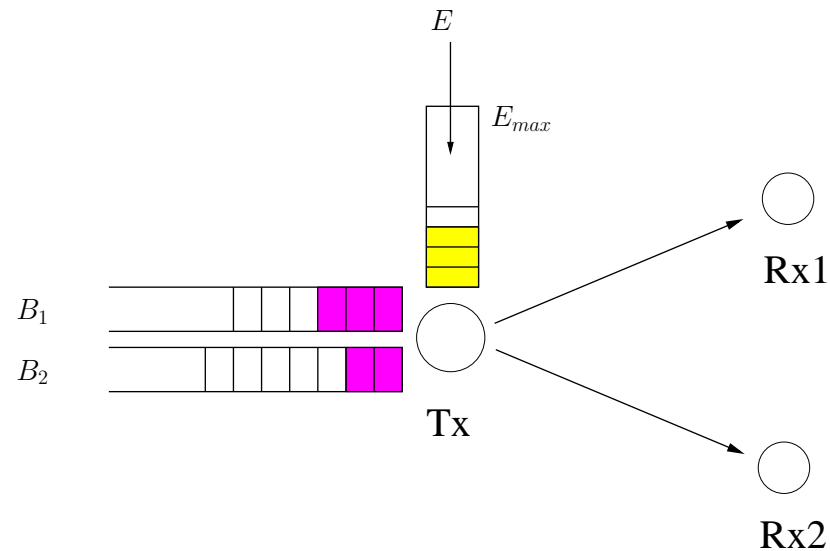$$P_c = \frac{\mu - 1}{\sigma^2 - \mu}$$

  where $\mu = \frac{\mu_2}{\mu_1}$ and $1 < \mu < \sigma^2$.

- If below P_c, then, only transmit to the stronger user

- Otherwise, stronger user's power share is P_c.

- Extreme cases:

  - If $\mu \leq 1$, only the stronger user's data is transmitted

  - If $\mu \geq \sigma^2$, only the weaker user's data is transmitted

The Structure of the Optimal Policy

E_0 E_1 E_2 E_3 E_4 E_5 E_6

T

$\sum_{j=0}^{i} E_j$

T

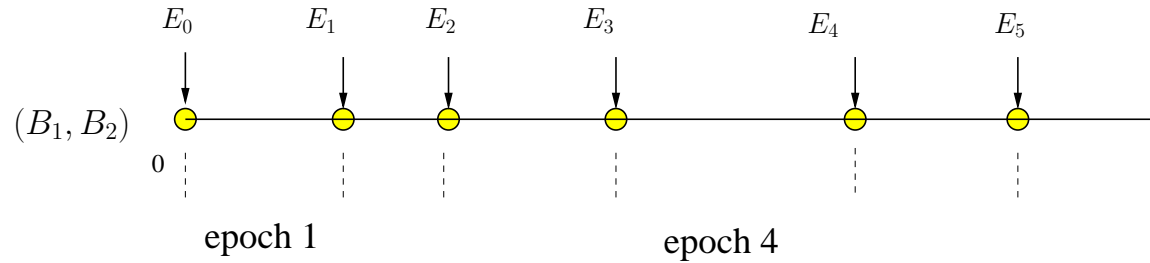P

P_c

T
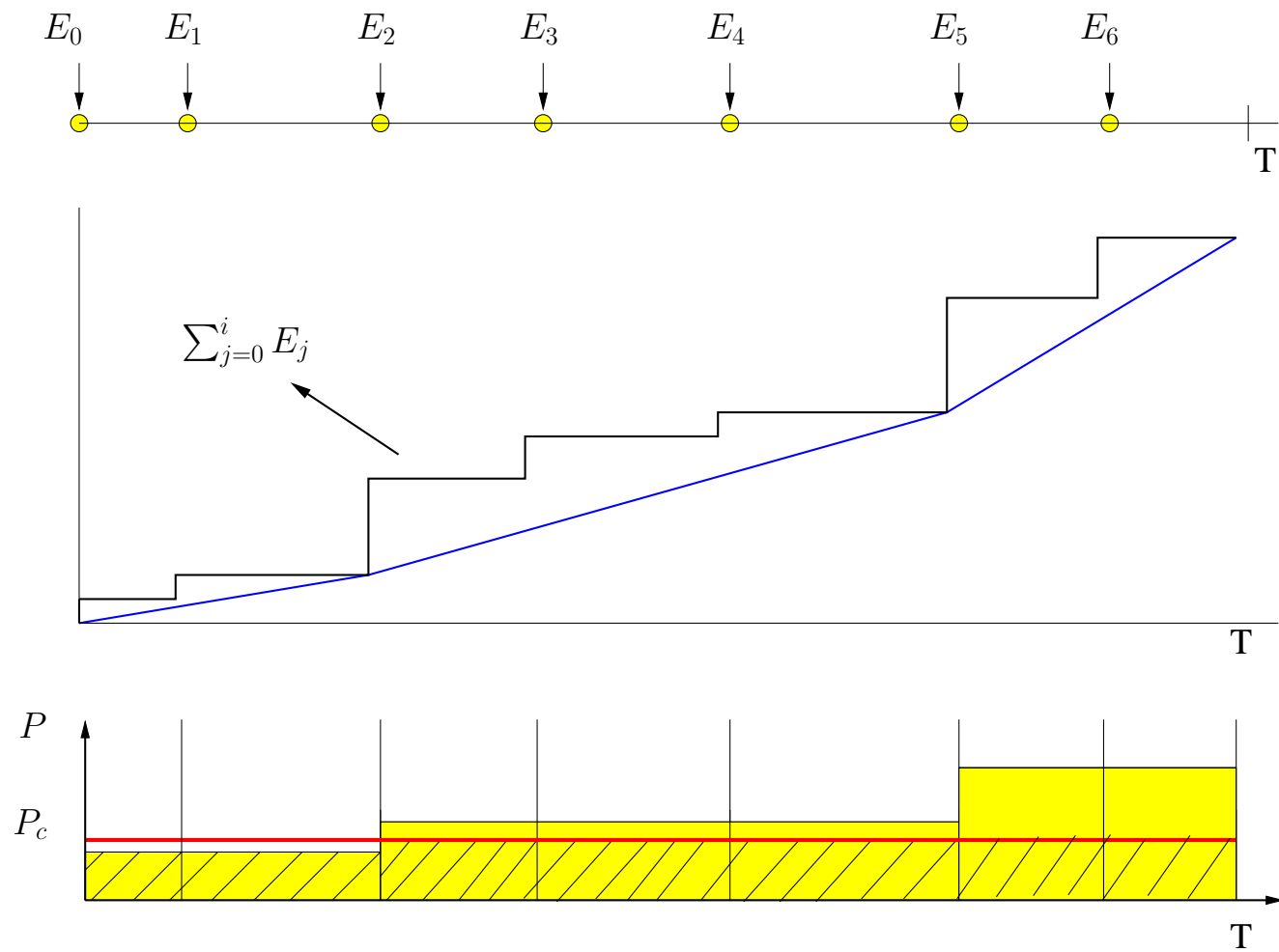
Broadcast Channel with Finite E_{max}

- (B_1, B_2) bits to be sent and battery capacity $E_{max} < \infty$

- AWGN broadcast channel:

$$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

- $N_1 \sim \mathcal{N}(0,1)$ and $N_2 \sim \mathcal{N}(0, \sigma^2)$ with $\sigma^2 > 1$

- 1st user stronger and 2nd user weaker
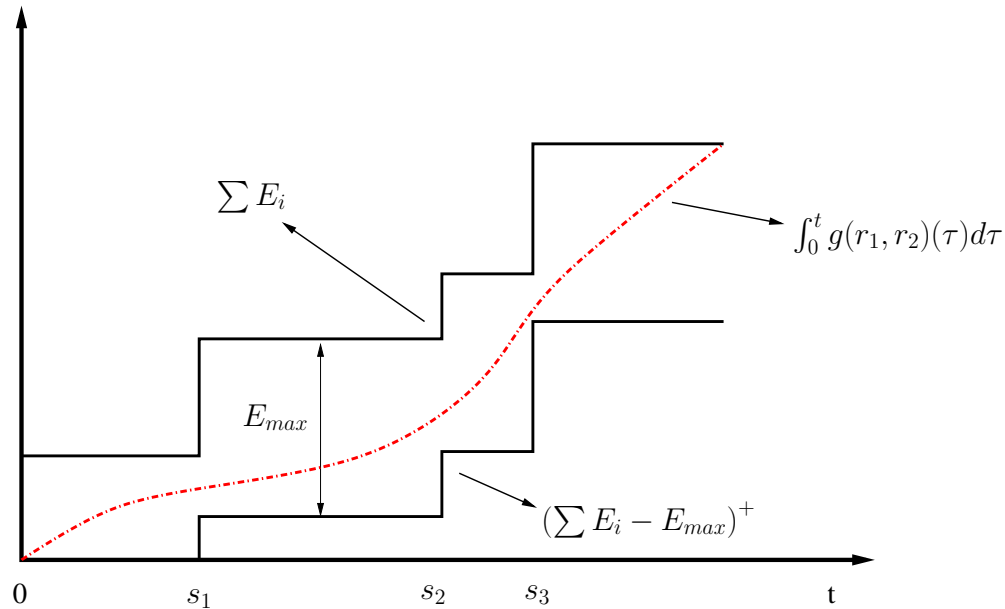
Broadcast Channel with Finite E_{max}

- Incoming energies are smaller than E_{max}: $E_i \leq E_{max}$

- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} g(r_1, r_2)(u)\,du \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

- No-energy-overflow condition: energy overflow (wasting) is suboptimal

$$\sum_{j=0}^{h(t)} E_j - \int_0^t g(r_1, r_2)(u)\,du \leq E_{max}, \quad \forall t$$
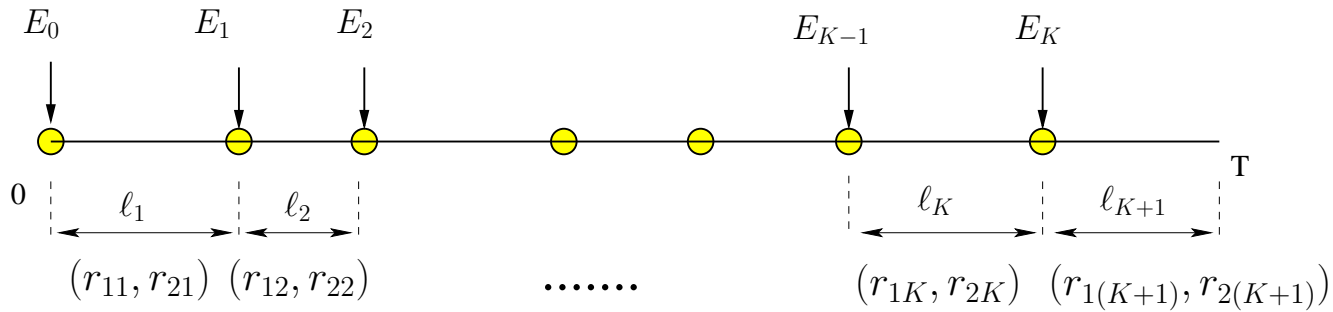
Constraints on the Power Policy

- **Energy causality** constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} g(r_1, r_2)(u)\,du \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

- **No-energy-overflow** condition: energy overflow (wasting) is suboptimal

$$\sum_{j=0}^{h(t)} E_j - \int_0^t g(r_1, r_2)(u)\,du \leq E_{max}, \quad \forall t$$

Find the Maximum Departure Region

- The transmission rates, and hence the transmission power, remain constant between energy harvests in any optimal policy

- The energy causality constraint reduces to constraints on (r_{1i}, r_{2i}):

$$\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$

- The no-energy-overflow condition:

$$\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq E_{max}, \qquad k = 1, \ldots, K$$

Finding the Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.

- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\max_{\mathbf{r}_1, \mathbf{r}_2} \ \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i$$

$$\text{s.t.} \ \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \ 1 \leq k \leq K+1$$

$$\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq E_{max}, \ 1 \leq k \leq K$$

Finding the Maximum Departure Region

- The Lagrangian function

$$\mathcal{L} = \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i - \sum_{k=1}^{K+1} \lambda_k \left(\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i - \sum_{i=0}^{k-1} E_i \right)$$

$$- \sum_{k=1}^{K} \eta_k \left(\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i - E_{max} \right) + \sum_{i=1}^{K+1} \gamma_{1i} r_{1i} + \sum_{i=1}^{K+1} \gamma_{2i} r_{2i}$$

- Total power in terms of Lagrange multipliers

$$P_i = \max \left\{ \frac{\mu_1}{\left(\sum_{k=i}^{K+1} \lambda_k - \sum_{k=i}^{K} \eta_k \right)} - 1, \frac{\mu_2}{\left(\sum_{k=i}^{K+1} \lambda_k - \sum_{k=i}^{K} \eta_k \right)} - \sigma^2 \right\}$$

Structure of the Optimal Policy

- Optimal total transmit power, $\{g(r_{1i}^*, r_{2i}^*)\}_{i=1}^{K+1}$, is independent of μ_1, μ_2.

- In particular, it is the same as the optimal single-user transmit power.

- The power shares follow a cut-off structure:

- Cut-off level P_c

$$P_c = \frac{\mu - 1}{\sigma^2 - \mu}$$

  where $\mu = \frac{\mu_2}{\mu_1}$ and $1 < \mu < \sigma^2$.

- If below P_c, then, only the stronger user

- Otherwise, stronger user's power share is P_c.

- Extreme cases:

  - If $\mu \le 1$, only the stronger user's data is transmitted
  - If $\mu \ge \sigma^2$, only the weaker user's data is transmitted

The Structure of the Optimal Policy
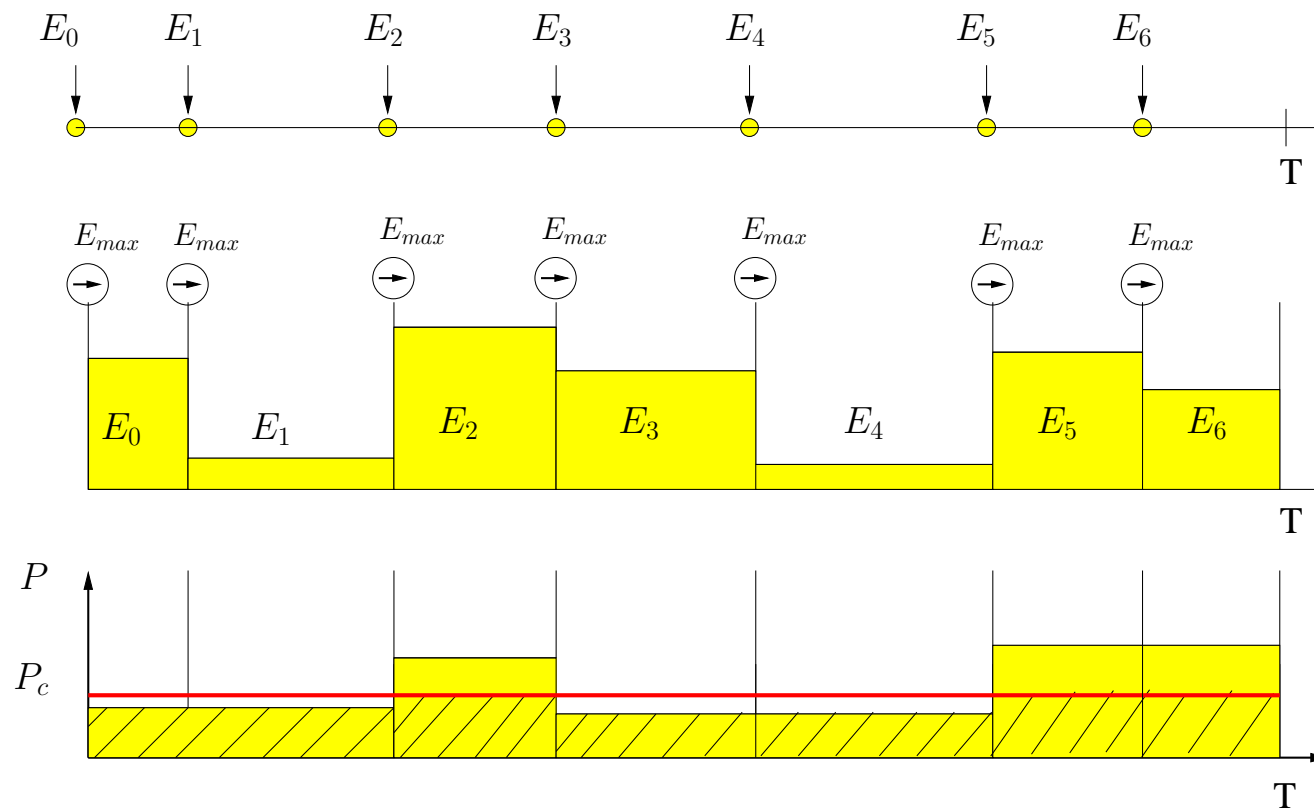
Conclusions for the Broadcasting Scenario

- Energy harvesting transmitter with infinite and finite capacity battery

- Maximize the departure region.

- Obtain the structure such as

    - the monotonicity of the transmit power

    - the cut-off power property

Optimal Packet Scheduling: Multiple Access Channel

- AWGN MAC channel $Y = X_1 + X_2 + Z$, $Z \sim N(0,1)$.

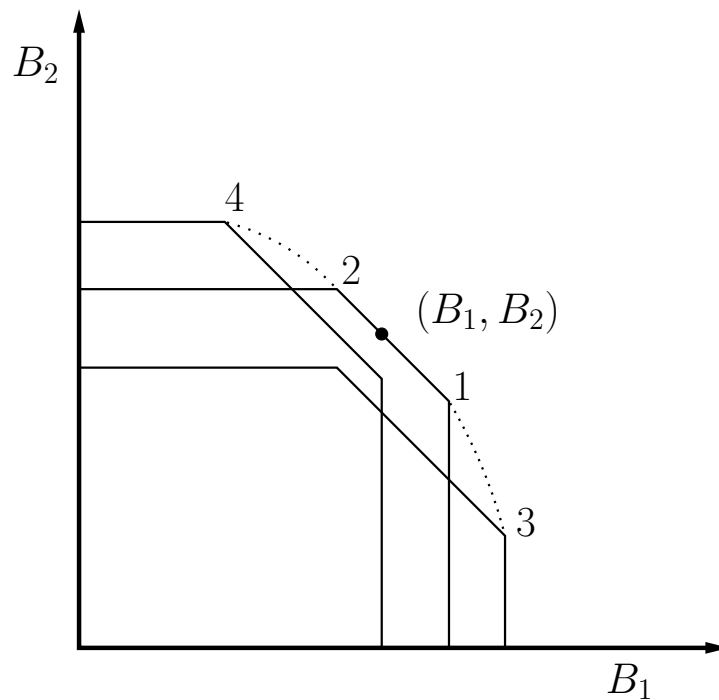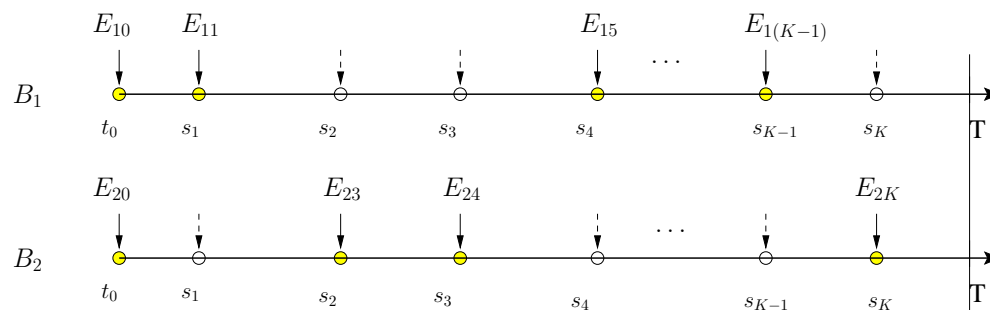- The capacity region is a pentagon denoted as $\mathcal{C}(P_1, P_2)$:

$$R_1 \leq f(P_1), \quad R_2 \leq f(P_2), \quad R_1 + R_2 \leq f(P_1 + P_2)$$

where $f(p) = \frac{1}{2} \log(1 + p)$.
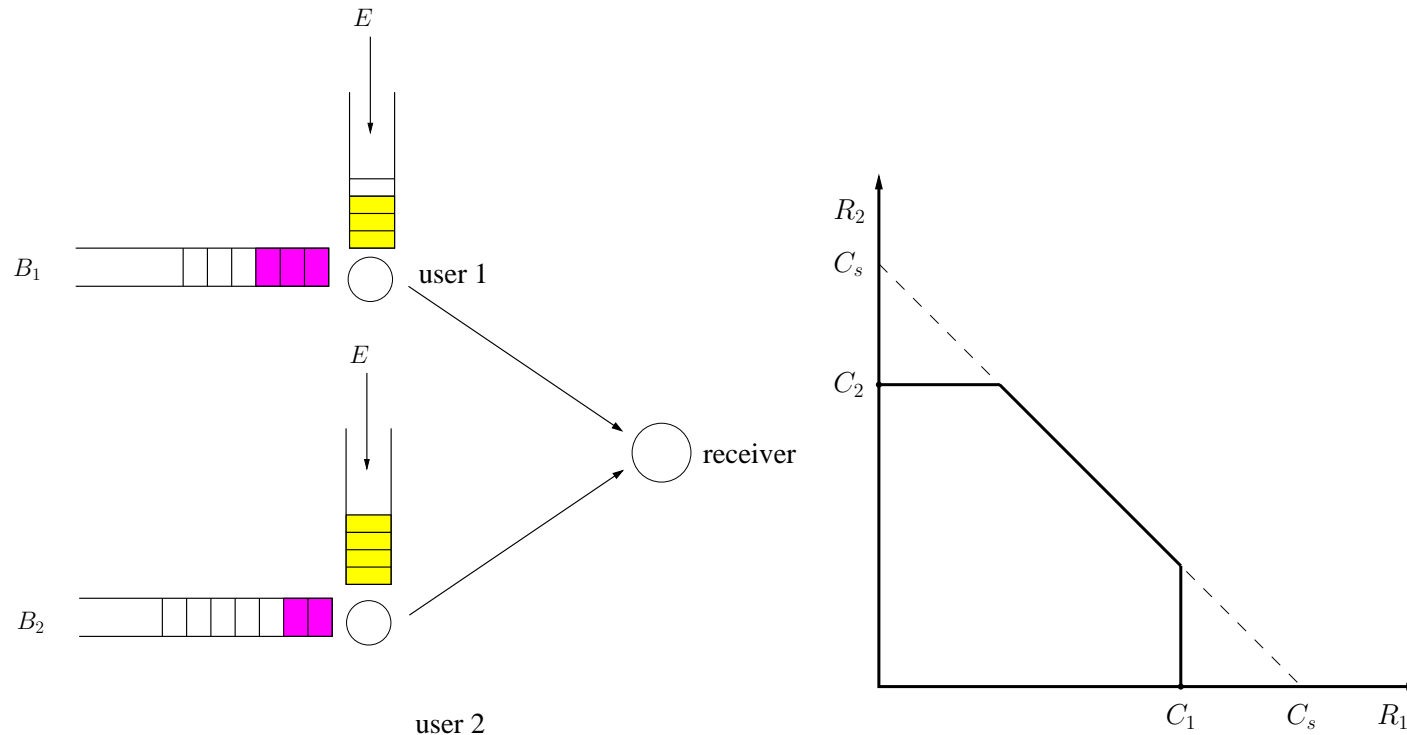
Problem Formulation

- Maximize departure region $\mathcal{D}(T)$ by time T.

