

# Optimal and Near-Optimal Online Strategies for Energy Harvesting Broadcast Channels

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**Abstract**—We consider an energy harvesting broadcast channel where a transmitter powered by energy harvested from the environment serves data to two receivers on the downlink. Energy harvests occur randomly over time as an independent and identically distributed (i.i.d.) random process; the battery at the transmitter in which the harvested energy is stored is of finite size. We focus on *online* transmission schemes where the transmitter knows the energy arrivals only causally as they happen. We first consider the case where the energy arrivals follow a Bernoulli distribution, where the incoming energy is either zero or it fills the battery completely. For this case, we determine the *optimum online* strategy that allocates power over time and between users optimally. We note that the optimum total transmit power is not equal to the optimum single-user transmit power as it depends on the receiver noise variance; this is unlike the offline problem, where the optimum total transmit power in the broadcast channel equals the optimum single-user transmit power. We then consider the case of general i.i.d. energy arrivals, and propose a sub-optimum strategy coined fractional power constant cut-off (FPCC) policy. We develop a lower bound for the performance of the proposed FPCC policy. In addition, we develop a universal upper bound for the broadcast channel capacity region that depends only on the average recharge rate. We show that the FPCC policy is *near-optimal* in that it yields rates that are within a constant gap from the developed upper bound, and therefore, from the actual capacity region, for all system parameters.

**Index Terms**—Broadcast channel, energy harvesting, near-optimal policy, online power control.

## I. INTRODUCTION

**WE** CONSIDER an energy harvesting broadcast channel, Fig. 1, in which a transmitter which harvests energy from nature at random times and amounts, serves data to two receivers on the downlink. The transmitter has two buffers, one for the incoming data and one for the harvested energy. The data buffer is infinitely backlogged. The energy buffer (battery), which is of finite size  $B$ , is recharged randomly by the energy harvesting process throughout the course of communication. We consider the *online* setting where the transmitter gets to know the energy arrivals (harvests) only

causally as they happen. The transmitter needs to determine a *transmission policy*, which chooses the total transmit power and also the amount of power allocated to serve each user's data, as a function of the available energy in the battery.

Energy harvesting communication has been the subject of intense research recently; see recent review articles on scheduling-theoretic [1], [2] and information-theoretic [3] approaches to energy harvesting communications. *Offline* power scheduling, where all energy arrivals are known non-causally ahead of time, has been studied in many different settings [4]–[32]. References [4]–[7] consider the single-user setting, where [4] develops a geometric approach for the case of an infinite-sized battery, [5] generalizes it to the case of a finite-sized battery, and [6] develops a directional water-filling algorithm for the case with fading. References [8]–[19] consider the offline scheduling problem in multi-user systems, in particular, [8]–[10] consider the broadcast channel, [11] and [12] consider the multiple access channel, [13] considers the interference channel, [14]–[16] consider the two-hop relay channel, [17] and [18] consider the two-hop relay channel with two parallel relays, i.e., the diamond channel, and [19] considers the bi-directional relay channel, i.e., the multiple access channel with user cooperation [33]. More general settings with battery imperfections are considered in [20] and [21], effects of processing costs are incorporated in [22]–[24] leading to bursty communication as in glue pouring [34]. Receiver side energy harvesting is considered in [25]–[30] where the receivers incur energy costs for decoding incoming data. Energy cooperation and sharing is incorporated into offline power scheduling in [31] and [32].

In contrast, *online* power scheduling, where energy harvests are known only causally, has been considered in fewer works and mostly for single-user systems so far [6], [7], [35]–[53]. In this case, there is a difficulty that arises due to the uncertainty about the future energy arrivals and the finiteness of the battery size. When the future energy arrivals are not known: if the energy is used too slowly, the resulting rate will be smaller and sufficient space will not be open for future energy arrivals into the battery resulting in wasting of energy; on the other hand, if the energy is used too fast, from the concavity of the rate-power relationship, the resulting rate will again be smaller and energy outages will occur due to frequent empty battery. In most cases, the online problem formulation results in algorithms relying mainly on dynamic programming and Markov decision process techniques [35]–[41]. References [6], [42], and [43] propose several sub-optimal schemes which do not rely on dynamic programming, [44]

Manuscript received January 27, 2016; revised May 16, 2016; accepted August 4, 2016. Date of publication September 20, 2016; date of current version December 29, 2016. This work was supported by NSF under Grant CNS 13-14733, Grant CCF 14-22111, Grant CCF 14-22129, and Grant CNS 15-26608. This paper was presented at the IEEE International Symposium on Information Theory, Barcelona, Spain, July 2016. (*Corresponding author: Sennur Ulukus.*)

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Digital Object Identifier 10.1109/JSAC.2016.2611858

studies competitive ratios of online strategies, [45] uses an adaptive stochastic control approach, [46] uses a Lyapunov optimization technique, [47] uses a linear programming approach, and [48] considers a multiple access channel setting with a storage dam model. As dynamic programming needs the knowledge of the underlying distribution, [49] uses a learning-theoretic approach to remove this assumption.

More recently, [50]–[53] (see also [43]) developed a unique framework to study online power scheduling in energy harvesting systems.<sup>1</sup> In particular, [51] and [52] consider a single-user channel with i.i.d. energy arrivals. They first study a special energy arrival process which is an i.i.d. Bernoulli random process where each arrival is either zero or it fills the battery completely. For this case, [51] and [52] develop an *optimal* online power control policy exploiting the *renewal* property of positive energy arrivals. They, then, propose a general sub-optimal power allocation method and prove it to be *near-optimal* (constant gap away from the optimal) for general i.i.d. energy arrivals.

In this paper, we consider an energy harvesting broadcast channel and develop *online* power scheduling algorithms for this channel model. Our work is most closely related to [8], [10], [51], and [52]. References [8] and [10] consider the energy harvesting broadcast channel and develop optimal *offline* power scheduling schemes for infinite-sized and finite-sized batteries, respectively. They show that the optimum *total* transmit power is equal to the optimum single-user power, which is constant between energy arrivals [4]. In addition, they show that there exists a *cut-off* power level: the stronger user is served with the cut-off power when possible, and the weaker user is served only with the remaining part of the power after the cut-off power is used for the stronger user; if the total power is less than the cut-off power, only the stronger user is served. On the other hand, [51] and [52] consider the single-user energy harvesting channel and develop online power scheduling algorithms. They show that near-optimal transmit power decreases over time. Our paper may be viewed as extending the offline broadcast setting of [8] and [10] to the case of online broadcast setting; or it may be viewed as extending the online single-user setting of [51] and [52] to the case of online broadcast setting.

In this paper, we first consider a special energy arrival process which is Bernoulli that either brings no energy or fills the battery completely. In this case, we solve for the *exactly optimum* online power scheduling strategy. We show that the optimal total transmit power between the energy arrivals is decreasing in time, and there exists a cut-off level below which all power is allocated to serve the stronger user, and only the power above which is allocated to serve the weaker user. Unlike [8] and [10], the optimum total transmit power is not equal to the optimum single-user power, as the optimum single-user power is not universal as in [4] since it depends

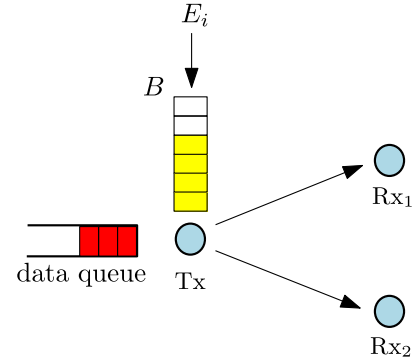


Fig. 1. System model: an energy harvesting broadcast channel.

on the receiver noise variance in this case. We determine the optimum online strategy to achieve any point on the boundary of the broadcast channel capacity region. In certain parts of the region, only a single user may be served, depending on the user priorities; in other parts, both users will be served. We show that, when both users are served, the stronger user is served for a duration no less than the weaker user is served; that is, the weaker user may be served for a portion of the stronger user's serving duration, however the opposite may not occur. We show that between the energy arrivals, whenever the stronger user's power allocation is decreasing, the weaker user's power allocation is zero; and whenever the stronger user's power allocation is equal to the cut-off power, the weaker user's power allocation is decreasing.

Next, inspired by the optimum solution for Bernoulli arrivals, we propose a sub-optimal strategy that is valid for all i.i.d. energy arrivals: fractional power constant cut-off (FPCC) policy. In FPCC, the transmitter uses a universal but sub-optimal fractional power policy, however, this power is allocated optimally to users based on a cut-off power. We develop a lower bound for the performance of the FPCC policy, and a universal upper bound for the capacity region of the energy harvesting broadcast channel which depends only on the average recharge rate. We show that the FPCC scheme is *near-optimal* in that it yields rates that are within a constant gap from the developed upper bound, and thus, from the actual capacity region, for all system parameters.

