

Delay-minimal Transmission for Average Power Constrained Multi-access Communications – Part II: Asymmetric Scenario

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Abstract

In part I of this two-part paper, we investigated the problem of minimizing the overall transmission delay of packets in a multi-access wireless communication system under a symmetric assumption. In this part II, we extend our approach to the general, asymmetric setting. We develop a procedure by which we determine the optimal solution analytically in a closed-form, although the optimal solution does not have a symmetric structure. In addition, in this part, we prove that the allocation procedures we developed for both symmetric and asymmetric settings are optimal. Furthermore, in this part II, we provide numerical examples for both symmetric and asymmetric settings.

I. INTRODUCTION

In part I of this two-part paper, we developed a transmission protocol, where users in a multiple-access system choose their probabilities of transmission based on their queue lengths. We showed that the optimal transmission scheme has a threshold structure, i.e., if the sum of the queue lengths exceeds a threshold, both users transmit a packet from their queues, and if the sum of the queue lengths is smaller than a threshold, only one user, which has the larger queue length transmits a packet from its queue, while the other user remains silent (equal queue length case is resolved by flip of a fair coin). Our analysis in part I was for a symmetric setting where the arrival rates to both queues, the average powers of the two transmitters, and the channel gains of the two users were identical. In this part II, we investigate the most general case where the arrival rates to the two queues θ_1, θ_2 , the average powers of the two transmitters P_{1avg}, P_{2avg} , and the channel gains of the two users h_1, h_2 are all arbitrary. The analysis of the problem in this asymmetric case is more complex, however, in this case also, we are able to derive a closed-form optimal solution. As we will show, the solution in this case has a threshold structure as well. However, the solution is not completely symmetric.

The first step of our two-step solution here is similar to that in part I. However, the second step where optimal y_n s and t_n s are assigned to x_{ij}^k s is different and significantly more complex, due to the asymmetry of the setting. The unknown variables x_{ij}^k s interact with each other through the transition equations. Finding an allocation to satisfy all of the equations simultaneously becomes rather difficult in this asymmetric setting. We choose to allocate y_n s and t_n s to a small number of states as in part I. A careful examination of the transition equations within groups reveals that the values of all of x_{ij}^k s are actually determined by the allocation of y_n between $x_{\frac{n}{2}+1, \frac{n}{2}-1}^1 + x_{\frac{n}{2}, \frac{n}{2}}^2$ and $x_{\frac{n}{2}-1, \frac{n}{2}+1}^2 + x_{\frac{n}{2}, \frac{n}{2}}^1$ when n is even, and the allocation of t_n between $x_{\frac{n+1}{2}, \frac{n-1}{2}}^3$ and $x_{\frac{n-1}{2}, \frac{n+1}{2}}^3$ when n is odd. Once we fix these sums, the values of all x_{ij}^k are fixed, and the transmission probabilities g_{ij}^k can be derived.

In addition, in this part II, we provide the mathematical proof for the optimality of the solutions derived in parts I and II. In this part, we also provide numerical examples for both symmetric and asymmetric settings. Related works and the development of the related literature [1]–[7] is

summarized in the introduction of part I of this two-part paper, which we do not repeat here, and proceed directly with our mathematical development for the asymmetric case.

II. SYSTEM MODEL

A. Physical Layer Model

We consider a discrete-time additive Gaussian noise multiple-access system with two transmitters and one receiver. The received signal is

$$Y = \sqrt{h_1}X_1 + \sqrt{h_2}X_2 + Z \quad (1)$$

where X_i is the signal of user i , $\sqrt{h_i}$ is the channel gain for user i , and Z is a Gaussian noise with zero mean and variance σ^2 . Here, h_1 and h_2 are real constants, with $h_1 \neq h_2$ in general.

In this two-user system, the multiple-access capacity region is [8]

$$R_1 \leq \frac{1}{2} \log \left(1 + \frac{h_1 P_1}{\sigma^2} \right) \quad (2)$$

$$R_2 \leq \frac{1}{2} \log \left(1 + \frac{h_2 P_2}{\sigma^2} \right) \quad (3)$$

$$R_1 + R_2 \leq \frac{1}{2} \log \left(1 + \frac{h_1 P_1 + h_2 P_2}{\sigma^2} \right) \quad (4)$$

where P_i is the transmit power for user i . Then, the region of feasible powers is

$$h_1 P_1 \geq \sigma^2 (2^{2R_1} - 1)$$

$$h_2 P_2 \geq \sigma^2 (2^{2R_2} - 1)$$

$$h_1 P_1 + h_2 P_2 \geq \sigma^2 (2^{2(R_1+R_2)} - 1) \quad (5)$$

The average power constraints for the two users are P_{1avg} and P_{2avg} .

B. Medium Access Control (MAC) Layer Model

In the MAC layer, we assume that packets arrive at the transmitters at a uniform size of B bits per packet. Let $a_1[n]$ and $a_2[n]$ denote the number of packets arriving at the first and the second transmitters, respectively, during time slot n ; see Figure 1. We assume that the packet

arrivals are i.i.d. from slot to slot, and the probabilities of arrivals are

$$\begin{aligned} Pr\{a_i[n] = 1\} &= \theta_i \\ Pr\{a_i[n] = 0\} &= 1 - \theta_i \end{aligned} \tag{6}$$

where θ_i is the arrival rate for user i , $i = 1, 2$.

We model the evolution of the queue lengths as a two-dimensional Markov chain. The two-dimensional Markov chain is shown in Figure 2. We follow the same definition of transmission probabilities g_{ij}^k , $k = 1, 2, 3$, $i = 0, \dots, N$, $j = 0, \dots, N$, as in part I. Then, the transition probabilities from one state to its neighbors are also functions of the arrival and departure probabilities. They are expressed in a similar form as in the symmetric case, except that we use θ_1 and θ_2 instead of θ in corresponding places. We assume that a stationary distribution exists. According to Little's law [9], the average delay in the system is

$$D = \frac{1}{\theta_1 + \theta_2} \sum_{i,j} \pi_{ij}(i + j) \tag{7}$$

where $\theta_1 + \theta_2$ is the average arrival rate for the system.

III. PROBLEM FORMULATION

First, let us consider the power consumption for the asymmetric setting. When only one user transmits, the transmit power for the active user needs to satisfy

$$h_i P_i \geq \sigma^2(2^{2R} - 1) \triangleq \alpha \tag{8}$$

where $R = B/\tau$. In order to minimize the power, the transmitted power for the active user should be α/h_i , depending on which user is transmitting. When both users transmit simultaneously, the received powers should additionally satisfy

$$h_1 P_1 + h_2 P_2 \geq \sigma^2(2^{4R} - 1) \triangleq \beta \tag{9}$$

The feasible transmitted power region is shown in Figure 3. Let us denote the received power pair as (β_1, β_2) . In order to minimize the transmit power, this pair should be on the dominant face of the feasible power region, i.e., $\beta_1 + \beta_2 = \beta$. Then, the corresponding transmit power pair is $(\beta_1/h_1, \beta_1/h_2)$. Note that different operating points need different sum of transmit powers.

Therefore, the problem in this asymmetric setting can be expressed as:

$$\min_{\mathbf{g}, \beta_1, \beta_2} \quad \frac{1}{\theta_1 + \theta_2} \sum_{i,j} \pi_{ij}(i + j) \quad (10)$$

$$\text{s.t.} \quad \frac{1}{h_1} \sum_{i,j} \pi_{ij}(g_{ij}^1 \alpha + g_{ij}^3 \beta_1) \leq P_{1avg} \quad (11)$$

$$\frac{1}{h_2} \sum_{i,j} \pi_{ij}(g_{ij}^2 \alpha + g_{ij}^3 \beta_2) \leq P_{2avg} \quad (12)$$

$$\boldsymbol{\pi} \mathbb{P}' = \boldsymbol{\pi} \quad (13)$$

$$\boldsymbol{\pi} \mathbf{1} = 1 \quad (14)$$

where \mathbb{P}' is the new transition matrix defined by transition probabilities.

