Optimization of CDMA Signature Sequences in Multipath Channels

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Abstract

We consider the problem of finding good signature sequences for interference avoidance in a CDMA multipath channel, for a given known channel state. We consider two cases. In the first case, we optimize the transmit powers as well as the signature sequences of the users; the optimality of the set is measured by the effective total squared correlation which is related to achievable SIRs. In the second case, we assume fixed received powers and optimize the signature sequences. In this case we consider two optimality criteria: total minimum mean square estimation error, and information theoretic sum capacity. For all three cost functions, we derive iterative algorithms for sequentially updating the users' signature sequences.

1. Introduction

Wireless channels are typically shared by many users, so that multiple access interference (MAI) is one of the main problems that information transmission through such channels faces. In the case of CDMA systems, several techniques have been proposed for dealing with interference. On the receiver side, there are advanced signal processing schemes, such as multiuser detection and receiver beamforming, that aim at *suppressing* MAI in the received signal. On the transmitter side, power control and signature sequence selection techniques can be employed to *avoid* MAI.

Recently, analytic solutions to the signature sequence selection problem have been found [5, 13, 14] for various optimality criteria. References [5] and [13] identify signature sequence sets that maximize the information theoretic *sum capacity* of a single-cell synchronous single-path CDMA system for the cases of equal received powers, and arbitrary (unequal) received powers, respectively. For the same system setting, [14] identifies the signature sequences and received powers that maximize the *user* capacity, i.e., the maximum number of users supportable at a given signal-to-interference (SIR) level for a fixed processing gain. Signature sequence sets having the least total squared correlation (TSC) turn out to be the optimum in all cases. References [8–10] introduced an iterative and distributed interference avoidance algorithm where users update their signature sequences sequentially in a way not to increase the TSC at each iteration. The algorithm converges to an optimum signature set. Recently, [11] identified the optimum signature sequence sets for single-cell, *asynchronous*, single-path CDMA systems and also provided iterative and distributed algorithms that converge to them.

Another impairment of wireless channels is that they are notoriously dispersive, due to the existence of multiple paths from the transmitter to the receiver. For this reason it is common to use RAKE receivers, which attempt to collect the energy from all the paths that the transmitted signal takes. Considerable work has been devoted to finding sets of sequences with good auto- and cross-correlation properties, to be used in such dispersive channels. Such sequences have typically uniformly low offset autocorrelation values. Although these sequences are a good choice in general, they need not be optimal for a given channel state. Since the RAKE receiver needs estimates of the channel coefficients, it is conceivable that these estimates could also be used to select a set of signatures that is best suited for the particular channel at hand.

In this paper, we consider the problem of signature sequence selection for a single-cell, synchronous CDMA system in the presence of multipath. We present signature sequence update algorithms for various optimality criteria. We consider two different cases. In the first case we optimize over the powers as well as the signature sequences to maximize the achievable common SIR target. In the second case, we assume that the received powers of the users are fixed and optimize over the signature sequences of the users to (a) minimize the total MMSE of the system, and (b) maximize the information theoretic sum capacity.

The problem of selecting good signature sequences for multipath CDMA channels has been previously considered in [1, 4], where iterative algorithms were proposed to minimize the MSE by adapting the signature sequences and receiver filters. Recently [3] considered a version of this problem where multipath channels of all users were assumed to be the same. In [6], a technique was presented to determine sequences and receivers that completely eliminate the MAI for unknown multipath channel under certain conditions.

2. System Model

We consider a single-cell system with K users and processing gain N. Each user modulates an N dimensional complex-valued signature sequence, $\mathbf{s}_k$. The transmission of user k goes through a linear multipath channel with L chip-synchronous paths, characterized by an impulse response $\mathbf{h}_k$. We assume a dense multipath profile, so that the received signal has a duration of $N+L-1$ chips. The received signal due to the unmodulated signature of user k is

$$r_k[t] = \sum_{l=1}^{L} h_k[l] s_k[t-l+1] \tag{1}$$

This can be expressed in matrix form,

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{s}_k \tag{2}$$

where

$$\mathbf{H}_k = \begin{bmatrix} h_k[1] & 0 & 0 & & 0 \\ h_k[2] & h_k[1] & 0 & \cdots & 0 \\ h_k[3] & h_k[2] & h_k[1] & & 0 \\ \vdots & \vdots & \vdots & \ddots & \\ h_k[L] & h_k[L-1] & h_k[L-2] & \ddots & h_k[1] \\ 0 & h_k[L] & h_k[L-1] & & h_k[2] \\ 0 & 0 & h_k[L] & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & \\ 0 & 0 & 0 & & h_k[L] \end{bmatrix} \tag{3}$$

The matrix $\mathbf{H}_k$ is $(N+L-1)\times N$. We assume that the users are synchronous and that $L \ll N$, so that there is no significant ISI, as in [3,4]. A small guard time may be inserted between successive bits to ensure this. Thus, a one-shot approach is sufficient for detection. The received signal is

$$\mathbf{r} = \sum_{k=1}^{K} \sqrt{p_k} b_k \mathbf{H}_k \mathbf{s}_k + \mathbf{n} \tag{4}$$

where b_k and p_k are the transmitted bit and power of user k, respectively. The vector $\mathbf{n}$ is AWGN with covariance $\sigma^2\mathbf{I}$. Without loss of generality we assume $h_k[1] \neq 0$.