## II. SYSTEM MODEL

We consider a two-user energy harvesting broadcast channel, see Fig. 1. The transmitter has a battery of size  $B$ . The time is slotted. At each time slot  $i$ ,  $E_i$  units of energy enters the battery (is harvested), where  $E_i$  is an i.i.d. random process. We denote the amount of energy in the battery at time  $i$  as  $b_i$ . We follow a transmit-first strategy, where in each slot data is transmitted first and then energy is harvested. The battery energy level evolves as:

$$b_i = \min\{B, b_{i-1} - P_{i-1} + E_i\} \quad (1)$$

where  $P_{i-1}$  is the energy of the symbol transmitted in slot  $i - 1$ , and it is limited by the amount of energy available in the battery in that slot, i.e.,  $P_{i-1} \leq b_{i-1}$ .

The underlying physical layer is a Gaussian broadcast channel, where the received signal at receiver  $k$  is  $Y_k = X + N_k$ ,

<sup>1</sup>In addition, [50] establishes a connection between the problems of online power control and information-theoretic capacity with causal knowledge of energy arrivals for single-user energy harvesting systems. While it is possible for such a relationship to hold in the broadcast channel setting as well, in this paper, we focus only on the online power control problem for the energy harvesting broadcast channel.

for  $k = 1, 2$ . Here,  $X$  is the transmitted signal and  $N_k$  are the Gaussian receiver noises with variances  $\sigma_k^2$ . Without loss of generality, let  $\sigma_1^2 < \sigma_2^2$ . The Gaussian broadcast channel is degraded. In this case, it is degraded in favor of the first user, i.e., the first user is the stronger user and the second user is the weaker user. The capacity region of the Gaussian broadcast channel in slot  $i$  is [54] (see [8]–[10]):

$$r_{1i} \leq \frac{1}{2} \log \left( 1 + \frac{\alpha_i P_i}{\sigma_1^2} \right) \quad (2)$$

$$r_{2i} \leq \frac{1}{2} \log \left( 1 + \frac{(1 - \alpha_i) P_i}{\alpha_i P_i + \sigma_2^2} \right) \quad (3)$$

The boundary of the capacity region is traced by sweeping  $\alpha_i$  in  $[0, 1]$ . On the boundary,  $X$  is Gaussian with power  $P_i$ , where  $\alpha_i P_i$  portion of this power is allocated to serve the data of the stronger user, and  $(1 - \alpha_i) P_i$  is allocated to serve the data of the weaker user. The stronger user experiences no interference as it can decode and subtract the weaker user's signal (see (2)), while the weaker user experiences the power allocated for the stronger user as interference (see (3)). On the boundary of the capacity region where (2) and (3) are satisfied with equality, we can write  $P_i$  in terms of the rates  $r_{1i}$  and  $r_{2i}$  as:

$$P_i = \sigma_1^2 e^{2(r_{1i} + r_{2i})} + (\sigma_2^2 - \sigma_1^2) e^{2r_{2i}} - \sigma_2^2 \triangleq g(r_{1i}, r_{2i}) \quad (4)$$

Therefore,  $g(r_{1i}, r_{2i})$  is the minimum total power needed to provide users with rates  $r_{1i}$  and  $r_{2i}$ .

Our goal in this paper is to characterize the optimal long-term throughput region of the system. We characterize this region by characterizing its tangent lines. Therefore, characterizing this region is equivalent to determining long-term weighted average throughput:

$$\Phi = \lim_{n \rightarrow \infty} \max_{p^n \in \hat{\mathcal{F}}^n} \mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^n \max_{\alpha_i \in [0,1]} (\mu_1 r_{1i} + \mu_2 r_{2i}) \right] \quad (5)$$

for all  $\mu_1, \mu_2 \in [0, 1]$ , where  $\hat{\mathcal{F}}^n$  denotes the feasible region for transmit powers subject to causal energy knowledge. From (1), the current battery state  $b_i$  depends on the previous slot through the action,  $P_{i-1}$ , and battery state,  $b_{i-1}$ , along with the current energy arrival  $E_i$ . The stage reward is  $\max_{\alpha_i \in [0,1]} (\mu_1 r_{1i} + \mu_2 r_{2i})$  and the admissible policies at each stage,  $P_i$ , are the values in  $[0, b_i]$  which depend only on the current battery state. Hence, it follows that the optimal policy exists and is Markovian see [40, Th. 6.4] and [55, Th. 4.4.2], respectively. The optimal Markov policy can then be found using dynamic programming by solving Bellman's equations [56, Ch. 4]. Hence, (5) can be expressed as:

$$\Phi = \lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^n (\mu_1 r_{1i}^* + \mu_2 r_{2i}^*) \right] \quad (6)$$

where  $r_{1i}$  and  $r_{2i}$  are replaced by  $r_{1i}^*$  and  $r_{2i}^*$ , respectively, due to the existence of the optimal Markovian policy.

While we will eventually consider an arbitrary i.i.d. energy arrival process  $E_i$ , initially, we will consider a special i.i.d. energy arrival process which is Bernoulli with a particular support set, in particular,  $E_i = 0$  with probability  $1 - p$ , and

$E_i = B$  with probability  $p$ . That is, the energy arrival process is such that, either no energy arrives, or when energy arrives, it fills the battery completely. This process will enable *renewals* when energy arrives. We will start with this special energy arrival process in Section III and consider the general arrivals in Section IV.

### III. OPTIMAL STRATEGY: CASE OF BERNOULLI ARRIVALS

Since broadcast rate region is convex, we characterize it by determining its tangent lines. Thus, we consider all weighted sum rates of the form  $\mu_1 r_1 + \mu_2 r_2$ , where  $\mu_1, \mu_2$  are both in  $[0, 1]$ , and are referred to as the *priorities* of the users. The average weighted sum rate is:

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^n (\mu_1 r_{1i}^* + \mu_2 r_{2i}^*) \right] \quad (7)$$

A non-zero energy arrival resets the system and forms a *renewal*. Then, from [57, Th. 3.6.1] (see also [58]):

$$\begin{aligned} \lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{i=1}^n (\mu_1 r_{1i}^* + \mu_2 r_{2i}^*) \right] \\ = \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^L (\mu_1 r_{1i}^* + \mu_2 r_{2i}^*) \right] \end{aligned} \quad (8)$$

$$= p \sum_{k=1}^{\infty} p(1-p)^{k-1} \sum_{i=1}^k (\mu_1 r_{1i}^* + \mu_2 r_{2i}^*) \quad (9)$$

$$= \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} p^2 (1-p)^{k-1} (\mu_1 r_{1i}^* + \mu_2 r_{2i}^*) \quad (10)$$

$$= \sum_{i=1}^{\infty} p(1-p)^{i-1} (\mu_1 r_{1i}^* + \mu_2 r_{2i}^*) \quad (11)$$

where  $L$  is the inter-arrival time, which is geometric with  $p$ . We used  $\mathbb{P}[L = k] = p(1-p)^{k-1}$ , and  $\mathbb{E}[L] = 1/p$  in (8)-(11). Hence, the rate and therefore power allocation problem becomes:

$$\begin{aligned} \max_{(r_{1i}, r_{2i})} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1} (\mu_1 r_{1i} + \mu_2 r_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} g(r_{1i}, r_{2i}) \leq B \\ & r_{1i}, r_{2i} \geq 0, \quad \forall i \end{aligned} \quad (12)$$

which is an optimization problem only in terms of the rates. In essence, this optimization problem aims to maximize the expected (weighted sum) transmitted rate before the next energy arrival. Therefore, the power allocation policy obtained as the solution of (12) corresponds to the optimum power allocation policy between two energy arrivals.

Here,  $\mu_1$  and  $\mu_2$  determine the point on the boundary of the capacity region, and also the power (and rate) schedule that achieves it. First, we will consider the case where one of the  $\mu_i$  is zero (without loss of generality  $\mu_2 = 0$ ). This will achieve the intercept of the boundary of the capacity region with one of the axes. This will also reduce our multi-user broadcast setting into the single-user setting of [51] and [52].

We present this setting in the next sub-section for completeness and also to emphasize some of the properties of the solution. We will consider the general case where  $\mu_1$  and  $\mu_2$  are non-zero in the subsequent sub-section.

