

# Optimal Energy Allocation for Energy Harvesting Transmitters With Hybrid Energy Storage and Processing Cost

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**Abstract**—We consider data transmission with an energy harvesting transmitter that has hybrid energy storage with a perfect super-capacitor (SC) and an inefficient battery. The SC has finite storage space while the battery has unlimited space. The transmitter can choose to store the harvested energy in the SC or in the battery. The energy is drained from the SC and the battery simultaneously. In this setting, we consider throughput optimal offline energy allocation problem over a point-to-point channel. In contrast to previous works, the hybrid energy storage model with finite and unlimited storage capacities imposes a generalized set of constraints on the transmission policy. As such, we show that the solution generalizes that for a single battery and is found by a sequential application of the directional water-filling algorithm. Next, we consider offline throughput maximization in the presence of an additive time-linear processing cost in the transmitter's circuitry. In this case, the transmitter has to additionally decide on the portions of the processing cost to be drained from the SC and the battery. Despite this additional complexity, we show that the solution is obtained by a sequential application of a directional glue pouring algorithm, parallel to the costless processing case. Finally, we provide numerical illustrations for optimal policies and performance comparisons with some heuristic online policies.

**Index Terms**—Energy harvesting, hybrid energy storage, processing power, throughput maximization, directional water-filling.

## I. INTRODUCTION

A key determinant of the performance of energy management policies in energy harvesting systems is the efficiency of energy storage. Energy storage units may foster imperfections such as leakage of the available energy and inefficiency due to other physical reasons. A well-known design method to boost the energy storage efficiency is to augment a super-capacitor (SC) to the existing battery and obtain a hybrid energy storage unit, see e.g., [1]–[4]. In this literature, it is common

knowledge that super-capacitors can store energy nearly ideally; however, they suffer from low energy storage capacities. On the other hand, batteries have large storage capacities while they suffer from inefficient energy storage. In this paper, we consider throughput optimal energy allocation for energy harvesting transmitters with such a hybrid energy storage unit.

In data transmission with such a device, aside from determining the transmit power level, the transmitter has to decide the portions of the incoming energy to be saved in the SC and the battery. While it is desirable to save incoming energy in the SC due to its perfect storage efficiency, the storage capacity limitation necessitates careful management of the energy saved in the SC. In this regard, the transmitter may wish to save energy in the inefficient battery rather than losing it. Therefore, the extra degree of freedom to choose the portions of incoming energy to save in different storage units significantly complicates the energy management problem. In this paper, we address this problem in an offline setting.

Offline throughput maximization for energy harvesting systems has recently received considerable interest [5]–[22]. In [5], the transmission completion time minimization problem is solved in energy harvesting systems with an unlimited capacity battery that operates over a static channel. The solution of this problem has later been extended for a finite capacity battery [6], fading channel [7], [8], broadcast channel [9]–[11], multiple access channel [12], interference channel [13] and relay channel [14], [15]. Offline throughput maximization for energy harvesting systems with leakage in energy storage was studied in [16]. In [17]–[19], offline optimal performance limits of multi-user wireless systems with energy transfer are studied. This literature has also been extended in [20], [21] for systems with processing costs, which is another common non-ideal behavior for these systems. Finally, [22] addresses offline throughput maximization for energy harvesting devices with energy storage losses.

Previous works on offline throughput maximization did not address the hybrid energy storage model; however, a two-unit storage model in this spirit has appeared in [23]. In this reference, the authors analyze a save-then-transmit protocol in energy harvesting wireless systems with main and secondary energy storage devices that operate over fading channels. The objective is to minimize the outage probability over a single variable, namely the save ratio. Using this analysis, some useful guidelines are given. Our work is different from [23] in that our objective is throughput maximization and we perform the optimization over a sequence of variables. Moreover, unlike our

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hybrid storage model, both of the storage devices have unlimited capacities in the model of [23].

In this paper, we investigate offline throughput maximization for an energy harvesting transmitter with hybrid energy storage. As emphasized in [5]–[22], energy arrivals impose causality constraints on the energy management policy. In addition, battery limitation imposes no-energy-overflow constraints [6], [7], [11]. As the rate-power relation is concave, energy allocation has to be made as constant as possible in time subject to the energy causality and no-energy-overflow constraints. In the presence of hybrid energy storage, the energy causality and no-energy-overflow constraints take a new form since the transmitter has to govern the internal energy dynamics of the storage unit in addition to the power levels drained from these devices. We capture the inefficiency of the battery by a factor  $\eta$  and solve the resulting offline throughput maximization problem.

A natural way of formulating this problem for the specified model is over the energies drained from the SC and the battery and the portion of the incoming energy to be saved in the SC. Instead, in the spirit of [2], we formulate the problem in terms of energies drained from the SC and the battery and energy transferred from the SC to the battery after initially storing all incoming energy in the SC as much as possible. This formulation reveals many commonalities of this problem with the previous works. This problem relates to sum-throughput maximization in a multiple access channel with energy harvesting transmitters [12] since energies drained from two queues contribute to transmission of a common data. Battery storage loss model is reminiscent of that in [22], [24] where the transmitter is allowed to save the incoming energy in a lossy battery or use it immediately for data transmission. Finally, one-way energy transfer from the SC to the battery relates to the problem considered in [18] where a two-user multiple access channel is considered with energy transfer from one node to the other.

Despite the coupling between the variables that represent energies drained from and transferred within the energy storage unit, we show that the problem can be solved by application of the directional water-filling algorithm [7] in multiple stages. In particular, we first forbid energy transfer from the SC to the battery and solve this restricted optimization problem. We show that this problem is solved by optimizing the SC allocation first and then the battery allocation given the SC allocation. Next, we allow energy transfer from the SC to the battery and show that the optimal allocation is obtained by directional water-filling in a setting transformed by the storage efficiency  $\eta$ . As a consequence, we obtain a generalization of the directional water-filling algorithm which yields useful insights on the structure of the optimal offline energy allocation in energy harvesting systems. Byproducts of this analysis are new insights about the optimal policies over the multiple access channel under finite battery constraints.

In the second part of the paper, we extend the offline throughput maximization problem to the case where a time-linear additive processing cost is present in the data transmission circuitry. It is well-known that circuit power consumption is non-negligible compared to the power spent for data transmission in small scale and short range applications [25]. We note that a considerable portion of energy harvesting communication applications falls into this category, and the

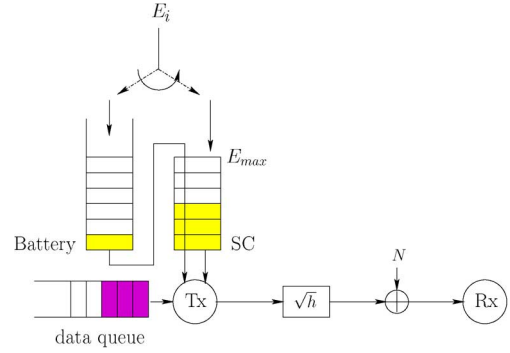


Fig. 1. System model with hybrid energy storage.

effects of circuit power have been investigated in previous works on energy harvesting communications [20], [21], [26], [27]. Among these works, the framework that is most pertinent to ours has been proposed in [21]. In contrast to [21], in our case, the transmitter has to additionally decide the portions of the energy cost drained from the SC and the battery in the presence of hybrid energy storage. Despite this additional complexity, we show that the solution of the throughput maximization problem with hybrid energy storage is obtained by a sequential application of an extended version of the directional glue pouring algorithm in [21]. To this end, we first construct an equivalent single epoch problem by introducing new time and power variables. In particular, we divide the available time for the SC and the battery and enforce SC and the battery to pay the energy cost in the corresponding time intervals. Moreover, we allow to drain energy from the SC only in its time interval while battery energy can be drained in both intervals. We show that this specific scheme yields a jointly optimal transmission and energy cost drainage scheme. We, then, generalize the single epoch analysis to multiple epochs and obtain an extension of the framework in [21] to the case of hybrid energy storage. In the final part of our paper, we illustrate optimal policies with and without processing cost in specific numerical studies and provide performance comparisons with heuristic policies in the online regime.

## II. SYSTEM MODEL

We consider a single-user additive Gaussian noise channel with an energy harvesting transmitter. The transmitter has three queues: a data queue and two energy queues. Two energy queues correspond to a hybrid energy storage unit composed of a battery and a super-capacitor (SC) as shown in Fig. 1. The battery has unlimited storage capacity whereas SC can store at most  $E_{max}$  units of energy. The battery is inefficient in the sense that the energy that can be drained from it is less than the amount that is stored. On the other hand, the SC is perfectly efficient in our model.<sup>1</sup> We assume infinite backlog in the data queue.

The physical layer is an AWGN channel with the input-output relation  $Y = \sqrt{h}X + N$  where  $h$  is the squared channel gain and  $N$  is Gaussian noise with zero-mean and unit-variance. Without loss of generality, we set  $h = 1$  throughout the communication.

<sup>1</sup>In real implementations, the SC leaks energy [1], [2]. In this paper, this imperfection of the SC is neglected.

We follow a continuous time model and the instantaneous rate is

$$r(t) = \frac{1}{2} \log(1 + p(t)) \quad (1)$$

At time  $t_i^e$ ,  $E_i$  amount of energy arrives.  $E_0^b$  and  $E_0^{sc}$  amounts of energies are available at the beginning in the battery and in the SC, respectively. In the following, we refer to the time interval between two energy arrivals as an epoch. More specifically, epoch  $i$  is the time interval  $[t_i^e, t_{i+1}^e)$  and the length of the epoch  $i$  is  $\ell_i = t_{i+1}^e - t_i^e$ .

Whenever energy  $E_i$  arrives at time  $t_i^e$ , the transmitter stores  $E_i^{sc}$  amount in the SC and  $E_i^b = E_i - E_i^{sc}$  amount in the battery. Since SC can store at most  $E_{max}$  units of energy,  $E_i^{sc}$  must be chosen such that no energy unnecessarily overflows. For this reason,  $E_i^{sc} \leq E_{max}$  must be satisfied. The efficiency of the battery is given by the parameter  $\eta$  where  $0 \leq \eta < 1$ : If  $E_i^b$  units of energy is stored in the battery, then  $\eta E_i^b$  units can be drained and  $(1 - \eta)E_i^b$  units are lost.<sup>2</sup> The available energy in the SC can be transferred to the battery for energy back-up purposes with zero time delay and energy loss. As a consequence, none of the arrived energy overflows; however, there is an energy loss due to inefficiency of the battery. The transmitter can instantaneously switch between the SC and the battery for maintaining the energy needed to drive its circuitry.<sup>3</sup> In view of this instantaneous switching capability, the circuitry is effectively driven by the superposition of the energies drained from the SC and the battery as depicted in Fig. 1.

A transmit power policy is denoted as  $p(t)$  over  $[0, T]$ .  $p(t)$  is constrained by the energy that can be drained from the hybrid storage system:

$$\int_0^{t_i^e} p(u)du \leq \sum_{j=0}^{i-1} E_j^{sc} + \eta E_j^b, \quad \forall i \quad (2)$$

where  $t_i^e$  in the upper limit of the integral is considered as  $t_i^e - \varepsilon$  for sufficiently small  $\varepsilon$ .

Moreover, we note that the power policy should cause no energy overflow in the SC. In order to express this constraint, we divide each incremental drained energy  $p(u)du$  as a linear combination of the energy drained from the SC,  $p^{sc}(u)du$ , and the energy drained from the battery,  $p^b(u)du$ . That is,  $p(u)du = p^{sc}(u)du + p^b(u)du$ . We are allowed to divide  $p(u)du$  into such components since the transmitter can instantaneously switch between the SC and the battery while driving the circuitry. No-energy-overflow constraint in the SC can now be expressed as follows:

$$\sum_{j=0}^i E_j^{sc} - \int_0^{t_i^e} p^{sc}(u)du \leq E_{max}, \quad \forall i \quad (3)$$

<sup>2</sup>This storage model for an imperfect battery is congruent to those models reported in, e.g., [3], [4], [24]. This imperfection does not model the effect of leakage or self-discharge, which could be neglected in the applications of interest, c.f. [4, Table II].