3. Summary of Results on Single-path CDMA

The problem of signature-sequence selection for a synchronous CDMA system with no multipath has received considerable attention [5, 8–10, 13, 14]. Various optimization criteria can be used to determine the best set of sequences. If the processing gain, N, is greater than or equal to the number of users, K, orthogonal sequences will be the solution for any sensible optimization criterion. Remarkably, it has been shown that also for the case $K > N$, many different criteria have the same solution, namely *Welch Bound Equality (WBE) sequences*. It was shown in [5] that WBE sequences maximize the information-theoretic sum capacity (for equal received powers), and in [14] that they maximize the user capacity. When the received powers of the users are the same, WBE sequences minimize the total MSE (using matched filters) [8–10] and total MMSE (MSE using MMSE filters) [14].

WBE sequences achieve Welch's lower bound on the total square correlation (TSC) of a set of sequences with equality (hence the name) [15]

$$\text{TSC} = \sum_{i=1}^{K}\sum_{j=1}^{K} (\mathbf{s}_i^\top \mathbf{s}_j)^2 \geq \frac{K^2}{N} \tag{5}$$

If $K < N$, the bound is loose, since for K orthonormal vectors, TSC $= K$. If, on the other hand, $K \geq N$, the bound is achieved iff $\mathbf{S}\mathbf{S}^\top = \frac{K}{N}\mathbf{I}_N$ [2], where $\mathbf{S}$ is the $N \times K$ matrix whose kth column is $\mathbf{s}_k$.

Thus, sequences that minimize TSC are optimal for all K and N. This observation motivated an iterative algorithm to find optimal sequences based on TSC minimization [8–10]. At each step, the algorithm replaces the signature of user k with the normalized MMSE filter for user k. The algorithm decreases the TSC at every step, and converges to a TSC-minimizing, hence optimum, signature set.

Recently, optimum signature sequences for single-cell, single-path, *asynchronous* CDMA systems have been identified and algorithms that converge to them have been provided in [11]. The optimum signature sequences minimize the total squared asynchronous correlation (TSAC) which depends on users' delay profile. The iterative algorithms proposed in [11] are based on the objective of decreasing the TSAC at every iteration by updating one signature sequence at a time.

4. Optimizing Powers and Signatures

The optimal choice of signature sequences depends on the receivers used. Consider, for instance, a "truncated" matched filter that ignores the last $L-1$ chips of the received signal. The effective signatures reduce then to $\mathbf{F}_k\mathbf{s}_k$, $k = 1,\ldots,K$, where $\mathbf{F}_k$ is the top $N\times N$ block of $\mathbf{H}_k$. These can be made to be orthogonal or WBE sequences (depending on K) in N dimensional space by an appropriate selection of the $\mathbf{s}_k$'s, since $h_k[1] \neq 0$ implies that the $\mathbf{F}_k$ matrices are invertible. Thus, single-path performance can be achieved in N dimensional space. The ISI-free multipath channel model adopted here provides an increase in the dimension of the problem from N to $N+L-1$. Our aim is to optimize the effective signal vectors in $N+L-1$ dimensional space by controlling the actual signal vectors in N dimensional space.

In this section, we assume normalized matched filter receivers, i.e., the receiver filter of user i is $\mathbf{c}_i = \mathbf{H}_i\mathbf{s}_i/\|\mathbf{H}_i\mathbf{s}_i\|$. In this case, the received power of user i, through its receiver filter is, $\tilde{p}_i = p_i(\mathbf{s}_i^\dagger \mathbf{H}_i^\dagger \mathbf{H}_i \mathbf{s}_i)$. We will use an approach similar to [11]. The SIR of user i can be written as

$$\mathrm{SIR}_i = \frac{\tilde{p}_i}{\sum_{j\neq i} \tilde{p}_j A_{ij} + \sigma^2} \tag{6}$$

where $A_{ii} = 0$ and for $i \neq j$,

$$A_{ij} = \left| \frac{\mathbf{s}_i^\dagger \mathbf{H}_i^\dagger \mathbf{H}_j \mathbf{s}_j}{\|\mathbf{H}_i\mathbf{s}_i\| \|\mathbf{H}_j\mathbf{s}_j\|} \right|^2 \tag{7}$$

It is worth noting that this SIR representation corresponds to an equivalent system

$$\mathbf{r} = \sum_{j=1}^{K} \sqrt{\tilde{p}_j} b_j \tilde{\mathbf{s}}_j + \mathbf{n} \tag{8}$$

where $\tilde{\mathbf{s}}_j = \mathbf{H}_j\mathbf{s}_j/\|\mathbf{H}_j\mathbf{s}_j\|$ is an equivalent unit-energy signature. Therefore, for $i \neq j$, $A_{ij} = |\tilde{\mathbf{s}}_i^\dagger \tilde{\mathbf{s}}_j|^2$.