Characterizing $\mathcal{D}(T)$

- Transmission rate remains constant between energy harvests.

- For any feasible transmit power sequences $\mathbf{p}_1$, $\mathbf{p}_2$ over $[0,T)$, the departure region is a pentagon defined as

$$B_1 \leq \sum_{n=1}^{N} f(p_{1n})l_n$$

$$B_2 \leq \sum_{n=1}^{N} f(p_{2n})l_n$$

$$B_1 + B_2 \leq \sum_{n=1}^{N} f(p_{1n} + p_{2n})l_n$$

- $\mathcal{D}(T)$ is a union of (B_1, B_2) and convex.

- The boundary points maximize $\mu_1 B_1 + \mu_2 B_2$ for some $\mu_1, \mu_2 \geq 0$.

$$\boxed{\mu_1 = \mu_2}$$

- The problem becomes $\max_{\mathbf{p}_1, \mathbf{p}_2} B_1 + B_2$.

- Sum of powers has same "majorization" property as in single-user.

- Merge energy arrivals of the users, get the optimal sum powers, $p_1, \ldots, p_n$

- Each feasible sequence of p_{1n} and p_{2n} gives a pentagon.

- Union of them is a larger pentagon: dominant faces on the same line.

- Need to identify the boundary of this larger pentagon.

Achieving Corner Points of the Boundary

- Maximize B_1 s.t. $B_1 + B_2$ is maximized at the same time $\Rightarrow$ point 1.

  – Equalize the transmit powers of the first user as much as possible

  – Additionally: both users' energy constraints are tight if sum power changes.
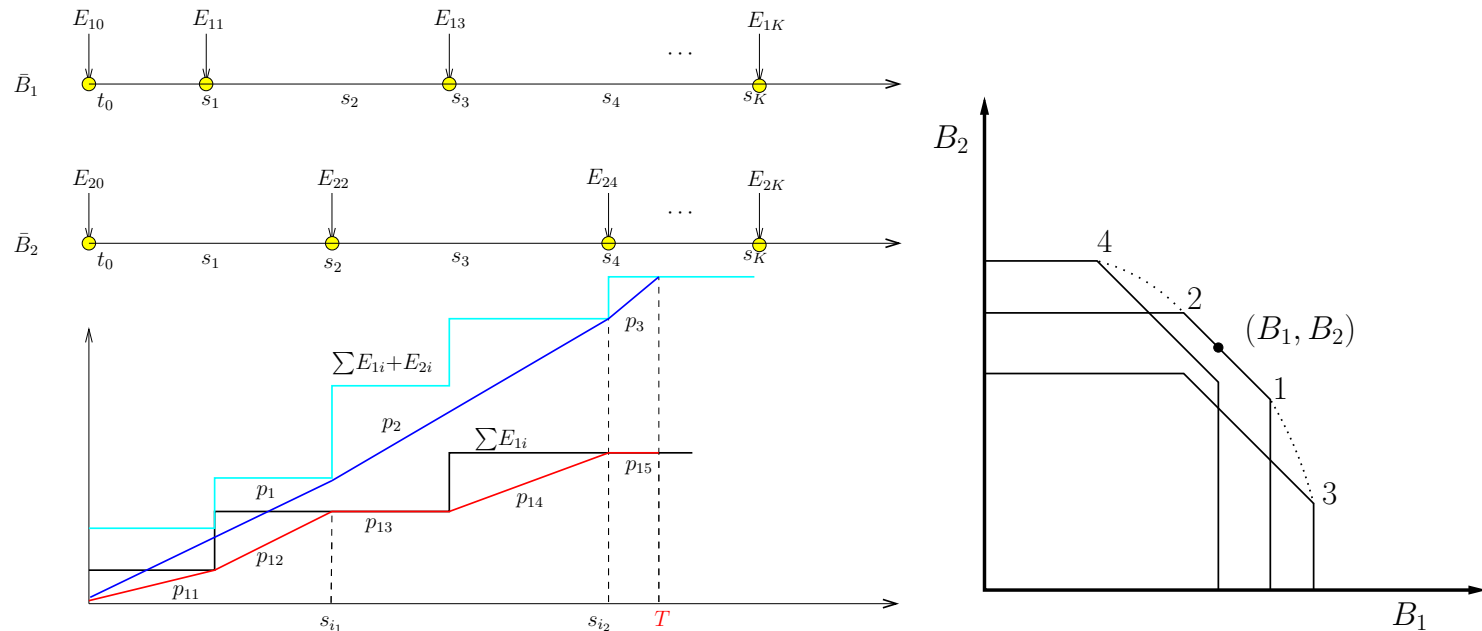
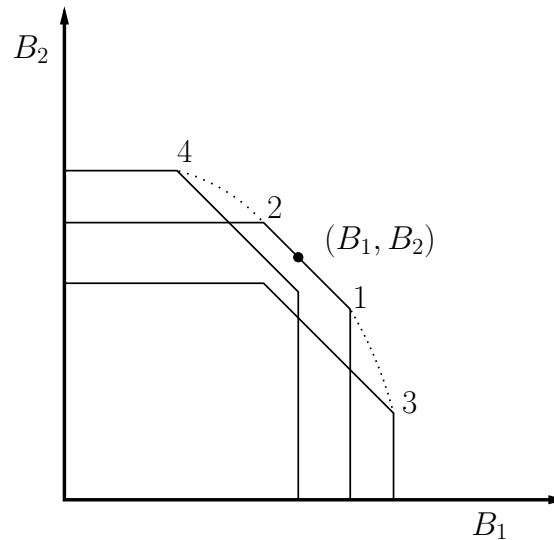$$\boxed{\mu_1 = 0 \textbf{ or } \mu_2 = 0}$$

- Maximize B_1 or $B_2 \Rightarrow$ a single-user scenario.

- Given p^*_{1n}, maximize B_2: backward/directional waterfilling with base level $p^*_{1n} \Rightarrow$ point 3.
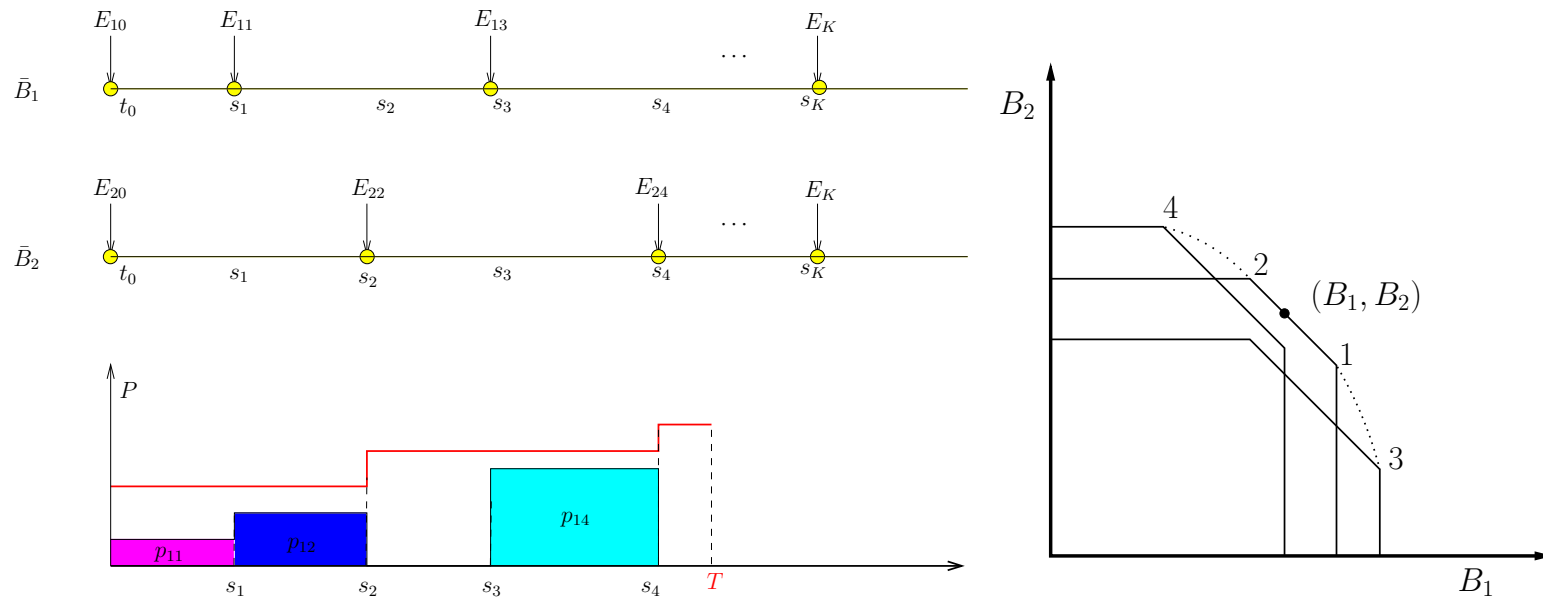
$$\boxed{\mu_1, \mu_2 > 0}$$

- Each boundary point corresponds to a corner point on some pentagon.

- $\mu_1 > \mu_2 \Rightarrow$ achieving points between point 1 and point 3:

$$\max_{\mathbf{p}_1, \mathbf{p}_2} \quad (\mu_1 - \mu_2) \sum_n f(p_{1n}) l_n + \mu_2 \sum_n f(p_{1n} + p_{2n}) l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \leq N$$

$$\sum_{n=1}^{j} p_{2n} l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad \forall j : 0 < j \leq N$$

35

Generalized Iterative Backward Waterfilling

- Solve the problem via generalized iterative backward waterfilling:

- Given $\mathbf{p}_2^*$, solve for $\mathbf{p}_1$:

$$\max_{\mathbf{p}_1} \quad (\mu_1 - \mu_2) \sum_{n=1}^{N} f(p_{1n})l_n + \mu_2 \sum_{n=1}^{N} f(p_{1n} + p_{2n}^*)l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n}l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \leq N$$

- Once $\mathbf{p}_1^*$ is obtained, we do a backward waterfilling for the second user.

- We perform the optimization for both users in an alternating way.

- The iterative algorithm converges to the global optimal solution.

# Conclusions for the Multiple Access Scenario

- Energy harvesting transmitters sending messages to a single access point.

- The problem: maximization of the departure region.

- Obtain the structure using generalized iterative waterfilling.

Interference Channel with an Energy Harvesting Transmitter
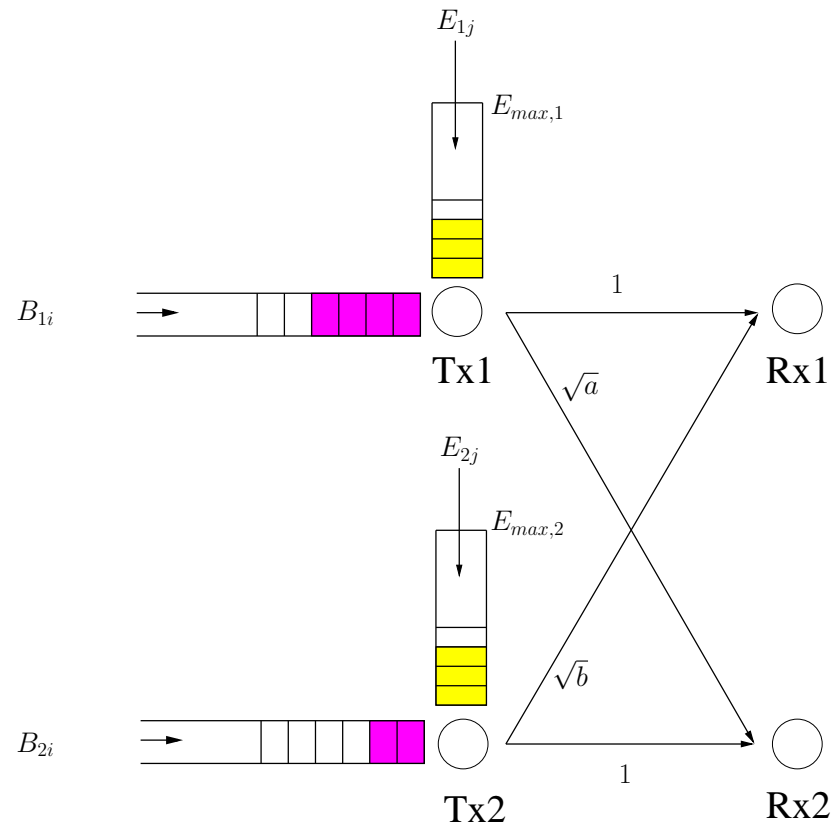
- Two transmitter-receiver pairs communicate in the same medium simultaneously.

- Energy arrives (is harvested) **during the communication session**.

- Batteries have finite storage capacities: $E_{max} < \infty$

- Objective: Maximize sum-rate of the users by **adapting to energy arrivals**

E_{1j}

$E_{max,1}$

B_{1i}

1

Tx1   Rx1

$\sqrt{a}$

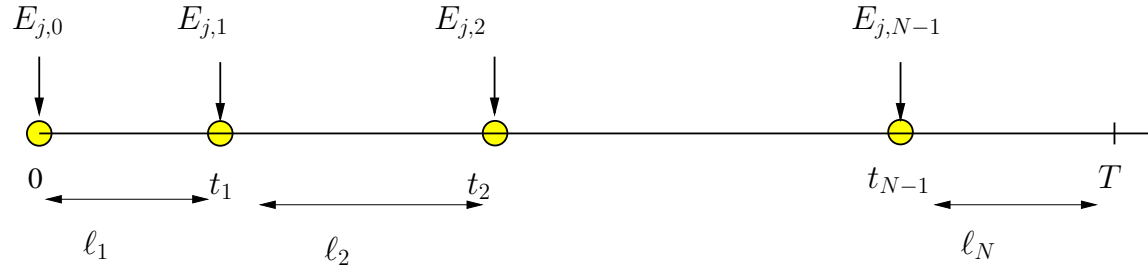E_{2j}

$E_{max,2}$

$\sqrt{b}$

B_{2i}

Tx2

1

Rx2
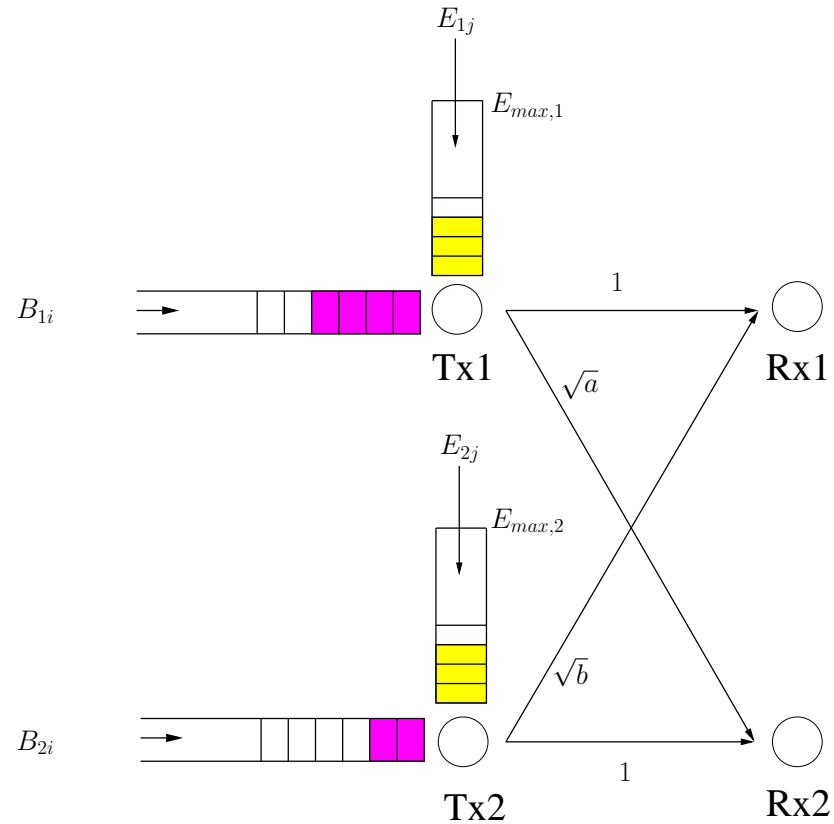
- AWGN interference channel:

$$Y_1 = X_1 + \sqrt{a}X_2 + N_1, \quad Y_2 = X_2 + \sqrt{b}X_1 + N_2$$

where $N_1 \sim \mathcal{N}(0,1)$, $N_2 \sim \mathcal{N}(0,1)$

- Sum-rate under $E[X_1^2] \leq p_1$ and $E[X_2^2] \leq p_2$ denoted as $r(p_1, p_2)$.

Energy Model

- Energy is *harvested* **during the course of communication**.

- We will consider offline policies.

- We have a slotted system with slot duration τ.

- Energy causality constraints in the Txs: energy that has not arrived cannot be used

$$\sum_{i=1}^{n} p_{j,i}\ell_i \leq \sum_{i=0}^{n-1} E_{j,i}, \quad n = 1,\ldots,N \text{ and } j = 1,2$$

- Battery limit constraints in the Txs: energy overflows are suboptimal:

$$\sum_{i=1}^{n}(p_{j,i}\ell_i - E_{j,i}) + E_{j,max} - E_{j,i+1} \geq 0, \quad n = 1,\ldots,N-1, \text{ and } j = 1,2$$

40

Sum-Rate Optimal Policy

- Sum-rate optimal policy is found by solving the following problem:

$$\max_{\mathbf{p}_1,\mathbf{p}_2} \sum_{i=1}^{K+1} r(p_{1i}, p_{2i})\ell_i$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_{j,i}\ell_i \leq \sum_{i=1}^{n} E_{j,i}, \ 1 \leq n \leq N$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n} p_{j,i}\ell_i \leq E_{max}, \ 1 \leq n \leq N$$

A General Iterative Solution

- For any achievable $r(p_1, p_2)$, there exists another achievable scheme with

  – $\hat{r}(p_1, p_2) > r(p_1, p_2)$

  – $\hat{r}(p_1, p_2)$ is jointly concave

- Solve the problem iteratively:

- Given $\mathbf{p}_2^*$, solve for $\mathbf{p}_1$:

$$\max_{\mathbf{p}_1} \quad \sum_{i=1}^{K+1} r(p_{1i}, p_{2i}^*)\ell_i$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_{1,i}\ell_i \leq \sum_{i=1}^{n} E_{1,i}, \ 1 \leq n \leq N$$

$$\sum_{i=1}^{n} E_{1,i} - \sum_{i=1}^{n} p_{1,i}\ell_i \leq E_{max}, \ 1 \leq n \leq N$$

- Once this solution is found, we fix it and solve for $\mathbf{p}_2^*$.

- The iterative algorithm converges to the global optimal solution.

Asymmetric Interference with $ab > 1$

- Let $a \geq 1$ and $b \leq 1$ with $ab > 1$.

$$r(p_1, p_2) = \frac{1}{2} \log \left(1 + \frac{p_1}{1 + ap_2} \right) + \frac{1}{2} \log \left(1 + p_2 \right)$$

- For fixed p_2, user 1 observes a fading level of $\frac{1}{1+ap_2}$.

- Use directional waterfilling for user 1's problem.

- For fixed p_1, user 2 has the generalized water level

$$\frac{\partial}{\partial p_2} r(p_1, p_2) = -\frac{ap_1}{2(1 + p_1 + ap_2)(1 + ap_2)} + \frac{1}{2(1 + ap_2)}$$

- Use generalized directional waterfilling for user 2's problem.

Asymmetric Interference with $ab < 1$

- Let $a \leq 1$ and $b \geq 1$ with $ab < 1$.

$$r(p_1, p_2) = \min\{\frac{1}{2}\log\left(1 + \frac{p_1}{1 + ap_2}\right) + \frac{1}{2}\log\left(1 + p_2\right), \frac{1}{2}\log\left(1 + bp_1 + p_2\right)\}$$

- Define $p_c = \frac{b-1}{1-ab}$.

- User 1 observes fading level $\frac{1}{1+ap_2}$ if $p_2 < p_c$ and $\frac{b}{1+p_2}$ otherwise.

- Use directional waterfilling for user 1's problem.

- Similarly, a generalized waterfilling algorithm solves user 2's problem.

Conclusions for the Interference Channel Scenario

- Energy harvesting transmitter-receiver pairs with finite capacity batteries.