<sup>3</sup>In real systems, the switching time between the battery and the SC is very small compared to epoch lengths of interest [2].

We note that the constraints in (2) and (3) generalize the energy causality and no-energy-overflow constraints in the single-stage energy storage models studied, e.g., in [7].

### III. OFFLINE THROUGHPUT MAXIMIZATION PROBLEM

In this section, we consider the throughput optimal offline energy allocation problem with a deadline  $T$ . Note that the power policy  $p(t) = p^{sc}(t) + p^b(t)$  has to be constant over each epoch, due to the concavity of the rate-power relation in (1). That is,  $p(t) = p_i$  over epoch  $i$ , the interval  $[t_i^e, t_{i+1}^e]$ . In principle, this does not imply that  $p^{sc}(t)$  and  $p^b(t)$  are individually constant over epoch  $i$ . However, we note that it suffices to assume  $p^{sc}(t) = p_i^{sc}$  and  $p^b(t) = p_i^b$  over epoch  $i$  due to the fact that time-varying  $p_i^{sc}(t)$  and  $p_i^b(t)$  with  $p_i = p_i^{sc}(t) + p_i^b(t)$  for all  $t \in [t_i^e, t_{i+1}^e)$  and  $\int_{t_i^e}^{t_{i+1}^e} p_i^{sc}(t)dt = p_i^{sc}(t_{i+1}^e - t_i^e)$ ,  $\int_{t_i^e}^{t_{i+1}^e} p_i^b(t)dt = p_i^b(t_{i+1}^e - t_i^e)$  would yield the same throughput. Therefore, the power policy is represented by the sequence  $p_i = p_i^{sc} + p_i^b$  where  $p_i^{sc}$  and  $p_i^b$  are the portions of the power drained from the SC and the battery, respectively, in epoch  $i$ .

Another component of the transmitter's policy is to determine the portions of the incoming energy  $E_i^{sc}$  and  $E_i^b$  to be saved in the SC and the battery, respectively, with  $E_i^{sc} + E_i^b = E_i$ . An equivalent formulation for finding  $E_i^{sc}$  and  $E_i^b$  is obtained as follows: Since the battery is inefficient ( $0 \leq \eta < 1$ ), we initially allocate incoming energy to the SC and the remaining energy to the battery while still allowing to transfer a portion of the energy in SC to the battery. One may be tempted to think that allocating  $\min\{E_i, E_{max}\}$  energy to the SC is optimal and further transferring energy from the SC to the battery is unnecessary; however, this is not the case. Indeed, transferring energy from the SC to the battery enables further smoothing the transmit power sequence as will be clear in the following sections. That is, even though saving energy in the battery results in some loss in energy, it enables us to store and distribute transmit power more equally over time. We denote the energy transfer power at epoch  $i$  as  $\delta_i$  with the convention that the transferred energy becomes available for use in epoch  $i + 1$ . The variables in the original problem formulation and its equivalent formulation are depicted in Fig. 2. In Fig. 2(a), the transmitter selects  $p_i^{sc}$ ,  $p_i^b$  and  $E_i^{sc}$  while in Fig. 2(b), the transmitter selects  $p_i^{sc}$ ,  $p_i^b$  and  $\delta_i$  where  $E_i^{sc} = \min\{E_i, E_{max}\}$  is fixed. One can obtain  $E_i^{sc}$  in the formulation in Fig. 2(a) from  $\delta_i$  in Fig. 2(b) and vice versa. We note that the equivalent formulation, while its use is not clear at this stage, will enable useful structural properties and a simple water-filling interpretation. This will be clear in Sections III-B and IV-D. These conveniences are not possible if the analysis is performed using the original formulation.

In view of (2), (3), we get the following constraints for all  $i$ :

$$\sum_{j=1}^i (p_j^{sc} \ell_j + \delta_j \ell_j) \leq \sum_{j=0}^{i-1} E_j^{sc} \quad (4)$$

$$\sum_{j=0}^i E_j^{sc} - \sum_{j=1}^i (p_j^{sc} \ell_j + \delta_j \ell_j) \leq E_{max} \quad (5)$$

$$\sum_{j=1}^i p_j^b \ell_j \leq \sum_{j=0}^{i-1} (\eta E_j^b + \eta \delta_j \ell_j) \quad (6)$$

$$p_i^{sc}, p_i^b, \delta_i \geq 0 \quad (7)$$

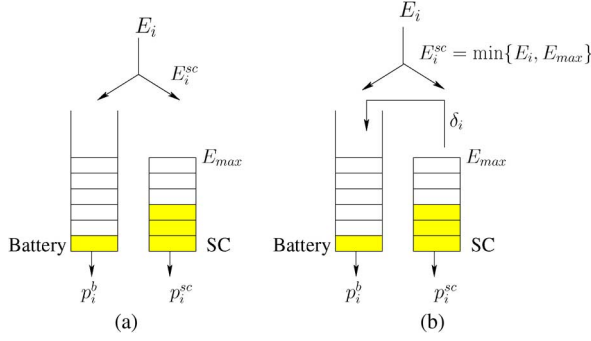


Fig. 2. The variables in the original problem formulation and its equivalent formulation followed in this paper. (a) Original formula; (b) Equivalent formulation.

where  $E_i^{sc} = \min\{E_i, E_{max}\}$  and  $E_i^b = (E_i - E_{max})^+$ . We set  $\delta_0 = 0$  and  $\delta_N = 0$  by convention. We remark that in the system model, energy transfer from the SC to the battery is not allowed. However, due to the offline nature, we have the freedom to allocate energy to the SC first and then transfer it to the battery. Moreover, one epoch delay in this energy transfer emphasizes the fact that if the energy in the SC in epoch  $i$  is transferred to the battery, that energy must be utilized starting from epoch  $i + 1$  as otherwise such an energy transfer cannot increase the throughput since the battery is inefficient.

Offline throughput maximization problem by deadline  $T$  with hybrid energy storage is:

$$\begin{aligned} \max_{p_i^{sc}, p_i^b, \delta_i \geq 0} \quad & \sum_{i=1}^N \frac{\ell_i}{2} \log(1 + p_i^{sc} + p_i^b) \\ \text{s.t.} \quad & (4) - (7) \end{aligned} \quad (8)$$

We note that the problem in (8) is a convex optimization problem and we can solve it using standard techniques [28]. The Lagrangian function for (8) is

$$\begin{aligned} \mathcal{L} = & - \sum_{i=1}^N \frac{\ell_i}{2} \log(1 + p_i^{sc} + p_i^b) \\ & + \sum_{i=1}^N \lambda_i \left( \sum_{j=1}^i (p_j^{sc} \ell_j + \delta_j \ell_j) - \sum_{j=0}^{i-1} E_j^{sc} \right) \\ & + \sum_{i=1}^{N-1} \mu_i \left( \sum_{j=0}^i E_j^{sc} - \sum_{j=1}^i (p_j^{sc} \ell_j + \delta_j \ell_j) - E_{max} \right) \\ & + \sum_{i=1}^N \nu_i \left( \sum_{j=1}^i p_j^b \ell_j - \sum_{j=0}^{i-1} (\eta E_j^b + \eta \delta_j \ell_j) \right) \\ & - \sum_{i=0}^N \gamma_i \delta_i - \sum_{i=1}^N \rho_{1i} p_i^{sc} - \sum_{i=1}^N \rho_{2i} p_i^b \end{aligned} \quad (9)$$

KKT optimality conditions for (8) are:

$$- \frac{1}{1 + p_i^{sc} + p_i^b} + \sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j - \rho_{1i} = 0, \quad \forall i \quad (10)$$

$$- \frac{1}{1 + p_i^{sc} + p_i^b} + \sum_{j=i}^N \nu_j - \rho_{2i} = 0, \quad \forall i \quad (11)$$

$$\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j - \eta \sum_{j=i+1}^N \nu_j - \gamma_i = 0, \quad \forall i \quad (12)$$

and the complementary slackness conditions are:

$$\lambda_i \left( \sum_{j=1}^i (p_j^{sc} \ell_j + \delta_j \ell_j) - \sum_{j=0}^{i-1} E_j^{sc} \right) = 0, \quad \forall i \quad (13)$$

$$\mu_i \left( \sum_{j=0}^i E_j^{sc} - \sum_{j=1}^i (p_j^{sc} \ell_j + \delta_j \ell_j) - E_{max} \right) = 0, \quad \forall i \quad (14)$$

$$\nu_i \left( \sum_{j=1}^i p_j^b \ell_j - \sum_{j=0}^{i-1} (\eta E_j^b + \eta \delta_j \ell_j) \right) = 0, \quad \forall i \quad (15)$$

$$\gamma_i \delta_i = \rho_{1i} p_i^{sc} = \rho_{2i} p_i^b = 0, \quad \forall i \quad (16)$$

We note that the optimization problem (8) may have many solutions. In order to get a solution, it suffices to find  $p_i^{sc*}, p_i^{b*}, \delta_i$  and Lagrange multipliers that satisfy (10)–(12) and (13)–(16). We observe the properties of the optimal  $p_i^{sc*}, p_i^{b*}$  and  $\delta_i^*$  in the following lemmas. We assume  $\eta < 1$  as for  $\eta = 1$ , there is no cost incurred due to saving energy in the battery and therefore energy can be blindly saved in the SC or the battery, yielding a single energy storage unit with unlimited space for which the solution is well-known [7].

**Lemma 1:** If  $p_i^{b*} \neq 0$ ,  $p_i^{sc*} + p_i^{b*}$  does not decrease in the passage from epoch  $i$  to epoch  $i + 1$ .

*Proof:* When  $p_i^{b*} \neq 0$ , we have  $\rho_{2i} = 0$ . By (11), we have  $p_i^{sc*} + p_i^{b*} = \frac{1}{\sum_{j=i}^N \nu_j} - 1$  and  $p_{i+1}^{sc*} + p_{i+1}^{b*} = \frac{1}{\sum_{j=i+1}^N \nu_j - \rho_{2(i+1)}} - 1$ . Since  $\nu_i \geq 0$  and  $\rho_{i+1} \geq 0$ , we conclude the desired result. ■

**Lemma 2:** If  $p_i^{sc*}, p_i^{b*} \neq 0$ , then  $\delta_i^* = 0$ .

*Proof:* If  $p_i^{sc*}, p_i^{b*} \neq 0$ , from (10) and (11), we have  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j = \sum_{j=i}^N \nu_j$ . Combining this with (12), we conclude that  $\gamma_i = \nu_i + (1 - \eta) \sum_{j=i+1}^N \nu_j > 0$  as  $\eta < 1$ . In view of the slackness condition  $\gamma_i \delta_i = 0$ , we get  $\delta_i^* = 0$ . ■

**Lemma 3:** If  $p_i^{sc*}, p_{i+1}^{sc*}, p_{i+1}^{b*} \neq 0$ ,  $p_i^{sc*} + p_i^{b*} \leq p_{i+1}^{sc*} + p_{i+1}^{b*}$ , then  $\delta_i^* = 0$ .

*Proof:* As  $p_i^{sc*}, p_{i+1}^{sc*}, p_{i+1}^{b*} \neq 0$ ,  $\rho_{1i} = \rho_{1(i+1)} = \rho_{2(i+1)} = 0$ . Therefore, by (10) and since  $p_i^{sc*} + p_i^{b*} \leq p_{i+1}^{sc*} + p_{i+1}^{b*}$ , we have  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j > \sum_{j=i+1}^N \lambda_j - \sum_{j=i+1}^{N-1} \mu_j$ . Moreover, since  $\rho_{2(i+1)} = 0$ , we have  $\sum_{j=i+1}^N \lambda_j - \sum_{j=i+1}^{N-1} \mu_j = \sum_{j=i+1}^N \nu_j$ . By (12),  $\gamma_i > 0$  and due to the slackness condition  $\gamma_i \delta_i = 0$ , we get  $\delta_i^* = 0$ . ■

Lemmas 1–3 reveal several useful properties of the optimal power sequences  $p_i^{sc*}$  and  $p_i^{b*}$  and their relation to the transfer power  $\delta_i^*$ . In view of these lemmas, we adopt the following strategy: Initially, we fix  $\delta_i = 0$  and find the optimal policy under this constraint. Note that  $\delta_i = 0$  is a good candidate for an optimal selection in view of Lemmas 2–3. If the resulting optimal policy is compatible with the KKT conditions, then we stop. Otherwise, we carefully update  $\delta_i$  so that the KKT conditions are satisfied.