A.  $\mu_1 > 0, \mu_2 = 0$

In this case, the problem in (12) specializes to:

$$\begin{aligned} \max_{\{r_{1i}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1} r_{1i} \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} \sigma_1^2 e^{2r_{1i}} - \sigma_1^2 \leq B \\ & r_{1i} \geq 0, \quad \forall i \end{aligned} \quad (13)$$

This problem is convex and is solved in [51] and [52]. The Lagrangian is:

$$\begin{aligned} \mathcal{L} = & - \sum_{i=1}^{\infty} p(1-p)^{i-1} r_{1i} + \lambda \left( \sum_{i=1}^{\infty} \sigma_1^2 e^{2r_{1i}} - \sigma_1^2 - B \right) \\ & - \sum_{i=1}^{\infty} v_i r_{1i} \end{aligned} \quad (14)$$

where  $\lambda, v_i \geq 0, \forall i$ . The necessary and sufficient KKT optimality conditions are:

$$-p(1-p)^{i-1} + \lambda \sigma_1^2 e^{2r_{1i}} - v_i = 0, \quad \forall i \quad (15)$$

Here, and also in all the subsequent KKT conditions, we absorb constants into Lagrange multipliers. For instance, in (15) a factor of 2 in the second term is absorbed into the Lagrange multiplier  $\lambda$ , i.e., we implicitly define a new Lagrange multiplier which is equal to  $2\lambda$ . Note that this does not affect the optimum solution or the analysis of the problem. When  $r_{1i} > 0$ , from complementary slackness  $v_i = 0$ , and we have:

$$e^{2r_{1i}} = \frac{p(1-p)^{i-1}}{\lambda \sigma_1^2} \quad (16)$$

Accordingly, from  $r_{1i} = \frac{1}{2} \log \left( 1 + \frac{P_i}{\sigma_1^2} \right)$ , when  $P_i > 0$ , we have:

$$P_i = \frac{p(1-p)^{i-1}}{\lambda} - \sigma_1^2 \quad (17)$$

We first note from (16)-(17) that the optimum rate and power decrease in time; they decrease strictly when  $p \in (0, 1)$ . Therefore, there exists a time  $i$  at which power  $P_i$  hits zero. Let us denote the last instance when  $P_i > 0$  as  $\tilde{N}$ . Therefore,  $\tilde{N}$  is the smallest integer such that,

$$p(1-p)^{\tilde{N}} < \lambda \sigma_1^2 \quad (18)$$

In addition, all power must be consumed, i.e.,  $\lambda > 0$ , as otherwise, we can increase one of the powers and improve the objective function. Thus,

$$\sum_{i=1}^{\tilde{N}} \left( \frac{p(1-p)^{i-1}}{\lambda} - \sigma_1^2 \right) = B \quad (19)$$

From (19), we have,

$$\lambda = \frac{1 - (1-p)^{\tilde{N}}}{B + \tilde{N} \sigma_1^2} \quad (20)$$

Inserting (20) into (18), we find the optimum  $\tilde{N}$  as the smallest integer satisfying,

$$\left[ p \left( \frac{B}{\sigma_1^2} + \tilde{N} \right) + 1 \right] (1-p)^{\tilde{N}} < 1 \quad (21)$$

We note from (21) that  $\tilde{N}$ , and therefore, the optimum power depends on the noise variance  $\sigma_1^2$ . Next, we show that, if the noise variance is larger, then the transmission duration is shorter. To that end, by rearranging (21), we obtain,

$$\frac{B}{\sigma_1^2} < \frac{1 - (1-p)^{\tilde{N}}}{p(1-p)^{\tilde{N}}} - \tilde{N} \quad (22)$$

Let us denote the right hand side of (22) as  $f(i) \triangleq \frac{1 - (1-p)^i}{p(1-p)^i} - i$ . Then,  $f(i)$  increases in  $i$  since,

$$f(i+1) - f(i) = \frac{1}{(1-p)^{i+1}} - 1 \geq 0 \quad (23)$$

Therefore, as  $\sigma_1^2$  increases, the left hand side of (22) decreases, and thus, the smallest value of  $\tilde{N}$  for which (22) is satisfied decreases.

We summarize the important properties of the optimum single-user transmission policy compactly in the following lemma, whose proof is developed above in this sub-section.

*Lemma 1: The optimal single-user online power allocation policy for i.i.d. Bernoulli energy arrivals: 1) decreases in time; 2) depends on the noise variance, i.e., is not universal; and 3) the transmission duration  $\tilde{N}$  decreases as the noise variance increases.*

B.  $\mu_1, \mu_2 > 0$

First, we note that, from the degradedness of the channel, if  $\mu_1 \geq \mu_2$  then  $r_{1i} > 0$  and  $r_{2i} = 0$ . Hence, we go back to the single-user power allocation as in the previous sub-section when  $\mu_1 \geq \mu_2$ . Therefore, the only remaining case to solve for is the case when  $\mu_1 < \mu_2$ .

The Lagrangian for the problem in (12) is:

$$\begin{aligned} \mathcal{L} = & - \sum_{i=1}^{\infty} p(1-p)^{i-1} (\mu_1 r_{1i} + \mu_2 r_{2i}) \\ & + \lambda \left( \sum_{i=1}^{\infty} \sigma_1^2 e^{2(r_{1i} + r_{2i})} + (\sigma_2^2 - \sigma_1^2) e^{2r_{2i}} - \sigma_2^2 - B \right) \\ & - \sum_{i=1}^{\infty} v_{1i} r_{1i} - \sum_{i=1}^{\infty} v_{2i} r_{2i} \end{aligned} \quad (24)$$

The necessary and sufficient KKT optimality conditions  $\forall i$  are:

$$-\mu_1 p(1-p)^{i-1} + \lambda \sigma_1^2 e^{2(r_{1i} + r_{2i})} - v_{1i} = 0 \quad (25)$$

$$\begin{aligned} & -\mu_2 p(1-p)^{i-1} + \lambda \left( \sigma_1^2 e^{2(r_{1i} + r_{2i})} + (\sigma_2^2 - \sigma_1^2) e^{2r_{2i}} \right) \\ & - v_{2i} = 0 \end{aligned} \quad (26)$$

Starting from (26) and using (4), we have

$$g(r_{1i}, r_{2i}) = \frac{\mu_2 p(1-p)^{i-1} + v_{2i}}{\lambda} - \sigma_2^2 \quad (27)$$

$$\geq \sigma_1^2 e^{2(r_{1i} + r_{2i})} - \sigma_1^2 \quad (28)$$

$$= \frac{\mu_1 p(1-p)^{i-1} + v_{1i}}{\lambda} - \sigma_1^2 \quad (29)$$

$$\geq \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2 \quad (30)$$

where the inequality in (28) is satisfied with equality when  $r_{2i} = 0$ , (29) follows from (25), and the inequality in (30) is satisfied with equality when  $r_{1i} > 0$ . Therefore, when  $r_{2i} > 0$ , we have

$$g(r_{1i}, r_{2i}) = \frac{\mu_2 p(1-p)^{i-1}}{\lambda} - \sigma_2^2 > \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2 \quad (31)$$

While, when  $r_{2i} = 0$  (which also implies that  $r_{1i} > 0$ ), we have

$$g(r_{1i}, r_{2i}) = \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2 > \frac{\mu_2 p(1-p)^{i-1}}{\lambda} - \sigma_2^2 \quad (32)$$

Therefore, we have

$$g(r_{1i}, r_{2i}) = \max\{u_i, v_i\} \quad (33)$$

where

$$u_i = \frac{\mu_2 p(1-p)^{i-1}}{\lambda} - \sigma_2^2 \quad (34)$$

$$v_i = \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2 \quad (35)$$

Hence, (33)-(35) give the general form of the optimum  $g(r_{1i}, r_{2i})$ , which is the optimum total transmit power,  $P_i^*$ , in the broadcast channel.

Next, we solve for the components of the optimum total transmit power allocated to serving the two users' data, i.e., we solve for  $\alpha_i$ , or in other words, for  $P_{1i}$  and  $P_{2i}$ , where  $P_{1i} = \alpha_i P_i$  and  $P_{2i} = (1 - \alpha_i) P_i$ , or equivalently the optimal rates  $r_{1i}$  and  $r_{2i}$ . To that end, let us assume that we have solved for the  $i$ th slot total transmit power allocation  $P_i^*$  for all  $i$  already. Then, within the  $i$ th slot, the optimization problem is:

$$\begin{aligned} \max_{\{r_{1i}, r_{2i}\}} \quad & p(1-p)^{i-1}(\mu_1 r_{1i} + \mu_2 r_{2i}) \\ \text{s.t.} \quad & g(r_{1i}, r_{2i}) \leq P_i^* \\ & r_{1i}, r_{2i} \geq 0 \end{aligned} \quad (36)$$

If  $\mu_1 \geq \mu_2$ , then from the degradedness of the channel, all the total power will be allocated to the message of user 1, i.e.,  $P_{1i} = P_i^*$ . For  $\mu_1 < \mu_2$ , using the KKT optimality conditions for (36), we obtain the following structure for the optimum solution:

$$P_c = \left( \frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+ \quad (37)$$

$$P_{1i} = \min\{P_c, P_i^*\} \quad (38)$$

$$P_{2i} = P_i^* - P_{1i} \quad (39)$$

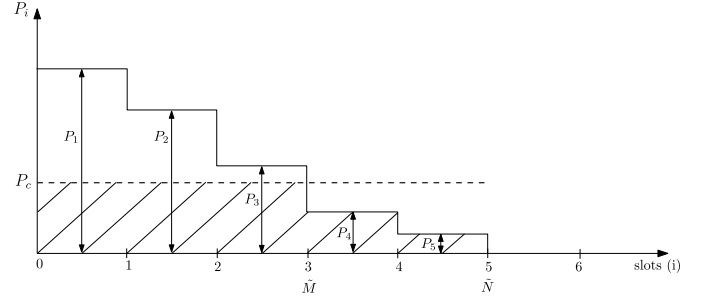


Fig. 2. Structure of the optimal policy. The shaded part is the portion of the power dedicated to user 1 (stronger user), and the unshaded part is dedicated for user 2 (weaker user). In this example,  $\tilde{M} = 3$  and  $\tilde{N} = 5$ .