IV. ANALYSIS OF THE PROBLEM

In this section, we use the definition of $x_{ij}^k \triangleq \pi_{ij} g_{ij}^k$. First, we consider the average power consumption when average power constraints for both users are large enough such that each user is able to transmit a packet during a slot whenever its queue is not empty. In this scenario, the corresponding Markov chain has four non-transient states, (0,0), (0,1), (1,0), (1,1), and the stationary distribution is

$$\pi_{00} = (1 - \theta_1)(1 - \theta_2), \quad \pi_{01} = \theta_2(1 - \theta_1), \quad \pi_{10} = \theta_1(1 - \theta_2), \quad \pi_{11} = \theta_1\theta_2 \quad (15)$$

The average power consumption for each queue is

$$P_{1csm} = \frac{1}{h_1}(\pi_{10}\alpha + \pi_{11}\beta_1) = \frac{1}{h_1}(\theta_1(1 - \theta_2)\alpha + \theta_1\theta_2\beta_1) \quad (16)$$

$$P_{2csm} = \frac{1}{h_2}(\pi_{01}\alpha + \pi_{11}\beta_2) = \frac{1}{h_2}(\theta_2(1 - \theta_1)\alpha + \theta_1\theta_2\beta_2) \quad (17)$$

We note that

$$P_{1csm}h_1 + P_{2csm}h_2 = (\theta_1 + \theta_2 - 2\theta_1\theta_2)\alpha + \theta_1\theta_2\beta \quad (18)$$

From Figure 3, we note that $\beta_1, \beta_2 \geq \alpha$, therefore, each individual term in (18) must additionally satisfy

$$P_{1csm} \geq \frac{1}{h_1} \theta_1 \alpha \quad (19)$$

$$P_{2csm} \geq \frac{1}{h_2} \theta_2 \alpha \quad (20)$$

Therefore, if the average power constraints P_{1avg} and P_{2avg} satisfy the following inequalities

$$P_{1avg} h_1 + P_{2avg} h_2 \geq (\theta_1 + \theta_2 - 2\theta_1 \theta_2) \alpha + \theta_1 \theta_2 \beta \quad (21)$$

$$P_{1avg} \geq \frac{1}{h_1} \theta_1 \alpha \quad (22)$$

$$P_{2avg} \geq \frac{1}{h_2} \theta_2 \alpha \quad (23)$$

then we can always find an operating point (β_1, β_2) such that $P_{1csm} \leq P_{1avg}$ and $P_{2csm} \leq P_{2avg}$, and we achieve the minimal possible delay in the system, which is one slot. The available power in this case is so large that the solution is trivial.

If

$$P_{1avg} h_1 + P_{2avg} h_2 < (\theta_1 + \theta_2 - 2\theta_1 \theta_2) \alpha + \theta_1 \theta_2 \beta \quad (24)$$

and P_{1avg} and P_{2avg} are large enough to prevent any overflows, both power constraints should be tight. Therefore, from (11)-(12), we have two equality power constraints,

$$\frac{1}{h_1} \sum_{i,j} (x_{ij}^1 \alpha + x_{ij}^3 \beta_1) = P_{1avg} \quad (25)$$

$$\frac{1}{h_2} \sum_{i,j} (x_{ij}^2 \alpha + x_{ij}^3 \beta_2) = P_{2avg} \quad (26)$$

Because the average arrival rate must be equal to the average departure rate when there is no overflow, we also have

$$\sum_{i,j} (x_{ij}^1 + x_{ij}^3) = \theta_1 \quad (27)$$

$$\sum_{i,j} (x_{ij}^2 + x_{ij}^3) = \theta_2 \quad (28)$$

Solving (25)-(28), we obtain

$$\beta_1 = \alpha + \frac{(\beta - 2\alpha)(P_{1avg}h_1 - \theta_1\alpha)}{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha} \quad (29)$$

$$\beta_2 = \alpha + \frac{(\beta - 2\alpha)(P_{2avg}h_2 - \theta_2\alpha)}{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha} \quad (30)$$

$$\sum_{i,j} x_{ij}^1 = \theta_1 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \quad (31)$$

$$\sum_{i,j} x_{ij}^2 = \theta_2 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \quad (32)$$

$$\sum_{i,j} x_{ij}^3 = \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \quad (33)$$

By jointly considering the normalization equation in (14), we also have

$$x_{00} = 1 - \frac{(\theta_1 + \theta_2)(\beta - \alpha) - (P_{1avg}h_1 + P_{2avg}h_2)}{\beta - 2\alpha} \quad (34)$$

Thus, we transform our optimization problem in (10)-(14) into

$$\min_{\mathbf{x}} \sum_{i,j} \left(\sum_{k=1}^3 x_{ij}^k (i + j) \right) \quad (35)$$

$$\text{s.t. } x_{00} = 1 - \frac{(\theta_1 + \theta_2)(\beta - \alpha) - (P_{1avg}h_1 + P_{2avg}h_2)}{\beta - 2\alpha} \quad (36)$$

$$\sum_{i,j} x_{ij}^1 = \theta_1 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \quad (37)$$

$$\sum_{i,j} x_{ij}^2 = \theta_2 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \quad (38)$$

$$\sum_{i,j} x_{ij}^3 = \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \quad (39)$$

$$\mathbb{Q}'\mathbf{x} = \mathbf{0} \quad (40)$$

where \mathbb{Q}' is the matrix derived from the new transition matrix \mathbb{P}' .

This is a linear programming problem. We again observe that, in the objective function, all of the x_{ij} s with the same sum of indices share the same weight. Therefore, as in part I, we group the states with the same sum of indices on the two-dimensional Markov chain, and we obtain

the group transition equations. For $n = 0, 1$, we have

$$x_{00}(\theta_1 + \theta_2 - \theta_1\theta_2) = (y_1 + t_2)(1 - \theta_1)(1 - \theta_2) \quad (41)$$

$$(x_{00} + y_1)\theta_1\theta_2 = (y_2 + t_3)(1 - \theta_1)(1 - \theta_2) + t_2(1 - \theta_1\theta_2) \quad (42)$$

and for $n = 2, 3, \dots, 2N - 2$, we have

$$y_n\theta_1\theta_2 = (y_{n+1} + t_{n+2})(1 - \theta_1)(1 - \theta_2) + t_{n+1}(1 - \theta_1\theta_2) \quad (43)$$

$$y_{2N-1}\theta_1\theta_2 = t_{2N}(1 - \theta_1\theta_2) \quad (44)$$

Figure 4 shows the transitions between diagonal groups for a system with $N = 3$.

We multiply both sides of the n -th equation in (41)-(44) with z^n and sum with respect to n to obtain

$$\begin{aligned} x_{00}(\theta_1 + \theta_2 - \theta_1\theta_2 + \theta_1\theta_2z) + (\theta_1\theta_2 - (1 - \theta_1)(1 - \theta_2)z^{-1}) \sum_{n=1}^{2N-1} y_n z^n \\ - ((1 - \theta_1\theta_2)z^{-1} + (1 - \theta_1)(1 - \theta_2)z^{-2}) \sum_{n=1}^{2N} t_n z^n = 0 \end{aligned} \quad (45)$$

Taking the derivative of (45) with respect to z and letting $z = 1$, we have

$$\begin{aligned} \sum_{n=1}^{2N} t_n n = \frac{1}{2 - \theta_1 - \theta_2} \left((\theta_1 + \theta_2 - 1) \left(\sum_{n=1}^{2N-1} y_n n \right) + (1 - \theta_1)(1 - \theta_2) \left(\sum_{n=1}^{2N-1} y_n \right) \right. \\ \left. + (1 - \theta_1\theta_2 + 2(1 - \theta_1)(1 - \theta_2)) \left(\sum_{n=1}^{2N-1} t_n \right) + x_{00}\theta_1\theta_2 \right) \end{aligned} \quad (46)$$

Using the definition of y_n , t_n and (46), the objective function becomes

$$\sum_{n=1}^{2N} (y_n + t_n)n = \frac{1}{2 - \theta_1 - \theta_2} \left(\sum_{n=1}^{2N-1} y_n n \right) + C \quad (47)$$

where C is a constant, and $\frac{1}{2 - \theta_1 - \theta_2}$ is positive. For future reference, let us define

$$\sum_{n=1}^{2N-1} y_n = \sum_{i,j} x_{ij}^1 + \sum_{i,j} x_{ij}^2 = \theta_1 + \theta_2 - \frac{2(P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha)}{\beta - 2\alpha} \triangleq \Psi' \quad (48)$$

Similar to the symmetric case, for this asymmetric scenario, minimizing the original objective function is equivalent to minimizing $\sum_{n=1}^{2N} y_n n$ while the sum of y_n s is fixed. Therefore, the optimization problem requires us to assign larger values to y_n s with smaller indices n , without violating the transition equation constraints.

V. THE MODIFIED OPTIMIZATION PROBLEM AND A TWO-STEP SOLUTION

We solve the optimization problem through two steps. In the first step, we will minimize $\sum_{n=1}^{2N} y_n n$ subject to (48), (36), and (41)-(44). In the second step, we will allocate y_n s and t_n s found from the first step to x_{ij}^k s such that the remaining independent transition equations are satisfied. We define

$$\eta' = \frac{\theta_1 + \theta_2 - \theta_1\theta_2}{(1 - \theta_1)(1 - \theta_2)}, \quad \delta' = \frac{\theta_1\theta_2}{(1 - \theta_1)(1 - \theta_2)}, \quad \rho' = \frac{1 - \theta_1\theta_2}{(1 - \theta_1)(1 - \theta_2)} \quad (49)$$

Then, we simply use η', δ', ρ' instead of η, δ, ρ in the allocation formulas for the symmetric case in part I to allocate Ψ' to y_n s. This completes the first step of our two-step approach.

In the symmetric scenario, we were able to allocate y_n s and t_n s to the states within each group in a symmetric way, and verified that this allocation was feasible. However, in this asymmetric scenario, we do not expect a symmetric structure for the allocation within groups to be feasible. In this setting, how to allocate y_n s and t_n s within each group becomes significantly more difficult.