### A. Optimal Policy for Fixed $\delta_i = 0$

For fixed  $\delta_i = 0$ , the problem becomes maximizing the throughput by the deadline subject to energy causality and finite SC  $E_{max}$  constraints only:

$$\begin{aligned} \max_{p_i^{sc}, p_i^b \geq 0} \quad & \sum_{i=1}^N \frac{\ell_i}{2} \log(1 + p_i^{sc} + p_i^b) \\ \text{s.t.} \quad & (4) - (7) \\ & \delta_i = 0, \quad \forall i \end{aligned} \quad (17)$$

where  $E_i^{sc} = \min\{E_i, E_{max}\}$  and  $E_i^b = (E_i - E_{max})^+$ . We note that (17) is equivalent to sum-throughput maximization in a two-user multiple access channel with finite and infinite capacity batteries. A simpler version of this problem where both users have infinite capacity battery is addressed in [12]. While the problem of sum-throughput maximization has a simple solution when batteries are unlimited by summing the energies of the users and performing single-user throughput maximization [12], the finite battery constraint in (17) prevents such a simple solution. As in the general problem in [12], the solution of (17) is found by iterative directional water-filling where infinitely many iterations are required in general.

Next, we show that due to the problem structure, we can find the solution of (17) only in two iterations. Note that the energy arrivals of the storage units are  $E_i^{sc} = \min\{E_i, E_{max}\}$  and  $E_i^b = (E_i - E_{max})^+$ . Energy is first allocated to the SC and the remaining energy is allocated to the battery. This specific allocation enables us to get the solution in two iterations. We state this result in the following lemma and provide the proof in Appendix.

**Lemma 4:** For fixed  $\delta_i = 0$ , let  $\hat{p}_i^{sc}$  be the outcome of directional water-filling given  $p_i^b = 0$ . Let  $\hat{p}_i^b$  be the outcome of directional water-filling given  $\hat{p}_i^{sc}$ . Then,  $\hat{p}_i^{sc}$  and  $\hat{p}_i^b$  are jointly optimal for (17).

We note that the claim in Lemma 4 would not be true if  $E_i^{sc}$  and  $E_i^b$  were allowed to take arbitrary values. Therefore, apart from providing a crucial step towards finding the solution of (8), the optimality result stated in Lemma 4 is an interesting case in the two-user multiple access channel with finite and infinite batteries where the optimal power sequences can be found only in two iterations.

We provide an illustration of the result of two iterations of directional water-filling in Fig. 3 where blue and red waters represent energies in the SC and the battery, respectively. In this specific example,  $E_i^{sc} = E_{max}$  only in epochs 1 and 4. We observe that the red water level is constant over epochs 1–3 and epochs 4–6. Moreover, in view of Lemma 1, whenever  $p_i^b$  is non-zero total power level increases. Note that the statement of Lemma 1, which is originally stated for the solution of (8), is also true for the solution of (17). This is due to the fact that Lemma 1 follows from the KKT condition in (11) and this condition still holds under the extra constraint  $\delta_i = 0$ .

### B. Determining the Optimal $\delta_i^*$

We note that for  $\hat{p}_i^{sc}$  and  $\hat{p}_i^b$ , there are Lagrange multipliers  $\lambda_i$ ,  $\mu_i$ ,  $\nu_i$ ,  $\rho_{1i}$  and  $\rho_{2i}$  that are compatible with (10) and (11). However, there may not exist  $\gamma_i \geq 0$  that are compatible with (12).

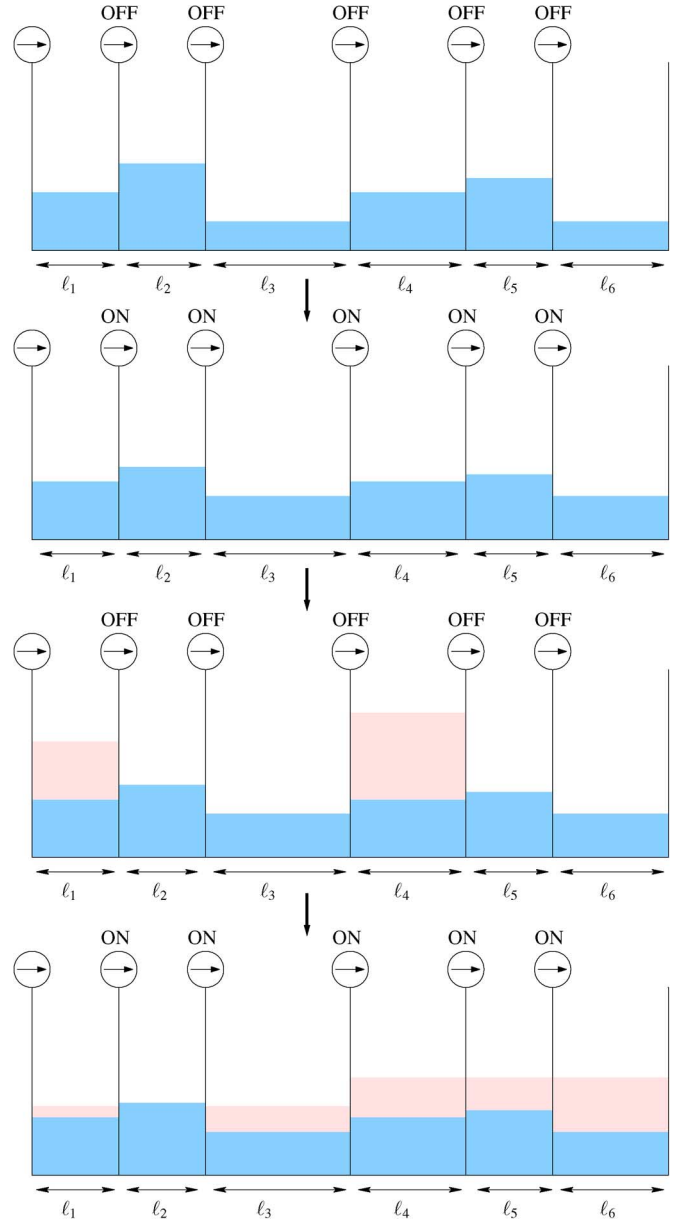


Fig. 3. An example of optimal power allocation for fixed  $\delta_i = 0$ .

In this section, we propose a method to update the allocations  $\hat{p}_i^{sc}$  and  $\hat{p}_i^b$  and the Lagrange multipliers  $\lambda_i, \mu_i, \nu_i, \rho_{1i}, \rho_{2i}$  that yield  $\delta_i^*$  and corresponding  $\gamma_i$  so that (10)–(12) and (13)–(16) are satisfied. For brevity, we restrict our treatment to the case where  $E_1^b > 0$  and  $E_i^b = 0$  for  $i = 2, \dots, N$ ; however, the arguments can be easily generalized. One can show that in this case,  $\nu_N > 0$  and  $\nu_i = 0$  for  $i = 1, \dots, N-1$ .

Note that if  $\hat{p}_i^b \neq 0$  for some  $i$ , resulting Lagrange multipliers yield  $\gamma_i \geq 0$ . In view of the KKT condition (12), we transform the directional water-filling setting as in Fig. 4: We multiply the water level and the bottom level by  $\frac{1}{\eta}$  at epochs where  $\hat{p}_i^b > 0$  and leave other epochs unchanged where the bottom level is 1. Moreover, if  $\gamma_i \geq 0$ , we set  $\delta_i^* = 0$  and transform the water level and the bottom level of that epoch. At epochs  $i$  with  $\gamma_i < 0$ , we wish to decrease  $\sum_{j=i}^N \nu_j$  and increase  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j$  so that  $\gamma_i$  approaches zero and the resulting allocations are compatible with (10)–(12) and (13)–(16). We next argue that if energy is



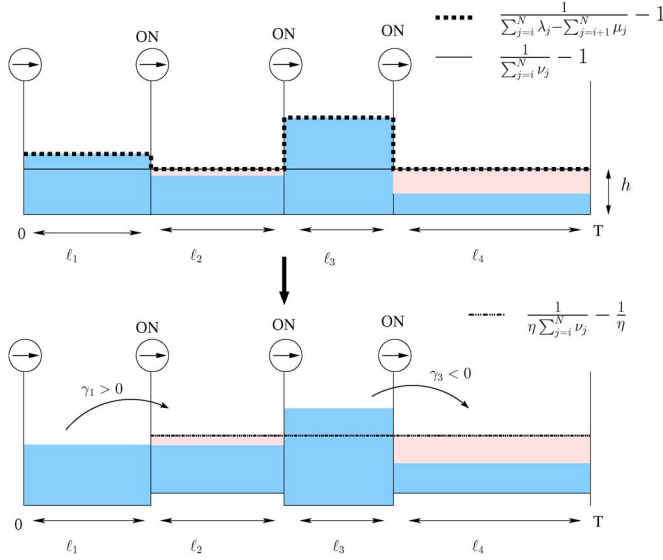


Fig. 4. Transforming the directional water-filling setting.

transferred from epochs  $i$  with  $\gamma_i < 0$  in a coordinated fashion, this is possible.

Recall that  $\nu_N > 0$  and  $\nu_i = 0$  for  $i = 1, \dots, N-1$ . We decrease  $\nu_N$  and increase  $\lambda_{\tilde{i}}, \mu_{\tilde{i}}$  and  $\sum_{j=\tilde{i}}^N \lambda_j - \sum_{j=\tilde{i}}^{N-1} \mu_j$  where  $\tilde{i}$  is the epoch index with the lowest  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j$ . This decreases the power level  $p_i^{sc}$  and increases the battery power level  $p_i^b$  at all epochs. Therefore, a non-zero energy transfer from epoch  $i$  occurs. As we decrease  $\nu_N$ ,  $\gamma_i$  also increases. In particular,  $\gamma_i$  may change sign from negative to positive in which case, we make sure that  $\delta_i^* = 0$  for that epoch and hence we transform the bottom levels and the water levels for those epochs as in Fig. 4. On the other hand,  $\sum_{j=\tilde{i}}^N \lambda_j - \sum_{j=\tilde{i}}^{N-1} \mu_j$  increases and it may hit the second lowest  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j$ . In this case, we start to increase  $\lambda_i, \mu_i$  and  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j$  in both of these epochs.

Note that this procedure corresponds to a coordinated energy transfer: We start energy transfer from the epoch  $\tilde{i}$  with the highest power level  $\hat{p}_{\tilde{i}}^{sc}$ . In the transformed setting, as we transfer  $\delta_i, \frac{1}{\eta}\delta_i$  units of water is added to the next epoch as shown in Fig. 5. If the power level of epoch  $\tilde{i}$  decreases to the level of the second highest power  $\hat{p}_{\tilde{i}}^{sc}$  with  $\gamma_i < 0$ , then energy is transferred simultaneously from these epochs. Causality conditions may forbid decreasing  $\nu_N$  after some level. This way, all epochs  $i$  which have initially  $\gamma_i < 0$  are updated so that  $\gamma_i \geq 0$  with  $\gamma_i = 0$  if  $\delta_i > 0$  and (10)–(12) and (13)–(16) are satisfied.

Note that when energy is transferred from the SC to the battery in epoch  $i$ , this energy spreads over future epochs  $i+1, \dots, N$ . Moreover, the energy that was transferred from epochs  $1, \dots, i-1$  in the second directional water-filling of Lemma 4 given  $\hat{p}_i^{sc}$  may flow back to these epochs. We, therefore, measure the transferred energy within the battery at each epoch by means of meters and negate it if energy flows in the opposite direction. This is reminiscent of the meters used for the two-way channel in [18], [19].