We provide the details of the derivation of (37)-(39) in the Appendix. In (37)-(39),  $P_c$  is the cut-off power level, which determines the maximum possible power to allocate to the message of user 1. We already know from the discussion before (37) that if  $\mu_1 \geq \mu_2$ , then all the total power will be allocated to the message of user 1, i.e.,  $P_{1i} = P_i^*$  and  $P_{2i} = 0$ . From (37)-(39), we see that, if  $\mu_2 \geq \frac{\sigma_2^2}{\sigma_1^2} \mu_1$ , then  $P_c = 0$ , and hence, no power will be allocated to the message of user 1, and all the power will be allocated for the message of user 2, i.e.,  $P_{1i} = 0$  and  $P_{2i} = P_i^*$ . For all the other cases, i.e., when  $\mu_1 < \mu_2 < \frac{\sigma_2^2}{\sigma_1^2} \mu_1$ ,  $P_c$  is positive and user 1 will be allocated the minimum of  $P_c$  and the total available power  $P_i^*$ , and user 2 will be allocated the remaining power.

From the development in this sub-section, we conclude the following observations: First, the optimum total transmit power,  $P_i^*$ , which is given by (33)-(35) is decreasing in time, as both  $u_i$  in (34) and  $v_i$  in (35) are decreasing, and  $g(r_{1i}, r_{2i})$  in (33) is the maximum of two decreasing sequences, which is decreasing. Second, the power allocated to the stronger user's message,  $P_{1i}$ , is either equal to  $P_c$  if  $P_i^* \geq P_c$  and therefore is constant, or is equal to  $P_i^*$  if  $P_i^* < P_c$  and therefore is decreasing. Thus, the power allocated to the stronger user's message is decreasing. Third, the power allocated to the weaker user's message,  $P_{2i}$ , is either decreasing or equal to zero; it is decreasing when  $P_{1i} = P_c$  and is equal to zero when  $P_{1i} = P_i^*$ . Note that, when the stronger user's power allocation is strictly decreasing, i.e., when  $P_{1i} = P_i^*$ , this happens towards the end of the transmission, and during this time the weaker user's power allocation is zero. This means that there exist numbers  $\tilde{M}$  and  $\tilde{N}$  with  $\tilde{M} < \tilde{N}$  such that powers allocated to both users are positive for slots  $i = 1, \dots, \tilde{M}$  and the power allocated only for the stronger user is positive for slots  $i = \tilde{M} + 1, \dots, \tilde{N}$ . This optimal policy is illustrated in Fig. 2.

We summarize the important properties of the optimum broadcast transmission policy compactly in the following lemma, whose proof is developed above in this sub-section.

**Lemma 2:** The optimal online power allocation policy for the broadcast channel with i.i.d. Bernoulli energy arrivals is as follows: 1) the total transmit power decreases in time; 2) the individual powers allocated to both users' messages decrease in time; 3) the stronger user's power allocation is positive for a duration longer than the duration for which the weaker user's power allocation is positive.

We now give the explicit solution for the optimum broadcast channel power schedule. When  $\mu_1 > \mu_2$  or  $\mu_2 \geq \frac{\sigma_2^2}{\sigma_1^2} \mu_1$ , the problem reduces to a single-user problem, and the method in the previous sub-section can be used. When  $\mu_1 < \mu_2 \leq \mu_1 \frac{\sigma_2^2}{\sigma_1^2}$ , both users are served according to the properties of the optimum solution described above. Hence, we need to solve for  $\tilde{M}$  and  $\tilde{N}$ . For slots  $i = 1, \dots, \tilde{M}$ , both users are served, and hence from (31), we have  $g(r_{1i}, r_{2i}) = u_i$ , for  $i = 1, \dots, \tilde{M}$ . For slots  $i = \tilde{M} + 1, \dots, \tilde{N}$ , only the stronger user is served, and hence from (32), we have  $g(r_{1i}, r_{2i}) = v_i$ . In addition, the total power constraint needs to be satisfied with equality and hence,  $\lambda$  should be chosen such that,

$$\sum_{i=1}^{\tilde{M}} u_i + \sum_{i=\tilde{M}+1}^{\tilde{N}} v_i = B \quad (40)$$

Using (34)-(35), we solve (40) for  $\lambda$  and obtain

$$\lambda = \frac{\mu_2 - (\mu_2 - \mu_1)(1-p)^{\tilde{M}} - \mu_1(1-p)^{\tilde{N}}}{B + \tilde{N}\sigma_1^2 + \tilde{M}(\sigma_2^2 - \sigma_1^2)} \quad (41)$$

Therefore, if the values of  $\tilde{M}$  and  $\tilde{N}$  are known,  $\lambda$  can be obtained from (41). The problem then becomes to find the values of  $\tilde{M}$  and  $\tilde{N}$ ; in fact, the problem is to solve for  $\tilde{M}$ ,  $\tilde{N}$  and  $\lambda$  jointly. The optimal  $\tilde{M} \leq \tilde{N}$  are the smallest integers such that the following conditions are satisfied

$$\mu_1 p(1-p)^{\tilde{N}} < \sigma_1^2 \lambda \quad (42)$$

$$\frac{\mu_2 p(1-p)^{\tilde{M}}}{\lambda} - \sigma_2^2 < P_c \quad (43)$$

where  $\lambda$  is given by (41). The first condition is similar to the single-user condition in (18) which ensures that there is no further slot that can be utilized after slot  $\tilde{N}$ , i.e., the power drops below zero after slot  $\tilde{N}$ . The second condition is to obtain the slot number  $\tilde{M}$  after which the power drops below  $P_c$ , hence the weaker user is no longer served. The solution to (41)-(43) is unique since it is the smallest pair of numbers satisfying these conditions. An example case where  $\tilde{M}$ ,  $\tilde{N}$  and  $P_c$  are marked is shown in Fig. 2.

#### IV. NEAR OPTIMAL STRATEGIES: GENERAL ENERGY ARRIVALS

In this section, we consider the case of general i.i.d. energy arrivals which are not necessarily Bernoulli. Let  $E_i$  be an arbitrary i.i.d. energy arrival process with average recharge rate  $\mathbb{E}[E_i] = \mu$ . In this case, finding the *exactly optimal* transmission scheme analytically seems intractable, as there is no *renewal* property as in Bernoulli arrivals. Nevertheless, we will determine a *nearly optimal* transmission scheme. Towards that end, we first propose a sub-optimal online scheme which depends only on the average recharge rate  $\mu$  and the variances of the receiver noises,  $\sigma_1^2, \sigma_2^2$  in the next sub-section. We then develop a lower bound on the performance of this policy for the case of Bernoulli arrivals. Next, we show that, under this scheme, the performance of Bernoulli energy arrivals provides a lower bound for the performance of any general i.i.d. energy arrival process. Finally, we develop a universal upper bound

which depends only on the average recharge rate and receiver noise variances (does not depend on the specific statistics of the energy arrival process), and show that the proposed sub-optimal online scheme is within a constant gap from the upper bound, and therefore, from the optimum online scheme.

##### A. Sub-Optimal Scheme: Fractional Power Constant Cut-Off (FPCC) Policy

We first define the policy for Bernoulli energy arrivals, and then generalize it to general energy arrivals. We note that for Bernoulli energy arrivals, the optimal total transmit power allocated decreases exponentially over time, see (33)-(35); and the total transmit power is divided among users according to a cut-off property, see (37)-(39). As in [51] and [52], this motivates us to construct a fractional power policy over time. Consider that in a Bernoulli arrival case, we had an energy arrival and the battery became full. Then, in the next slot we allocate a  $p$  fraction of the available energy for transmission, i.e.,  $Bp$ . This reduces the available energy in the battery to  $B(1-p)$ . In the next slot, we allocate a  $p$  fraction of the available energy for transmission, i.e.,  $Bp(1-p)$ , and so on so forth. This gives a total power allocation policy which is  $P_i = Bp(1-p)^{i-1}$  in slot  $i$ , which is different from the optimum described in (33)-(35), but preserves the *fractional* structure. Next, we follow the exact optimum partition of this sub-optimal total transmit power in all slots among the two users as in (37)-(39). That is, we allocate  $P_{1i} = \min\{P_c, Bp(1-p)^{i-1}\}$  for user 1, and  $P_{2i} = Bp(1-p)^{i-1} - P_{1i}$  for user 2. Note that, this results in a *universal* allocation of total transmit power, which does not depend on priorities  $\mu_1, \mu_2$  (unlike the optimum (33)-(35)), while the allocation of individual powers is optimum as in (37)-(39) and depends on  $\mu_1, \mu_2$ .