First, we use a simple example to illustrate the procedure of allocation within each group, then, we generalize the procedure to arbitrary cases. In this simple example, we assume that $N = 4$.

Assume that after the group allocation, we obtained y_1, \dots, y_5 and $t_5, t_6 \neq 0$, and the rest of the y_n s and t_n s are equal to zero. In order to keep the allocation simple, when we assign y_3, y_5, t_5 in each group, we assign them only to two states: $(1, 2), (2, 1)$ and $(2, 3), (3, 2)$, respectively; while we assign y_4 to three states: $(1, 3), (2, 2), (3, 1)$, and we assign t_6 to a single state $(3, 3)$. Figure 5 illustrates the allocation pattern within groups. We do not assign any values to the states with dotted circles. The dotted states will be transient states after the allocation. We need to guarantee that the nonzero-valued states only transit to other nonzero-valued states. This requires us to set $x_{12}^1 = x_{21}^2 = x_{23}^1 = x_{32}^2 = 0$, and $x_{13}^1 = x_{13}^3 = x_{31}^2 = x_{31}^3 = 0$. The valid transitions are represented as arrows in Figure 5. We can see that the transitions are within the positive recurrent class.

Then, let us examine each group and find transition equations to be satisfied for each state.

For states $(0, 1), (0, 2), (1, 2), (1, 3), (2, 3)$, the transition equations to be satisfied are

$$\begin{aligned}
x_{01}^2(1 - \theta_2(1 - \theta_1)) &= (x_{00} + x_{10}^1)\theta_2(1 - \theta_1) + (x_{02}^2 + x_{11}^1)(1 - \theta_1)(1 - \theta_2) \\
x_{02}^2(1 - \theta_2(1 - \theta_1)) &= x_{11}^1\theta_2(1 - \theta_1) \\
x_{12}^2(1 - \theta_2(1 - \theta_1)) &= (x_{02}^2 + x_{11}^1)\theta_1\theta_2 + x_{21}^1\theta_2(1 - \theta_1) \\
&\quad + (x_{13}^2 + x_{22}^1 + x_{23}^3)(1 - \theta_1)(1 - \theta_2) \\
x_{13}^2(1 - \theta_2(1 - \theta_1)) &= (x_{11}^1 + x_{23}^3)\theta_2(1 - \theta_1) \\
x_{23}^2(1 - \theta_2(1 - \theta_1)) + x_{23}^3(1 - \theta_1\theta_2) &= (x_{13}^2 + x_{22}^1)\theta_1\theta_2 + (x_{32}^1 + x_{33}^3)\theta_2(1 - \theta_1) \tag{50}
\end{aligned}$$

We have five more similar transition equations for states $(0, 1), (0, 2), (1, 2), (1, 3), (2, 3)$. All the unknown variables are interacting with each other through these equations. How to find an allocation satisfying all of these equations simultaneously becomes rather difficult. After simple manipulations, equations in (50) become equivalent to

$$\begin{aligned}
x_{01}^2 &= (x_{00} + x_{10}^1 + x_{01}^2)\theta_2(1 - \theta_1) + (x_{02}^2 + x_{11}^1)(1 - \theta_1)(1 - \theta_2) \\
x_{02}^2 &= (x_{11}^1 + x_{02}^2)\theta_2(1 - \theta_1) \\
x_{12}^2 &= (x_{02}^2 + x_{11}^1)\theta_1\theta_2 + (x_{12}^2 + x_{21}^1)\theta_2(1 - \theta_1) + (x_{13}^2 + x_{22}^1 + x_{23}^3)(1 - \theta_1)(1 - \theta_2) \\
x_{13}^2 &= (x_{22}^1 + x_{13}^2 + x_{23}^3)\theta_2(1 - \theta_1) \\
x_{23}^2 &= (x_{13}^2 + x_{22}^1)\theta_1\theta_2 + (x_{32}^1 + x_{23}^2 + x_{33}^3)\theta_2(1 - \theta_1) - x_{23}^3(1 - \theta_1\theta_2) \tag{51}
\end{aligned}$$

Observing the right hand sides of (51), we note that, $x_{00}, x_{10}^1 + x_{10}^2, x_{12}^2 + x_{21}^1, x_{32}^1 + x_{23}^2, x_{33}^3$ are known, therefore, the allocation for states $(0, 1), (0, 2), (1, 2), (1, 3), (2, 3)$ depends only on the values of $x_{02}^2 + x_{11}^1, x_{22}^1 + x_{13}^2$, and x_{23}^3 . Similarly, the allocation for states $(1, 0), (2, 0), (2, 1), (3, 1), (3, 2)$ also depends on the values of $x_{20}^1 + x_{11}^2, x_{22}^2 + x_{31}^1$, and x_{32}^3 only. Since

$$y_2 = (x_{02}^2 + x_{11}^1) + (x_{20}^1 + x_{11}^2) \tag{52}$$

$$y_4 = (x_{22}^1 + x_{13}^2) + (x_{22}^2 + x_{31}^1) \tag{53}$$

$$t_5 = x_{23}^3 + x_{32}^3 \tag{54}$$

the allocation actually depends on how we split y_2, y_4 and t_5 between $(x_{02}^2 + x_{11}^1)$ and $(x_{20}^1 + x_{11}^2)$,

$(x_{22}^1 + x_{13}^2)$ and $(x_{22}^2 + x_{31}^1)$, x_{23}^3 and x_{32}^3 , respectively. Once we fix the values of $x_{02}^2 + x_{11}^1$, $x_{22}^1 + x_{13}^2$, and x_{23}^3 , we obtain the values of all of the states, completing the allocation. We note that there is more than one feasible allocation within groups, and for each feasible allocation, all of the transition equations are satisfied, and the power constraints are satisfied as well. In order to keep the solution simple, we let

$$x_{02}^2 + x_{11}^1 = y_2/2 \quad (55)$$

$$x_{22}^1 + x_{13}^2 = y_4/2 \quad (56)$$

$$x_{23}^3 = t_5/2 \quad (57)$$

Plugging these into (51), we get

$$\begin{aligned} x_{01}^2 &= (x_{00} + y_1)\theta_2(1 - \theta_1) + \frac{1}{2}y_2(1 - \theta_1)(1 - \theta_2) \\ x_{02}^2 &= \frac{1}{2}y_2\theta_2(1 - \theta_1) \\ x_{12}^2 &= \frac{1}{2}y_2\theta_1\theta_2 + y_3\theta_2(1 - \theta_1) + \frac{1}{2}(y_4 + t_5)(1 - \theta_1)(1 - \theta_2) \\ x_{13}^2 &= \frac{1}{2}(y_4 + t_5)\theta_2(1 - \theta_1) \\ x_{23}^2 &= \frac{1}{2}y_4\theta_1\theta_2 + (y_5 + t_6)\theta_2(1 - \theta_1) - \frac{1}{2}t_5(1 - \theta_1\theta_2) \end{aligned} \quad (58)$$

Going back to (55)-(56), we obtain

$$\begin{aligned} x_{11}^1 &= \frac{1}{2}y_2(1 - \theta_2(1 - \theta_1)) \\ x_{22}^1 &= \frac{1}{2}y_4 - \frac{1}{2}(y_4 + t_5)\theta_2(1 - \theta_1) \end{aligned} \quad (59)$$

Since $y_n \geq t_{n+1}\rho'/\delta'$, we can easily verify that $x_{23}^2 \geq 0$, $x_{22}^1 \geq 0$. The allocation for the remaining half of the states has a similar structure. Thus, each state has a positive value, and the allocation is feasible.

Once we obtain the values of x_{ij}^k s, we can compute the transmission probabilities using $g_{ij}^k =$

$\frac{x_{ij}^k}{\sum_{k=1}^3 x_{ij}^k}$. Here, we have

$$g_{11}^1 = \frac{1 - \theta_2(1 - \theta_1)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)} \quad (60)$$

$$g_{11}^2 = \frac{1 - \theta_1(1 - \theta_2)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)} \quad (61)$$

$$g_{22}^1 = \frac{y_4 - (y_4 + t_5)\theta_2(1 - \theta_1)}{2y_4 - (y_4 + t_5)(\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2))} \quad (62)$$

$$g_{22}^2 = \frac{y_4 - (y_4 + t_5)\theta_1(1 - \theta_2)}{2y_4 - (y_4 + t_5)(\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2))} \quad (63)$$

which are not equal to $1/2$ in general. However, a threshold structure still exists. In this example, the threshold is 5. When the sum of the two queue lengths is greater than 5, both users transmit during a slot. When the sum of the two queue lengths is less than 5, only one user with longer queue transmits during a slot; in this case, if both queue lengths are the same, users transmit according to probabilities in (60)-(63).