### C. Discussion

When  $\delta_i = 0$ , in general the first directional water-filling yields a non-monotone power sequence  $\hat{p}_i^{sc}$  due to the finite

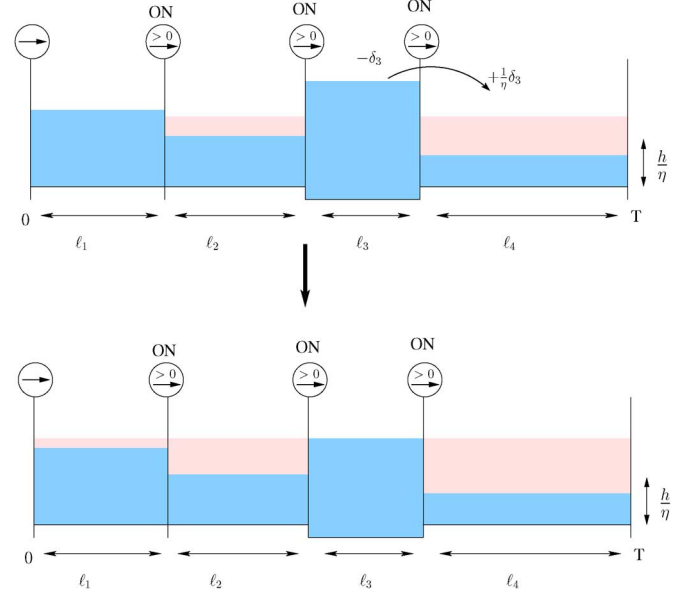


Fig. 5. The water flow in the transformed directional water-filling setting.

storage limit  $E_{max}$ . The second directional water-filling fills the gaps due to non-monotonicity of  $\hat{p}_i^{sc}$  and ameliorates the non-monotonicity of the total power level  $\hat{p}_i^{sc} + \hat{p}_i^b$ . The second stage of the algorithm further smooths out the non-monotonicity of the total power by transferring energy from the SC to the battery in epochs where power is sharply high. Therefore, the cumulative effect of the two-stage algorithm is to collectively transfer energy from the past to the future in both storage devices and make the total power level as constant as possible subject to energy causality and finite SC capacity limit constraints. The extent to which this transfer is continued is determined in a transformed directional water-filling setting where the key parameter is the storage efficiency  $\eta$ .

We remark that for  $\eta = 1$ , the outcome of the algorithm is the same as the power policy yielded by single-user directional water-filling applied to the energy arrivals  $E_i$  with unlimited battery capacity. This is due to the fact that storing energy in the battery or the SC does not cause a performance difference in this case and hence the same performance is achieved if all energy is allocated to the battery only. We also remark that for  $\eta = 0$ , the algorithm stops after the first directional water-filling since battery is never used in this case. Therefore, the algorithm reduces to the classical directional water-filling with  $E_{max}$  constraint in [7]. We remark that even when the energy arrivals are always smaller than the SC capacity, i.e., even when  $E_i \leq E_{max}$  for all  $i$ , the presence of the battery improves the throughput performance as the battery enables smoothing out the variations in the transmit power.

Finally, we note that the results can be straightforwardly generalized when the energy transfer from the SC to the battery is lossy: Assume that when  $\delta_i$  energy is transferred from the SC, only  $\Theta\delta_i$  energy can be saved in the battery where  $\Theta \in [0, 1]$ . In this case, the solution is found by following similar steps. The first step remains unchanged: We first fix  $\delta_i = 0$  and find the solution. In the second step, we need to transform the directional water-filling setting with  $\eta\Theta$  in stead of  $\eta$ .

#### IV. OFFLINE THROUGHPUT MAXIMIZATION WITH PROCESSING COST

In this section, we consider the case in which the transmitter's circuitry causes an additive time-linear processing cost in data transmission. In particular, the processing cost could be viewed as a constant circuit power  $\epsilon$  whenever it is active. Hence, for a transmit power policy  $p(t)$ , the total power consumption is  $p(t) + \epsilon \mathbf{1}_{p(t)>0}$  where  $\epsilon$  is in energy units per time units.

The energy causality and no-energy-overflow constraints in (2) and (3) extend naturally to the case of non-negligible processing power and can be expressed as:

$$\int_0^{t_i^e} (p(u) + \epsilon \mathbf{1}_{p(u)>0}) du \leq \sum_{j=0}^{i-1} E_j^{sc} + \eta E_j^b, \quad \forall i \quad (18)$$

$$\sum_{j=0}^i E_j^{sc} - \int_0^{t_i^e} (p^{sc}(u) + \epsilon^{sc} \mathbf{1}_{p(u)>0}) du \leq E_{max}, \quad \forall i \quad (19)$$

where  $\epsilon_i^{sc}$  and  $\epsilon_i^b$  are the portions of the processing power drained from the SC and the battery, respectively, in epoch  $i$ :  $\epsilon = \epsilon_i^{sc} + \epsilon_i^b$ .

##### A. The Case of a Single Epoch

We start by considering the single epoch case. Assume that  $E^{sc}$  and  $E^b$  units of energy are available before the start of transmission in the SC and the battery, respectively, and let the deadline be set to infinity. We have the following optimization problem:

$$\max_{t, p^{sc}(\cdot), p^b(\cdot)} \int_0^t \frac{1}{2} \log(1 + p^{sc}(u) + p^b(u)) du \quad (20)$$

where  $p^{sc}(u)$  and  $p^b(u)$  are the powers drained from the SC and the battery during the  $0 \leq u \leq t$  time interval. The energy constraints for (20) are:  $\int_0^t (p^{sc}(u) + \epsilon^{sc}) du \leq E^{sc}$  and  $\int_0^t (p^b(u) + \epsilon^b) du \leq \eta E^b$  where  $\epsilon^{sc} + \epsilon^b = \epsilon$ . We remark that the single epoch analysis in [21], [29] does not immediately apply to our problem since our problem involves two power variables and the transmitter incurs a processing cost when either one (or both) of these power variables is non-zero and the processing energy can be drained from two different energy storage devices.

We observe that due to the concavity of the  $\log(\cdot)$  function,  $p^{sc}(u) + p^b(u)$  must remain constant whenever  $p^{sc}(u) + p^b(u) > 0$  and such an allocation is always feasible since the energies  $E^{sc}$  and  $E^b$  are assumed to be available before the transmission starts. This, in turn, implies that the transmission duration  $t$  is  $t = \frac{E^{sc} + \eta E^b}{p^{sc} + p^b + \epsilon}$  where  $p^{sc}$  and  $p^b$  are constant powers drained from the SC and the battery during the  $0 \leq u \leq t$  interval. Hence, the objective function in (20) is expressed as a single-variable function of  $p^{sc} + p^b$ :  $\frac{E^{sc} + \eta E^b}{p^{sc} + p^b + \epsilon} \frac{1}{2} \log(1 + p^{sc} + p^b)$ . Equating its derivative to zero, we obtain the following equation (c.f. [21], [29]):

$$\frac{\log(1 + p^*)}{(p^* + \epsilon)} = \frac{1}{1 + p^*} \quad (21)$$

Let  $p^*$  be the solution of the equation in (21). Then,  $p^{sc*}$  and  $p^{b*}$  are the solutions of (20) if  $p^{sc*} + p^{b*} = p^*$ . Note that  $p^*$

is the unique solution of (21), which parametrically depends on  $\epsilon$  and is independent of  $E^{sc}$  and  $E^b$  [21], [29]. Moreover, we note that the selections of  $p^{sc*}$  and  $p^{b*}$  are not unique and they determine  $\epsilon^{sc}$  and  $\epsilon^b$ . In particular, we have

$$\epsilon^{sc} = \frac{E^{sc}}{E^{sc} + \eta E^b} (p^* + \epsilon) - p^{sc*} \quad (22)$$

$$\epsilon^b = \frac{\eta E^b}{E^{sc} + \eta E^b} (p^* + \epsilon) - p^{b*} \quad (23)$$

Now, let us impose a deadline  $t \leq T$  to the problem in (20). If the deadline  $T$  satisfies  $T \geq \frac{E^{sc} + \eta E^b}{p^* + \epsilon}$ , the solution is the same as the solution with an infinite deadline. On the other hand, if  $T \leq \frac{E^{sc} + \eta E^b}{p^* + \epsilon}$ , then  $p^{sc*} + p^{b*} = \frac{E^{sc} + \eta E^b}{T} - \epsilon$  and  $\epsilon^{sc}$ ,  $\epsilon^b$  are determined as:

$$\epsilon^{sc} = \frac{E^{sc}}{T} - p^{sc*} \quad (24)$$

$$\epsilon^b = \frac{\eta E^b}{T} - p^{b*} \quad (25)$$

In the infinite deadline case, one possible selection is  $p^{sc*} = \frac{E^{sc}}{E^{sc} + \eta E^b} p^*$  and  $p^{b*} = \frac{\eta E^b}{E^{sc} + \eta E^b} p^*$ ;  $\epsilon^{sc}$  and  $\epsilon^b$  are determined according to (22), (23). This selection facilitates an alternative view of the problem: If in the first  $t^{sc} = \frac{E^{sc}}{p^* + \epsilon}$  time units,  $p^{sc} = p^*$ ,  $p^b = 0$  and in the following  $t^b = \frac{\eta E^b}{p^* + \epsilon}$  time units,  $p^b = p^*$  and  $p^{sc} = 0$ , the optimal throughput for (20) is achieved. Moreover, the processing energy is drained from the SC and the battery with power  $\epsilon$  only when they are active. This selection has the following counterpart if the deadline is finite: When  $\frac{E^{sc}}{p^* + \epsilon} \leq T \leq \frac{E^{sc} + \eta E^b}{p^* + \epsilon}$ ,  $p^{sc} = p^*$  over the first  $t^{sc} = \frac{E^{sc}}{p^* + \epsilon}$  time units and  $p^b$  is determined by water-filling  $\eta E^b$  units of energy over  $[0, T]$  interval given  $p^{sc}$  and no processing cost from the battery in the first  $t^{sc}$  units. Secondly, if  $T < \frac{E^{sc}}{p^* + \epsilon}$ ,  $p^{sc} = \frac{E^{sc}}{T} - \epsilon$  and  $p^b = \frac{\eta E^b}{T}$  over  $[0, T]$ .

This alternative view of the problem suggests that a solution for (20) can be found by solving

$$\max_{t^{sc}, t^b, p^{sc}, p_1^b, p_2^b} \frac{t^{sc}}{2} \log(1 + p^{sc} + p_1^b) + \frac{t^b}{2} \log(1 + p_2^b) \quad (26)$$

where the energy constraints are  $t^{sc}(p^{sc} + \epsilon) \leq E^{sc}$  and  $t^{sc} p_1^b + t^b(p_2^b + \epsilon) \leq \eta E^b$  along with the deadline  $t^{sc} + t^b \leq T$ . Note that the processing energy is drained from the SC in the first  $t^{sc}$  units and from the battery in the remaining time units. The problem (26) has a unique solution<sup>4</sup>  $t^{sc*}, t^{b*}, p^{sc*}, p_1^{b*}, p_2^{b*}$ . To see this note that all of the time and energy constraints must be satisfied with equality and whenever  $t^{b*} > 0$ , we must have  $p^{sc*} + p_1^{b*} = p_2^{b*} = \frac{E^{sc} + \eta E^b}{T} - \epsilon$ , which along with the time and energy constraints, determine the variables in (26) uniquely. Similarly, if  $t^{b*} = 0$ , then  $p^{sc*} = \frac{E^{sc}}{T} - \epsilon$ ,  $p_1^{b*} = \frac{\eta E^b}{T}$  and  $p_2^{b*}$  can be selected arbitrarily. Note that using the unique solution  $t^{sc*}, t^{b*}, p^{sc*}, p_1^{b*}, p_2^{b*}$  of (26), we can get a solution of (20) by setting the SC power as  $\frac{t^{sc*}}{t^{sc*} + t^{b*}} p^{sc*}$  and the battery power as  $\frac{t^{b*}}{t^{sc*} + t^{b*}} p_1^{b*} + \frac{t^{b*}}{t^{sc*} + t^{b*}} p_2^{b*}$ . Moreover,  $\frac{t^{sc*}}{t^{sc*} + t^{b*}} \epsilon$  units of processing energy is drained from the SC and the remaining

<sup>4</sup>If  $t^{b*} = 0$ ,  $p_2^{b*}$  can be selected arbitrarily; however, this does not violate the uniqueness of the solution.

processing energy is drained from the battery. We note that in an optimal solution of (26),  $p_1^{b*} = 0$  whenever  $t_i^{sc*} + t_i^{b*} < T$ .