For general energy arrivals, we allocate a fraction  $q = \mu/B$  of the available energy in the battery for transmission, i.e., if the energy available in the battery in slot  $i$  is  $b_i$ , we choose the total transmit power in that slot as  $P_i = qb_i$ . Then, we partition the total transmit power between the two users as in the optimum scheme in (37)-(39), i.e., we allocate  $P_{1i} = \min\{P_c, qb_i\}$  for user 1, and  $P_{2i} = qb_i - P_{1i}$  for user 2.

##### B. A Lower Bound on the Proposed Online Policy

We first develop a lower bound for the proposed FPCC policy for the case of Bernoulli arrivals. The power allocated to the stronger user is  $P_{1i} = \min\{P_c, Bp(1-p)^{i-1}\}$ . Let us define a deterministic integer  $i^*$  as

$$i^* \triangleq \max\{i \in \mathbb{N} : P_c \leq Bp(1-p)^{i-1}\} \quad (44)$$

If  $P_c \leq pB$ , then,  $i^*$  represents the last slot until which the stronger user's power share is  $P_c$ ; after  $i^*$ , the stronger user gets the entire power. We further define a random variable  $K$  as

$$K \triangleq \min\{i^*, L\} \quad (45)$$

where  $L$  is a geometric random variable with parameter  $p$  as used in (8)-(11). First, we give a lower bound for  $\frac{\mathbb{E}[K]}{\mathbb{E}[L]}$  in the following lemma.

*Lemma 3:* The quantity  $\frac{\mathbb{E}[K]}{\mathbb{E}[L]}$  is lower bounded by  $(1 - \frac{P_c}{pB})$ .

*Proof:* Note that  $K$  takes values in  $[1, i^*]$  with pmf  $\mathbb{P}[K = k] = p(1-p)^{k-1}$  for  $k = 1, \dots, i^* - 1$ , and  $\mathbb{P}[K = i^*] = (1-p)^{i^*-1}$ . This follows since whenever  $k < i^*$ , we have  $\mathbb{P}[K = k] = \mathbb{P}[L = k]$ , which is the pmf of  $L$  which is geometric with parameter  $p$ ; and,  $K = i^*$  when  $L \geq i^*$ . Then,

$$\mathbb{E}[K] = \sum_{i=1}^{i^*-1} ip(1-p)^{i-1} + i^*(1-p)^{i^*-1} \quad (46)$$

$$= \frac{1}{p} (1 - (1-p)^{i^*}) \quad (47)$$

Then, noting  $\mathbb{E}[L] = 1/p$ , we have

$$\frac{\mathbb{E}[K]}{\mathbb{E}[L]} = 1 - (1-p)^{i^*} \geq \left(1 - \frac{P_c}{pB}\right) \quad (48)$$

where the inequality follows because by the definition in (44)  $i^*$  satisfies  $P_c > Bp(1-p)^{i^*}$ . ■

Next, in the following lemma, we derive a lower bound for the rate region achievable with the FPCC policy for all i.i.d. Bernoulli energy arrivals.

*Lemma 4:* The achievable rate region with the FPCC policy for any i.i.d. Bernoulli energy arrival process is lower bounded as follows,

$$r_1 \geq \frac{1}{2} \log \left( 1 + \frac{\alpha\mu}{\sigma_1^2} \right) - 0.72 \quad (49)$$

$$r_2 \geq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha\mu + \sigma_2^2} \right) - 0.99 \quad (50)$$

for some  $\alpha \in [0, 1]$ , where  $\mu = \mathbb{E}[E_i]$  is the average recharge rate.

*Proof:* When  $\mu_1 \geq \mu_2$ , the entire power is allocated to the message of the first user, and when  $\mu_2 \geq \mu_1 \frac{\sigma_2^2}{\sigma_1^2}$ , the entire power is allocated to the message of the second user. In these two cases, the system reduces to a single-user system [52]. These conditions are valid for both the optimum power allocation and the sub-optimum power allocation of FPCC. In addition, specific to FPCC, due to the sub-optimal fractional power allocation, from  $P_{1i} = \min\{P_c, Bp(1-p)^{i-1}\}$ , if  $P_c > Bp$ , then  $P_c > Bp(1-p)^{i-1}$  for all  $i$ , and the stronger user gets all the power all the time, and the system again reduces to a single-user system [52]. Using (37), this last case happens when  $\mu_1 \geq \frac{\sigma_1^2 + Bp}{\sigma_2^2 + Bp} \mu_2$ . Therefore, in the following, we only consider the remaining case, which is  $\frac{\sigma_2^2 + Bp}{\sigma_1^2 + Bp} \mu_1 < \mu_2 < \frac{\sigma_2^2}{\sigma_1^2} \mu_1$ . In this case,  $0 < P_c < Bp$ , and  $i^* \geq 1$ .

First, we consider the first user's rate,

$$r_1 = \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) + \sum_{i=K+1}^L \frac{1}{2} \log \left( 1 + \frac{Bp(1-p)^{i-1}}{\sigma_1^2} \right) \right] \quad (51)$$

$$\geq \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) + \sum_{i=K+1}^L \left( \frac{1}{2} \log \left( 1 + \frac{Bp}{\sigma_1^2} \right) + \frac{i-1}{2} \log(1-p) \right) \right] \quad (52)$$

$$\geq \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) + \sum_{i=K+1}^L \frac{1}{2} \log \left( 1 + \frac{Bp}{\sigma_1^2} \right) + \sum_{i=1}^L \frac{i-1}{2} \log(1-p) \right] \quad (53)$$

$$\geq \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) + \sum_{i=K+1}^L \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) + \sum_{i=1}^L \frac{i-1}{2} \log(1-p) \right] \quad (54)$$

$$= \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^L \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) + \sum_{i=1}^L \frac{i-1}{2} \log(1-p) \right] \quad (55)$$

$$= \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) + \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \frac{L(L-1)}{4} \log(1-p) \right] \quad (56)$$

$$\geq \frac{1}{2} \log \left( 1 + \frac{P_c}{\sigma_1^2} \right) - 0.72 \quad (57)$$

where (52) follows because  $\log(a+x)$  is monotone in  $x$ , (53) follows since  $\log(1-p)$  is negative, (54) follows since  $P_c \leq Bp$ , and (57) follows by bounding the last term numerically as in [52].

Next, we consider the second user's rate,

$$r_2 = \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( 1 + \frac{Bp(1-p)^{i-1} - P_c}{P_c + \sigma_2^2} \right) \right] \quad (58)$$

$$= \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( \frac{Bp(1-p)^{i-1} + \sigma_2^2}{P_c + \sigma_2^2} \right) \right] \quad (59)$$

$$\geq \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( \frac{(1-p)^{i-1} (Bp + \sigma_2^2)}{P_c + \sigma_2^2} \right) \right] \quad (60)$$

$$= \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( \frac{Bp + \sigma_2^2}{P_c + \sigma_2^2} \right) + \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{i-1}{2} \log(1-p) \right] \right] \quad (61)$$

$$\geq \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( \frac{Bp + \sigma_2^2}{P_c + \sigma_2^2} \right) + \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^L \frac{i-1}{2} \log(1-p) \right] \right] \quad (62)$$

$$= \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( \frac{Bp + \sigma_2^2}{P_c + \sigma_2^2} \right) + \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \frac{L(L-1)}{4} \log(1-p) \right] \right] \quad (63)$$

$$\geq \frac{1}{\mathbb{E}[L]} \mathbb{E} \left[ \sum_{i=1}^K \frac{1}{2} \log \left( \frac{Bp + \sigma_2^2}{P_c + \sigma_2^2} \right) \right] - 0.72 \quad (64)$$

$$= \frac{\mathbb{E}[K]}{\mathbb{E}[L]} \frac{1}{2} \log \left( \frac{Bp + \sigma_2^2}{P_c + \sigma_2^2} \right) - 0.72 \quad (65)$$

$$\geq \left( 1 - \frac{P_c}{pB} \right) \frac{1}{2} \log \left( 1 + \frac{Bp - P_c}{P_c + \sigma_2^2} \right) - 0.72 \quad (66)$$

where (60) follows because  $\log(a+x)$  is monotone in  $x$ , (62) follows since  $\log(1-p)$  is negative, (64) follows by bounding the last term numerically as in [52], and (66) follows from Lemma 3.