While we generalize this example to an arbitrary setting, we follow the same basic allocation pattern. If n is odd, we assign y_n and t_n only to two states $(\frac{n+1}{2}, \frac{n-1}{2})$ and $(\frac{n-1}{2}, \frac{n+1}{2})$; if n is even, we assign y_n to three states: $(\frac{n}{2} + 1, \frac{n}{2} - 1)$, $(\frac{n}{2}, \frac{n}{2})$, $(\frac{n}{2} - 1, \frac{n}{2} + 1)$, and we assign t_n to a single state $(\frac{n}{2}, \frac{n}{2})$. We illustrate the allocation pattern in Figure 6. We need to make sure that the transitions only happen within the positive recurrent class. Therefore, when n is odd, we let $x_{\frac{n-1}{2}, \frac{n+1}{2}}^1 = x_{\frac{n+1}{2}, \frac{n-1}{2}}^2 = 0$; when n is even, we let $x_{\frac{n}{2}-1, \frac{n}{2}+1}^1 = x_{\frac{n}{2}+1, \frac{n}{2}-1}^2 = 0$. Then, let us examine the transition equations for the states. For $n = 1$, we have

$$x_{01}^2(1 - \theta_2(1 - \theta_1)) = (x_{00} + x_{10}^1 + x_{11}^3)\theta_2(1 - \theta_1) + (x_{02}^2 + x_{11}^1 + x_{12}^3)(1 - \theta_1)(1 - \theta_2) \quad (64)$$

For $n = 2, 3, \dots$, if n is even, the transitions between states are illustrated in Figure 7. The transition equation for state $(\frac{n}{2} - 1, \frac{n}{2} + 1)$ is

$$x_{\frac{n}{2}-1, \frac{n}{2}+1}^2(1 - \theta_2(1 - \theta_1)) = (x_{\frac{n}{2}, \frac{n}{2}}^1 + x_{\frac{n}{2}, \frac{n}{2}+1}^3)\theta_2(1 - \theta_1) \quad (65)$$

If n is odd, the transitions between states are illustrated in Figure 8. The transition equation for

state $(\frac{n-1}{2}, \frac{n+1}{2})$ is

$$x_{\frac{n-1}{2}, \frac{n+1}{2}}^2(1 - \theta_2(1 - \theta_1)) + x_{\frac{n-1}{2}, \frac{n+1}{2}}^3(1 - \theta_1\theta_2) = (x_{\frac{n-3}{2}, \frac{n+1}{2}}^2 + x_{\frac{n-1}{2}, \frac{n-1}{2}}^1)\theta_1\theta_2 \quad (66)$$

$$+ (x_{\frac{n+1}{2}, \frac{n-1}{2}}^1 + x_{\frac{n+1}{2}, \frac{n+1}{2}}^3)\theta_2(1 - \theta_1) + (x_{\frac{n+1}{2}, \frac{n+1}{2}}^1 + x_{\frac{n-1}{2}, \frac{n+3}{2}}^2 + x_{\frac{n+1}{2}, \frac{n+3}{2}}^3)(1 - \theta_1)(1 - \theta_2)$$

After a transformation, (64) is equivalent to

$$x_{01}^2 = (x_{00} + x_{10}^1 + x_{01}^2 + x_{11}^3)\theta_2(1 - \theta_1) + (x_{02}^2 + x_{11}^1 + x_{12}^3)(1 - \theta_1)(1 - \theta_2) \quad (67)$$

where x_{00} is known, $x_{10}^1 + x_{01}^2 = y_1$, $x_{11}^3 = t_2$.

For $n = 2, 3, \dots$, when n is even, (65) is equivalent to

$$x_{\frac{n}{2}-1, \frac{n}{2}+1}^2 = (x_{\frac{n}{2}, \frac{n}{2}}^1 + x_{\frac{n}{2}-1, \frac{n}{2}+1}^2 + x_{\frac{n}{2}, \frac{n}{2}+1}^3)\theta_2(1 - \theta_1) \quad (68)$$

and when n is odd, (66) is equivalent to

$$x_{\frac{n-1}{2}, \frac{n+1}{2}}^2 = (x_{\frac{n-3}{2}, \frac{n+1}{2}}^2 + x_{\frac{n-1}{2}, \frac{n-1}{2}}^1)\theta_1\theta_2 + (x_{\frac{n+1}{2}, \frac{n-1}{2}}^1 + x_{\frac{n-1}{2}, \frac{n+1}{2}}^2 + x_{\frac{n+1}{2}, \frac{n+1}{2}}^3)\theta_2(1 - \theta_1)$$

$$+ (x_{\frac{n+1}{2}, \frac{n+1}{2}}^1 + x_{\frac{n-1}{2}, \frac{n+3}{2}}^2 + x_{\frac{n+1}{2}, \frac{n+3}{2}}^3)(1 - \theta_1)(1 - \theta_2) - x_{\frac{n-1}{2}, \frac{n+1}{2}}^3(1 - \theta_1\theta_2) \quad (69)$$

where $x_{\frac{n+1}{2}, \frac{n-1}{2}}^1 + x_{\frac{n-1}{2}, \frac{n+1}{2}}^2 = y_n$, $x_{\frac{n+1}{2}, \frac{n+1}{2}}^3 = t_{n+1}$.

The transition equations for the remaining half of the recurrent states can be expressed in a similar form. Therefore, the values of x_{ij}^k s are determined only by the allocation of y_n between $x_{\frac{n}{2}+1, \frac{n}{2}-1}^1 + x_{\frac{n}{2}, \frac{n}{2}}^2$ and $x_{\frac{n}{2}-1, \frac{n}{2}+1}^2 + x_{\frac{n}{2}, \frac{n}{2}}^1$ when n is even, and the allocation of t_n to $x_{\frac{n+1}{2}, \frac{n-1}{2}}^3$ and $x_{\frac{n-1}{2}, \frac{n+1}{2}}^3$ when n is odd. If we let

$$x_{\frac{n}{2}, \frac{n}{2}}^1 + x_{\frac{n}{2}-1, \frac{n}{2}+1}^2 = y_n/2, \quad \text{when } n \text{ is even} \quad (70)$$

$$x_{\frac{n-1}{2}, \frac{n+1}{2}}^3 = t_n/2, \quad \text{when } n \text{ is odd} \quad (71)$$

and solve equations (67)-(69), then, for $n = 1$, we obtain

$$x_{01}^2 = (x_{00} + y_1 + t_2)\theta_2(1 - \theta_1) + \frac{1}{2}(y_2 + t_3)(1 - \theta_1)(1 - \theta_2)$$

$$x_{10}^1 = (x_{00} + y_1 + t_2)\theta_1(1 - \theta_2) + \frac{1}{2}(y_2 + t_3)(1 - \theta_1)(1 - \theta_2) \quad (72)$$

For $n = 2, 3, \dots$, if n is even, we get

$$x_{\frac{n}{2}-1, \frac{n}{2}+1}^2 = \frac{1}{2}(y_n + t_{n+1})\theta_2(1 - \theta_1) \quad (73)$$

$$x_{\frac{n}{2}+1, \frac{n}{2}-1}^1 = \frac{1}{2}(y_n + t_{n+1})\theta_1(1 - \theta_2) \quad (74)$$

$$x_{\frac{n}{2}, \frac{n}{2}}^1 = \frac{1}{2}y_n - \frac{1}{2}(y_n + t_{n+1})\theta_2(1 - \theta_1) \quad (75)$$

$$x_{\frac{n}{2}, \frac{n}{2}}^2 = \frac{1}{2}y_n - \frac{1}{2}(y_n + t_{n+1})\theta_1(1 - \theta_2) \quad (76)$$

and if n is odd, we have

$$\begin{aligned} x_{\frac{n-1}{2}, \frac{n+1}{2}}^2 &= \frac{1}{2}y_{n-1}\theta_1\theta_2 + (y_n + t_{n+1})\theta_2(1 - \theta_1) \\ &\quad + \frac{1}{2}(y_{n+1} + t_{n+2})(1 - \theta_1)(1 - \theta_2) - \frac{1}{2}t_n(1 - \theta_1\theta_2) \end{aligned} \quad (77)$$

$$\begin{aligned} x_{\frac{n+1}{2}, \frac{n-1}{2}}^1 &= \frac{1}{2}y_{n-1}\theta_1\theta_2 + (y_n + t_{n+1})\theta_1(1 - \theta_2) \\ &\quad + \frac{1}{2}(y_{n+1} + t_{n+2})(1 - \theta_1)(1 - \theta_2) - \frac{1}{2}t_n(1 - \theta_1\theta_2) \end{aligned} \quad (78)$$

This completes the allocation. Note that $t_n \neq 0$ only when n is equal to $n^* + 1$, $n^* + 2$, and/or $n^* + 3$, depending on the value of Δ . When $t_{n+1} = 0$, it automatically disappears from the right hand sides of (72)-(78). From the group transition equations, we have $y_n \geq t_{n+1}\rho'/\delta'$, and it is easy to verify that all states have nonnegative assignments and the transition equations are also satisfied in this case. Therefore, there always exists a feasible allocation to satisfy all of the transition equations with y_n s defined through this allocation scheme. We also note that when $\theta_1 = \theta_2$, our assignment here coincides with the assignment in the symmetric scenario given in part I.