This specific allocation is not necessary for the optimality in (20) and one may suggest different optimal allocations. However, we will see in the following section that this allocation enables us to extend the analysis in Section III and interpret the solutions properly.

### B. The Case of Multiple Epochs

As the rate-power relation is concave and the processing cost is additive and independent of the transmit power level, the transmit power policy  $p(t)$  has to be constant during each epoch  $i$  as long as  $p(t) > 0$ ; see also [21], [29]. Therefore, we get the following constraints for all  $i$ :

$$\sum_{j=1}^i ((p_j^{sc} + \epsilon) t_j^{sc} + \delta_j t_j^{sc}) \leq \sum_{j=0}^{i-1} E_j^{sc} \quad (27)$$

$$\sum_{j=0}^i E_j^{sc} - \sum_{j=1}^i ((p_j^{sc} + \epsilon) t_j^{sc} + \delta_j t_j^{sc}) \leq E_{max} \quad (28)$$

$$\sum_{j=1}^i (p_{1j}^{b,sc} + (p_{2j}^b + \epsilon) t_j^b) \leq \sum_{j=0}^{i-1} (\eta E_j^b + \eta \delta_j t_j^{sc}) \quad (29)$$

$$p_i^{sc}, p_{1i}^b, p_{2i}^b, \delta_i \geq 0 \quad (30)$$

where  $E_i^{sc} = \min(E_i, E_{max})$  and  $E_i^b = (E_i - E_{max})^+$ . We set  $\delta_0 = 0$  and  $\delta_N = 0$  by convention.  $t_i = t_i^{sc} + t_i^b$  is the time portion of epoch  $i$  in which the transmitter is active. Thus,  $0 \leq t_i \leq \ell_i$ . We note that the constraint set in (27)–(30) is not convex. To circumvent this difficulty, we introduce a change of variables:  $\alpha_i \triangleq p_i^{sc} t_i^{sc}$ ,  $\beta_i \triangleq p_{2i}^b t_i^b$ ,  $\theta_i \triangleq p_{1i}^{b,sc} t_i^{sc}$  and  $\gamma_i \triangleq \delta_i t_i^{sc}$ . The constraint set in terms of the new variables is:

$$\sum_{j=1}^i (\alpha_j + \epsilon t_j^{sc} + \gamma_j) \leq \sum_{j=0}^{i-1} E_j^{sc} \quad (31)$$

$$\sum_{j=0}^i E_j^{sc} - \sum_{j=1}^i (\alpha_j + \epsilon t_j^{sc} + \gamma_j) \leq E_{max} \quad (32)$$

$$\sum_{j=1}^i (\theta_j + \beta_j + \epsilon t_j^b) \leq \sum_{j=0}^{i-1} (\eta E_j^b + \eta \gamma_j) \quad (33)$$

$$0 \leq t_i^{sc} + t_i^b \leq \ell_i, \quad (34)$$

$$t_i^{sc}, t_i^b, \alpha_i, \beta_i, \theta_i, \gamma_i \geq 0 \quad (35)$$

Offline throughput maximization problem in the new variable set is:

$$\begin{aligned} \max_{\alpha_i, \beta_i, \theta_i, t_i^{sc}, t_i^b, \gamma_i \geq 0} & \sum_{i=1}^N \frac{t_i^{sc}}{2} \log \left( 1 + \frac{\alpha_i}{t_i^{sc}} + \frac{\theta_i}{t_i^{sc}} \right) \\ & + \frac{t_i^b}{2} \log \left( 1 + \frac{\beta_i}{t_i^b} \right) \\ \text{s.t.} & \quad (31)–(35) \end{aligned} \quad (36)$$

The concavity of the objective function in (36) follows from the convexity preservation of the perspective operation [28]. Note that the function  $\frac{t_i}{2} \log \left( 1 + \frac{\alpha_i}{t_i} + \frac{\beta_i}{t_i} \right)$  is the perspective of the

strictly concave function  $\frac{1}{2} \log(1 + \alpha_i + \beta_i)$ . The Lagrangian for (36) is as follows:

$$\begin{aligned} \mathcal{L} = & - \sum_{i=1}^N \left[ \frac{t_i^{sc}}{2} \log \left( 1 + \frac{\alpha_i}{t_i^{sc}} + \frac{\theta_i}{t_i^{sc}} \right) + \frac{t_i^b}{2} \log \left( 1 + \frac{\beta_i}{t_i^b} \right) \right] \\ & + \sum_{i=1}^N \lambda_i \left[ \sum_{j=1}^i (\alpha_j + \epsilon t_j^{sc} + \gamma_j) - \sum_{j=0}^{i-1} E_j^{sc} \right] \\ & + \sum_{i=1}^{N-1} \mu_i \left[ \sum_{j=0}^i E_j^{sc} - \sum_{j=1}^i (\alpha_j + \epsilon t_j^{sc} + \gamma_j) - E_{max} \right] \\ & + \sum_{i=1}^N \nu_i \left[ \sum_{j=1}^i (\theta_j + \beta_j + \epsilon t_j^b) - \sum_{j=0}^{i-1} (\eta E_j^b + \eta \gamma_j) \right] \\ & - \sum_{i=1}^N \rho_{1i} \alpha_i - \sum_{i=1}^N \rho_{2i} \theta_i - \sum_{i=1}^N \rho_{3i} \beta_i - \sum_{i=0}^N \xi_i \gamma_i \\ & - \sum_{i=1}^N \sigma_{1i} t_i^{sc} - \sum_{i=1}^N \sigma_{2i} t_i^b + \sum_{i=1}^N z_i (t_i^{sc} + t_i^b - \ell_i) \end{aligned} \quad (37)$$

where  $\lambda_i, \mu_i, \nu_i, \rho_{1i}, \rho_{2i}, \rho_{3i}, \xi_i, \sigma_{1i}, \sigma_{2i}$  and  $z_i$  are the Lagrange multipliers. The KKT optimality conditions for (36) are:

$$-\frac{t_i^{sc}}{t_i^{sc} + \alpha_i + \theta_i} + \sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j - \rho_{1i} = 0 \quad (38)$$

$$-\frac{t_i^{sc}}{t_i^{sc} + \alpha_i + \theta_i} + \sum_{j=i}^N \nu_j - \rho_{2i} = 0 \quad (39)$$

$$-\frac{t_i^b}{t_i^b + \beta_i} + \sum_{j=i}^N \nu_j - \rho_{3i} = 0 \quad (40)$$

$$\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j - \eta \sum_{j=i+1}^N \nu_j - \xi_i = 0 \quad (41)$$

$$\begin{aligned} & \frac{\alpha_i + \theta_i}{t_i^{sc} + \alpha_i + \theta_i} - \log \left[ \frac{t_i^{sc} + \alpha_i + \theta_i}{t_i^{sc}} \right] \\ & + \epsilon \left[ \sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j \right] - \sigma_{1i} + z_i = 0 \end{aligned} \quad (42)$$

$$\frac{\beta_i}{t_i^b + \beta_i} - \log \left[ \frac{t_i^b + \beta_i}{t_i^b} \right] + \epsilon \sum_{j=i}^N \nu_j - \sigma_{2i} + z_i = 0 \quad (43)$$

and the corresponding complementary slackness conditions are:

$$\lambda_i \left[ \sum_{j=1}^i (\alpha_j + \epsilon t_j^{sc} + \gamma_j) - \sum_{j=0}^{i-1} E_j^{sc} \right] = 0 \quad (44)$$

$$\mu_i \left[ \sum_{j=0}^i E_j^{sc} - \sum_{j=1}^i (\alpha_j + \epsilon t_j^{sc} + \gamma_j) - E_{max} \right] = 0 \quad (45)$$

$$\nu_i \left[ \sum_{j=1}^i (\theta_j + \beta_j + \epsilon t_j^b) - \sum_{j=0}^{i-1} (\eta E_j^b + \eta \gamma_j) \right] = 0 \quad (46)$$

$$\rho_{1i} \alpha_i = \rho_{2i} \theta_i = \rho_{3i} \beta_i = \xi_i \gamma_i = 0 \quad (47)$$

$$\sigma_{1i} t_i^{sc} = \sigma_{2i} t_i^b = z_i (t_i^{sc} + t_i^b - \ell_i) = 0 \quad (48)$$



We note that the optimization problem in (36) may have many solutions. To find a solution, it suffices to find  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$  and Lagrange multipliers that are consistent with (38)–(43) and (44)–(48). This, in turn, yields optimal transmit power sequences  $p_i^{sc}$ ,  $p_{1i}^b$  and  $p_{2i}^b$  along with time sequences  $t_i^{sc}$  and  $t_i^b$ . Based on our analysis of a single epoch, we observe some properties of an optimal solution for (36) in the following lemmas.

**Lemma 5:** If  $0 < t_i^{sc*} < \ell_i$ , then  $\delta_i^* = 0$ .

*Proof:* For the case when  $t_i^{b*} = 0$ , we have  $p_{1i}^{b*} = p_{2i}^{b*} = 0$ . Hence,  $\sigma_{1i} = 0$ ,  $z_i = 0$  and  $\nu_i = 0$ . By (42),  $\frac{\log(1+p_i^{sc*})}{p_i^{sc*}+\epsilon} = \frac{1}{1+p_i^{sc*}}$  and therefore  $p_i^{sc*} = p^*$ . By (38) and (41), we get  $\xi_i > 0$  and hence  $\gamma_i = 0$  and  $\delta_i^* = 0$ .

When  $t_i^{b*} > 0$ , we have  $p_{2i}^{b*} > 0$ . From the slackness conditions in (47), (48),  $\sigma_{1i} = \sigma_{2i} = \rho_{1i} = \rho_{2i} = 0$ . By (38),  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j = \frac{1}{1+p_i^{sc*}}$ . By (39),  $\sum_{j=i}^N \nu_j = \frac{1}{1+p_{2i}^{b*}}$ . Using this in (41), we have  $\xi_i = (1-\eta) \left( \frac{1}{1+p_i^{sc*}} \right) + \eta\nu_i$ . By (46),  $\nu_i \geq 0$  and hence  $\xi_i > 0$  and together with the slackness condition  $\xi_i\gamma_i = 0$ , we get  $\delta_i^* = 0$ . ■

**Lemma 6:** If  $t_i^{sc*} + t_i^{b*} = \ell_i$  and  $p_{1i}^{b*} \neq 0$ , then  $\delta_i^* = 0$ .

*Proof:* Note that  $t_i^{sc*} > 0$  as energy is first allocated to the SC. Hence,  $p_i^{sc*} > 0$  and  $p_{1i}^{b*} > 0$ . By the slackness condition in (47),  $\rho_{1i} = \rho_{2i} = 0$ . From (38), (39), we have  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j = \frac{1}{1+p_i^{sc*}+p_{1i}^{b*}} = \sum_{j=i}^N \nu_j$ . Using this in (41), we have  $\xi_i = (1-\eta) \left( \frac{1}{1+p_i^{sc*}+p_{1i}^{b*}} \right) + \eta\nu_i > 0$  as  $\nu_i \geq 0$  and  $0 < \eta < 1$ . This, from the corresponding slackness condition, implies  $\gamma_i = 0$ . As  $t_i^{sc*} > 0$ , we get  $\delta_i^* = 0$ . ■

**Lemma 7:** If  $t_i^{sc*} + t_i^{b*} = \ell_i$ ,  $t_i^{b*} \neq 0$  and  $p_{1i}^{b*} \neq 0$ , then  $p_i^{sc*} + p_{1i}^{b*} = p_{2i}^{b*} \geq p^*$ .