- Maximize the sum-rate of the communication.

- Sum-rate is a jointly concave function of powers.

- Iterative generalized directional water-filling algorithm.

- Specific cases such as asymmetric interference with $ab < 1$ and $ab > 1$.

- Extension to bit arrivals is available.

Two-Hop Communication with Energy Harvesting Nodes
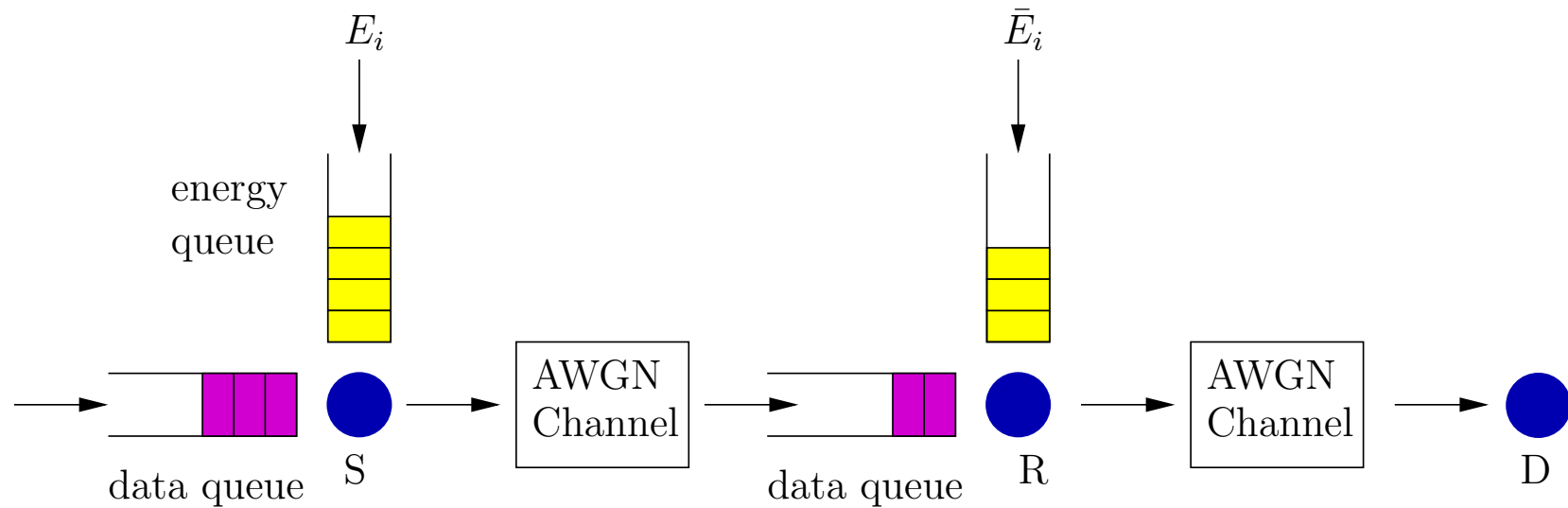
- Source (S) sends messages to the destination (D) via a relay (R).

- Source and relay uses energy harvested from the environment.

- Source adapts its transmission to the energy profiles of both nodes.

- Relay adapts its transmission to the data stream from the source and its energy profile.

- Objective: maximize end-to-end throughput

Channel Model
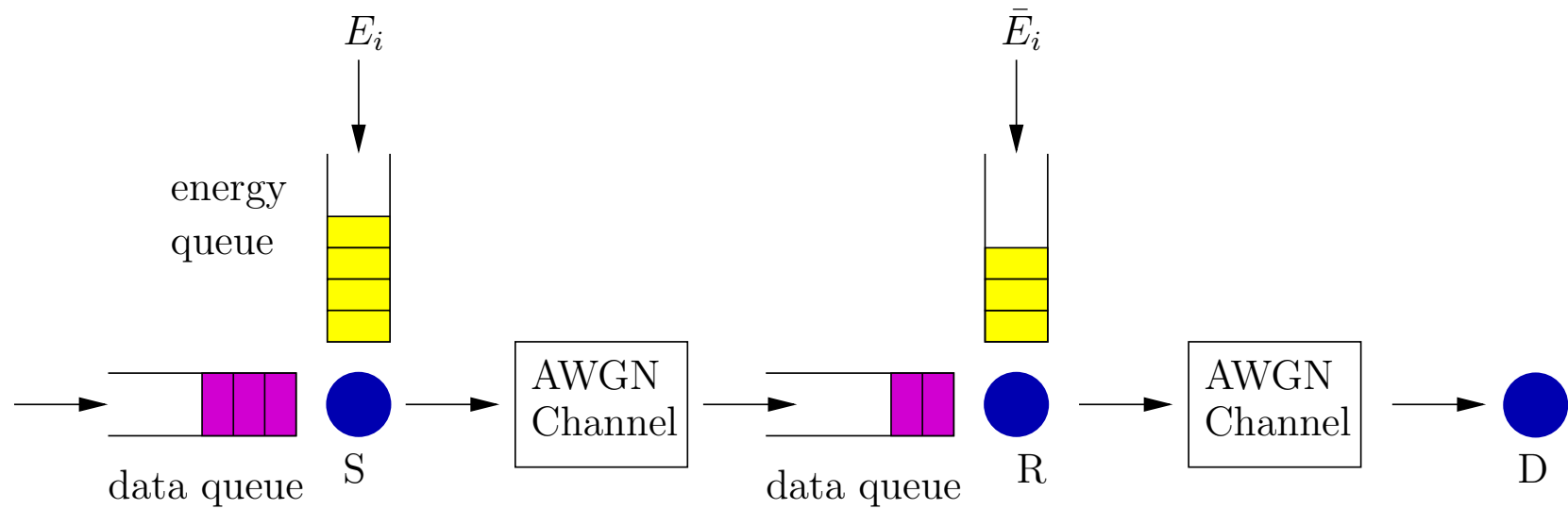
- Channel between S and R is AWGN with gain h_s:

$$r^s(p_s) = \frac{1}{2} \log\left(1 + h_s p_s\right)$$

- Channel between R and D is AWGN with gain h_r:

$$r^r(p_r) = \frac{1}{2} \log\left(1 + h_r p_r\right)$$

- Relay operates in full duplex mode.

Energy Model
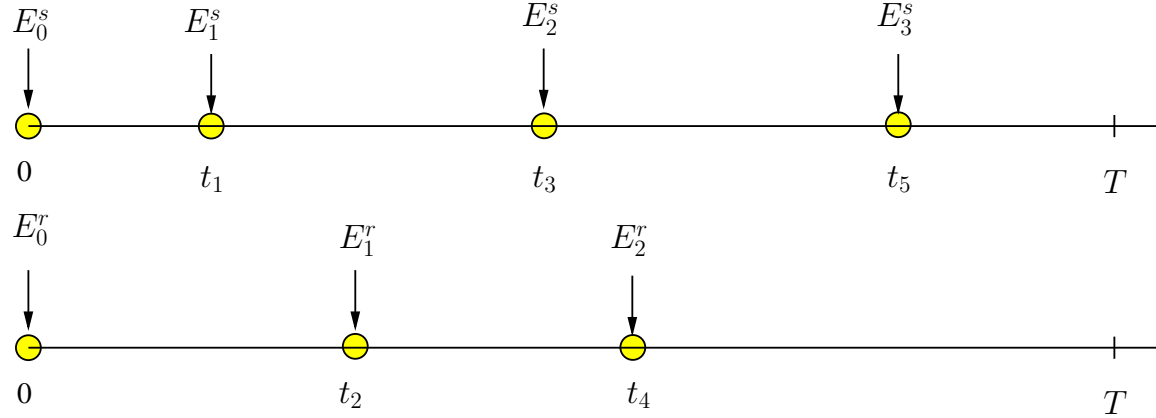
- Energy is *harvested* **during the communication**. We consider offline policies.

- Energy causality constraints in the nodes: energy that has not arrived cannot be used

$$\int_0^t p_s(\tau)d\tau \le \sum_{i=0}^{n-1} E_i^s, \quad n = 1, \ldots, N$$

$$\int_0^t p_r(\tau)d\tau \le \sum_{i=0}^{n-1} E_i^r, \quad n = 1, \ldots, N$$

- Data causality constraints in the relay: data that has not arrived cannot be forwarded.

$$\int_0^t r^r(p_r(\tau))d\tau \le \int_0^t r^p(p_s(\tau))d\tau, \quad \forall t \in [0, T]$$

Finding Optimal Policies of the Nodes

- Maximize end-to-end throughput

$$\max \int_0^T r^r(p_r(\tau))d\tau$$

$$\text{s.t.} \int_0^t p_s(\tau)d\tau \leq \sum_{i=0}^{n-1} E_i^s, \quad n = 1,\ldots,N$$

$$\int_0^t p_r(\tau)d\tau \leq \sum_{i=0}^{n-1} E_i^r, \quad n = 1,\ldots,N$$

$$\int_0^t r^r(p_r(\tau))d\tau \leq \int_0^t r^p(p_s(\tau))d\tau, \quad \forall t \in [0,T]$$

- Optimal policies are not unique.

- There is a separation-based optimal policy.

Separation-Based Optimal Policy

- Source maximizes its throughput **without regard to relay energy profile:**

$$\max \int_0^T r^s(p_s(\tau))d\tau$$

$$\text{s.t.} \int_0^t p_s(\tau)d\tau \leq \sum_{i=0}^{n-1} E_i^s, \quad n = 1, \ldots, N$$

- Relay maximizes its throughput **according to the optimal source data stream:**

$$\max \int_0^T r^r(p_r(\tau))d\tau$$

$$\text{s.t.} \int_0^t p_r(\tau)d\tau \leq \sum_{i=0}^{n-1} E_i^r, \quad n = 1, \ldots, N$$

$$\int_0^t r^r(p_r(\tau))d\tau \leq \int_0^t r^p(p_s(\tau))d\tau, \quad \forall t \in [0, T]$$

- Both problems are single-user throughput maximization problems.

- This policy is not energy minimal.

Structure of Separation-Based Optimal Policy

E_0^s E_1^s E_2^s E_3^s

T

E_0^r E_1^r E_2^r E_3^r E_4^r E_5^r E_6^r

T

$\int_0^t B_s(\tau)d\tau$

$\sum E_i^r$

Optimal Policy

T
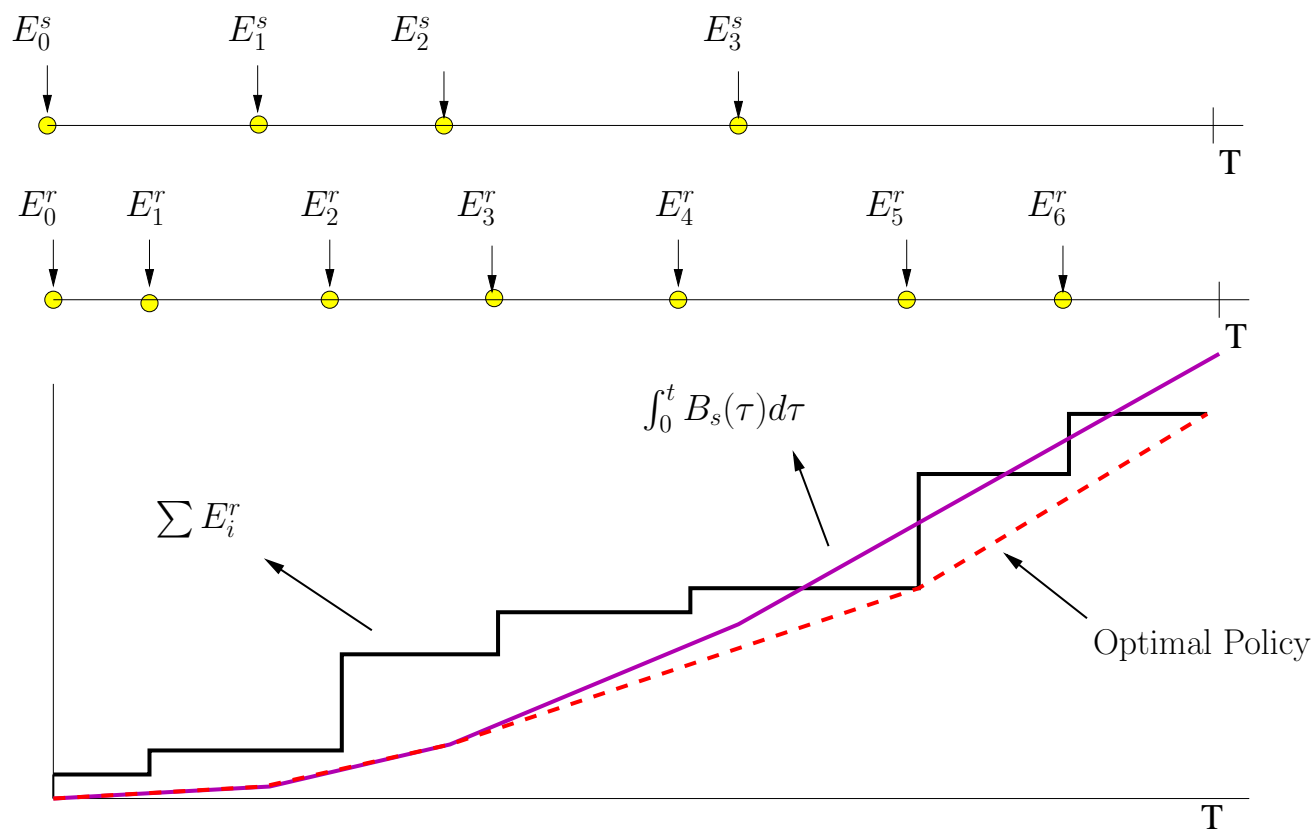
Conclusions for the Two-Hop Communication Scenario

- Energy harvesting source and relay with infinite capacity batteries.

- Maximize the end-to-end throughput.

- Optimal policy is not unique.

- An optimal policy is obtained based on a separation principle:

  – Both source and relay perform single-user optimizations.

  – It is not *energy minimal*.

- There is no simple extension for the finite battery case.

Wireless Energy Transfer

- Newly emerging technologies have enabled us to perform wireless energy transfer efficiently.

- Inductive coupling can be used to wirelessly transfer energy.
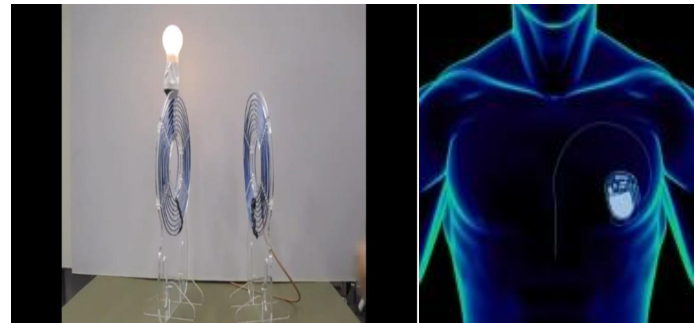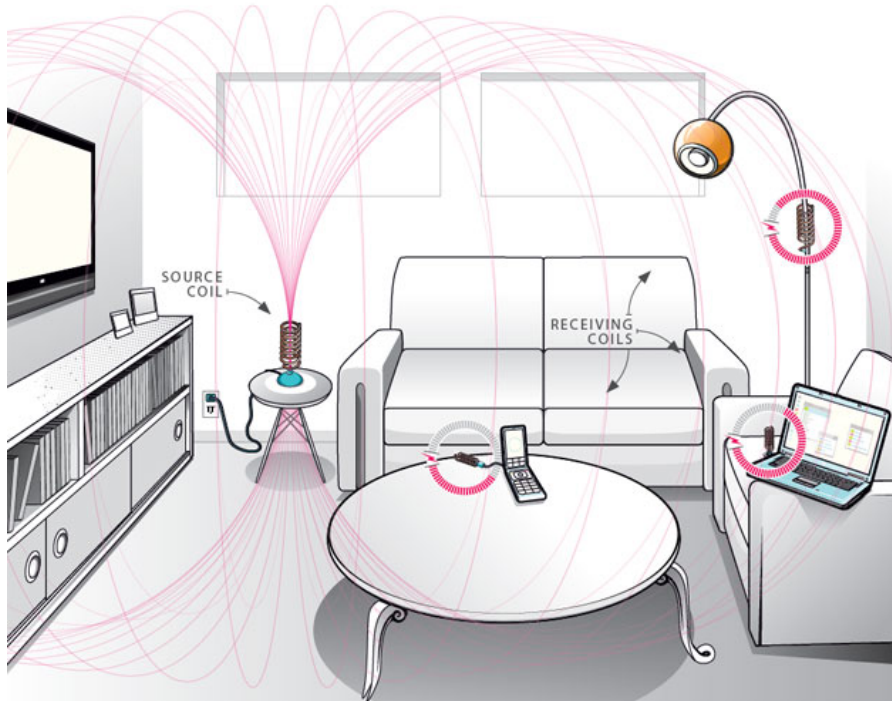
Energy Cooperation in Multi-user Energy Harvesting Communications

- Wireless energy transfer is a new cooperation paradigm: energy cooperation.
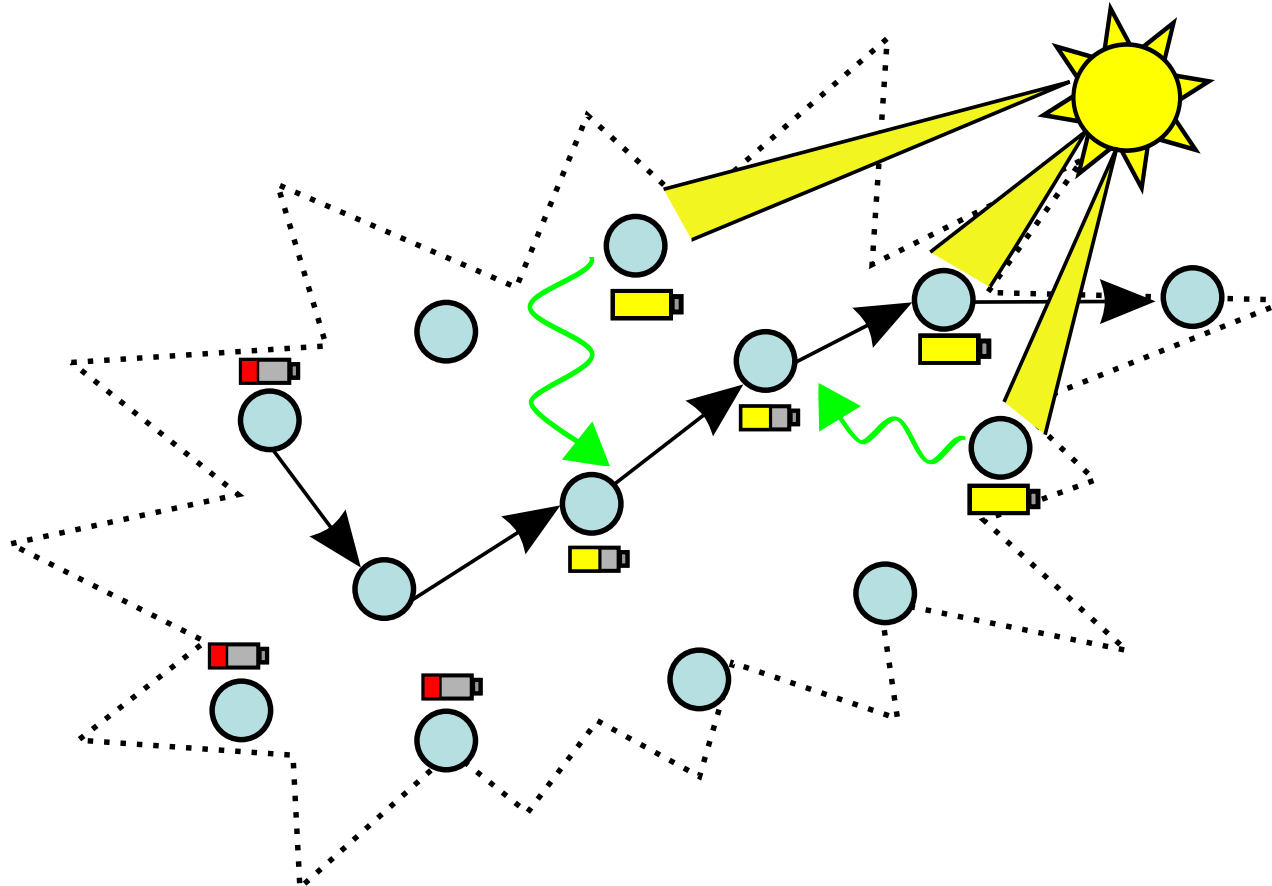
- *Energy cooperation:* An energy harvesting network where the nodes can share their energy as well as their information.