Since  $P_c < Bp$ , by substituting  $P_c = \alpha pB$  with some  $\alpha \in [0, 1]$ , and denoting  $\mu = Bp$ , from (57) and (66), we obtain the simultaneous lower bounds for the two rates,

$$r_1 \geq \frac{1}{2} \log \left( 1 + \frac{\alpha \mu}{\sigma_1^2} \right) - 0.72 \quad (67)$$

$$r_2 \geq \frac{(1-\alpha)}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) - 0.72 \quad (68)$$

Next, we develop a further lower bound for the rate of user 2 as,

$$r_2 \geq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) - \frac{\alpha}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) - 0.72 \quad (69)$$

$$\geq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) - \frac{\alpha}{2} \log \left( \frac{1}{\alpha} \right) - 0.72 \quad (70)$$

$$\geq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) - 0.99 \quad (71)$$

where (70) follows since the second term increases in  $\mu$ , hence, a lower bound is obtained by letting  $\mu \rightarrow \infty$ , and (71) follows by upper bounding the term  $\frac{\alpha}{2} \log_2 \left( \frac{1}{\alpha} \right)$  numerically to 0.265. This, combined with the 0.72 bound, gives a constant bound of 0.99. ■

The next step in lower bounding the achievable rates for general i.i.d. arrivals is to show that the Bernoulli energy arrivals give the lowest rate over all i.i.d. energy arrivals with the same mean. This was proved for the single-user case in [52]. We invoke this result in [52] together with a concavity result from [10] to prove the following lemma.

**Lemma 5:** *For the FPCC policy, any i.i.d. energy arrival process yields an achievable rate region no smaller than that of the Bernoulli energy arrivals with the same mean.*

**Proof:** For the FPCC policy, the achievable weighted sum rate,  $J(g^n, E, x, \mu_1, \mu_2)$ , under any energy arrival process  $E$ , initial battery state  $x$ , transmission policy  $g^n = \{qb_i\}_{i=1}^n$ , and for a given  $\mu_1, \mu_2$  is given by,

$$J(g^n, E, x, \mu_1, \mu_2) = \frac{1}{n} \mathbb{E} \left[ \sum_{i=1}^n f(qb_i, \mu_1, \mu_2) \middle| b_1 = x \right] \quad (72)$$

where  $f(g_i, \mu_1, \mu_2) \triangleq \max_{\alpha_i} \mu_1 r_1(\alpha_i, g_i) + \mu_2 r_2(\alpha_i, g_i)$ ,  $b_i$  is the battery state in slot  $i$ , and  $q$  is the fraction of

power transmitted. It was shown in [10, Lemma 2], that after optimizing  $\alpha_i$ , the function which is only in terms of the total power,  $f(g_i, \mu_1, \mu_2)$  is strictly concave in the transmit power  $g_i$ . Hence, we can apply [52, Lemma 2], and following similar steps to [52, Proposition 4], we conclude that the rate region for Bernoulli arrivals provides a lower bound for all other i.i.d. energy arrivals. ■

Finally, we give a universal lower bound for the proposed FPCC policy under any i.i.d. energy arrival process in the following theorem. The lower bound depends only on the average recharge rate, but not on the statistics, of the energy harvesting process.

**Theorem 1:** *The achievable rate region with the FPCC policy for any arbitrary i.i.d. energy arrival process is lower bounded as follows,*

$$r_1 \geq \frac{1}{2} \log \left( 1 + \frac{\alpha \mu}{\sigma_1^2} \right) - 0.72 \quad (73)$$

$$r_2 \geq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) - 0.99 \quad (74)$$

for some  $\alpha \in [0, 1]$ , where  $\mu = \mathbb{E}[E_i]$  is the average recharge rate.

The proof of Theorem 1 follows from combining Lemma 4 and Lemma 5.

### C. An Upper Bound for Online Policies

Here, we develop an upper bound for the performance of all online scheduling algorithms only in terms of the average recharge rate.

**Theorem 2:** *The optimal online achievable rate region is upper bounded as follows,*

$$r_1 \leq \frac{1}{2} \log \left( 1 + \frac{\alpha \mu}{\sigma_1^2} \right) \quad (75)$$

$$r_2 \leq \frac{1}{2} \log \left( 1 + \frac{(1-\alpha)\mu}{\alpha \mu + \sigma_2^2} \right) \quad (76)$$

for some  $\alpha \in [0, 1]$ , where  $\mu = \mathbb{E}[E_i]$  is the average recharge rate.

**Proof:** First, we note that the achievable rate region for the optimum online algorithm is upper bounded by the achievable rate region with the optimum offline algorithm, where all of the energy arrival information is known ahead of time. In addition, the achievable rate region with finite-sized battery is upper bounded by the achievable rate region with an unlimited-sized battery. For the offline problem, eliminating the *no-energy-overflow* constraints due to the finite battery size, the feasible set for the total transmit power control policy  $g^n$  is

$$\mathcal{F}^n \triangleq \left\{ \{g_i\}_{i=1}^n : \frac{1}{m} \sum_{i=1}^m g_i \leq \frac{1}{m} \left( \sum_{i=1}^m E_i + B \right), \forall m = 1, \dots, n \right\} \quad (77)$$

where we have added  $B$  to the right hand side of (77) to allow for the optimistic scenario that the system has started with a full battery at the beginning (while the upper bound does not



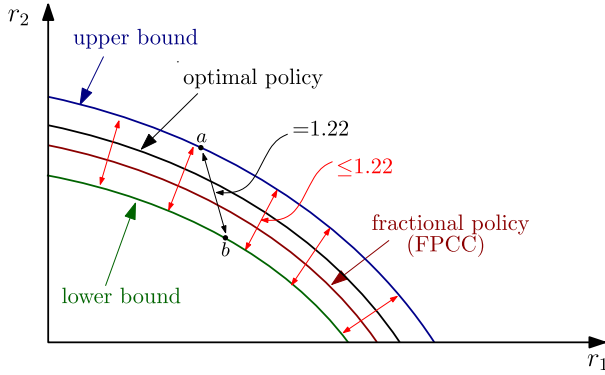


Fig. 3. Illustration of the bounds on the optimal online policy and the proposed online policy FPCC. The distance between the upper and lower bound is less than 1.22.  $a$  and  $b$  are two points on the upper and lower bounds, respectively, with the same  $\alpha$ .

depend on the initial battery state). Then the offline weighted sum rate with weights  $\mu_1, \mu_2$  is,

$$R_{off} \triangleq \lim_{n \rightarrow \infty} \max_{\{g_i\}_{i=1}^n \in \mathcal{F}^n} \frac{1}{n} \sum_{i=1}^n f(g_i, \mu_1, \mu_2) \quad (78)$$

where  $f(g_i, \mu_1, \mu_2)$  is the maximized weighted sum rate only in terms of the total transmit power  $g_i$  in slot  $i$ , after maximization with respect to partitioning of the power to users, i.e.,  $f(g_i, \mu_1, \mu_2) \triangleq \max_{\alpha_i} \mu_1 r_1(\alpha_i, g_i) + \mu_2 r_2(\alpha_i, g_i)$ . We further upper bound this rate as,

$$R_{off} \leq \lim_{n \rightarrow \infty} \max_{\{g_i\}_{i=1}^n \in \mathcal{F}^n} f\left(\frac{1}{n} \sum_{i=1}^n g_i, \mu_1, \mu_2\right) \quad (79)$$

$$\leq f(\mu, \mu_1, \mu_2) \quad (80)$$

where the first inequality follows due to the concavity of  $f(g_i, \mu_1, \mu_2)$  in  $g_i$  [10, Lemma 2]. The second inequality follows by relaxing the feasible set in (77) by removing all but the last constraint when  $m = n$ :  $\frac{1}{n} \sum_{i=1}^n g_i \leq \frac{1}{n} (\sum_{i=1}^n E_i + B)$ , and by noting that from strong law of large numbers  $\frac{1}{n} \sum_{i=1}^n E_i \rightarrow \mu$  almost surely, and the remaining  $\frac{1}{n} B$  terms goes to zero as  $n$  tends to infinity. Since, this is valid for all  $\mu_1, \mu_2$ , then  $f(\mu, \mu_1, \mu_2)$  traces the boundary of the capacity region of the broadcast channel with average power constraint  $\mu$ . ■

The Euclidean distance between any two points with the same  $\alpha$  on the upper and lower bounds is equal to  $\sqrt{0.72^2 + 0.99^2} = 1.22$ . Since the distance between the two points with the same  $\alpha$  can be no less than the distance between the two bounds, the distance between the two bounds is less than or equal to 1.22. Hence, combining Theorem 1 and Theorem 2, we conclude that the proposed online FPCC policy yields rates which are within a constant gap from the universal upper bound, and therefore, from the optimum online policy, for all system parameters. We show the relations developed in Fig. 3.

## V. NUMERICAL RESULTS

In this section, we illustrate the results obtained in this paper using several numerical examples.