*Proof:* By (47), we have  $\rho_{1i} = \rho_{2i} = \rho_{3i} = 0$ . From (39) (40), we have  $\sum_{j=i}^N \nu_j = \frac{1}{1+p_i^{sc*}+p_{1i}^{b*}} = \frac{1}{1+p_{2i}^{b*}}$ . The second equality will be satisfied only when  $p_i^{sc*} + p_{1i}^{b*} = p_{2i}^{b*}$ . Next, since  $t_i^{sc*} + t_i^{b*} = \ell_i$ , from the slackness condition in (48),  $z_i \geq 0$ . Also, from (38), we have  $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j = \frac{1}{1+p_i^{sc*}+p_{1i}^{b*}}$ . Combining this with (42), and by the fact that  $z_i \geq 0$ , we get  $\log(1+p_i^{sc*}+p_{1i}^{b*}) \geq \frac{\epsilon+p_i^{sc*}+p_{1i}^{b*}}{1+p_i^{sc*}+p_{1i}^{b*}}$ , which holds only when  $p_i^{sc*} + p_{1i}^{b*} \geq p^*$  where  $p^*$  is the threshold power level. ■

Lemmas 5–7 provide useful properties of the optimal power allocation in the presence of additive processing cost  $\epsilon$ . In particular, we first determine a threshold power level  $p^*$  based only on  $\epsilon$ , and determine the energy flow in time accordingly. In view of these properties, we continue our analysis for fixed  $\delta_i = 0$  case in the following section. If the resulting power sequences are consistent with the optimality constraints, then we stop. Otherwise, we allow energy transfer from the SC to the battery using some additional steps.

### C. Optimal Policy for Fixed $\delta_i = 0$

For fixed  $\delta_i = 0$ , the problem is the following:

$$\begin{aligned} \max_{\alpha_i, \beta_i, \theta_i, t_i^{sc}, t_i^b \geq 0} \quad & \sum_{i=1}^N \frac{t_i^{sc}}{2} \log \left( 1 + \frac{\alpha_i}{t_i^{sc}} + \frac{\theta_i}{t_i^{sc}} \right) + \frac{t_i^b}{2} \log \left( 1 + \frac{\beta_i}{t_i^b} \right) \\ \text{s.t.} \quad & (31) - (35) \\ & \delta_i = 0, \quad \forall i \end{aligned} \quad (49)$$

Parallel to Lemma 4, we next show in the following lemma that the solution of (49) is found by applying the directional glue-pouring algorithm in [21] only twice.

**Lemma 8:** For fixed  $\delta_i = 0$ , let  $\hat{p}_i^{sc}$  and  $\hat{t}_i^{sc}$  be the outcome of directional glue-pouring given  $p_i^b = 0$ . Let  $\hat{p}_{1i}^b$ ,  $\hat{p}_{2i}^b$  and  $\hat{t}_i^b$  be the outcome of directional glue-pouring given  $\hat{p}_i^{sc}$  and no processing cost from the battery over the first  $\hat{t}_i^{sc}$  time units. Then,  $\hat{p}_i^{sc}$ ,  $\hat{p}_{1i}^b$ ,  $\hat{p}_{2i}^b$ ,  $\hat{t}_i^{sc}$  and  $\hat{t}_i^b$  are jointly optimal for (49).

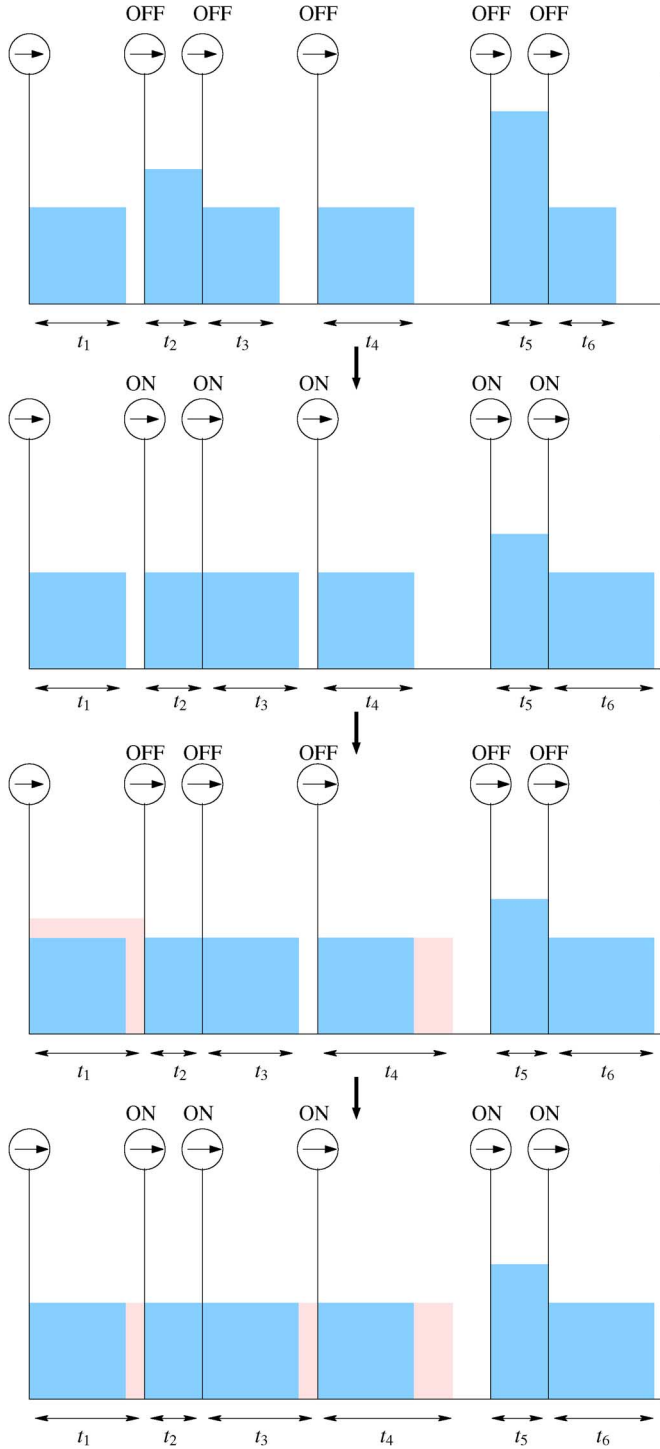
The proof of Lemma 8 follows similarly to the proof of Lemma 4 and is skipped here; see [30] for a detailed proof. We present an illustration of the two iterations of the directional glue-pouring algorithm in Fig. 6, where blue and red glues represent energies in the SC and the battery, respectively. In this example,  $E_i^{sc} = E_{max}$  in epochs 1, 4 and 5. In the upper two figures in Fig. 6, we show the first directional glue pouring where  $\hat{p}_i^{sc}$  and  $\hat{t}_i^{sc}$  are obtained given  $p_{1i}^b = 0$ ,  $p_{2i}^b = 0$ . Note that if the epoch length is sufficiently large,  $p_i^{sc*}$  is kept at the threshold level  $p^*$  as long as possible and is set to zero for the rest of the epoch. In the second iteration,  $\hat{p}_i^{sc}$  and  $\hat{t}_i^{sc}$  are fixed and we pour  $\eta E_i^b$  on top of these power levels. We note that the second iteration of the directional glue-pouring algorithm is a generalized version of the one in [21] in that the processing cost drained from the battery in the initial  $\hat{t}_i^{sc}$  time units of each epoch  $i$  is zero. As a result of the second iteration, we obtain  $\hat{p}_{1i}^b$ ,  $\hat{p}_{2i}^b$  and  $\hat{t}_i^b$ . These two iterations yield an optimal allocation for (49).

### D. Determining the Optimal $\delta_i^*$

In the previous section, we have seen that for  $\hat{p}_i^{sc}$ ,  $\hat{p}_{1i}^b$ ,  $\hat{p}_{2i}^b$ ,  $\hat{t}_i^{sc}$  and  $\hat{t}_i^b$ , there exist Lagrange multipliers  $\lambda_i$ ,  $\mu_i$ ,  $\nu_i$ ,  $\rho_{1i}$ ,  $\rho_{2i}$  and  $\rho_{3i}$  that satisfy (38)–(43); however, it is not clear if there exist  $\xi_i$  that satisfy (41). In this section, we propose a method to update  $\hat{p}_i^{sc}$ ,  $\hat{p}_{1i}^b$ ,  $\hat{p}_{2i}^b$ ,  $\hat{t}_i^{sc}$ ,  $\hat{t}_i^b$  and the corresponding Lagrange multipliers so that we obtain  $\xi_i$  and  $\delta_i^*$  such that (38)–(43) are satisfied. For brevity and clarity of explanation, we assume without loss of generality that  $p_i^{sc}$  is higher than the threshold level  $p^*$  for  $i = 1$  and equal to  $p^*$  for  $i = 2, \dots, N$ .

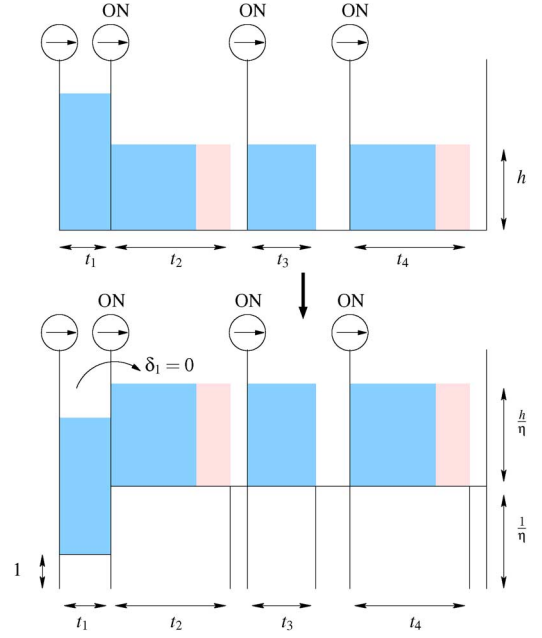
By Lemmas 5 and 6,  $\delta_i^* = 0$  if  $t_i^{sc*} < \ell_i$  or  $p_{1i}^{b*} > 0$ . Indeed,  $\delta_i > 0$ , only if  $\hat{p}_i^{sc} > p^*$ . Thus, we first consider to update the values of  $z_i$  for those epochs where  $z_i = 0$ . The energy for these epochs comes from those previous epochs where  $z_i \geq 0$  and  $p_{1i}^{b*} = 0$ .

In order to find  $\delta_i^*$ , we transform the energy and water levels of isolated epochs as in Section III-B. We set the bottom levels of epochs with  $\hat{p}_i^{sc} > p^*$  and for the remaining epochs, we set the bottom level to  $\frac{1}{\eta}$  and multiply the water level by  $\frac{1}{\eta}$ . In this transformed setting, if the water level is higher in an epoch where  $\hat{p}_i^{sc} > p^*$  compared to the next epoch, then we transfer  $\delta_i$  units of water from the SC in this epoch and  $\frac{1}{\eta}\delta_i$  units of water is added to the battery in the next epoch. This way, we transfer the energy in a systematic way. In the particular case when  $p_{1i}^{sc} > p^*$  and  $p_i^{sc} = p^*$  for  $i = 2, \dots, N$ , energy is transferred from epoch 1 to epochs  $i = 2, \dots, N$ . Note that  $z_1 > 0$  and  $z_i = 0$  for  $i = 2, \dots, N$  for this particular allocation. When energy is transferred,  $\lambda_i$ ,  $\mu_i$  are increased and  $\nu_i$  remains unchanged until  $\hat{t}_i^{sc} + t_i^b = \ell_i$  provided that sufficiently large energy is transferred. If the water level in epoch 1 is still higher than  $p^*$ , we start transferring energy to the next epoch in the transformed setting. We also note that the transferred energy can be utilized


 Fig. 6. Optimal allocation for fixed  $\delta_i = 0$ .

in later epochs as long as the power is kept at  $p^*$  and hence the optimal allocation is not unique. Once  $t_i^{sc} + t_i^b = \ell_i$ , we have to make  $z_i > 0$  due to the slackness condition in (47) and raise the transmit power levels  $p_{1i}^b$  above zero and  $p_{2i}^b$  above the threshold level  $p^*$ .