Gaussian Two-Hop Relay Channel with Energy Cooperation
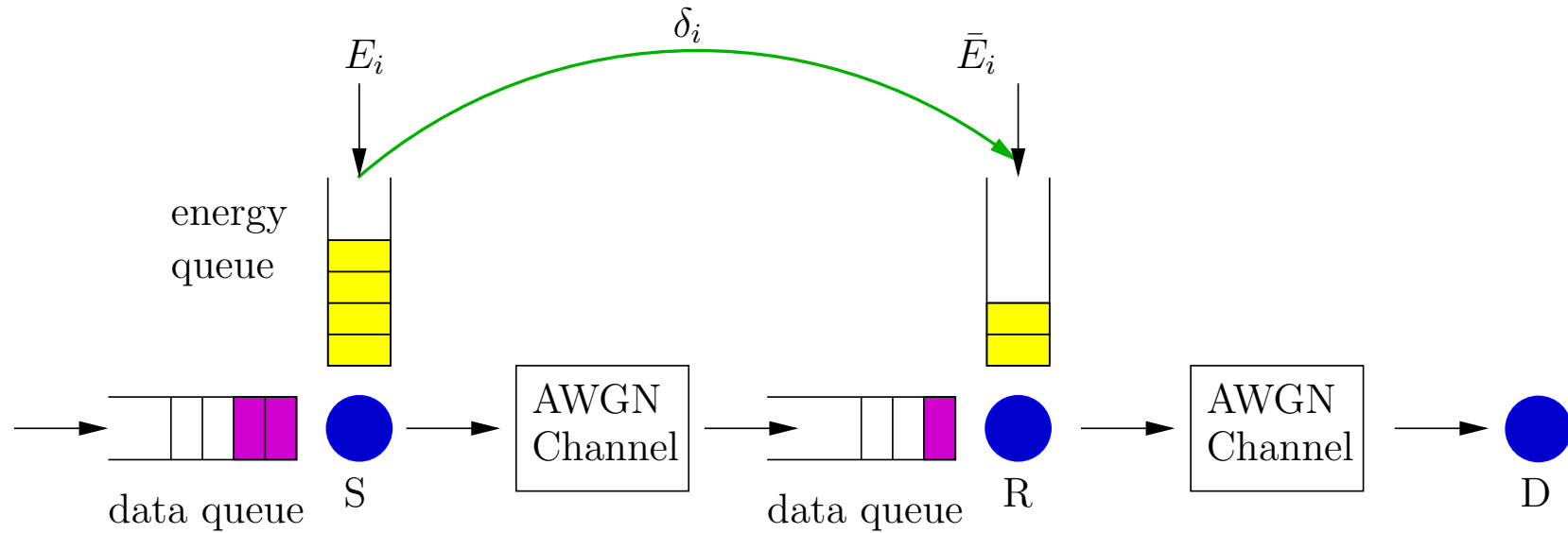
- Energy harvesting source and relay with deterministic energy arrivals E_i, $\bar{E}_i$.

- Wireless energy transfer unit that allows the source to transfer some of its energy to the relay (with $0 \leq \alpha \leq 1$ efficiency).

- Unlimited data and energy buffers at the source and the relay.

- New energy arrivals at every slot i, $1 \leq i \leq T$.

- The source transfers δ_i energy to the relay at slot i.

- Relay receives $\alpha \delta_i$ of this transferred energy at the next slot.

Two Hop Relay Channel without Energy Cooperation
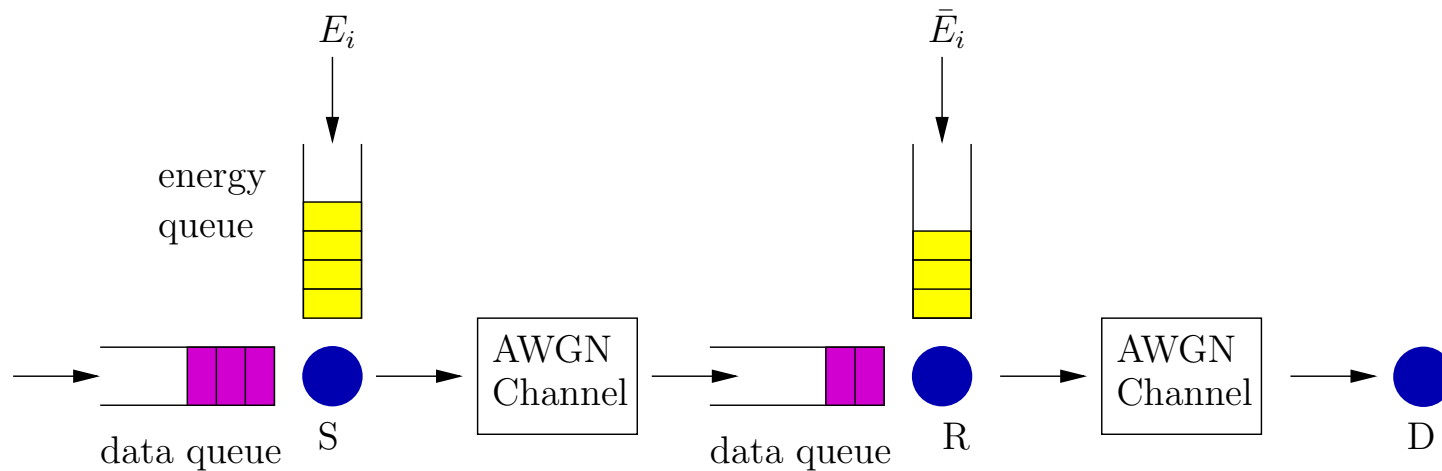
- Optimal source/relay profile is a separable policy.

- Source performs single user throughput maximization with respect to its own energy arrivals.

- Relay forwards as many of the received bits as possible, satisfying data causality and energy causality.

Two Hop Relay Channel without Energy Cooperation
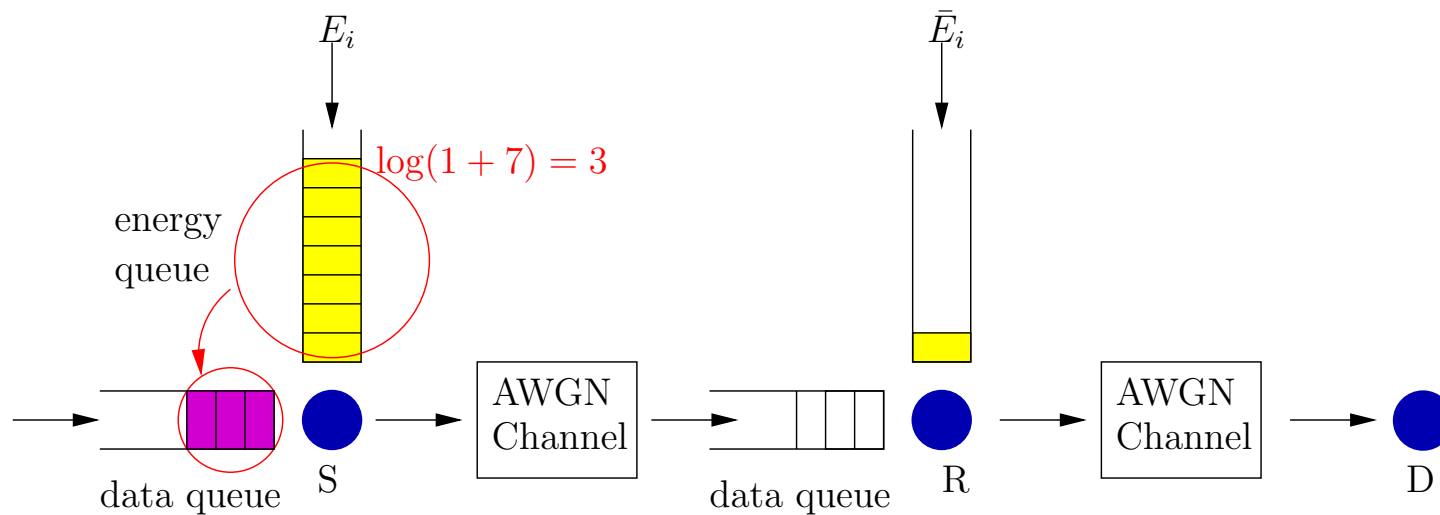
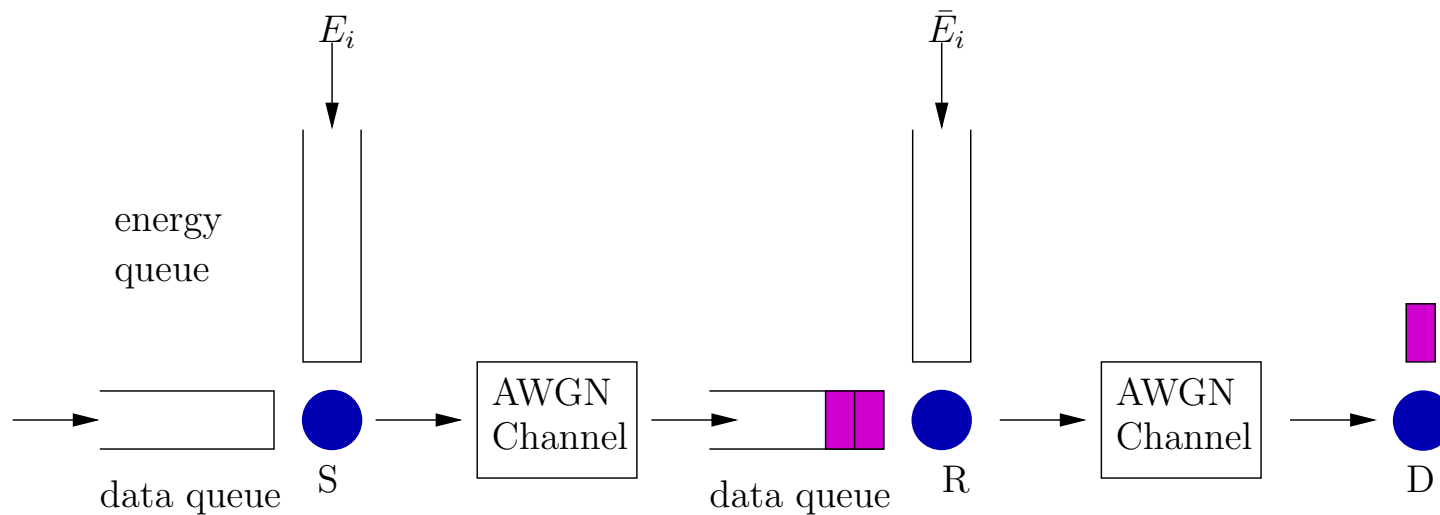- **Separable policy**, source maximizes its own throughput.
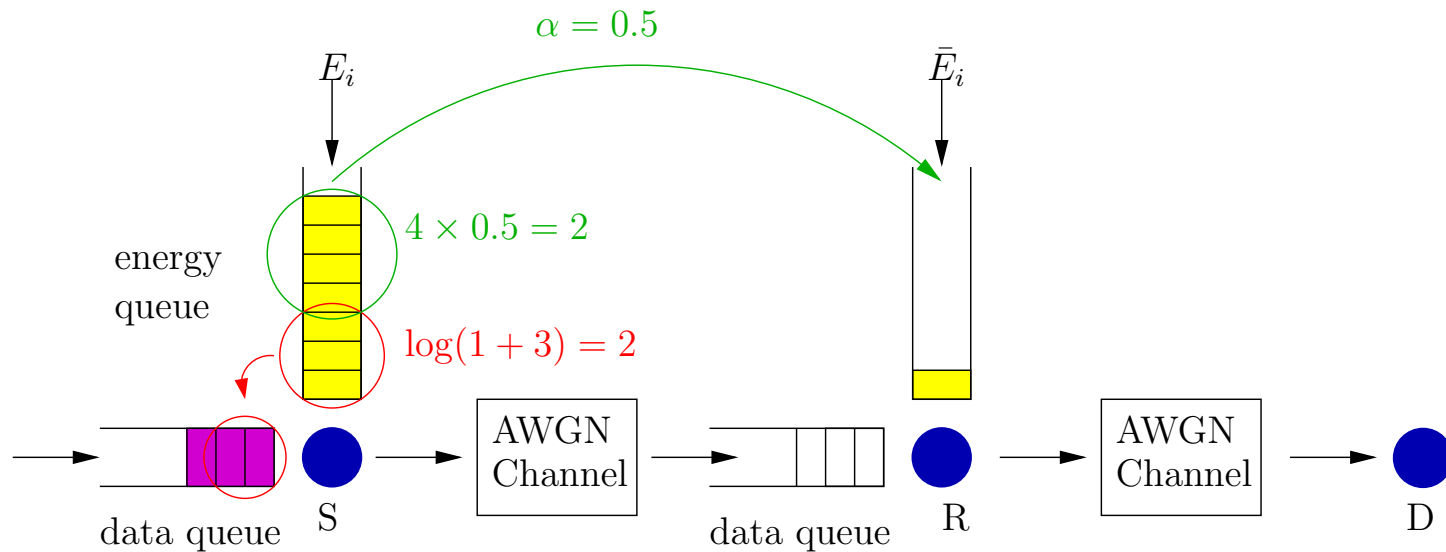
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Two Hop Relay Channel without Energy Cooperation
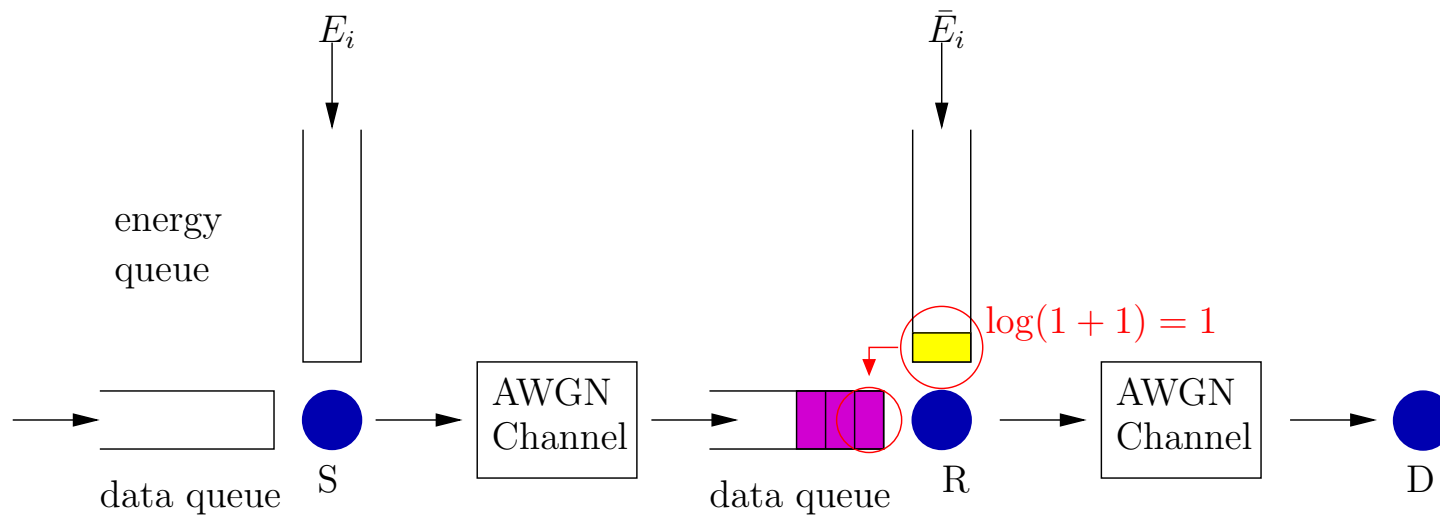
- Separable policy, source maximizes its own throughput.

- Relay tries to send as much as it can.

-

-

Two Hop Relay Channel without Energy Cooperation

- Separable policy, source maximizes its own throughput.

- Relay tries to send as much as it can.

- 1 bit sent to destination, 2 bits remaining at the relay.

- End-to-end throughput is 1 bit.

Two Hop Relay Channel with Energy Cooperation

$\alpha = 0.5$

E_i $\bar{E}_i$

$4 \times 0.5 = 2$

energy
queue

$\log(1 + 3) = 2$

data queue   S   AWGN Channel   data queue   R   AWGN Channel   D

- Source sends less data, but some energy to assist the relay.

-

-

-

Two Hop Relay Channel with Energy Cooperation
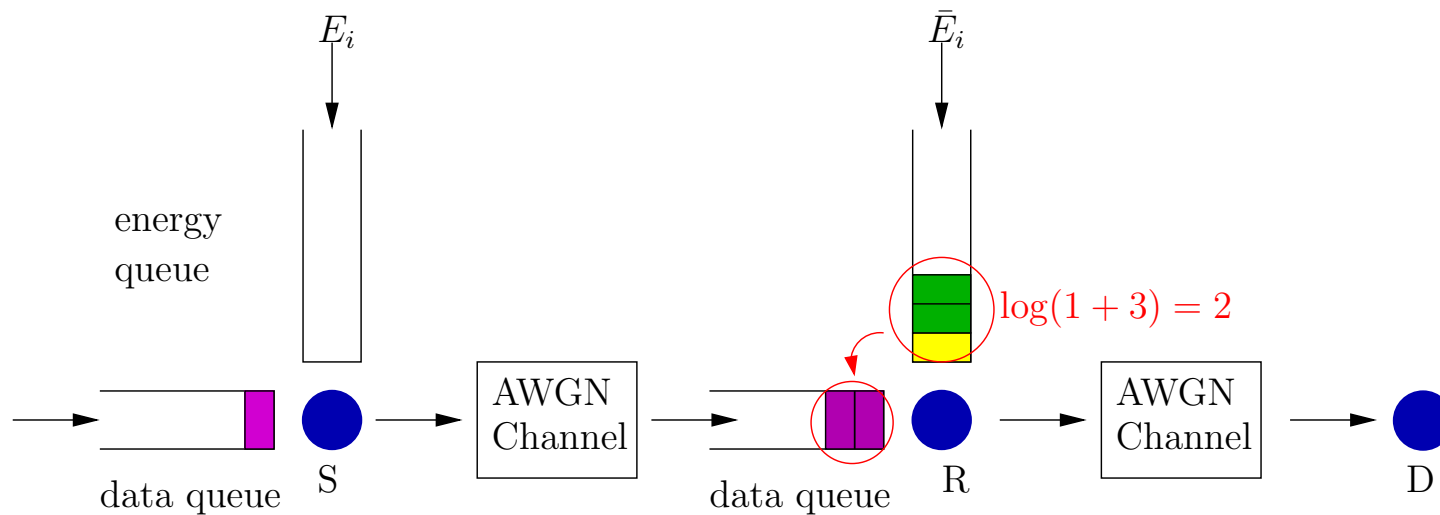
E_i

$\bar{E}_i$

energy queue

$\log(1+3)=2$

AWGN Channel

AWGN Channel

data queue   S

data queue   R

D

- Source sends less data, but some energy to assist the relay.

- Relay uses this extra energy to forward more data.

-

-

Two Hop Relay Channel with Energy Cooperation
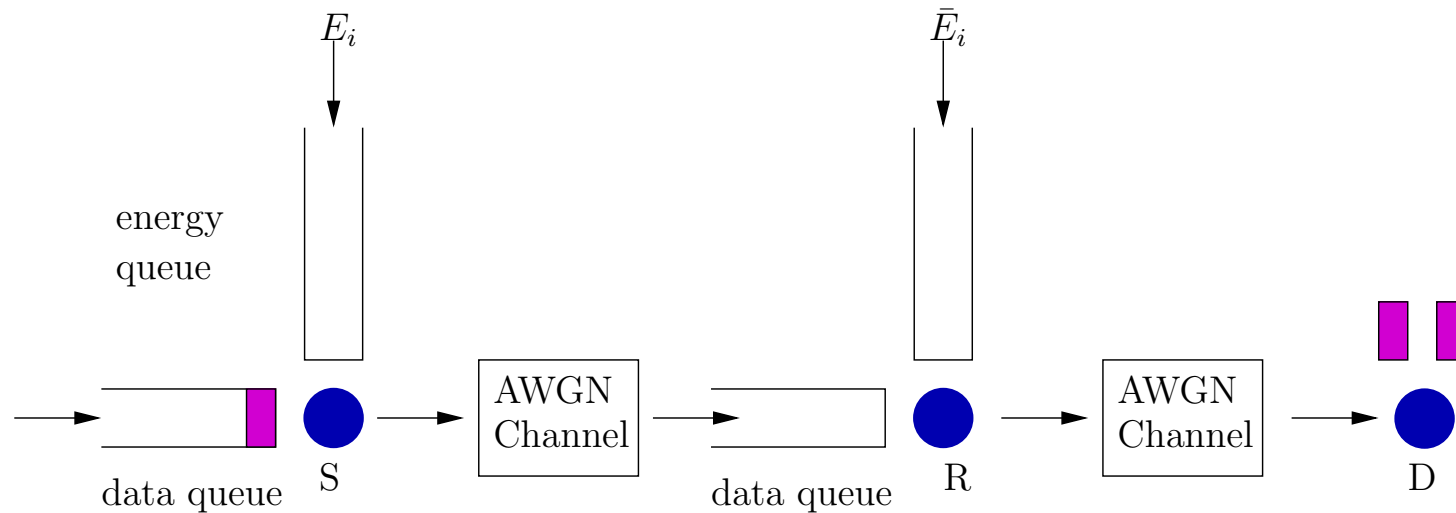
E_i

$\bar{E}_i$

energy
queue

AWGN
Channel

AWGN
Channel

data queue   S

data queue   R

D

- Source sends less data, but some energy to assist the relay.

- Relay uses this extra energy to forward more data.

- 2 bits sent to destination, 0 bits remaining at the relay.

- End-to-end throughput is 2 bits.

End-to-end Throughput Maximization

- Maximize end-to-end throughput

$$\max \sum_{i=1}^{T} \frac{1}{2} \log\left(1 + \bar{P}_i\right)$$

$$\text{s.t.} \sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} (E_i - \delta_i), \quad \forall k$$

$$\sum_{i=1}^{k} \bar{P}_i \leq \sum_{i=1}^{k} (\bar{E}_i + \alpha \delta_i), \quad \forall k$$

$$\sum_{i=1}^{k} \frac{1}{2} \log\left(1 + \bar{P}_i\right) \leq \sum_{i=1}^{k} \frac{1}{2} \log\left(1 + P_i\right), \quad \forall k$$

- subject to:

  - Data causality at the relay node

  - Energy causality at both nodes

  - (Possibly) non-zero energy transfers

- Solution could be identified only in special cases.

Case I: Higher Initial Relay Energy

Figure axes: vertical axis labeled $\sum E_i$, $\sum_{i=1}^{T} E_i$, $\sum_{i=1}^{T} \bar{E}_i$; tick marks $\bar{E}_1 + \bar{E}_2$, $E_1 + E_2$, $\bar{E}_1$, E_1. Horizontal axis labeled slot number i with ticks 1, 2, $\ldots$, $\tilde{i}$, $\tilde{i}+1$, $\ldots$, T.