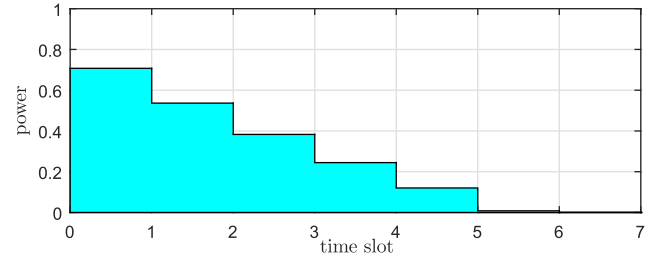


Fig. 4. Optimum single-user power allocation for i.i.d. Bernoulli arrivals. Here, the receiver noise variance is  $\sigma_1^2 = 1$ .

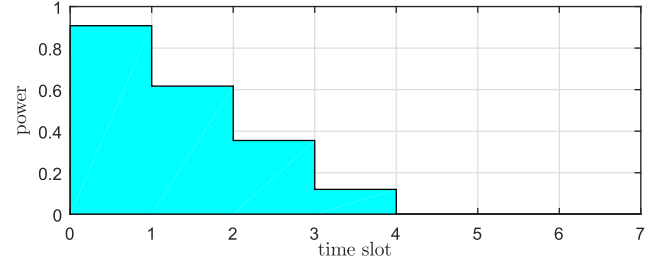


Fig. 5. Optimum single-user power allocation for i.i.d. Bernoulli arrivals. Here, the receiver noise variance is  $\sigma_1^2 = 2$ .

We first show the effect of receiver noise variance on the optimum online power allocation for the single-user case with i.i.d. Bernoulli arrivals. We let  $B = 2$  and  $p = 0.1$ . The problem is stated in (13). We first solve it for  $\sigma_1^2 = 1$  and plot the optimum power allocation in Fig. 4, and then solve it for  $\sigma_1^2 = 2$  and plot the optimum power allocation in Fig. 5. We observe from Figs. 4 and 5, that 1) the optimum power decreases over time, 2) depends on the noise variance, and 3) the transmission duration decreases as the noise variance increases: when  $\sigma_1^2 = 1$ , the total power is transmitted in  $\tilde{N} = 6$  slots, while when  $\sigma_1^2 = 2$ , the total power is transmitted in  $\tilde{N} = 4$  slots. That is, the power allocation shrinks towards the earlier slots as the noise variance increases. This also shows that the single-user solution is not universal as it depends on the receiver noise variance; this is unlike the case for the offline problem [4], where the solution is the same for all receiver noise variances, in fact, it is the same for all concave functions.

Next, we consider the broadcast channel with i.i.d. Bernoulli arrivals, and find the optimum power allocations: the optimum total power allocation  $P_i$  and its optimum distribution to users  $P_{1i}$  and  $P_{2i}$ . In this broadcast channel, we let  $\sigma_1^2 = 1$  and  $\sigma_2^2 = 2$ , and  $B = 2$  and  $p = 0.1$ . In Figs. 6 and 7, we plot the optimum power allocations for two different points on the boundary of the capacity region corresponding to two different  $\mu_1, \mu_2$  pairs. In both figures,  $\mu_1, \mu_2$  are such that  $\mu_1 < \mu_2 < \frac{\sigma_2^2}{\sigma_1^2}$  so that both users are allocated power and both users achieve non-zero rates. In Fig. 6,  $\mu_1 = 1, \mu_2 = 1.8$  and Fig. 7,  $\mu_1 = 1, \mu_2 = 1.7$ ; that is, in Fig. 7 the second user's priority is decreased. The problem is stated in (12). We solve it using the optimum total transmit power in (33)-(35) together with  $\lambda$  in (41), and the optimum power shares of the users in (37)-(39) and the transmission durations of the users  $\tilde{M}$  and  $\tilde{N}$ .

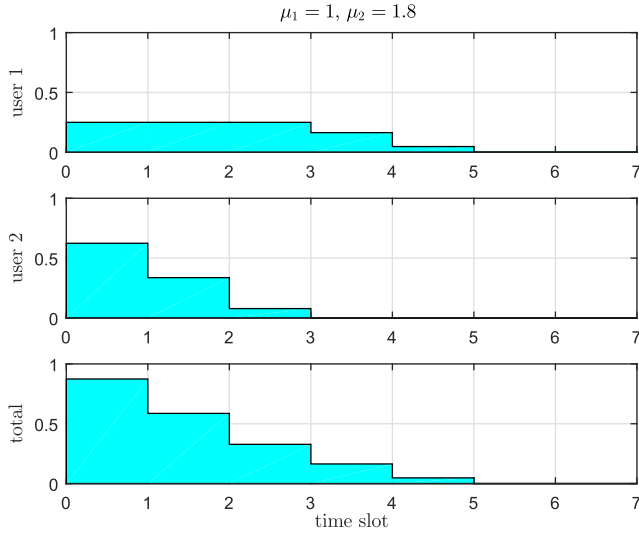


Fig. 6. Optimal online power allocation for the broadcast channel with i.i.d. Bernoulli arrivals. Here,  $\mu_1 < \mu_2 \leq \mu_1 \frac{\sigma_2^2}{\sigma_1^2}$ , both user rates are positive:  $\mu_1 = 1$ ,  $\mu_2 = 1.8$ .

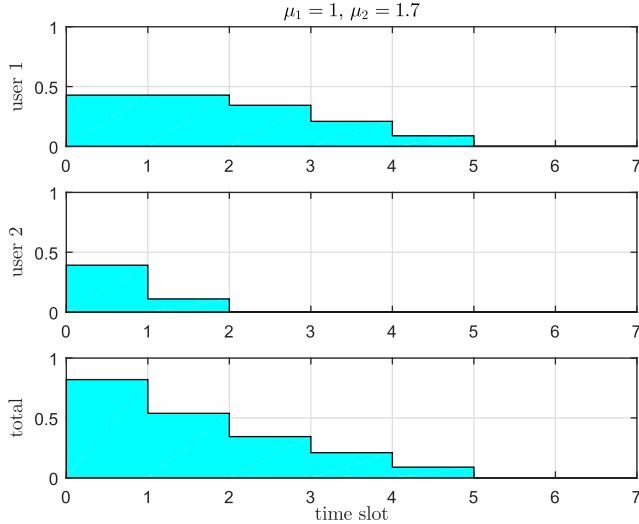


Fig. 7. Optimal online power allocation for the broadcast channel with i.i.d. Bernoulli arrivals. Here,  $\mu_1 < \mu_2 \leq \mu_1 \frac{\sigma_2^2}{\sigma_1^2}$ , both user rates are positive:  $\mu_1 = 1$ ,  $\mu_2 = 1.7$ .

in (42)-(43). As proved in Lemma 2, we observe from Figs. 6 and 7, that 1) the optimum total transmit power decreases over time, 2) the individual powers allocated to users decrease over time as well, 3) the stronger (first) user's power allocation is positive for a longer duration than that for the weaker (second) user. We also note that when the first user's power is constant (slots 1, 2, 3 in Fig. 6 and slots 1, 2 in Fig. 7), the second user's power is decreasing; and when the first user's power is decreasing (slots 4, 5 in Fig. 6 and slots 3, 4, 5 in Fig. 7), the second user's power is zero. We note that in Fig. 6,  $\tilde{M} = 3$  and  $\tilde{N} = 5$ , and Fig. 7,  $\tilde{M} = 2$  and  $\tilde{N} = 5$ . We note that as  $\mu_2$  decreases from the setting of Fig. 6 to the setting of Fig. 7, the second user's power allocation decreases.

Next, we consider the achievable rates as a function of the battery size  $B$  and energy arrival probability  $p$  for the case

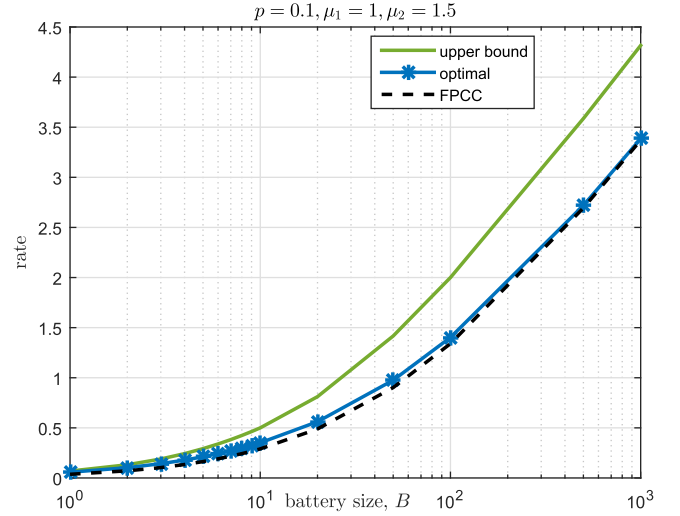


Fig. 8. Achievable weighted sum rate for the optimum online and sub-optimum FPCC together with the upper bound as a function of the battery size  $B$  for a fixed energy arrival probability  $p$  for i.i.d. Bernoulli arrivals.