We also note that if the power level of epoch 1 is lower than that in other epochs in the transformed setting, then  $\delta_i = 0$ ,  $i = 1, \dots, N$ . An example of such a scenario is shown in Fig. 7. Even though  $p_i^{sc}$  is higher than  $p^*$ , the water level in epoch 1 is


 Fig. 7. Determining  $\delta_i$  in the transformed setting.

lower than those of other epochs in the transformed settings. Therefore, there is no transfer from SC to the battery in this scenario.

Once  $t_i^{sc} + t_i^b = \ell_i$  for all  $i$ , if the water level in epoch 1 is still higher than the levels in other epochs, we continue transferring energy. However, resulting water levels are now determined by classical directional water-filling [7] over the whole epoch length  $\ell_i$  since no additional processing cost is incurred.

Finally, we note that when energy is transferred from SC to the battery in epoch  $i$ , it spreads to the future epochs  $i + 1, \dots, N$ . The energy level in some epochs may go above the threshold level  $p^*$  as a result of this transfer. On the other hand, some energy that was already transferred may have to flow back to the battery in epochs  $j < i$ , resulting in a two-way flow of energy within the storage elements. To keep track of the amount of energy transferred in both directions, we measure the flow of energy across each epoch by means of meters and negate any energy that flows backward [18], [19]. An example of this back-flow is illustrated in Fig. 8 where energy is transferred from epoch 1 to 2 and from epoch 3 to 5. The meters across these epochs have positive values. When energy is transferred from the SC to the battery in epoch 4, it causes energy to flow back, and meters show zero value.

## V. NUMERICAL RESULTS

In this section, we numerically study the optimal offline transmission policy in the specified hybrid energy storage model. We consider an additive white Gaussian noise channel with bandwidth  $W = 1$  MHz and noise spectral density  $N_0 = 10^{-19}$  W/Hz. The path loss between the transmitter and the receiver is 100 dB. This results in an instantaneous rate-power relation

$$r(t) = W \log(1 + p_{sc}(t) + p_b(t)) \quad (50)$$

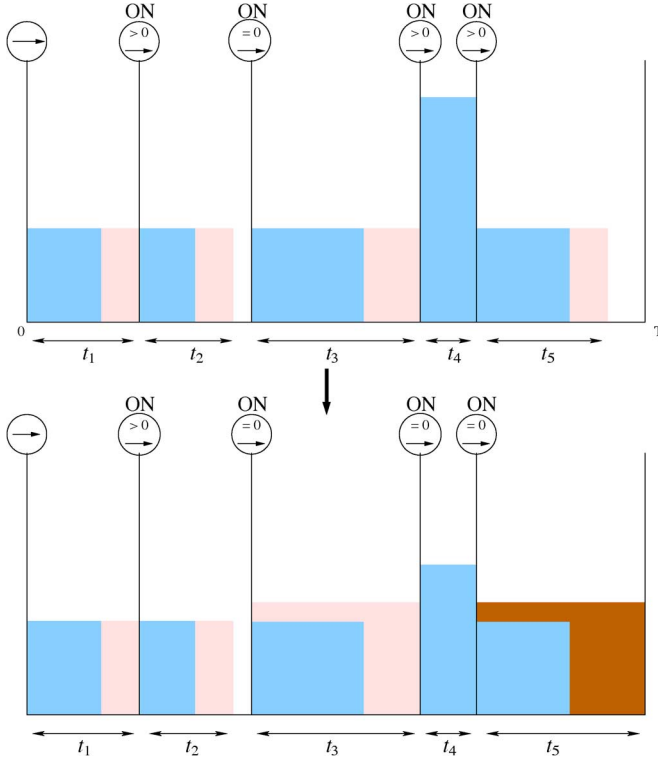


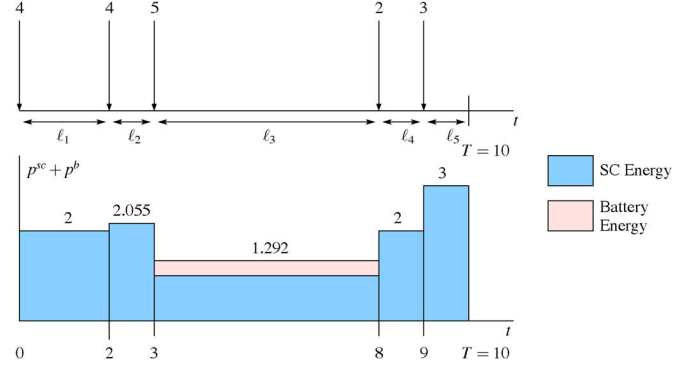
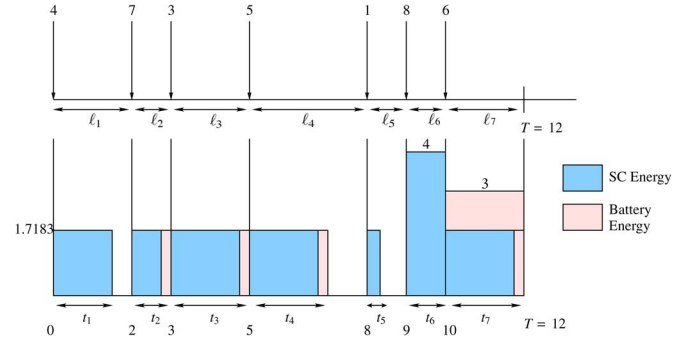
Fig. 8. Demonstration of water backflow and energy meters.

where  $p_{sc}(t)$  and  $p_b(t)$  are the instantaneous transmit powers drained from the SC and the battery, respectively. In particular,  $r(t)$  is in Mbps and  $p_{sc}$  and  $p_b$  are in mW.

#### A. Deterministic Energy Arrivals

We start with illustrations of optimal policies under deterministic energy arrivals. The SC has a storage capacity of  $E_{max} = 5$  mJ. The battery has infinite storage with efficiency  $\eta = 0.75$ . The specific realization of the energy arrivals is  $E = [4, 5, 2, 3]$  mJ at times  $t = [2, 3, 8, 9]$  sec. In addition,  $E_0^{sc} = 4$  mJ,  $E_0^b = 0$ . The deadline constraint is  $T = 10$  sec. We show the energy arrivals and the resulting optimal transmission policy for this case in Fig. 9. Note that in this example, the energy arrival amounts are less than  $E_{max}$  at each epoch and hence  $E_i^b = 0$ . However, the freedom to save energy in the battery strictly increases the throughput as it enables to spread energy in time. Specifically, in this example, the battery enables to transfer energy from epoch 2 to epoch 3 and this increases throughput. Indeed, if there was only the SC available as storage device the optimal throughput would only be 7.0385 Mbits; however, when the battery is also available, the optimum throughput is 7.1743 Mbits.

Next, we consider the effect of processing power where we fix  $\epsilon = 1$  mJ/sec. The energy arrival sequence is  $E = [7, 3, 5, 1, 8, 6]$  mJ at times  $t = [2, 3, 5, 8, 9, 10]$  sec with initial energies  $E_0^{sc} = 4$  mJ and  $E_0^b = 0$ . The energy arrivals and the resulting optimal transmission policy are depicted in Fig. 10. We note that the transmission times may be less than the corresponding epoch lengths as discussed in Section IV.

Fig. 9. Optimal transmit powers for hybrid storage with  $E = [4, 4, 5, 2, 3]$  mJ at times  $t = [2, 3, 8, 9]$  sec,  $\eta = 0.75$ ,  $E_{max} = 5$  mJ and  $T = 10$  sec.Fig. 10. Optimal transmit powers for  $E = [4, 7, 3, 5, 1, 8, 6]$  mJ at times  $t = [2, 3, 5, 8, 9, 10]$  sec,  $\eta = 0.75$ ,  $E_{max} = 5$  mJ,  $\epsilon = 1$  and  $T = 12$  sec.

#### B. Stochastic Energy Arrivals

In this section, we consider stochastic energy arrivals. We compare the performance of the optimal offline policy with those of three heuristic event-based online policies. In particular, these policies take action only when an energy arrival event occurs.

We note that for optimal operation, an online policy has to first fill the space in the SC due to its perfect storage efficiency and then save the remaining energy in the battery. Due to the same reason, power must be drained from the SC first and then from the battery if the energy in the SC is ran out. This way, the space available in the SC for future energy arrivals is maximized. Hence, specifying the total power level  $p = p^b + p^{sc}$  at each time is sufficient to describe the online policy. Without losing optimality, we can restrict the policies to satisfy  $p^{sc}(t)p^b(t) = 0$ .

1) *Constant Power Policy*: This policy transmits with a constant power equal to the average recharge rate,  $\mathbb{E}[E_i]$ . The transmission continues until the hybrid storage unit runs out of energy. This policy uses the mean value of the energy arrival process.

2) *Energy Adaptive Transmission Policy*: This policy transmits with power equal to the instantaneously available energy at each energy arrival instant, i.e.  $p_i = E_{current}$ . Note that available energy is the sum of energies in the SC and the battery:  $E_{current} = E^{sc} + \eta E^b$ . Similar to the constant power policy, the transmitter remains active as long as the power level  $p_i$  can be maintained and otherwise it is silent.

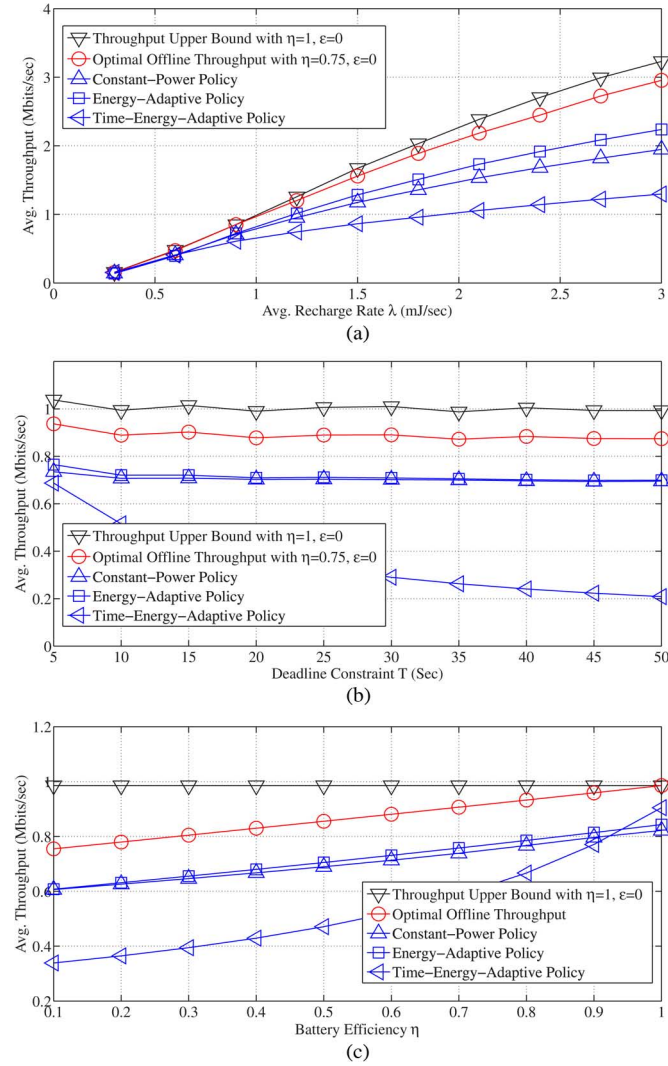


Fig. 11. Performances of the proposed policies with ideal processing power. (a) Varying energy arrival rates  $\lambda$  with  $\eta = 0.6$ ,  $E_{max} = 2$  mJ and  $T = 10$  sec. (b) Varying transmission deadline constraint  $T$  with  $\eta = 0.6$ ,  $E_{max} = 1$  mJ and  $\lambda = 1$  mJ/sec. (c) Varying battery efficiency  $\eta$  with  $E_{max} = 1$  mJ,  $\lambda = 1$  mJ/sec and  $T = 10$  sec.