Legend:
— Source energy arrivals E_i
-- Relay energy arrivals $\bar{E}_i$
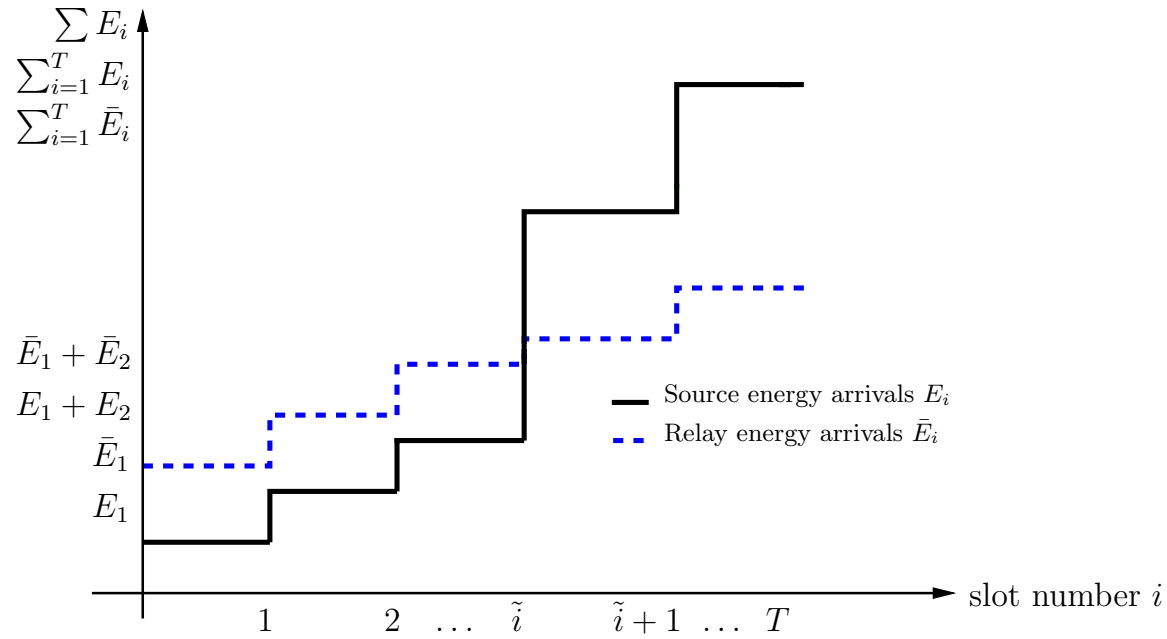
- Higher initial relay energy and single intersection with source energy curve

- Covers the case when the source is energy harvesting and all relay energy is available initially

Case I: Higher Initial Relay Energy

Legend:
- Source energy arrivals E_i
- Relay energy arrivals $\bar{E}_i$
- $(\alpha E_i + \bar{E}_i)/(1+\alpha)$
- Min of E_i and $(\alpha E_i + \bar{E}_i)/(1+\alpha)$
- Optimal policy
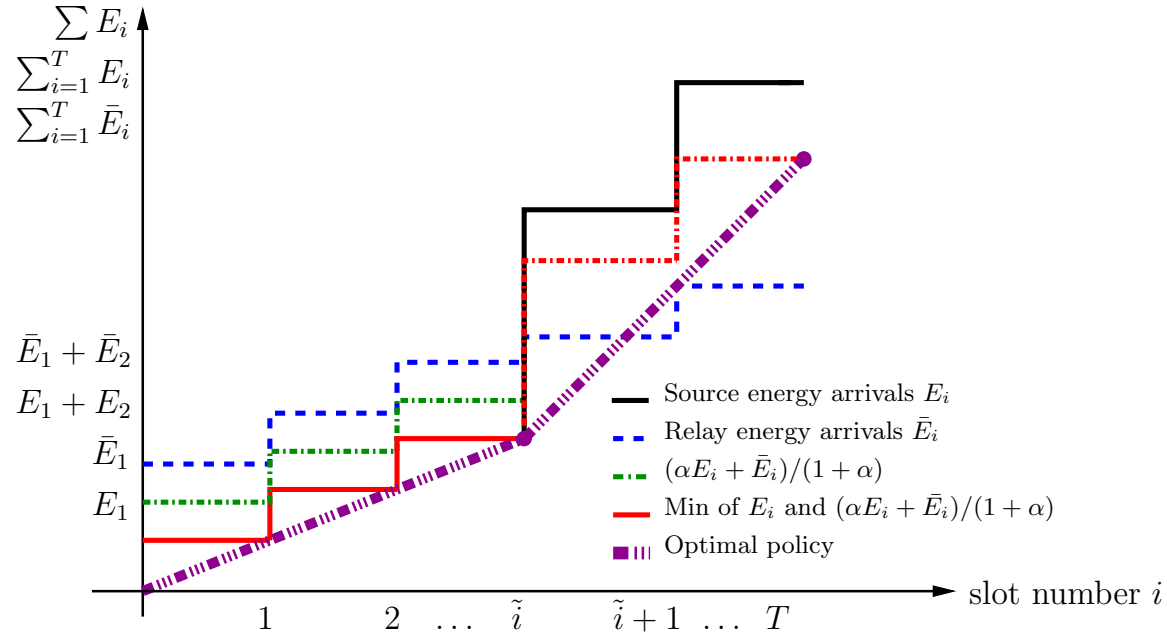
- Since source energy is low initially, no energy transfer until the intersection

- Form a new energy profile $\min\left(\frac{\alpha E_i + \bar{E}_i}{\alpha+1}, E_i\right)$ and maximize throughput

- Source and relay powers are matched to ensure relay data queue is empty.

Case II: Non energy harvesting source and energy harvesting relay
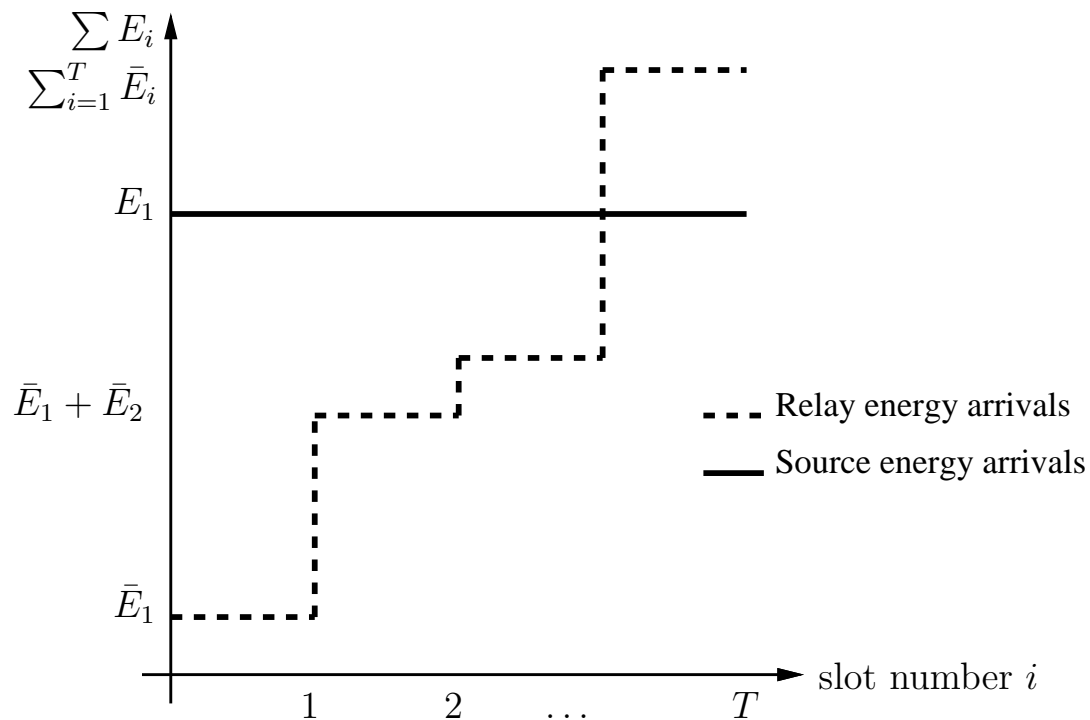
- All source energy is available initially

- Relay is energy harvesting

66

Case II: Non energy harvesting source and energy harvesting relay
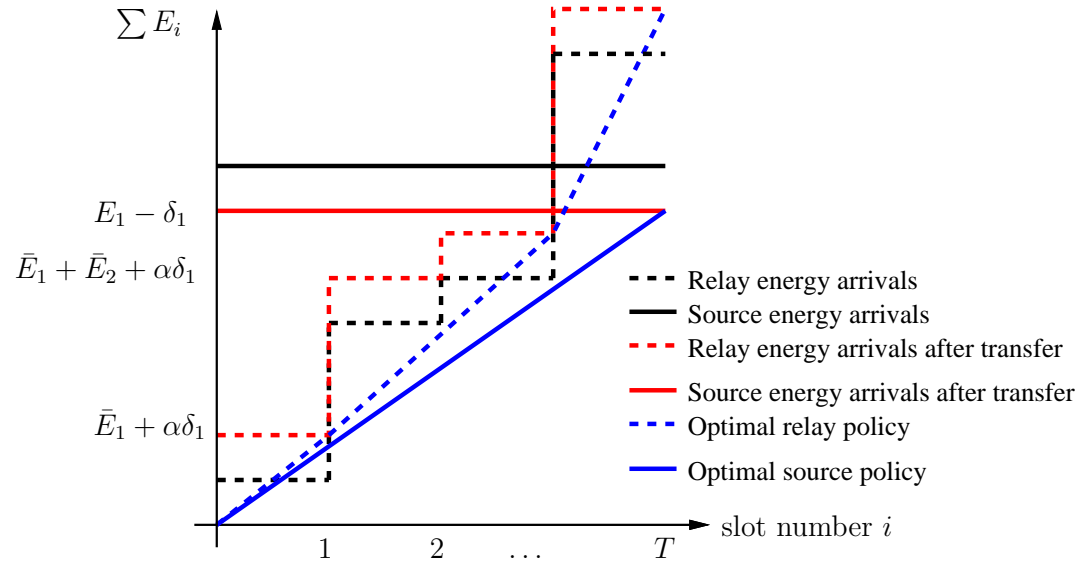
- Transferring energy at a slot can only increase relay powers after that slot.

- Since source is not energy harvesting, energy transfer at first slot is optimal.

$$f(\bar{E}_1 + \delta_1^*, \bar{E}_2, \ldots, \bar{E}_T) = \frac{T}{2} \log\left(1 + \frac{E_1 - \delta_1^*}{T}\right)$$

- $f(\bar{E}_1, \ldots, \bar{E}_T)$ is the maximum number of bits for arrivals $(\bar{E}_1, \ldots, \bar{E}_T)$.

Gaussian Two Way Channel with Energy Cooperation

- Energy harvesting users with deterministic energy arrivals E_i, $\bar{E}_i$

- One-way wireless energy transfer with efficiency $0 < \alpha < 1$.
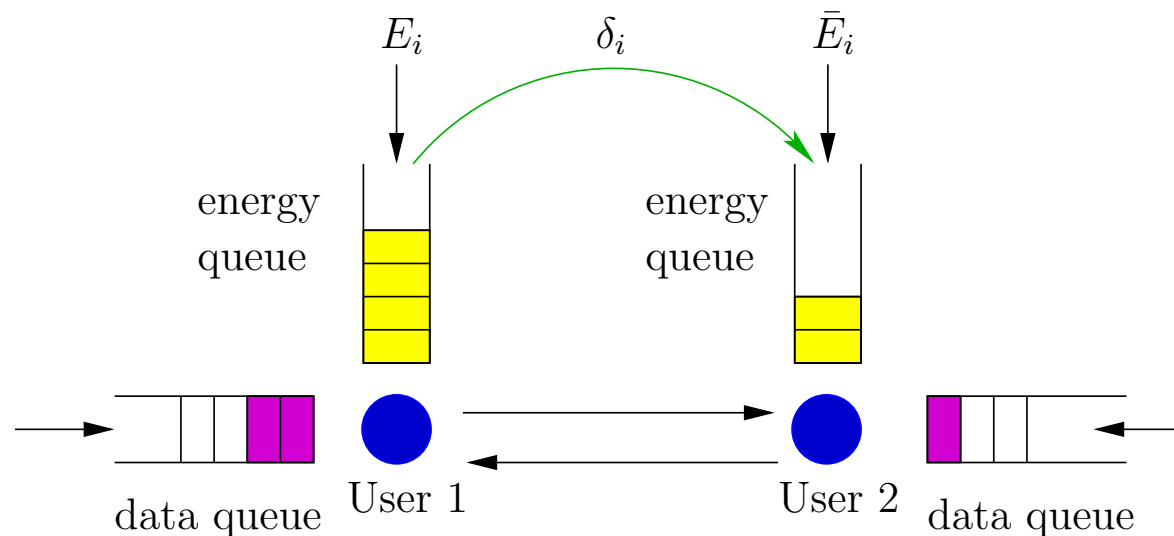
- Physical layer is a Gaussian two-way channel:

$$Y_1 = X_1 + X_2 + N_1$$

$$Y_2 = X_1 + X_2 + N_2$$

N_1, N_2 are Gaussian noises with zero mean and unit power.

Capacity Region
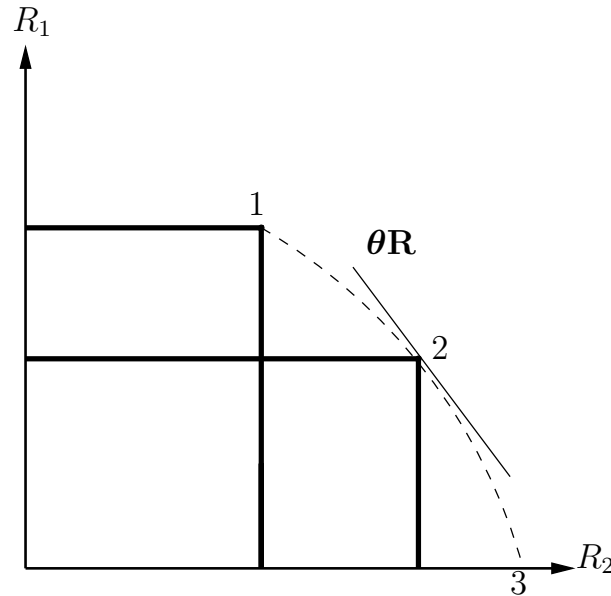
- Convex region, boundary is characterized by solving

$$\max_{\bar{P}_i, P_i, \delta_i} \quad \sum_{i=1}^{T} \theta_1 \frac{1}{2} \log\left(1 + P_i\right) + \theta_2 \frac{1}{2} \log\left(1 + \bar{P}_i\right)$$

$$\text{s.t.} \quad (\boldsymbol{\delta}, \mathbf{P}, \bar{\mathbf{P}}) \in \mathcal{F}$$

- Point 1 is achieved by $\boldsymbol{\delta} = 0$: **no energy transfer.**

- Point 3 is achieved by $\boldsymbol{\delta} = \mathbf{E}$: **full energy transfer.**

Water-filling Approach

- The solution to the optimization problem is given by *generalized two-dimensional directional water-filling algorithm*.

- Transfer energy from one user to another while maintaining optimal allocation in time.

- Spread the energy as much as possible in time and user dimensions.

- Now we give a numerical example for $\theta_1 = \theta_2$ and $\alpha = 1$.

Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

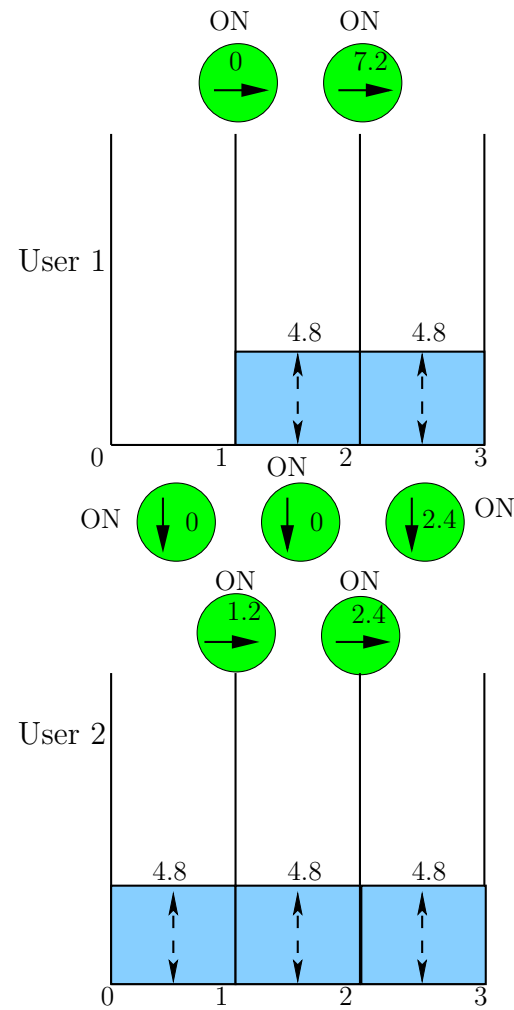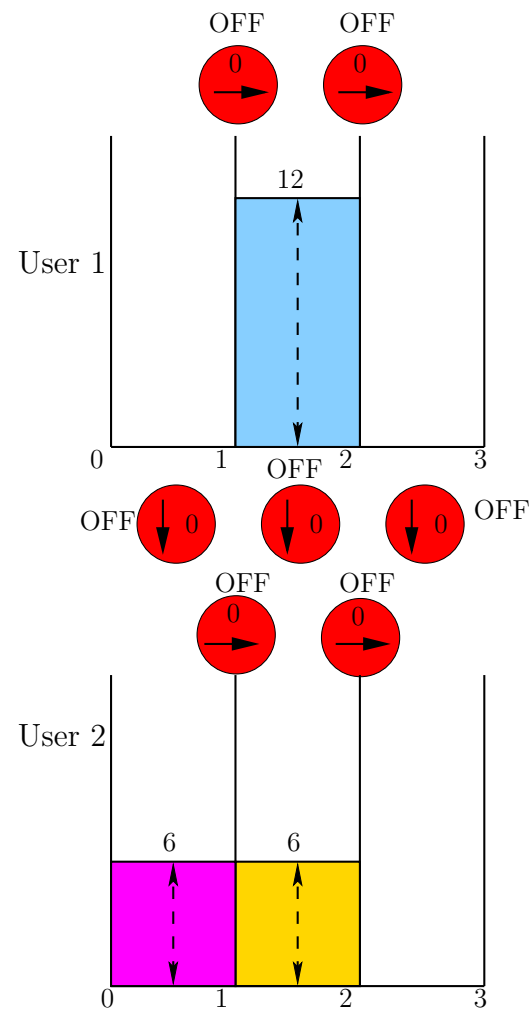# Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

Numerical Example

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Numerical Example

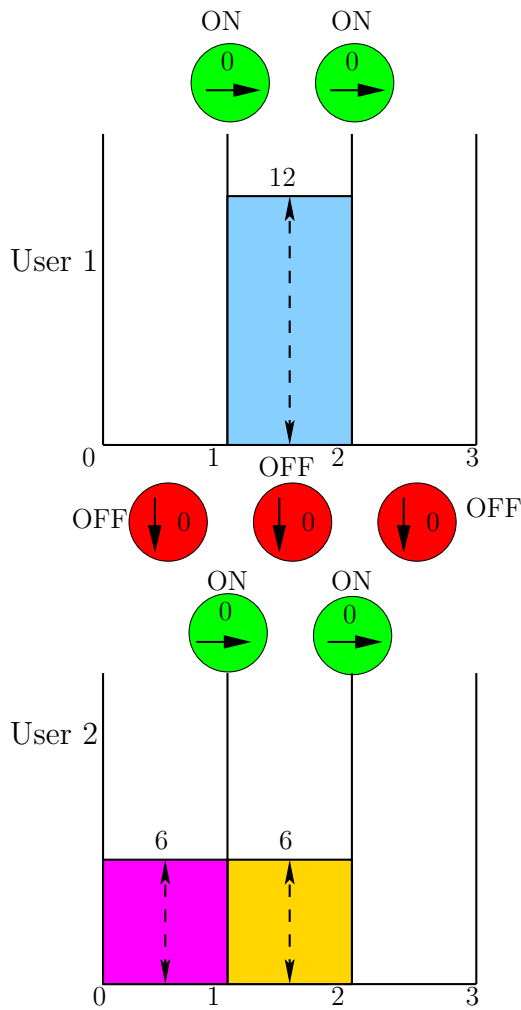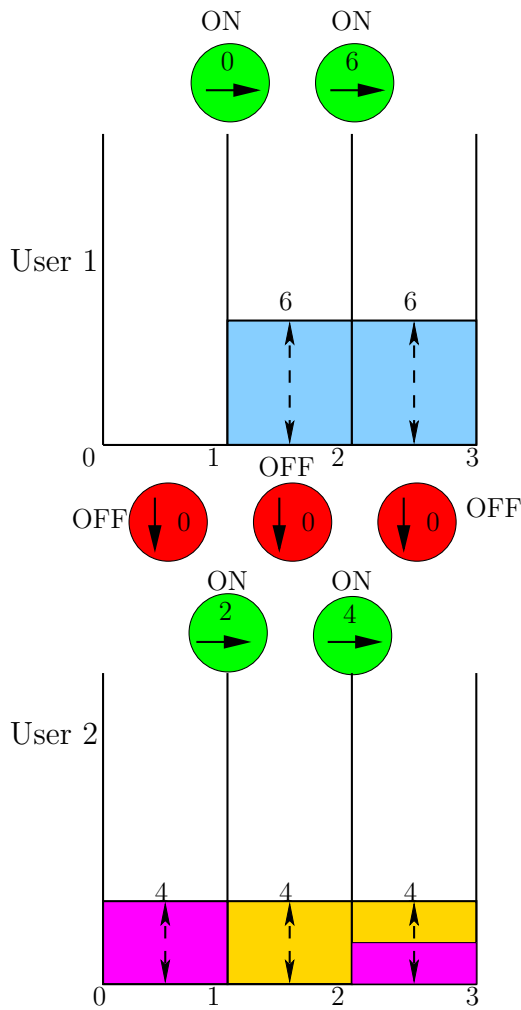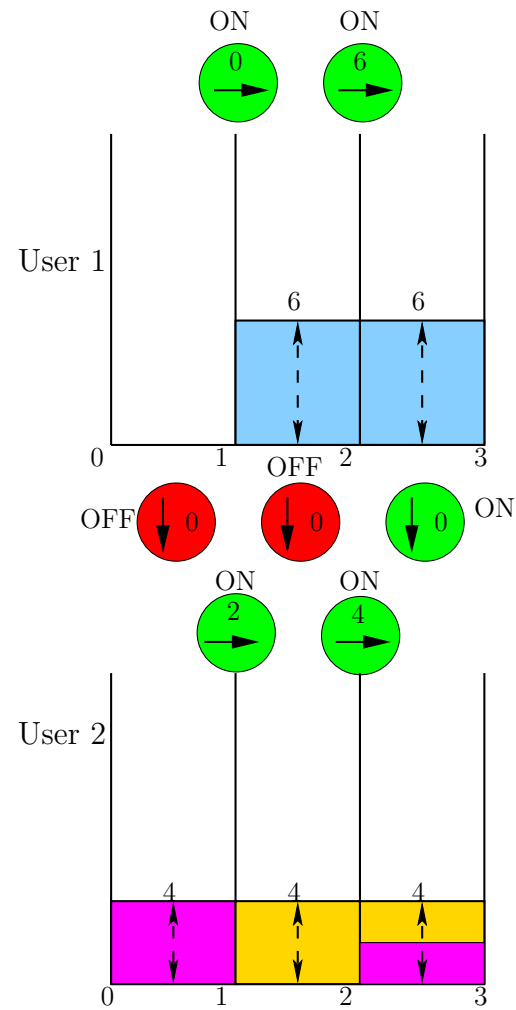$\mathbf{E} = [0, 12, 0] \text{ mJ } \bar{\mathbf{E}} = [6, 6, 0] \text{ mJ}$