The transition probabilities can be computed once we determine the allocation for each state. From our allocation, we note that even in the asymmetric scenario, there still exists a threshold number \bar{n} , where \bar{n} is the largest group index n such that $y_n \neq 0$. We have $t_n > 0$ only when $n \geq \bar{n}$. Since $g_{ij}^k = \frac{x_{ij}^k}{\sum_{k=1}^3 x_{ij}^k}$, we have $g_{ij}^3 = 1$ when $n > \bar{n}$. When $n < \bar{n}$, we have $g_{ij}^1 = 1$ if

$i > j$ and $g_{ij}^2 = 1$ if $i < j$. Then, for $n \leq \bar{n}$, and n is even, we have

$$g_{n/2, n/2}^1 = \frac{y_n - (y_n + t_{n+1})\theta_2(1 - \theta_1)}{2y_n - (y_n + t_{n+1})(\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2)) + t_n} \quad (79)$$

$$g_{n/2, n/2}^2 = \frac{y_n - (y_n + t_{n+1})\theta_1(1 - \theta_2)}{2y_n - (y_n + t_{n+1})(\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2)) + t_n} \quad (80)$$

$$g_{n/2, n/2}^3 = \frac{t_n}{2y_n - (y_n + t_{n+1})(\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2)) + t_n} \quad (81)$$

If $t_n, t_{n+1} = 0$, which happens when $n < \bar{n} - 1$, (79)-(81) reduce to

$$g_{n/2, n/2}^1 = \frac{1 - \theta_2(1 - \theta_1)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)} \quad (82)$$

$$g_{n/2, n/2}^2 = \frac{1 - \theta_1(1 - \theta_2)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)} \quad (83)$$

Therefore, if the sum of the two queue lengths is greater than \bar{n} , both users should transmit one packet during the slot. If the sum of the two queue lengths is less than \bar{n} , only the user with the longer queue transmits one packet in the slot and the other user remains silent; if in this case both queues have the same length, then the probability that the first user transmits one packet while the second one keeps silent is $\frac{1 - \theta_2(1 - \theta_1)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)}$, and the probability that the second user transmits one packet while the first one keeps silent is $\frac{1 - \theta_1(1 - \theta_2)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)}$.

VI. THE OPTIMALITY OF THE ALLOCATION SCHEME

In this section, we will prove the optimality of the group allocation scheme. Since the group allocation procedure for the symmetric setting and the asymmetric setting are similar, once we prove the optimality for the symmetric setting, the optimality of the group allocation for the asymmetric setting will follow. Therefore, here we will prove that the allocation presented in the symmetric scenario in part I is optimal.

In an optimization problem, where the inequality constraints are convex and the equality constraints are affine, if x^* is such that there exists a set of Lagrange multipliers which together with x^* satisfy the KKT conditions, then x^* is a global minimizer for the problem [10] [11]. In our problem, we have a linear objective function and linear constraints. Therefore, if we prove that the point achieved by the assignment in part I (and here above) satisfies the KKT conditions, then we can say that it is the global minimizer for our problem.

Theorem 1: The allocation scheme minimizes the average delay of packets in the system.

Proof: In the allocation scheme, if $\Delta = y_{n^*}\delta\rho/(\delta + \rho)$, then it is easy to prove that the resulting allocation is optimal, since every $y_n, n < n^*$ achieves its maximum possible value. However, this is not the case when $\Delta \neq y_{n^*}\delta\rho/(\delta + \rho)$, because the second to last nonzero y_n does not achieve its maximum. In the following, we prove that our allocation is optimal for this case as well.

Define $\mathbf{y} = [y_1, y_2, \dots, y_{2N-1}, t_2, \dots, t_{N-1}, t_{2N}]$. Then, the linear equality constraints, including the $2N$ group transition equations and the sum constraint can be written as a $(2N + 1) \times 2(2N - 1)$ matrix form as follows

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ -\delta & 1 & 0 & \cdots & 0 & \rho & 1 & 0 & \cdots & 0 \\ 0 & -\delta & 1 & \cdots & 0 & 0 & \rho & 1 & \cdots & 0 \\ & \vdots & & \ddots & \vdots & & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & -\delta & 0 & 0 & 0 & \cdots & \rho \\ 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \mathbf{y}^T = \begin{pmatrix} x_{00}\eta \\ x_{00}\delta \\ 0 \\ \vdots \\ 0 \\ 0 \\ \Psi \end{pmatrix} \quad (84)$$

which we write equivalently as,

$$\mathbb{A}\mathbf{y}^T = \mathbf{b} \quad (85)$$

by defining

$$\mathbb{A} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & \rho + \delta & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & (\rho + \delta)\delta & \rho + \delta & 1 & \cdots & 0 \\ \vdots & & \ddots & \vdots & & & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 & (\rho + \delta)\delta^{2N-3} & (\rho + \delta)\delta^{2N-4} & (\rho + \delta)\delta^{2N-5} & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 & (\rho + \delta)\delta^{2N-2} & (\rho + \delta)\delta^{2N-3} & (\rho + \delta)\delta^{2N-4} & \cdots & \rho + \delta \\ 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \quad (86)$$

and

$$\mathbf{b} = \begin{pmatrix} x_{00}\eta \\ x_{00}\delta(1+\eta) \\ x_{00}\delta^2(1+\eta) \\ \vdots \\ x_{00}\delta^{2N-2}(1+\eta) \\ x_{00}\delta^{2N-1}(1+\eta) \\ \Psi \end{pmatrix} \quad (87)$$

The Lagrangian is expressed as

$$L(\mathbf{y}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathbf{c}^T \mathbf{y} - \boldsymbol{\lambda}^T (\mathbb{A} \mathbf{y} - \mathbf{b}) - \boldsymbol{\mu}^T \mathbf{y} \quad (88)$$

where $\mathbf{c} = [1, 2, \dots, 2N-1, 0, 0, \dots, 0]$, $\boldsymbol{\lambda} \in \mathbf{R}^{2N+1}$ and $\boldsymbol{\mu} \in \mathbf{R}^{4N-2}$.

We need to prove that there exists a set of $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}^*$ associated with our allocation \mathbf{y}^* , such that they satisfy

$$\boldsymbol{\mu}^* \geq \mathbf{0} \quad (89)$$

$$\mathbf{y}^* \geq \mathbf{0}, \quad \mathbb{A} \mathbf{y}^{*T} = \mathbf{b} \quad (90)$$

$$\mathbf{c} = \mathbb{A}^T \boldsymbol{\lambda}^* + \boldsymbol{\mu}^* \quad (91)$$

$$\boldsymbol{\mu}^{*T} \mathbf{y}^* = 0 \quad (92)$$

Consider the \mathbf{y} we obtained with the algorithm. Let us consider the case when $\Delta < y_{n^*} \delta \rho / (\delta + \rho)$ first. The allocation indicates that $y_n > 0$ only when $n = 1, 2, \dots, n^* + 1$, and $t_n > 0$ only when $n = n^* + 1, n^* + 2$. Because of the complementary slackness in (92), we obtain

$$\mu_n = 0, \quad n = 1, 2, \dots, n^* + 1, n^* + 2N - 1, n^* + 2N \quad (93)$$

Plugging this into (91), and solving the equations, we have

$$\lambda_n = \frac{1}{\rho + 1} + n - n^* - 1, \quad n = 1, 2, \dots, n^* + 1 \quad (94)$$

$$\lambda_{2N+1} = \frac{\rho}{\rho + 1} + n^* \quad (95)$$

$$\mu_{n+2N-2} = - \left(\lambda_{n-1} + (\rho + \delta) \sum_{i=n}^{n^*-1} \lambda_i \delta^{i-n} + \rho \delta^{n^*-n} \lambda_{n^*} \right), \quad n = 2, 3, \dots, n^* \quad (96)$$

Thus, we have $\lambda_n < 0$ when $n \leq n^*$, which guarantees the positiveness of $\{\mu_n\}_{n=2N}^{n^*+2N-2}$. We also have

$$\sum_{i=n^*+2}^{2N} \lambda_i \delta^{i-n^*-2} = - \frac{1}{(\rho + \delta)(\rho + 1)} \quad (97)$$

and

$$\mu_n = \frac{1}{\rho + 1} + n - n^* - 1 - \lambda_n, \quad n = n^* + 2, \dots, 2N - 1 \quad (98)$$

$$\mu_n = - \left(\lambda_{n-1} + (\rho + \delta) \sum_{i=n}^{2N} \lambda_i \delta^{i-n} \right), \quad n = n^* + 2N + 1, \dots, 4N - 2 \quad (99)$$

We can always find a set of negative $\{\lambda_i\}_{i=n^*+2}^{2N}$ to satisfy (97). Since they are all negative, this guarantees that $\{\mu_n\}_{n=n^*+2}^{2N-1}$ and $\{\mu_n\}_{n=n^*+2N+1}^{4N-2}$ are positive. Therefore, at the point \mathbf{y}^* , we can always find a set of multipliers satisfying all of the KKT constraints. This proves that the allocation our algorithm gives is a global minimizer. \square

VII. NUMERICAL EXAMPLES

Here we give simple examples to show how our allocation scheme works. We choose $N = 10$, i.e., each queue has a buffer of size 10 packets. Therefore, the joint queue states is represented by an 11×11 Markov chain.