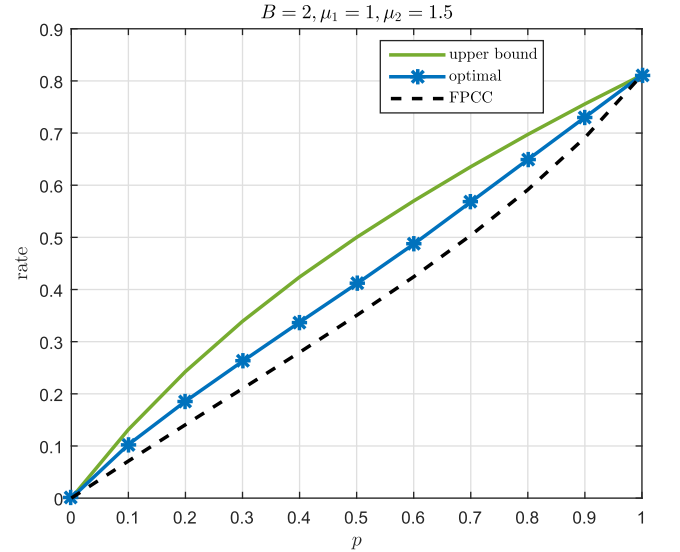


Fig. 9. Achievable weighted sum rate for the optimum online and sub-optimum FPCC together with the upper bound as a function of the energy arrival probability  $p$  for a fixed battery size  $B$  for i.i.d. Bernoulli arrivals.

of i.i.d. Bernoulli arrivals. We consider a fixed set of weights:  $\mu_1 = 1$  and  $\mu_2 = 1.5$ . In Fig. 8, we plot the achievable weighted sum rate with the optimal online solution and the sub-optimum FPCC scheme together with the upper bound as a function of the battery size  $B$  for a fixed energy arrival probability of  $p = 0.1$ . We observe that FPCC performs close to the optimal online. In Fig. 9, we plot the achievable rates as a function of the energy arrival probability  $p$  for a fixed battery size  $B = 2$ . We again observe FPCC perform close to the optimal online.

In Fig. 10, we plot the entire achievable and upper bound regions for the broadcast channel with i.i.d. Bernoulli energy arrivals  $B = 5$ ,  $p = 0.5$ , and  $\sigma_1^2 = 1$ ,  $\sigma_2^2 = 5$ . The arrows denote the movement of achievable rate pairs from the optimal policy to the sub-optimum FPCC. In particular, the optimal

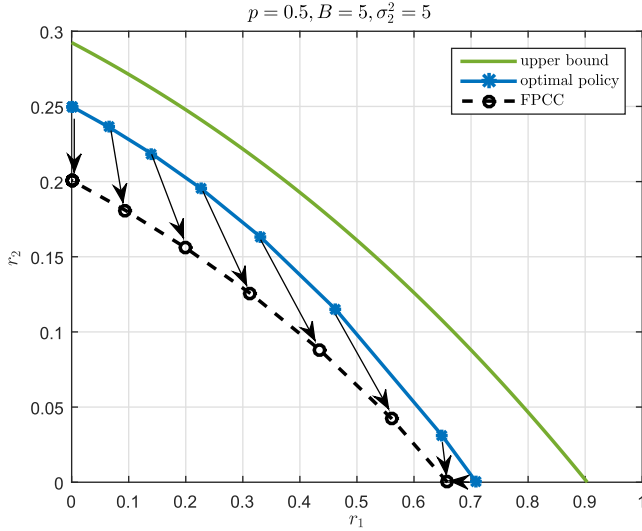


Fig. 10. Rate regions with the optimum online and sub-optimum FPCC together with the upper bound for i.i.d. Bernoulli arrivals.

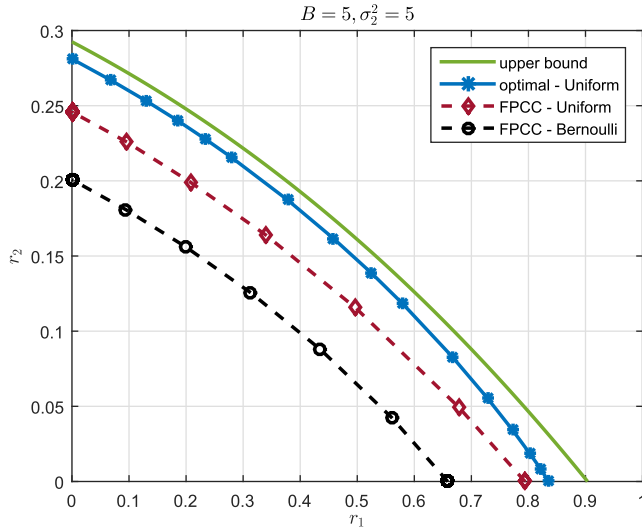


Fig. 11. Achievable rate regions for the sub-optimum FPCC for i.i.d. uniform arrivals together with i.i.d. Bernoulli arrivals with the same average recharge rate. In addition, the optimum achievable rate region for i.i.d. uniform found by dynamic programming, and the upper bound.

policy curve is traced with changing  $\mu_1, \mu_2$ , equivalently by changing  $P_c$ . The arrows point to the achievable rates when the optimal total transmit power is replaced with the fractional power policy for the same cut-off power  $P_c$ .

Finally, we consider an example of general i.i.d. energy arrivals by considering a (continuous) uniform probability distribution for the energy arrivals in  $[0, B]$ . Therefore, the average recharge rate is  $\mu = B/2$ . In Fig. 11, we plot the rate regions with sub-optimum FPCC and the optimum policy which is found by using dynamic programming for this uniform energy arrivals. We also show the achievable rate region with a corresponding Bernoulli arrivals; for this case energies arrive in amounts 0 and  $B$  with probabilities  $p = 0.5$  and  $1 - p = 0.5$ . As proved in Lemma 5, the case of Bernoulli arrivals with the same average recharge rate yields a smaller achievable rate region with the FPCC scheme.

## VI. CONCLUSIONS

We studied the optimal online transmission policies for the broadcast channel with an energy harvesting transmitter. We noted that, unlike the offline setting, the online optimal policy depends on the noise variance even in the single-user case. For Bernoulli arrivals, we showed that the optimal online total transmit power decreases in time. Depending on the priorities of the users, and hence the operating point on the boundary of the rate region, only one of the users may be served, in which case the problem reduces to a single-user problem. When both users are served simultaneously, then the weaker user may be served for only a subset of the duration that the stronger user is served. Depending on the user priorities, the stronger user is either not allocated any power throughout the transmission duration or it is allocated power for the entire duration. We showed that, as in the offline problem, there exists a cut-off power which is dedicated for the stronger user; the cut-off level depends on the operating point on the rate region. We showed that whenever the stronger user's allocated power is decreasing, the weaker user's allocated power is zero; and whenever the stronger user's allocated power is constant, the weaker user's allocated power is decreasing. Next, we considered the general i.i.d. energy arrivals. We proposed a sub-optimum online algorithm, FPCC, where the total transmit power follows a fractional allocation, and the individual user powers are cut-off based. The proposed scheme does not depend on the energy arrival distribution. We obtained bounds on the performance of the FPCC policy for any general i.i.d. energy arrivals, and showed that it is within a constant gap from the developed upper bound, therefore, from the optimum online policy.

## APPENDIX

### SOLUTION OF PROBLEM (36)

We can equivalently write problem (36) as:

$$\max_{\alpha \in [0,1]} \frac{\mu_1}{2} \log \left( 1 + \frac{\alpha P_i^*}{\sigma_1^2} \right) + \frac{\mu_2}{2} \log \left( 1 + \frac{(1-\alpha)P_i^*}{\alpha P_i^* + \sigma_2^2} \right) \quad (81)$$

where  $P_{1i} = \alpha P_i^*$  and  $P_{2i} = (1-\alpha)P_i^*$ . Problem (81) can be further rewritten as:

$$\max_{\alpha \in [0,1]} \left[ \frac{\mu_1}{2} \log \left( 1 + \frac{\alpha P_i^*}{\sigma_1^2} \right) - \frac{\mu_2}{2} \log (\alpha P_i^* + \sigma_2^2) + \frac{\mu_2}{2} \log (P_i^* + \sigma_2^2) \right] \quad (82)$$

Differentiating the objective function of (82) with respect  $\alpha$  and equating to zero we have:

$$\alpha P_i^* = \frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \quad (83)$$

We further need to impose the constraint  $\alpha \in [0, 1]$ , i.e., we need to have  $0 \leq \alpha P_i^* \leq P_i^*$ . This gives:

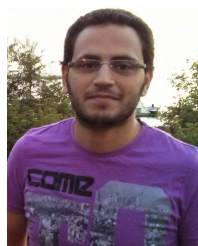
$$\alpha P_i^* = \min \left\{ P_i^*, \left( \frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+ \right\} \quad (84)$$

Denoting  $P_c = \left( \frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+$  gives the expressions in (37)–(39).

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