Numerical Example

$\mathbf{E} = [0, 12, 0] \text{ mJ } \bar{\mathbf{E}} = [6, 6, 0] \text{ mJ}$

Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\mathbf{\bar{E}} = [6, 6, 0]$ mJ

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$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

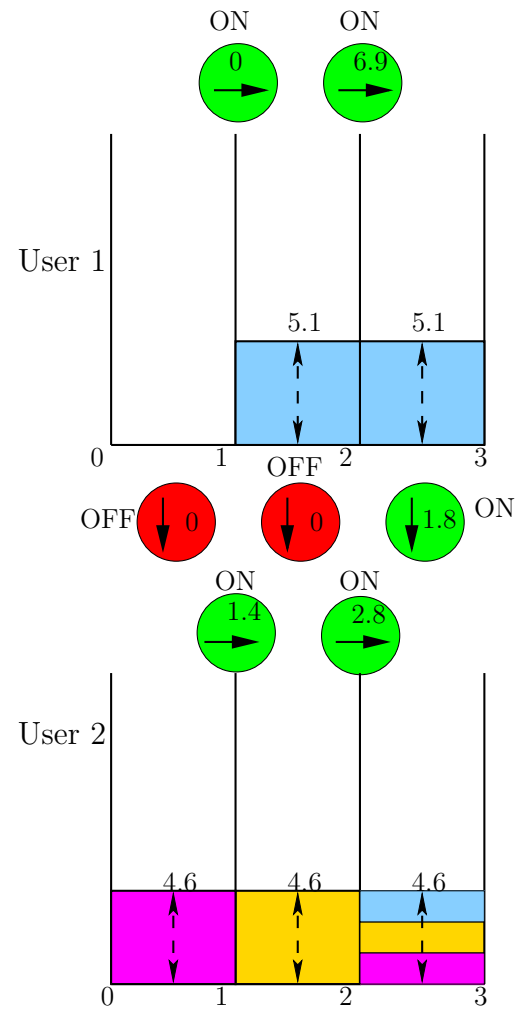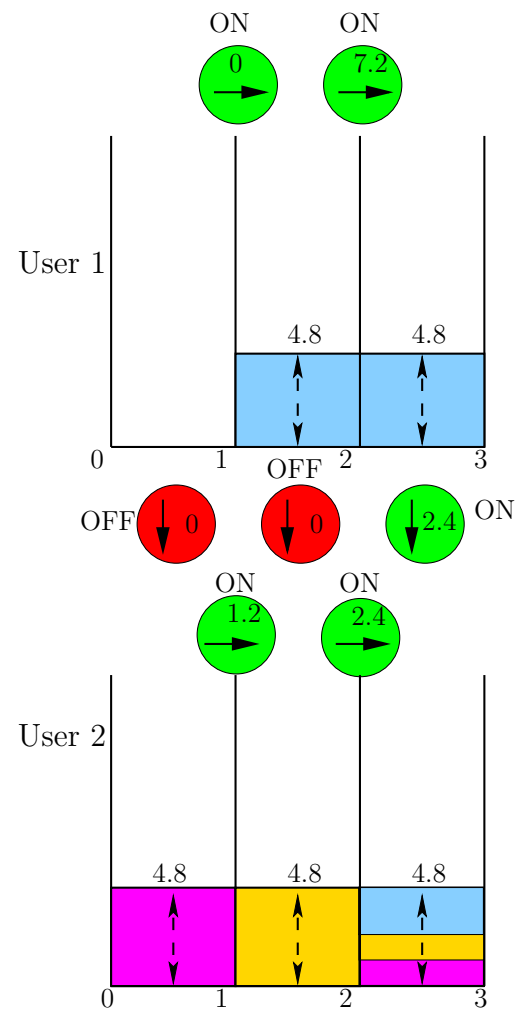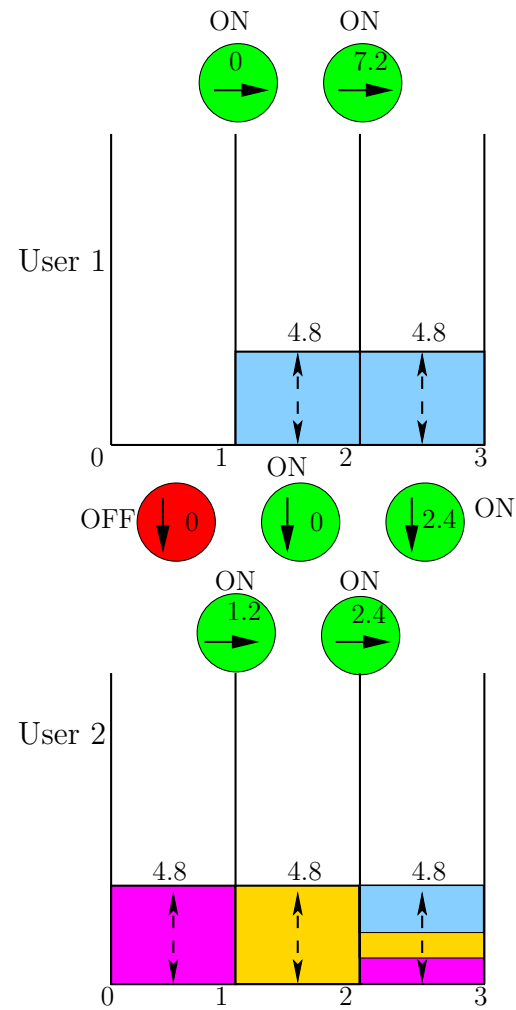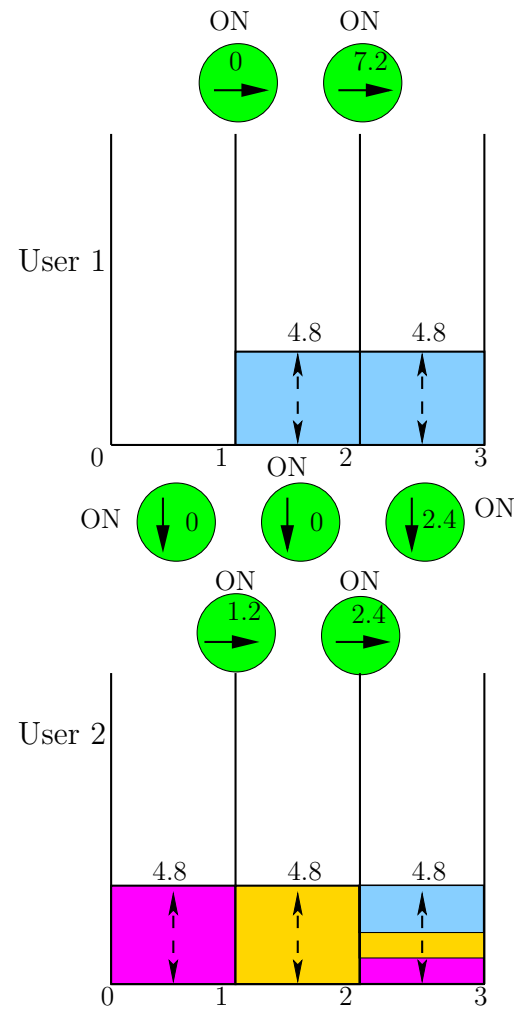# Numerical Example

$\mathbf{E} = [0, 12, 0] \text{ mJ } \bar{\mathbf{E}} = [6, 6, 0] \text{ mJ}$

Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

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Numerical Example
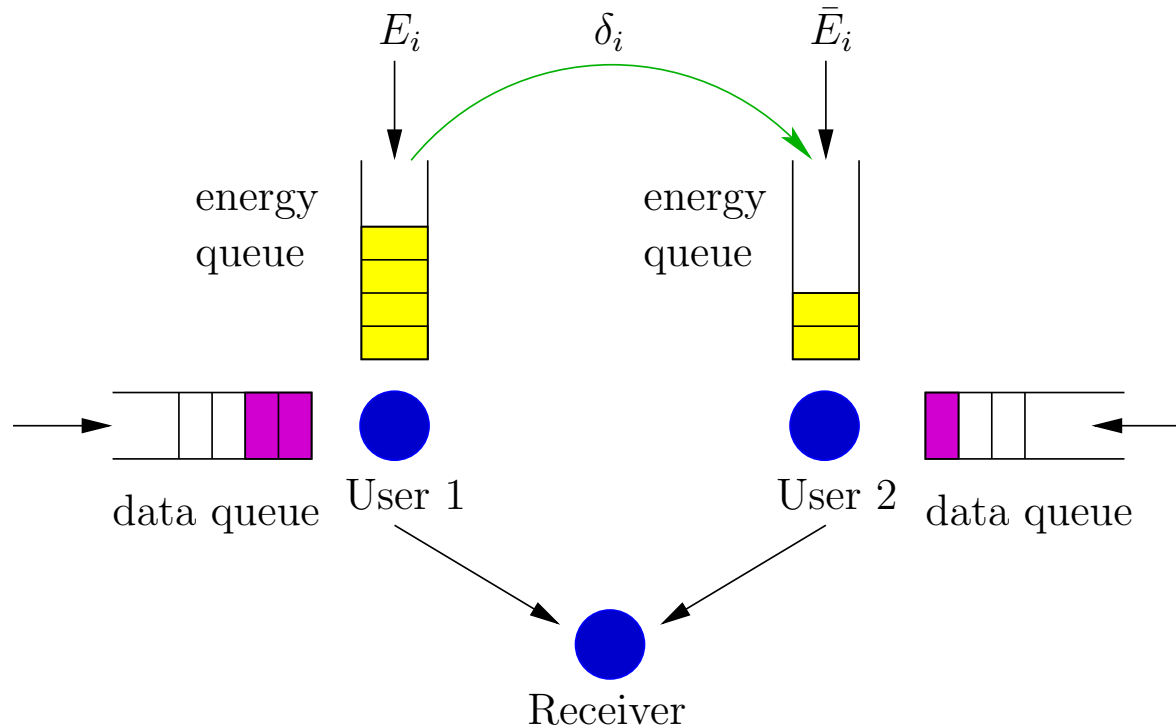
$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

Numerical Example

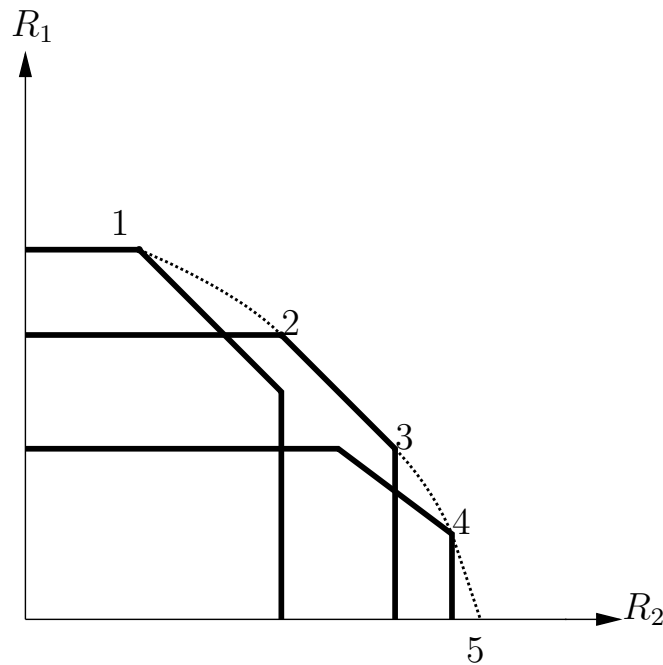$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

Two User Gaussian MAC with Energy Cooperation

- Energy harvesting users with deterministic energy arrivals E_i, $\bar{E}_i$

- One-way wireless energy transfer with efficiency $0 < \alpha < 1$.
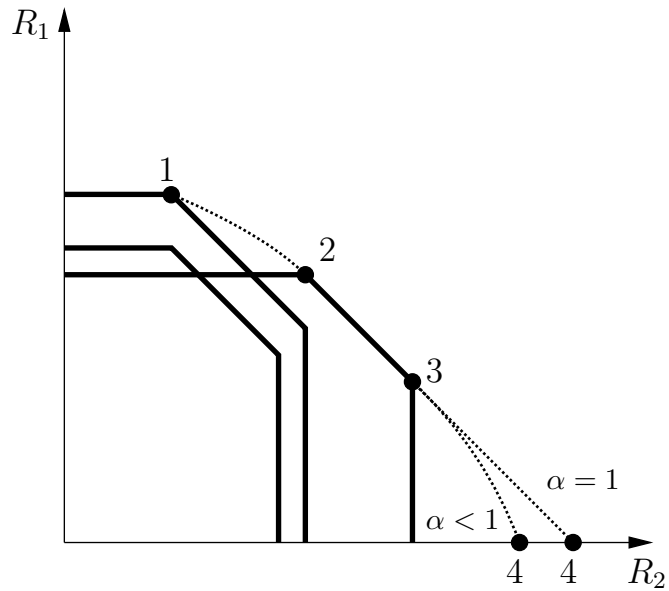
Capacity Region

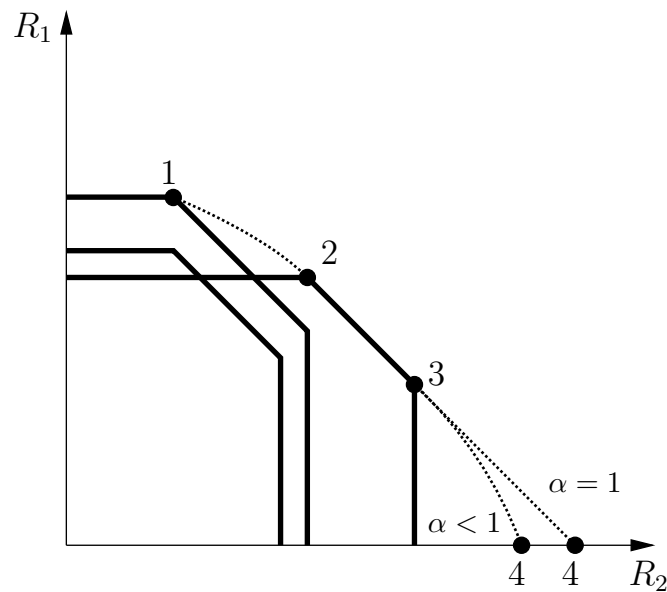- Convex region, boundary is characterized as $\max_{\mathbf{R} \in \mathcal{C}^M} \boldsymbol{\theta} \mathbf{R}$, $\boldsymbol{\theta} \geq 0$

- We investigate $\theta_1 \geq \theta_2$ and $\theta_1 < \theta_2$ separately.

The Case $\theta_1 \geq \theta_2$

- **In the optimal solution, no energy is transferred.**

- Solution is found by generalized backward directional water-filling algorithm.

The Case $\theta_1 < \theta_2, \alpha < 1$

- Point 4 is achieved by **full energy transfer.**

- Energy transfer is necessary to achieve points between 3 and 4.

The Case $\theta_1 < \theta_2$, $\alpha = 1$
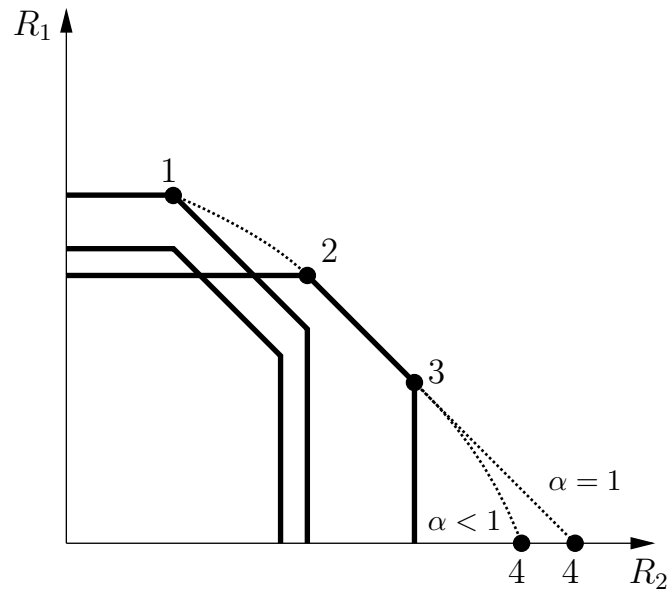
- When $\alpha = 1$, boundary points between 3 and 4 are linear.

- $2, 3$ and 4 are all sum rate optimal.

Conclusions for Energy Cooperation Scenarios

- Energy harvesting users with infinite capacity batteries.

- Energy transfer capability in an orthogonal channel in one way.

- Energy transfer provides a new degree of freedom to smooth out the energy profiles.

- Optimal policies identified for Gaussian two-hop relay, two-way and MAC channels.

- End-to-end throughput maximization for the two-hop relay channel.

- Capacity regions for two-way and MAC channels.

Information Theory of Single-User Energy Harvesting Communication

- Energy is not available up front, arrives randomly in time.

- Energy can be saved in the battery for future use.

- Transmission is interrupted if battery energy is run out.