First, we consider the symmetric scenario. We assume the arrival rate $\theta = 1/2$, and the power levels $\alpha = 1$, $\beta = 3$. Therefore, we have $\eta = 3$, $\delta = 1$, $\rho = 3$. From the analysis, we know that if $P_{avg} \geq 5/8$, the average delay is one slot, which is the minimal possible delay in the system.

If $P_{avg} = 9/16$, we have $x_{00} = 1/8$, $\sum_{i,j} x_{ij}^1 = \sum_{i,j} x_{ij}^2 = 3/8$, $\sum_{i,j} x_{ij}^3 = 1/8$. Therefore, $\Psi = 3/4$. Following our allocation scheme, we have $y_1 = 3/8$, $y_2 = 3/8$, $t_3 = 1/8$. Then, we need to allocate these within groups.

We start with y_1 . Because of the symmetry of the setting, we simply let $x_{10}^1 = x_{01}^2 = y_1/2 = 3/16$, $x_{12}^3 = x_{21}^3 = t_3/2 = 1/16$. Then, we consider y_2 . We also let $x_{20}^1 = x_{02}^2$, $x_{11}^1 = x_{11}^2$. This symmetric allocation guarantees that the flow equations for states $(0, 1)$ and $(1, 0)$ are satisfied. The values of x_{20}^1 and x_{11}^1 also depend on the allocation of t_3 . The state $(2, 0)$ must satisfy the transition equation

$$x_{20}^1 (\theta(1 - \theta) + \theta^2 + (1 - \theta)^2) = (x_{11}^2 + x_{21}^3)\theta(1 - \theta)$$

Together with the symmetric allocation, we have

$$x_{20}^1 + x_{11}^2 = y_2/2 = 3/16$$

Solving these equations, we get the allocation for the second group as

$$x_{20}^1 = x_{02}^2 = 1/16$$

$$x_{11}^2 = x_{11}^1 = 1/8$$

We see that the two values are positive, thus feasible. Then, the transmission probabilities are $g_{11}^1 = g_{11}^2 = 1/2$, $g_{12}^3 = g_{21}^3 = 1$. The threshold of the sum of the queue lengths is 2 in this case. If the sum of the queue lengths is greater than 2, both users transmit, if the sum of the queue lengths is less than or equal to 2, only the user with the longer queue transmits and the other user remains silent; if both queues have one packet in their queues, each queue transmits with probability $1/2$ while the other queue remains silent.

If $P_{avg} = 17/32$, we have $x_{00} = 1/16$, $\sum_{i,j} x_{ij}^1 = \sum_{i,j} x_{ij}^2 = 7/16$, $\sum_{i,j} x_{ij}^3 = 1/16$. Therefore, $\Psi = 7/8$. Following our allocation scheme, we have $y_1 = 3/16$, $y_2 = y_3 = 1/4$, $y_4 = 3/16$, $t_5 = 1/16$. Then, we assign these within groups. For y_1 , we simply let $x_{10}^1 = x_{01}^2 = y_1/2 = 1/32$. Then, considering to allocate y_2 , we have $x_{20}^1 = x_{02}^2 = 1/32$, $x_{11}^2 = x_{11}^1 = 3/32$. After completing the allocation, we have $x_{21}^1 = x_{12}^2 = 1/8$, $x_{31}^1 = x_{13}^2 = 1/32$, $x_{22}^2 = x_{22}^1 = 1/16$, $x_{23}^3 = x_{32}^3 = 1/32$. The transmission probabilities are $g_{11}^1 = g_{11}^2 = g_{22}^1 = g_{22}^2 = 1/2$, $g_{10}^1 = g_{01}^2 = g_{20}^1 = g_{02}^2 = g_{21}^1 = g_{12}^2 = g_{13}^1 = g_{31}^2 = g_{32}^3 = g_{23}^3 = 1$. The threshold of the sum of the queue lengths is 4 in this case. If the sum of the queue lengths is greater than 4, both users transmit, if the sum of the queue lengths is less than or equal to 4, only the user with the longer queue

transmits and the other user remains silent; if both queues have equal length, which is either 1 or 2 in this case, each queue transmits with probability 1/2 while the other queue remains silent.

We compute the average delay as a function of average power for $\theta = 0.5$, $\theta = 0.48$ and $\theta = 0.46$, and plot them in Figure 9. We observe that it is a piecewise linear function, and each linear segment corresponds to the same threshold value. This is because based on our optimal allocation scheme, for a fixed threshold value, the objective function is a linear function in x_{00} , thus it is linear in P_{avg} . If P_{avg} increases, D_{avg} decreases, and the threshold decreases as well. The minimum value of P_{avg} on each curve corresponds to the maximum threshold, which is 19 in this example. This is also the minimum amount of average power required to prevent any overflows. We also observe that the delay-power tradeoff curve is convex, which is consistent with the result in [4]. We note that although these three values of θ are close to each other, the average delay varies dramatically. This is because the average delay is not a linear function of θ .

For the asymmetric scenario, we assume $\theta_1 = 1/2$, $\theta_2 = 1/3$, then $\eta' = 2$, $\delta' = 1/2$, $\rho' = 5/2$. We assume $h_1 = 1$, $h_2 = 2$. From (21), we know that if $P_{1avg}h_1 + P_{2avg}h_2 \geq 1$, $P_{1avg} \geq 1/2$, $P_{2avg} \geq 2/3$, then each user can always transmit a packet whenever its queue is not empty, and the average delay is one slot.

If $P_{1avg} = 19/36$, $P_{2avg} = 13/18$, then $P_{1avg}h_1 + P_{2avg}h_2 = 8/9$. Plugging these into (29)-(36), we have $\beta_1 = 1/2$, $\beta_2 = 1/2$, $\sum_{i,j}^1 x_{ij}^1 = 4/9$, $\sum_{i,j}^2 x_{ij}^1 = 5/18$, $\sum_{i,j}^3 x_{ij}^1 = 1/18$, $x_{00} = 2/9$. Then, $\Psi = 13/18$. Following the group allocation scheme, we have $y_1 = 4/9$, $y_2 = 5/18$, $t_3 = 1/18$. Then, we need to assign them within groups. From (72)-(78), we get $x_{01}^2 = 1/6$, $x_{10}^1 = 5/18$, $x_{02}^2 = 1/36$, $x_{11}^1 = 4/36$, $x_{11}^2 = 3/36$, $x_{20}^1 = 2/36$, and $x_{12}^3 = x_{21}^3 = 1/18$. The transmission probabilities are $g_{11}^1 = 4/7$, $g_{11}^2 = 3/7$, $g_{10}^1 = g_{01}^2 = g_{20}^1 = g_{02}^2 = g_{12}^3 = g_{21}^3 = 1$. The threshold is 2. If the sum of the queue lengths is greater than 2, both users transmit, if the sum of the queue lengths is less than or equal to 2, only the user with the longer queue transmits and the other user remains silent; if both queues have one packet in their queues, the first queue transmits with probability 4/7, and the second queue transmits with probability 3/7.

VIII. CONCLUSIONS

We investigated the average delay minimization problem for a two-user multiple-access system with average power constraints for the general asymmetric scenario, where users have arbitrary powers, channel gains, and arrival rates. We considered a discrete-time model. In each slot, the arrivals at each queue follow a Bernoulli distribution, and we transmit at most one packet from each queue with some probability. Our objective is to find the optimal set of departure probabilities. We modeled the problem as a two-dimensional Markov chain, and minimized the average delay through controlling the departure probabilities in each time slot. We transformed the problem into a linear programming problem and found the optimal solution analytically. The optimal policy has a threshold structure. Whenever the sum of the queue lengths exceeds a threshold, both queues transmit one packet during the slot, otherwise, only one of the queues, which is longer, transmits one packet during the slot and the other queue remains silent; if both queues have the same length, only one of the queues transmits with a probability which depends on the arrival rates to both queues while the other queue remains silent.

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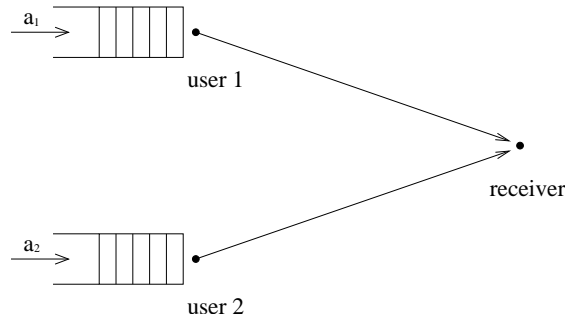


Fig. 1. System model.