3) *Time-Energy Adaptive Transmission Policy*: A variant of the energy adaptive transmission policy is obtained by adapting the transmission power to the total energy level and the time remaining till the deadline  $T$ . The power level is determined by  $p_i = \frac{E_{current}}{T-s_i}$ , where  $s_i$  is the time of the most recent energy arrival.

We also consider comparing the performances of the policies with upper bounds. In the no processing energy case, we consider the offline optimal throughput when the battery efficiency is  $\eta = 1$  as an upper bound. Note that this is essentially the offline optimal throughput with an infinite storage, whose solution is known due to [5]. In the nonzero processing energy case, we consider the offline throughput with zero processing energy as an upper bound.

We select the energy arrivals as a compound Poisson process with uniform density  $f_e$  over the interval  $[0, 2P_{avg}]$  where  $P_{avg}$  is the average power. We perform simulations for 500 randomly

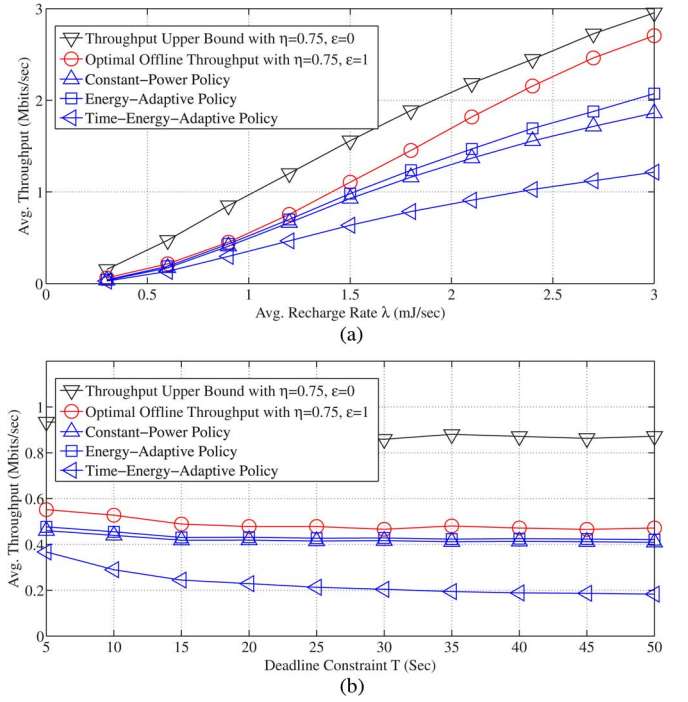


Fig. 12. Performances of the proposed policies with non-ideal processing power. (a) Varying energy arrival rates  $\lambda$  with non-ideal processing power using  $\eta = 0.6$ ,  $E_{max} = 2$  mJ,  $\epsilon = 1$  mJ/sec and  $T = 10$  sec. (b) Varying deadline constraint  $T$  with non-ideal processing power using  $\eta = 0.6$ ,  $E_{max} = 1$  mJ,  $\epsilon = 1$  mJ/sec and  $\lambda = 1$  mJ/sec.

generated realizations of the energy arrivals. The rate  $\lambda_e$  of the Poisson marking process is taken to be 1/sec so that the average recharge rate  $E[E_i]$  is equal to  $P_{avg}$  throughout the simulations.

We start by examining the performance of hybrid storage system with zero processing cost. We simulate different scenarios by varying the energy arrival rate, battery efficiency and transmission deadline constraint. As a baseline, we choose the storage capacity of SC as  $E_{max} = 1$  mJ, the battery efficiency as  $\eta = 0.6$  and the deadline constraint as  $T = 10$  sec. We vary these values as necessary. In Fig. 11(a), we show the average throughput with respect to the average recharge rate  $P_{avg}$ . We observe monotone increases in the performances of the policies as energy recharge rate increases. Similar comparisons are made in Figs. 11(b) and 11(c) with respect to varying battery efficiency and deadline. Note that time-energy adaptive performs well in small deadlines; however, as the deadline is increased the loss incurred due to saving energy in the battery significantly deteriorates its performance.

Next, we consider the average throughput performances of the transmission policies with hybrid energy storage and processing cost  $\epsilon = 1$  mJ/sec. We obtain performance comparisons of the policies with respect to varying energy recharge rate and transmission deadline constraints and present resulting plots in Figs. 12(a) and 12(b). Moreover, we compare the performances of the policies with another upper bound, which is the optimal offline throughput with zero processing energy. We observe that processing cost significantly diminishes the throughput particularly in the high energy arrival regime.

## VI. CONCLUSION

In this paper, we consider data transmission with an energy harvesting transmitter that has a hybrid energy storage unit composed of an inefficient battery and a perfect super-capacitor (SC). The SC has finite storage capacity whereas the battery has unlimited capacity. We solve the offline throughput maximization problem for such an energy harvesting transmitter. In order to optimize performance in an energy harvesting transmitter with such a hybrid energy storage unit, internal energy dynamics of the overall energy storage unit has to be properly adjusted. We solve this energy management problem by applying directional water-filling multiple times. This solution generalizes the directional water-filling algorithm in [7] and provides useful insights on the optimal time-energy flow subject to energy causality and battery limit constraints in the presence of hybrid energy storage. Next, we extend the solution of the offline throughput maximization problem with hybrid energy storage to the case when a time-linear additive processing cost is also present when the transmitter is active. We show that throughput maximization problem in the hybrid energy storage setting can be solved by a sequential application of the directional glue-pouring algorithm [21]. Finally, we present numerical illustrations of the optimal policies and performance comparisons with heuristic online transmission policies.

## APPENDIX PROOF OF LEMMA 4

To prove the asserted optimality, it suffices to show that for the power levels  $\hat{p}_i^{sc}$  and  $\hat{p}_i^b$ , there are Lagrange multipliers  $\lambda_i$ ,  $\mu_i$ ,  $\nu_i$ ,  $\rho_{1i}$ ,  $\rho_{2i}$  that are consistent with (10), (11) and (13)–(16). Note that we ignored (12) as  $\delta_i = 0$  fixed. Consider the first directional water-filling that yields  $\hat{p}_i^{sc} > 0$  sequence. Let  $i_n$  be the epoch indices such that  $E_{i_n} = E_{max}$ . We remark that the directional water-filling determines the energy allocation between the epochs  $i = i_n, i_n + 1, \dots, i_{n+1} - 1$  independent of the other epochs. For the sequence  $\hat{p}_i^{sc}, i = i_n, i_n + 1, \dots, i_{n+1} - 1$ , there exist  $\lambda_i$  and  $\mu_i$  such that

$$\frac{1}{1 + \hat{p}_i^{sc}} = \sum_{k=i}^{i_{n+1}} \lambda_k - \sum_{k=i}^{i_{n+1}-1} \mu_k \quad (51)$$

Note that since  $\hat{p}_i^{sc} > 0$ ,  $\rho_{1i} = 0$ . Therefore, for  $i = i_n + 1, \dots, i_{n+1} - 1$ , energy causality and no-energy-overflow conditions cannot be simultaneously active, implying that  $\lambda_i \mu_i = 0$ . In particular,  $\hat{p}_i^{sc}$  increases when  $\lambda_i > 0$  and decreases when  $\mu_i > 0$ .

In the second directional water-filling,  $\hat{p}_i^{sc}$  are given and the outcomes are  $\hat{p}_i^b$ ,  $\nu_i$  and  $\rho_{2i}$ . Note that  $E_{i_n}^b \geq 0$  and  $E_i^b = 0$  for  $i = i_n + 1, \dots, i_{n+1} - 1$ . Therefore, the water levels in the second directional water-filling must be constant in between these intervals, i.e.,  $\nu_i = 0$  for  $i = i_n, \dots, i_{n+1} - 2$  and  $\nu_i > 0$  for  $i = i_{n+1} - 1$  such that

$$\frac{1}{1 + \hat{p}_i^{sc} + \hat{p}_i^b} = \nu_{i_{n+1}} - \rho_{2i} \quad (52)$$

for  $i = i_n, i_n + 1, \dots, i_{n+1} - 1$ . Due to the complementary slackness conditions in (15),  $\rho_{2i} \geq 0$  if  $\hat{p}_i^b = 0$  and otherwise  $\rho_{2i} = 0$ . We note that with the  $\hat{p}_i^b$  found from (52), Lagrange

multipliers  $\lambda_i, \mu_i$  in (51) do not satisfy (10) while they satisfy the corresponding slackness conditions in (13), (14). However, current selection of variables satisfy (11).

We next argue that  $\lambda_i, \mu_i$  can be updated so that (10) is satisfied while still satisfying the slackness conditions. In particular, we can combine (51) and (52) and find  $\tilde{\lambda}, \tilde{\mu}$  such that for  $i = i_n, i_n + 1, \dots, i_{n+1} - 1$ :

$$\frac{1}{1 + \hat{p}_i^{sc} + \hat{p}_i^b} = \min \left\{ \sum_{k=i}^{i_{n+1}} \lambda_k - \sum_{k=i}^{i_{n+1}-1} \mu_k, \nu_{i_{n+1}} \right\} \quad (53)$$

$$= \sum_{k=i}^{i_{n+1}} \tilde{\lambda}_k - \sum_{k=i}^{i_{n+1}-1} \tilde{\mu}_k \quad (54)$$

where, if  $\hat{p}_{i-1}^{sc} + \hat{p}_{i-1}^b < \hat{p}_i^{sc} + \hat{p}_i^b$ :

$$\tilde{\lambda}_{i-1} = \frac{1}{1 + \hat{p}_{i-1}^{sc} + \hat{p}_{i-1}^b} - \frac{1}{1 + \hat{p}_i^{sc} + \hat{p}_i^b}, \quad \tilde{\mu}_{i-1} = 0 \quad (55)$$

If  $\hat{p}_{i-1}^{sc} + \hat{p}_{i-1}^b > \hat{p}_i^{sc} + \hat{p}_i^b$ :

$$\tilde{\mu}_{i-1} = \frac{1}{1 + \hat{p}_i^{sc} + \hat{p}_i^b} - \frac{1}{1 + \hat{p}_{i-1}^{sc} + \hat{p}_{i-1}^b}, \quad \tilde{\lambda}_{i-1} = 0 \quad (56)$$

and  $\tilde{\lambda}_{i_n} = \frac{1}{1 + \hat{p}_{i_n}^{sc} + \hat{p}_{i_n}^b}$  and  $\tilde{\lambda}_i = \tilde{\mu}_i = 0$  otherwise. In view of (53), we observe that over the epochs  $i = i_n, \dots, i_{n+1} - 1$ , if  $\hat{p}_i^{sc} + \hat{p}_i^b < \hat{p}_{i+1}^{sc} + \hat{p}_{i+1}^b$ , then  $\lambda_i > 0$ ,  $\mu_i = 0$  and hence  $\hat{p}_i^{sc} < \hat{p}_{i+1}^{sc}$ . Similarly, if  $\hat{p}_i^{sc} + \hat{p}_i^b > \hat{p}_{i+1}^{sc} + \hat{p}_{i+1}^b$ , then  $\mu_i > 0$ ,  $\lambda_i = 0$  and hence  $\hat{p}_i^{sc} > \hat{p}_{i+1}^{sc}$ . Therefore,  $\tilde{\lambda}_i$  and  $\tilde{\mu}_i$  have the following property: if  $\tilde{\lambda}_i > 0$  then  $\lambda_i > 0$  and if  $\tilde{\mu}_i > 0$  then  $\mu_i > 0$ . Hence,  $\tilde{\lambda}_i$  and  $\tilde{\mu}_i$  satisfy (10) as well as (13), (14). This proves the existence of Lagrange multipliers that satisfy (10), (11) as well as (13)–(16) and hence the outcomes of two successive directional water-fillings  $\hat{p}_i^{sc}, \hat{p}_i^b$  are jointly optimal.

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