- What is the highest achievable rate?
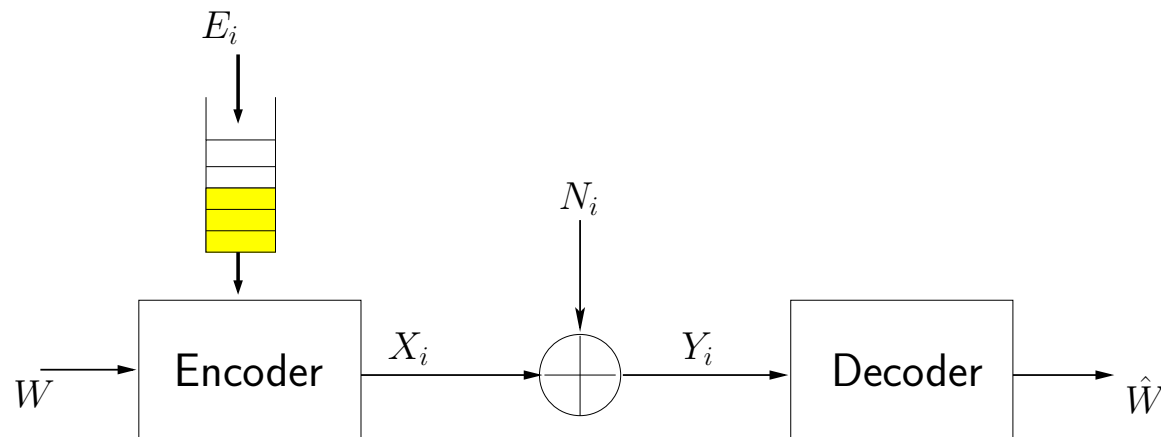
Classical AWGN Channel
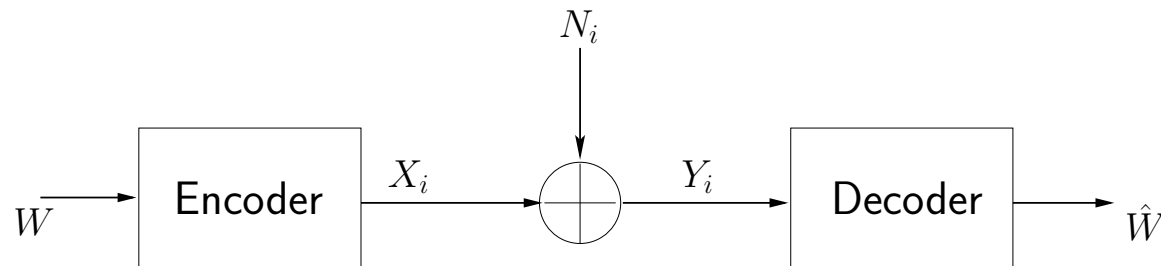
- AWGN channel:

$$Y = X + N$$

- Average power constraint:

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq P$$

- AWGN capacity formula with an average power constraint P:

$$C = \frac{1}{2}\log_2\left(1 + P\right)$$
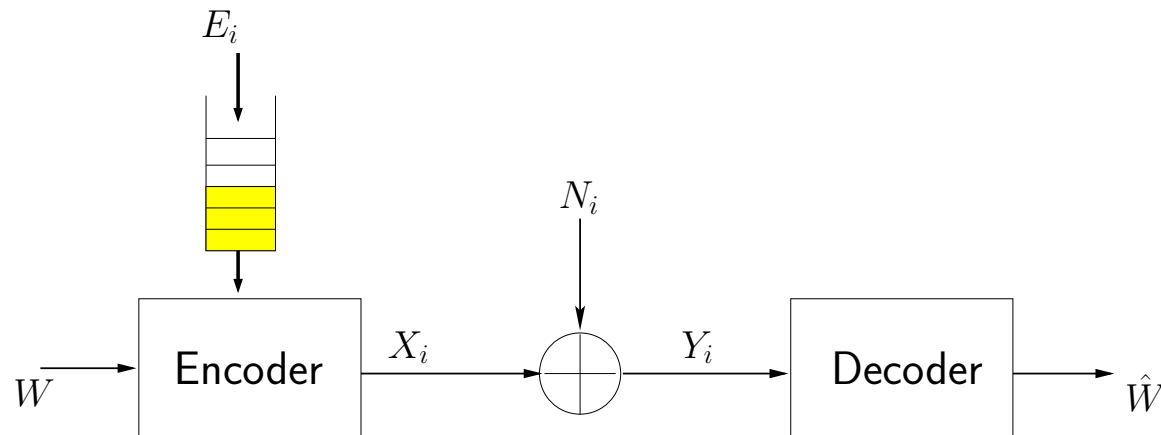
Achievability in the Classical AWGN Channel

- Generate codebook with i.i.d. Gaussians with zero-mean, variance $P - \varepsilon$.
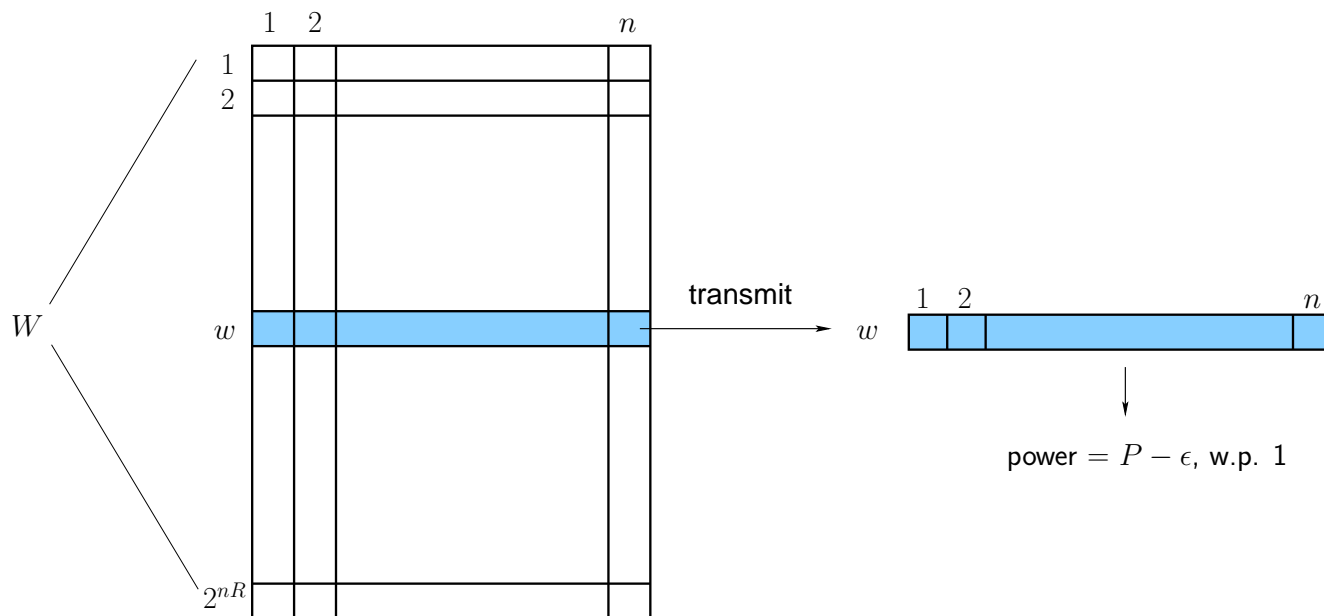
- By SLLN, codewords so generated obey the power constraint w.p. 1,

$$\frac{1}{n} \sum_{i=1}^{n} X_i^2 \rightarrow P - \varepsilon, \quad \text{w.p. 1}$$

# Energy Harvesting AWGN Channel Model ($E_{max} = \infty$)

- Code symbols are constrained to the battery energy at each channel use:

$$\sum_{i=1}^{k} X_i^2 \leq \sum_{i=1}^{k} E_i, \qquad k = 1, 2, \ldots, n$$

- Energy harvesting: n constraints.

- Average power constraint: a single constraint, $k = n$.

- $E[E_i] = P$: average recharge rate.

- Battery storage capacity is infinite.

Achievability in the Energy Harvesting AWGN Channel: Major Concerns

- If we generate an i.i.d. Gaussian codebook with zero-mean, variance $P - \varepsilon$.
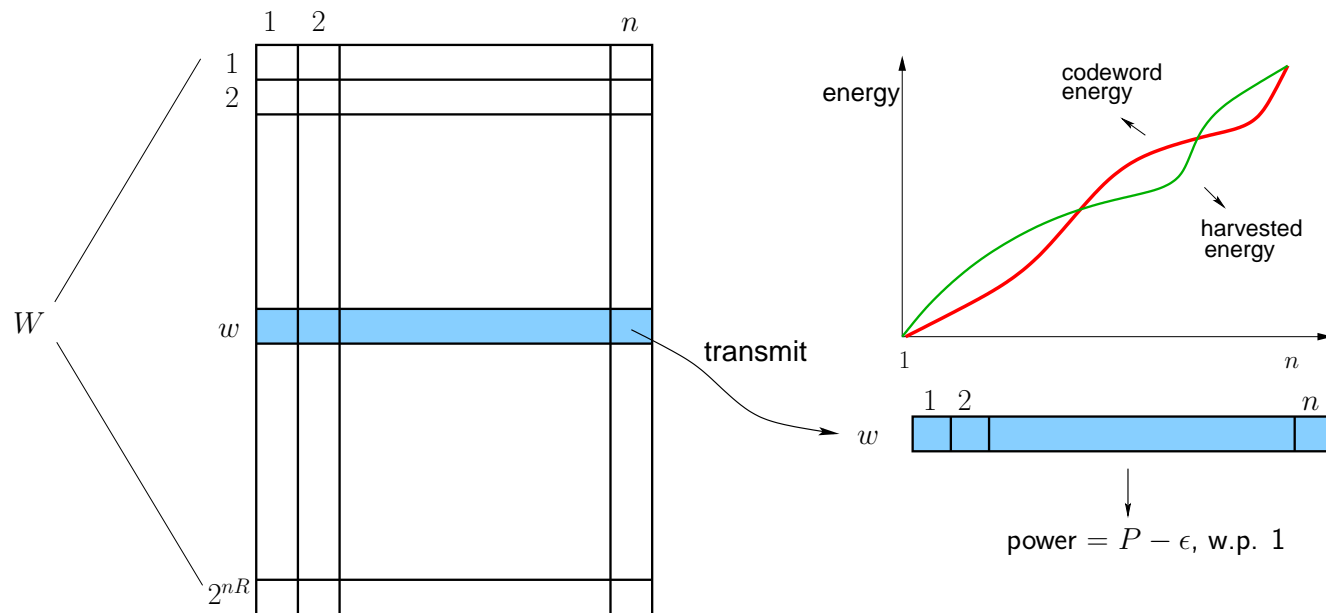
- How do we design the codebook such that:

  - all codewords are energy-feasible for all channel uses.

- Do we need energy arrival state information:

  - causally, non-causally or not at all, at the transmitter and/or receiver.

The Capacity with Energy Harvesting
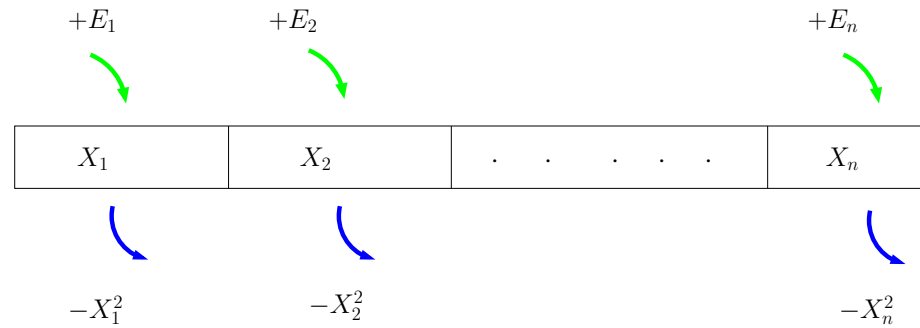
- Upper bound: Average power constrained AWGN capacity:

$$C \leq \frac{1}{2} \log \left(1 + P \right)$$

- This is an upper bound because:

  - Average power constraint imposes a single constraint:

$$\frac{1}{n} \sum_{i=1}^{n} X_i^2 \leq \frac{1}{n} \sum_{i=1}^{n} E_i \to P \quad \text{(by SLLN)}$$

  - While energy harvesting imposes n constraints:

$$\sum_{i=1}^{n} X_i^2 \leq \sum_{i=1}^{n} E_i, \qquad k = 1, \ldots, n$$

- Our contribution: This bound can be achieved.

Achieving the Capacity

- Probability of error: decoding error and violation of energy constraints

- **A first approach:**

  Design a codebook that obeys all n energy constraints.

- **An alternative approach:**

  Design a simple codebook and show the insignificance of energy shortages.

- We will follow the second approach.

- Two achievable schemes:

  1) Save-and-Transmit Scheme

  2) Best-Effort-Transmit Scheme

Save-and-Transmit Scheme

- Save energy in the first $h(n)$ channel uses, do not transmit.

- In the remaining $n - h(n)$ channel uses, send i.i.d. Gaussian signals.

- Saving period of $h(n)$ channel uses makes the remaining symbols feasible.

- Choose $h(n) \in o(n)$ so that saving incurs no loss in rate, i.e., $h(n)/n \to 0$.

- Rates $< \frac{1}{2}\log(1+P)$ are achievable.
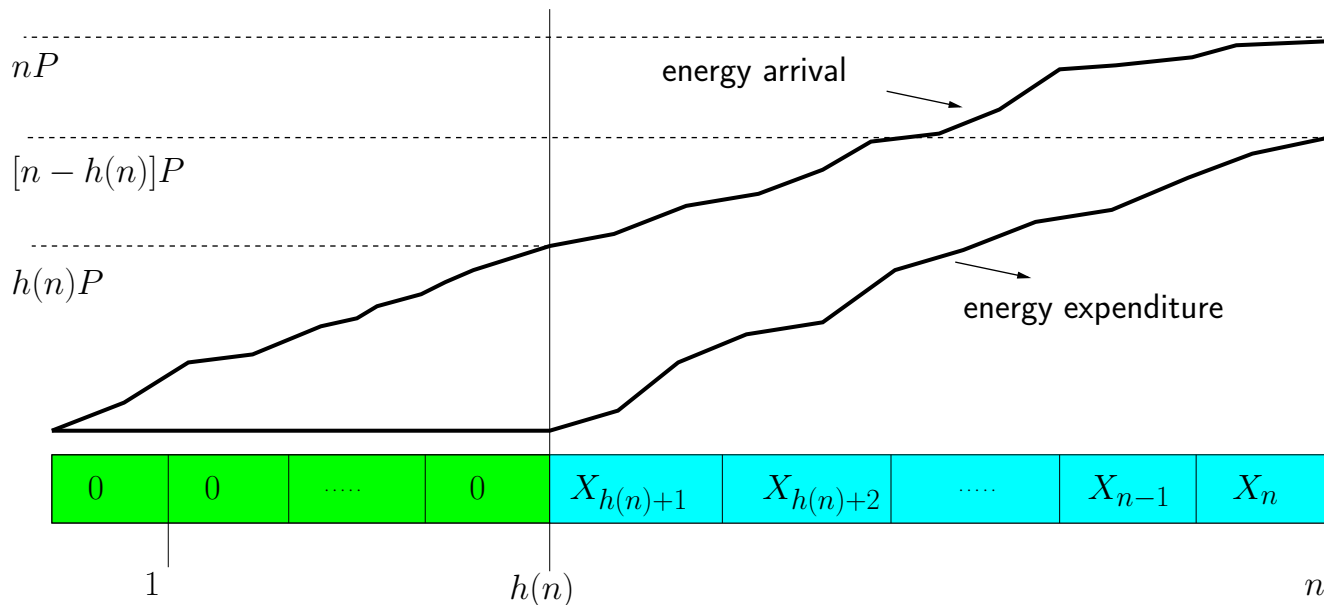
Best-Effort-Transmit Scheme

- X_i: i.i.d. Gaussian.

- $S(i)$: battery energy in the ith channel use.

- If $S(i) \geq X_i^2$, put X_i otherwise put 0 to the channel.

- Mismatch between the codewords and the transmitted symbols.

- Battery energy updates:

$$S(i+1) = S(i) + E_i - X_i^2 \mathbf{1}(S(i) \geq X_i^2)$$

- Since $E[X_i^2] = P - \varepsilon$, only finitely many symbols are infeasible.

- Finitely many mismatches. Inconsequential for joint typical decoding.

- Rates $< \frac{1}{2} \log(1 + P)$ are achievable.

Energy Harvesting AWGN Channel Model ($E_{max} = 0$)
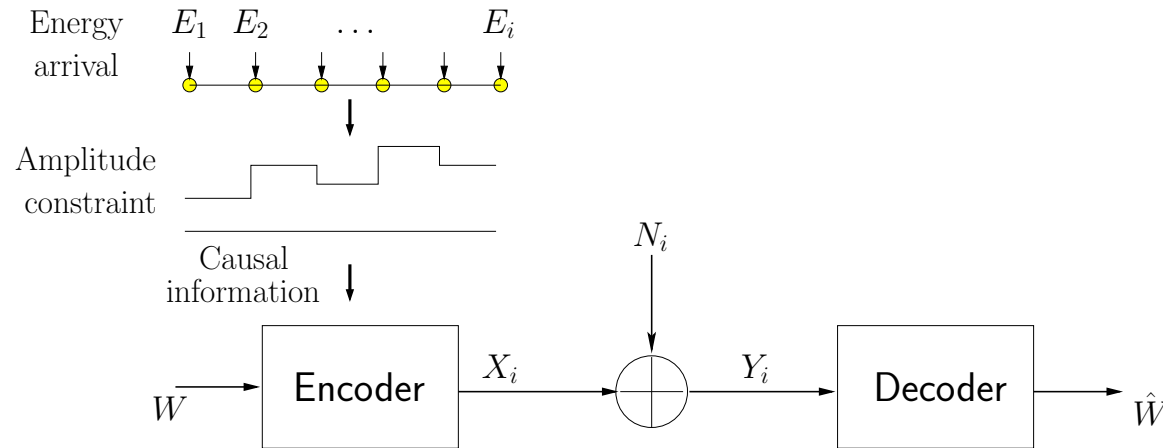
- At the ith channel use, i.i.d. E_i energy arrives

$$|X_i| \leq \sqrt{E_i}$$

- Alphabet $\mathcal{E}$ of energies is finite. For simplicity, binary: $\mathcal{E} = \{E_1, E_2\}$

- The transmitter knows energy arrivals **causally**.

- The receiver **does not know** energy arrivals.

The Channel Model

- **A state dependent channel with side information at the transmitter.**

- At realization E of the energy arrivals, the channel is

$$p(y|x,E) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}, \qquad |x| \leq \sqrt{E}$$

- **Combination of**

  - **Smith's** static amplitude constrained AWGN channel

  - **Shannon's** channel with side information at the transmitter
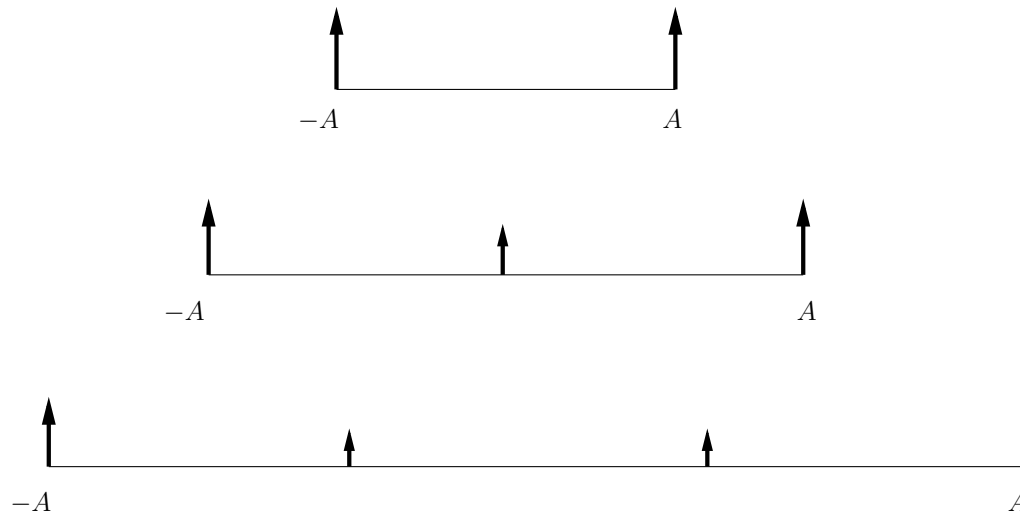
Smith's Amplitude Constrained AWGN Channel

- In 1971, Smith studied **static** amplitude constraints:

$$p(y|x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}, \qquad |x| \leq A$$

- At each channel use, channel symbol is amplitude constrained to A.

$$C_{Sm}(A) = \max_{|X| \leq A} I(X;Y)$$

- This is a convex functional optimization problem.

- The capacity achieving input distribution is **discrete.**

Shannon's Channels with Side Information at the Transmitter

- The state-dependent channel $p(y|x,s)$, $s \in \mathcal{S}$

- i.i.d. states with $P(s = s_i) = p_{s_i}$.

- s is available causally at the transmitter, not available at the receiver.
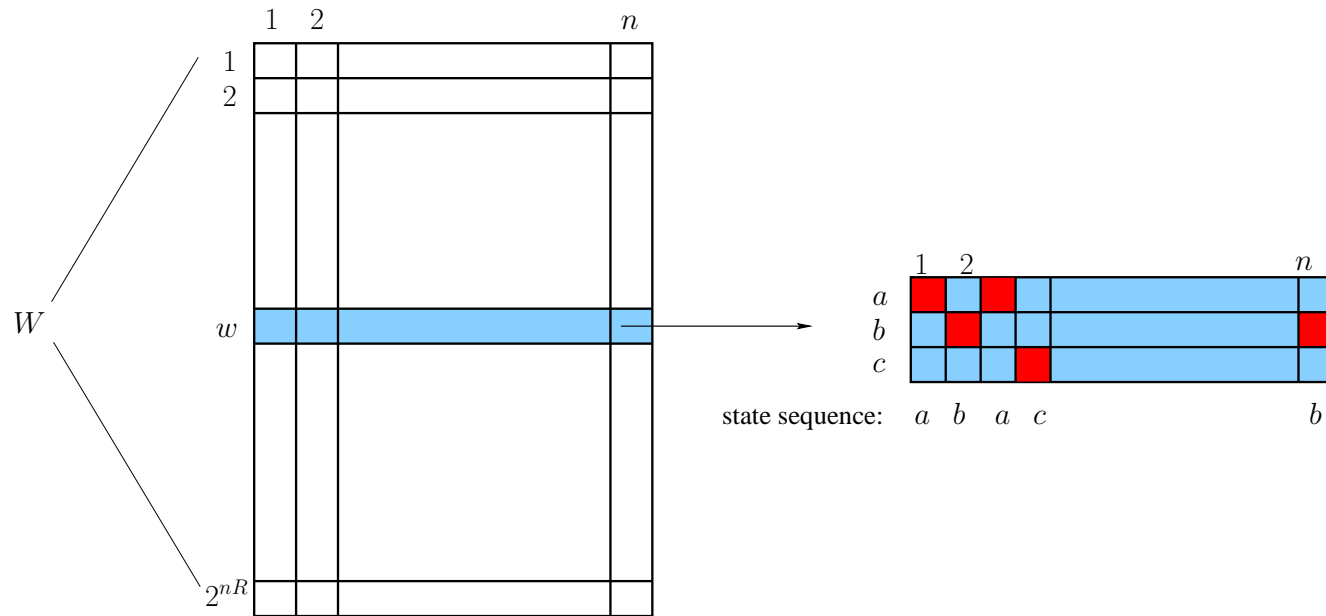
- Shannon proved in 1958 that

$$C_{Sh} = \max_{p(T)} I(T;Y)$$

- T is the extended input $T = [T_1, \ldots, T_{|\mathcal{S}|}]$ with

$$p(y|t = (t_1, \ldots, t_{|\mathcal{S}|})) = \sum_{i=1}^{|\mathcal{S}|} p_{s_i} p(y|t_i, s_i)$$

Shannon's Channels with Side Information at the Transmitter

- Shannon strategy: codewords are $|\mathcal{S}| \times n$ matrices.

state sequence: a b a c b

Capacity of AWGN Channel with Time-Varying Amplitude Constraints

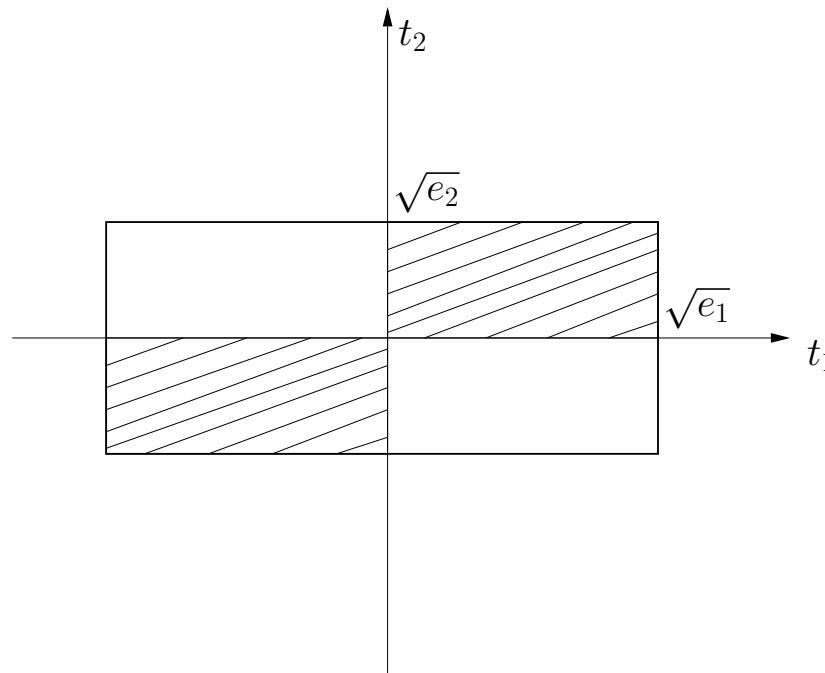- Applying Shannon's result,

$$C_{Sh} = \max_{p(T)} I(T;Y)$$

- $T = [T_1, T_2]$

$$p(y|t_1,t_2) = \underbrace{\frac{p_1}{\sqrt{2\pi}} e^{-\frac{(y-t_1)^2}{2}}}_{|t_1| \leq \sqrt{E_1}} + \underbrace{\frac{p_2}{\sqrt{2\pi}} e^{-\frac{(y-t_2)^2}{2}}}_{|t_2| \leq \sqrt{E_2}}$$

- If E is observed, the channel symbol needs to satisfy $|X| \leq \sqrt{E}$.