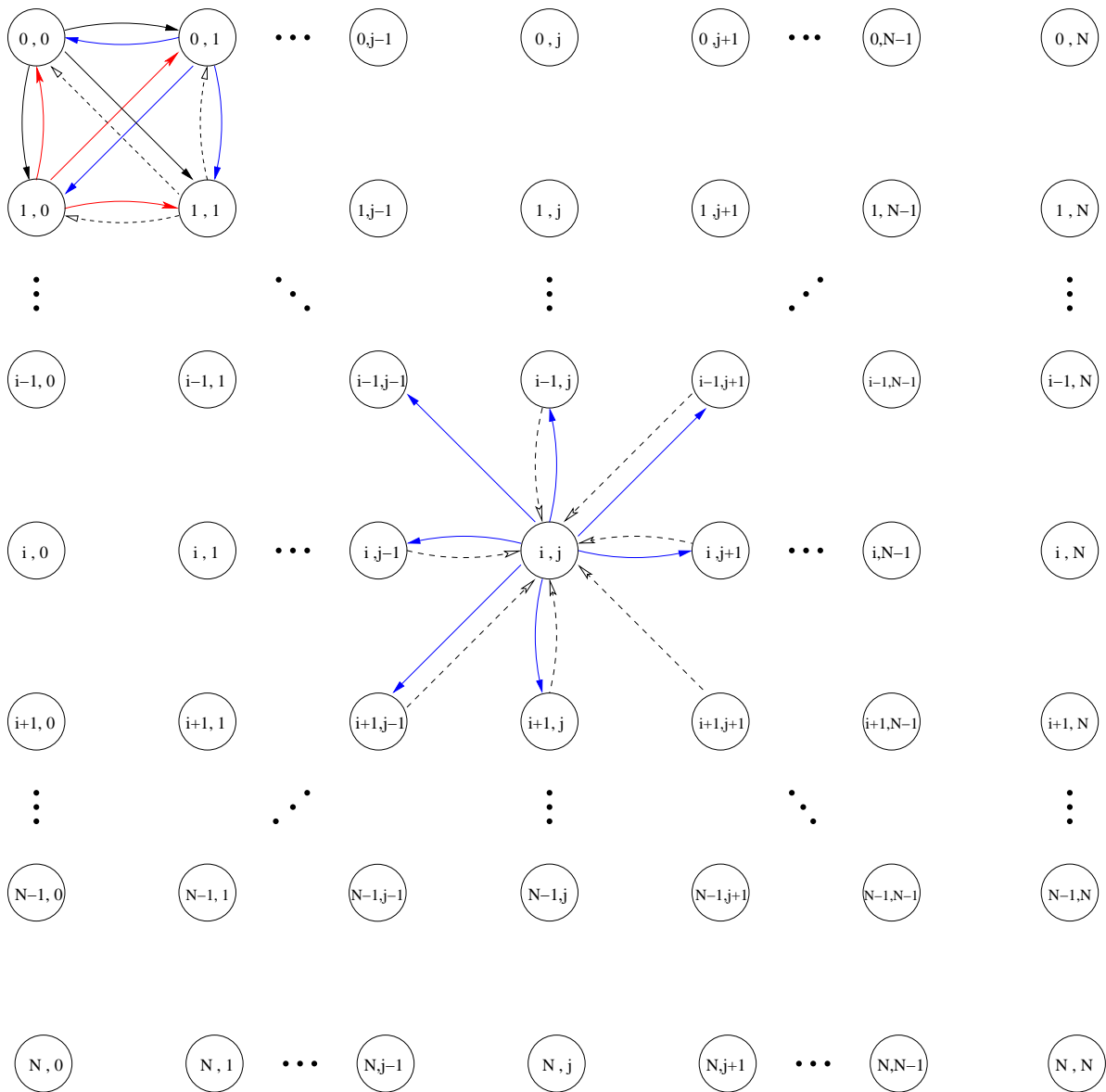


Fig. 2. Two-dimensional Markov chain.

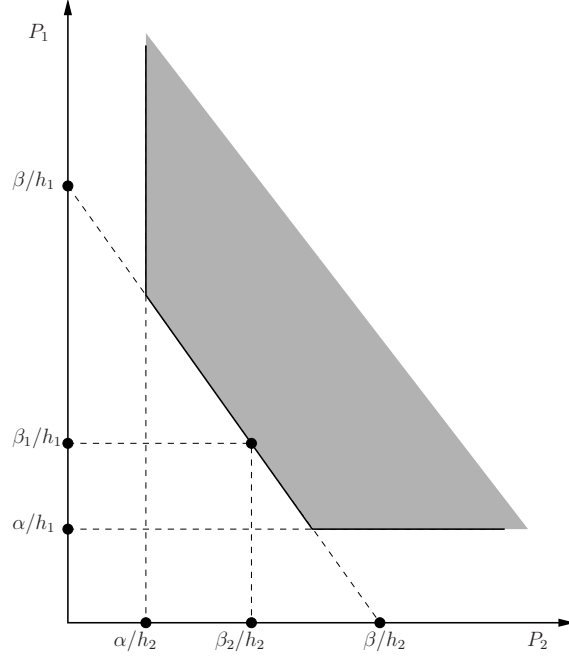


Fig. 3. Feasible power region.

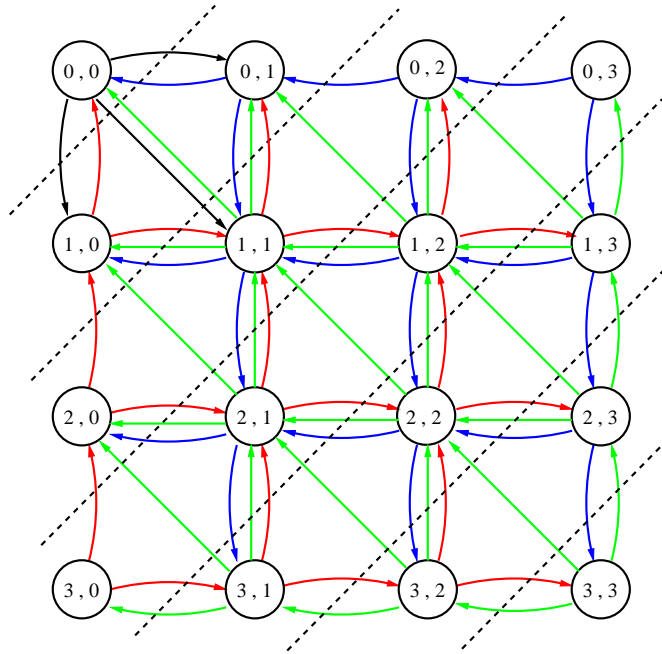


Fig. 4. The transitions between diagonal groups when $N = 3$.

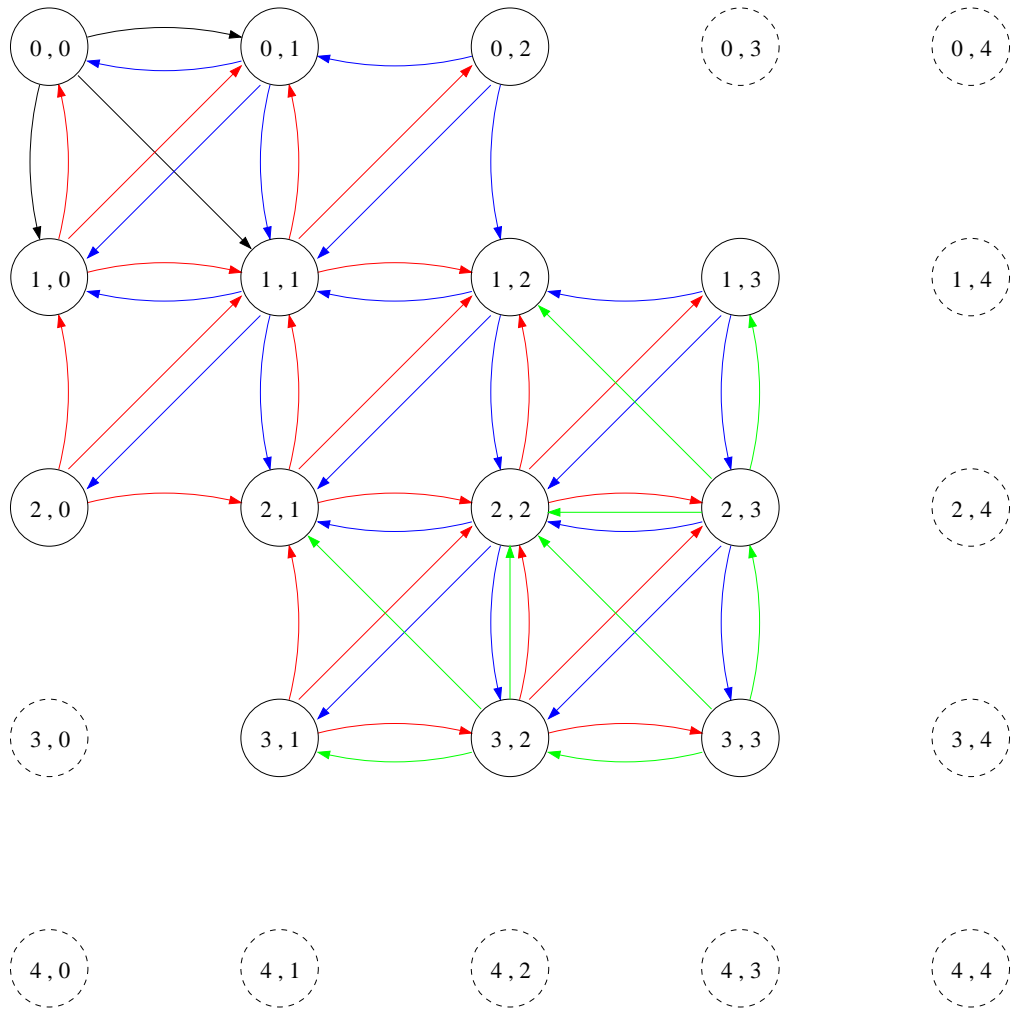


Fig. 5. Example: allocation within groups when $N = 4$.