- The capacity achieving distribution is **discrete.**

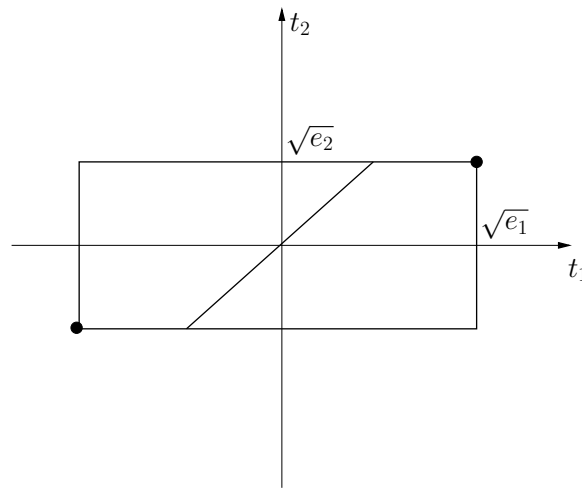- $[T_1, T_2]$ takes values from a finite set in $\mathbb{R}^2$.

Structure of the Optimal Mass Points

- Symmetric with respect to $(0,0)$

- Constrained to the shaded area
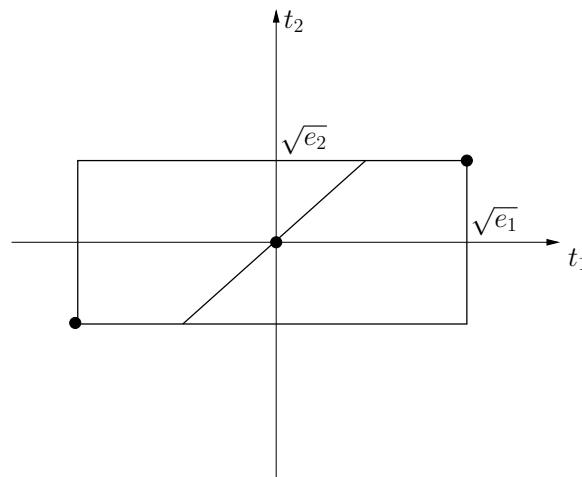
- $a_1 = \sqrt{E_1}$ and $a_2 = \sqrt{E_2}$

Structure of the Optimal Mass Points
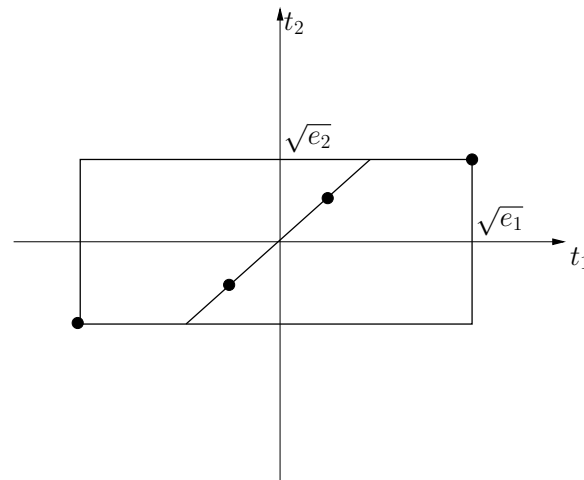
- If a_1 and a_2 are sufficiently small: binary

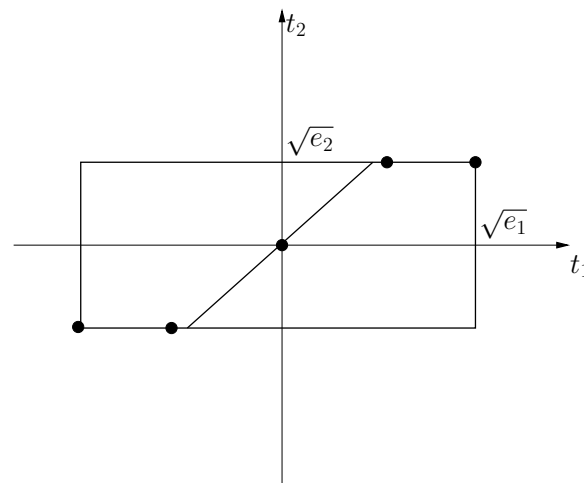- If a_1 and a_2 are increased: ternary

Structure of the Optimal Mass Points
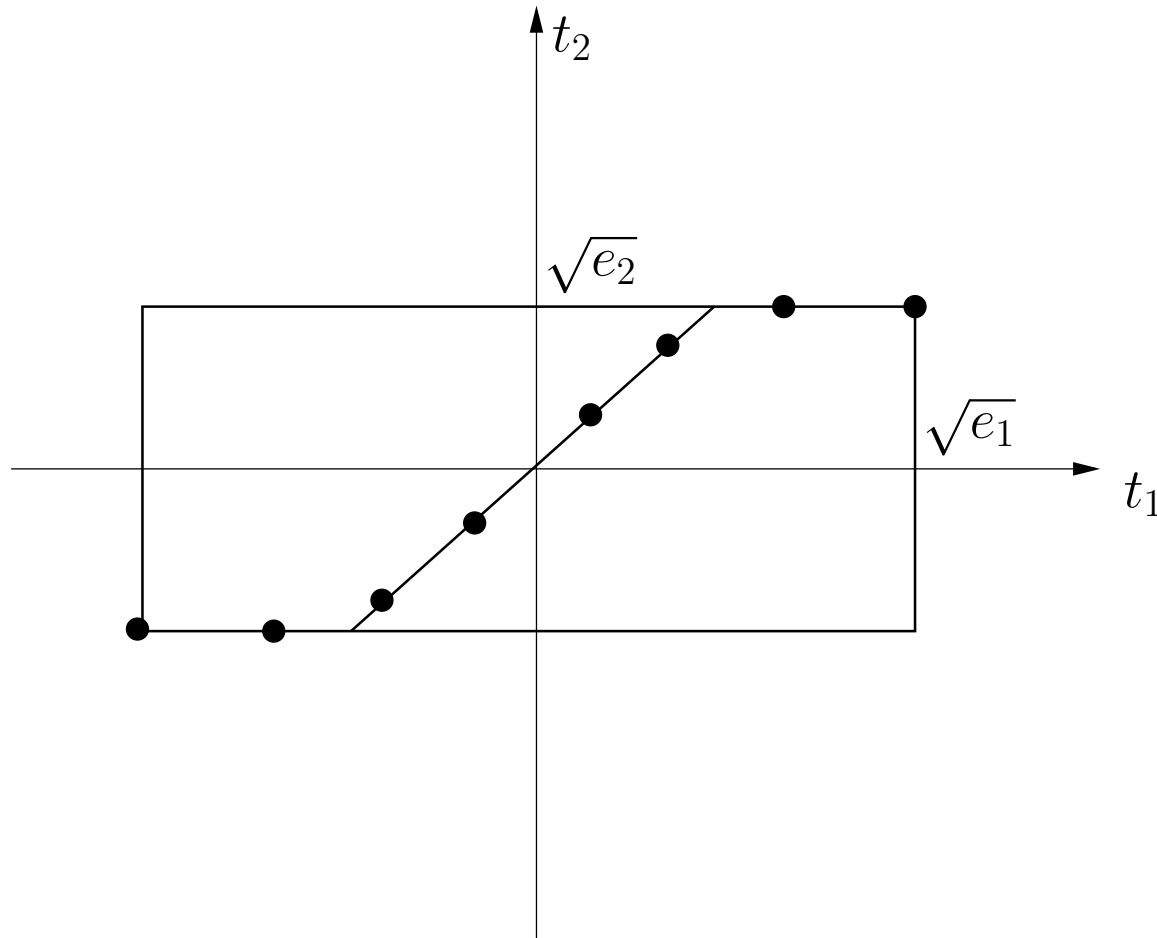
- If a_1 and a_2 are increased: quaternary

- If a_1 and a_2 are increased: quintuple

Experimental Observation of the Optimal Mass Points

- Experiments are based on verification of the necessary optimality conditions.

Other Scenarios for AWGN Channel with Energy Arrivals

- If the energy arrival information is available at both sides

$$C_{si@both} = p_1 C_{Sm}(a_1) + p_2 C_{Sm}(a_2)$$

- $a_1 = \sqrt{E_1}$ and $a_2 = \sqrt{E_2}$.

- If there is an infinite capacity battery:

$$C = \frac{1}{2} \log_2 \left(1 + p_1 E_1 + p_2 E_2\right)$$
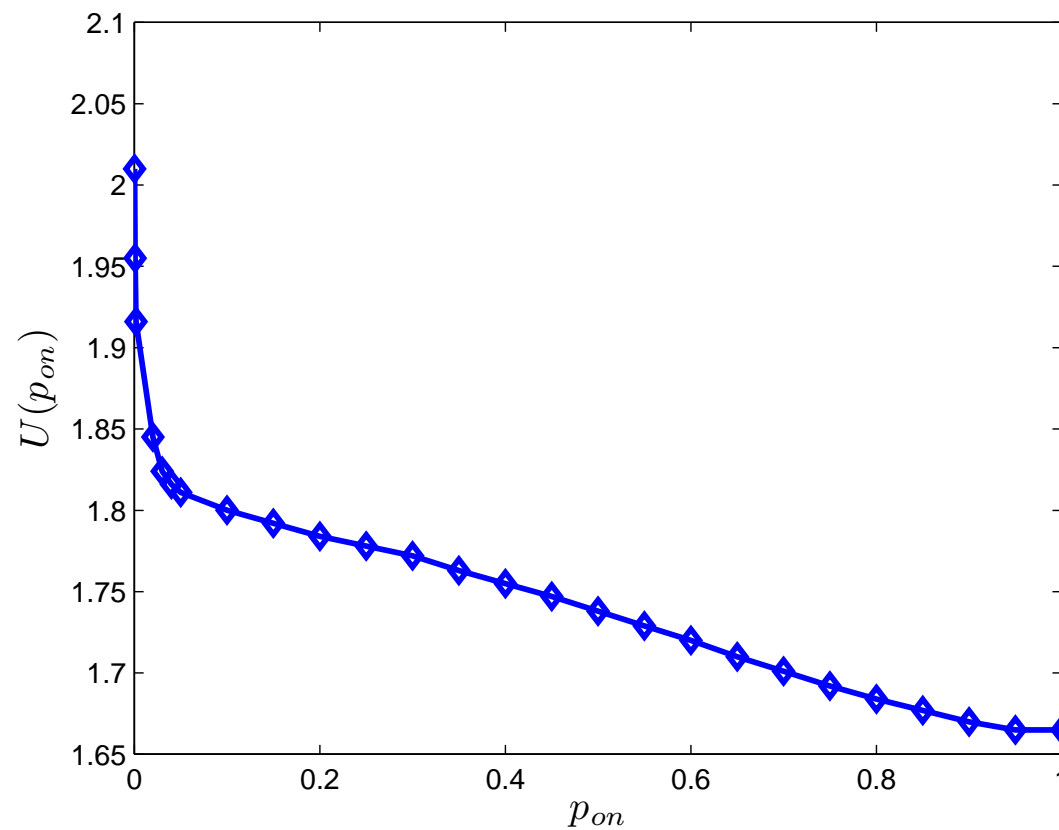
AWGN Channel with On-Off Energy Arrivals

- On-off energy arrivals: $E_1 = 0$, $E_2 = E > 0$.

- Energy E arrives with probability p_{on}.

- No energy arrives with probability $p_2 = 1 - p_{on}$

$$p(y|t) = \frac{p_{on}}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} + \frac{1 - p_{on}}{\sqrt{2\pi}} e^{-\frac{(y-t)^2}{2}}, \qquad |t| \leq \sqrt{E}$$

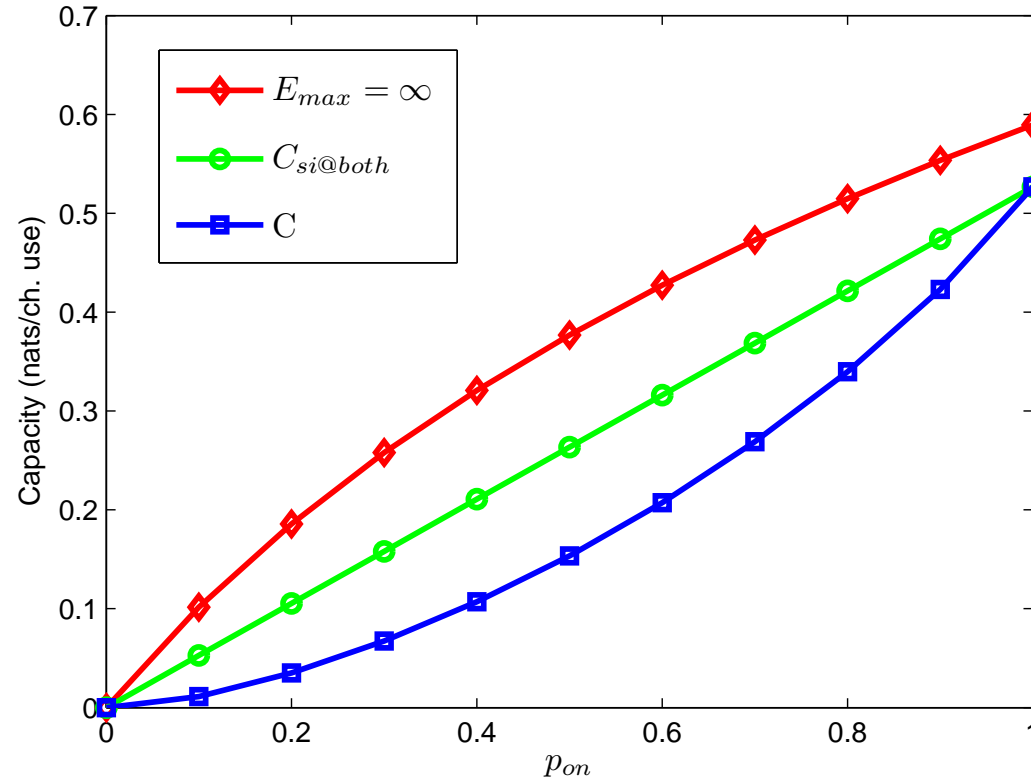- p_{on} determines the power of transmission.

AWGN Channel with On-Off Energy Arrivals

- p_{on} affects the turning point from binary to ternary.

- Define $U(p_{on})$ as the amplitude at which the turn occurs.
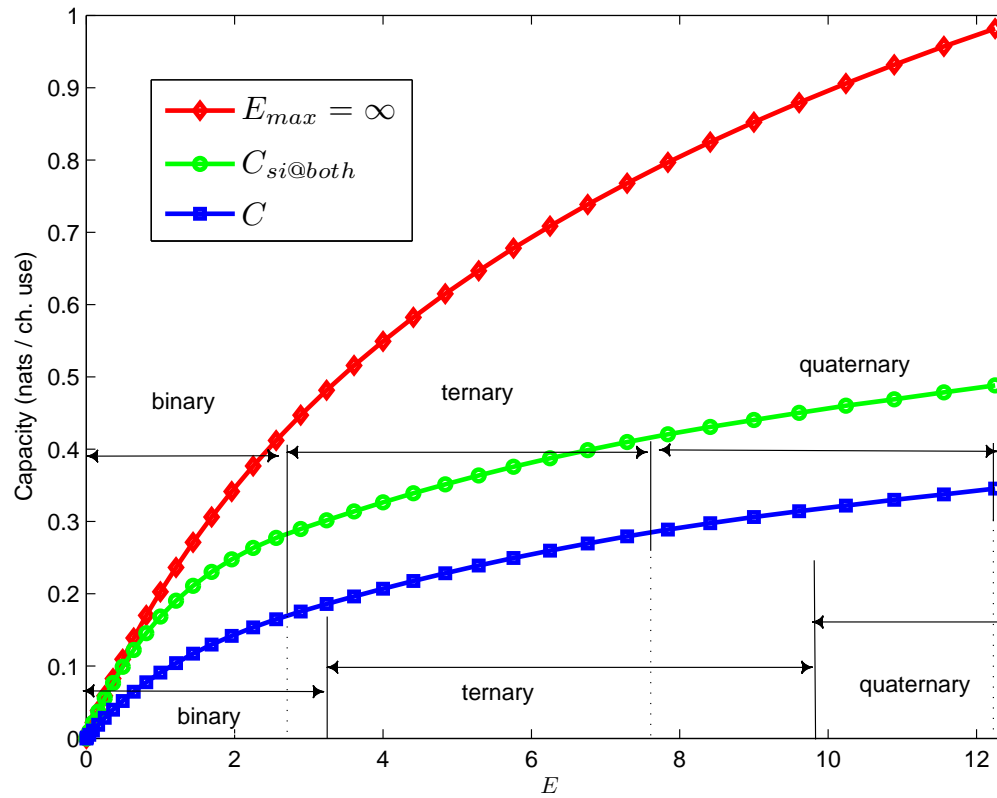
AWGN Channel with On-Off Energy Arrivals

- $E = 2.25$ and p_{on} is varying.

- Capacity achieving distributions are always binary.

AWGN Channel with On-Off Energy Arrivals

- $p_{on} = 0.5$ and E is varying.

- Capacity achieving distributions are also varying.

- When state information is available at both sides, same as Smith.

Conclusions

- Capacity of energy harvesting AWGN channel under two extremes.

- When battery capacity is $E_{max} = \infty$:

  – Transmitter/receiver do not need energy arrival information.

  – Equal to the AWGN channel capacity with average power $E[E_i] = P$.

  – Save-and-Transmit Scheme and Best-Effort-Transmit Scheme

- When battery capacity is $E_{max} = 0$:

  – Transmitter has causal energy information, receiver has no information.

  – Smith's static amplitude constraints and Shannon's causal side information

  – Discrete signaling is optimal.

- Open problem: When battery capacity E_{max} is finite.

Acknowledgements

- NSF CNS 0964632

- Slides: Omur Ozel

References

[1] J. Yang, O. Ozel and S. Ulukus, Broadcasting with an Energy Harvesting Rechargeable Transmitter, IEEE Transactions on Wireless Communications, February, 2012.

[2] J. Yang and S. Ulukus, Optimal Packet Scheduling in a Multiple Access Channel with Energy Harvesting Transmitters, JCN Special Issue on Energy Harvesting in Wireless Networks, April 2012.

[3] O. Ozel, J. Yang and S. Ulukus, Optimal Broadcast Scheduling for an Energy Harvesting Rechargeable Transmitter with a Finite Capacity Battery, IEEE Transactions on Wireless Communications, June 2012.

[4] K. Tutuncuoglu and A. Yener, Sum-Rate Optimal Power Policies for Energy Harvesting Transmitters in an Interference Channel, JCN Special Issue on Energy Harvesting in Wireless Networks, April 2012.

[5] D. Gunduz and B. Devillers, Two-Hop Communication with Energy Harvesting, IEEE CAMSAP, San Juan, PR, December 2011.

References

[6] B. Gurakan, O. Ozel. J. Yang and S. Ulukus, Energy Cooperation in Energy Harvesting Wireless Communications, IEEE ISIT, Cambridge, MA, July 2012.

[7] B. Gurakan, O. Ozel. J. Yang and S. Ulukus, Two-Way and Multiple Access Energy Harvesting Systems with Energy Cooperation, Asilomar Conference, Pacific Grove, CA, November 2012.

[8] B. Gurakan, O. Ozel. J. Yang and S. Ulukus, Energy Cooperation in Energy Harvesting Two-Way Communications, IEEE ICC, Budapest, Hungary, June 2013.

[9] O. Ozel and S. Ulukus, Achieving AWGN Capacity under Stochastic Energy Harvesting, IEEE Transactions on Information Theory, October 2012.

[10] O. Ozel and S. Ulukus, AWGN Channel under Time-Varying Amplitude Constraints with Causal Information at the Transmitter, Asilomar Conference, Pacific Grove, CA, November 2011.