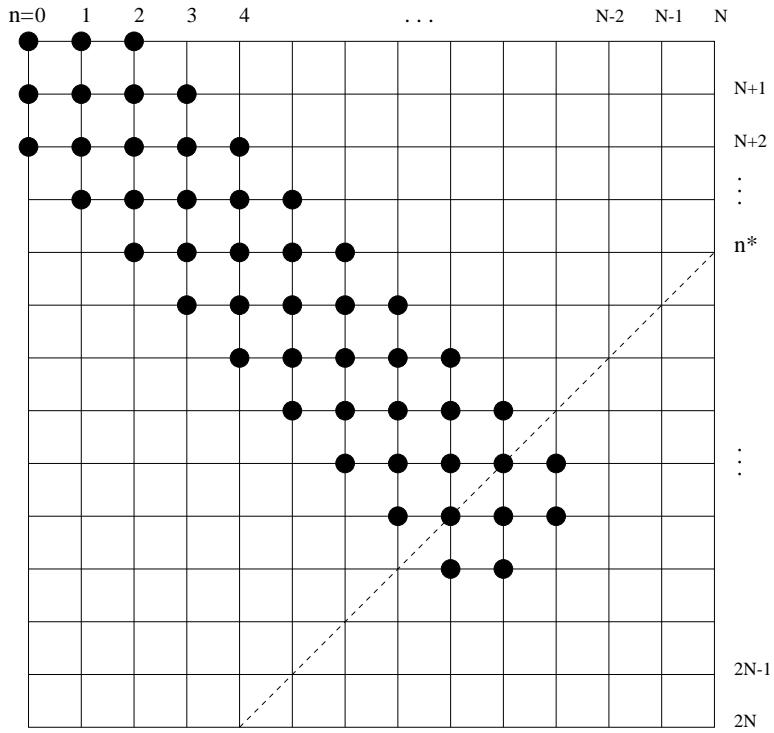


Fig. 6. Allocation pattern within groups.

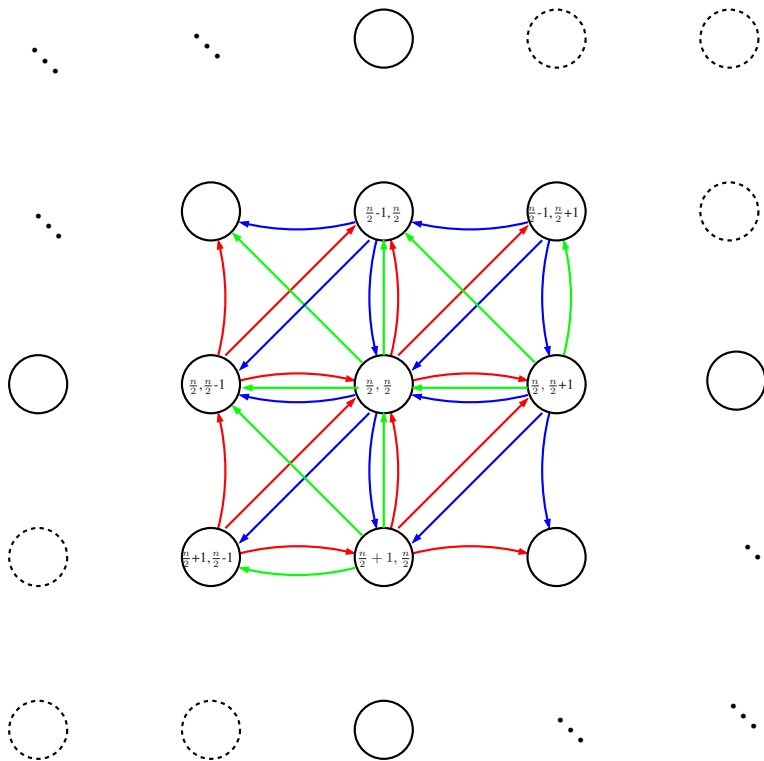


Fig. 7. The transitions between states when n is even.

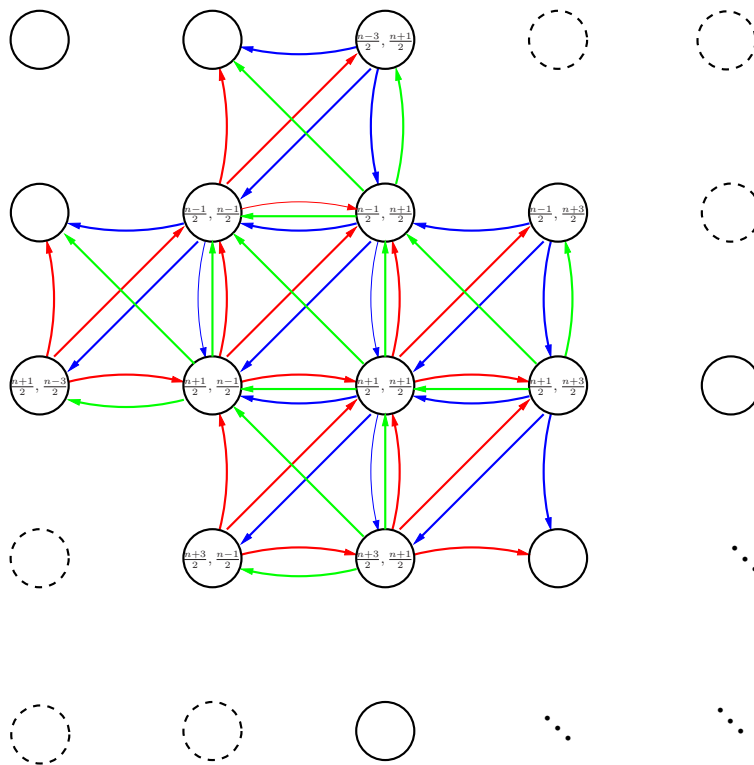


Fig. 8. The transitions between states when n is odd.

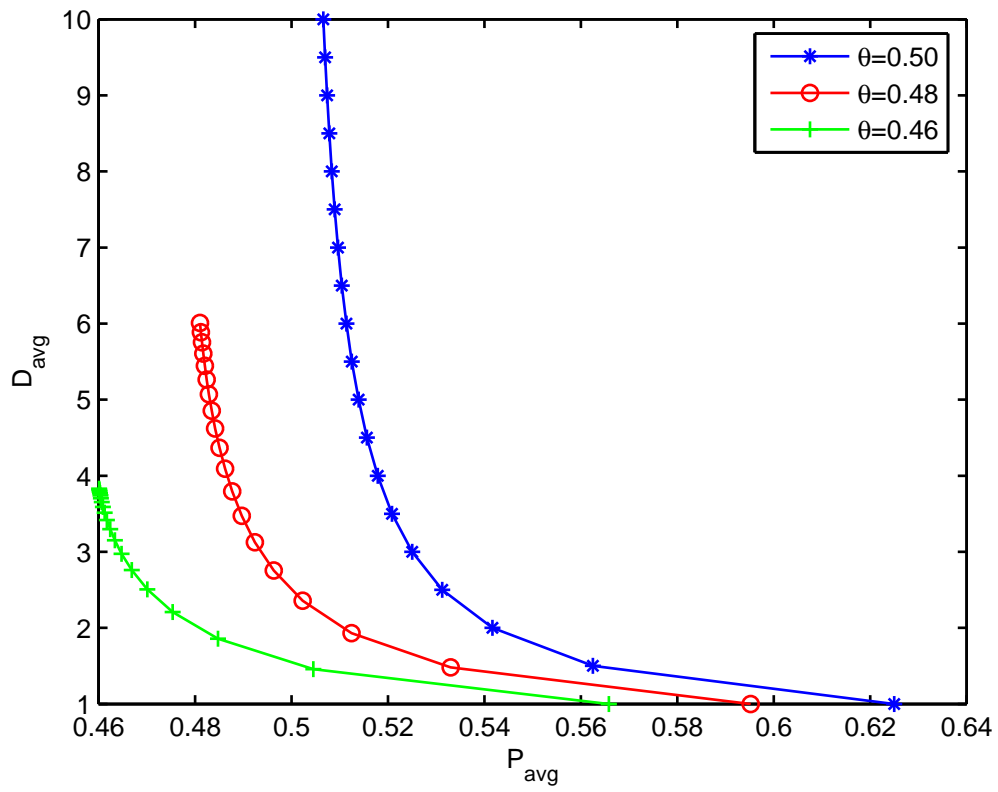


Fig. 9. The average delay versus average power in the symmetric scenario.