Consider a situation in which every user requires a target SIR of at least β. That is, for each user i, we wish to guarantee that $\mathrm{SIR}_i \geq \beta$. This can equivalently be written in a vector form as

$$\tilde{\mathbf{p}} \geq \beta \mathbf{A} \tilde{\mathbf{p}} + \beta\sigma^2 \mathbf{1} \tag{9}$$

where $\mathbf{1}$ is a vector all 1's. It is well known that, if the target SIR β is feasible, then the optimal (componentwise smallest) power allocation $\tilde{\mathbf{p}}^*$ satisfies (9) with equality,

$$\tilde{\mathbf{p}}^* = \beta\sigma^2 (\mathbf{I} - \beta\mathbf{A})^{-1} \mathbf{1} \tag{10}$$

It can be shown using a result in non-negative matrix theory [7] that the components of $\tilde{\mathbf{p}}^*$ are non-negative iff $\beta < 1/\rho(\mathbf{A})$, where $\rho(\mathbf{A})$ is the largest (also called Perron-Frobenius) eigenvalue of $\mathbf{A}$. Note that $\rho(\mathbf{A})$ depends on the channel matrices $\{\mathbf{H}_i\}_{i=1}^K$ and the signature sequences $\{\mathbf{s}_i\}_{i=1}^K$. In order to maximize the common achievable SIR, the signature sequences should be chosen such that $\rho(\mathbf{A})$ is as small as possible for given $\{\mathbf{H}_i\}_{i=1}^K$.

It is not easy to determine the dependence of $\rho(\mathbf{A})$ on individual signature sequences, so that its direct minimization is nontrivial. However, one can relate $\rho(\mathbf{A})$ to the *effective* TSC,

$$\mathrm{TSC}_{\mathrm{eff}} = \sum_{i,j} |\tilde{\mathbf{s}}_i^\dagger \tilde{\mathbf{s}}_j|^2 = \sum_{i,j} A_{ij} + K \tag{11}$$

by noting that

$$\frac{1}{K}\sum_{i,j} A_{ij} \leq \rho(\mathbf{A}) \leq \max_i \sum_j A_{ij} \leq \frac{1}{2}\sum_{i,j} A_{ij} \tag{12}$$

Since we want to minimize $\rho(\mathbf{A})$, and since $\rho(\mathbf{A})$ is upper and lower bounded by multiples of $\sum_{i,j} A_{ij}$, we see heuristically that it is a good idea to minimize $\sum_{i,j} A_{ij}$, hence equivalently $\mathrm{TSC}_{\mathrm{eff}}$ by (11). Thus, we seek an algorithm that minimizes $\mathrm{TSC}_{\mathrm{eff}}$ by updating one signature sequence at a time. In order to do this, we first isolate the terms that depend on $\mathbf{s}_k$ in $\mathrm{TSC}_{\mathrm{eff}}$:

$$\begin{aligned} \mathrm{TSC}_{\mathrm{eff}} &= 2\sum_{j\neq k} |\tilde{\mathbf{s}}_k^\dagger \tilde{\mathbf{s}}_j|^2 + K + \gamma_k & (13) \\ &= 2\frac{\mathbf{s}_k^\dagger \mathbf{H}_k^\dagger \tilde{\mathbf{Z}}_k \mathbf{H}_k \mathbf{s}_k}{\mathbf{s}_k^\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{s}_k} + K + \gamma_k & (14) \end{aligned}$$

where $\gamma_k = \sum_{i\neq k}\sum_{j\neq k} A_{ij}$ represents squared correlation terms that do not depend on $\mathbf{s}_k$, and $\tilde{\mathbf{Z}}_k$ is defined as

$$\tilde{\mathbf{Z}}_k = \sum_{j\neq k} \frac{\mathbf{H}_j \mathbf{s}_j \mathbf{s}_j^\dagger \mathbf{H}_j^\dagger}{\|\mathbf{H}_j \mathbf{s}_j\|^2} \tag{15}$$

It can be shown that $\mathrm{TSC}_{\mathrm{eff}}$ can be (maximally) decreased if $\mathbf{s}_k$ is replaced by $\mathbf{c}_k$

$$\mathbf{c}_k = \frac{1}{(\mathbf{v}^\dagger \mathbf{D}^{-1} \mathbf{v})^{1/2}} \mathbf{U}\mathbf{D}^{-1/2}\mathbf{v} \tag{16}$$

where $\mathbf{U}$ and $\mathbf{D}$ are the matrices with the eigenvectors and eigenvalues of the matrix $\mathbf{H}_k^\dagger \mathbf{H}_k$, and $\mathbf{v}$ is the eigenvector of $\mathbf{D}^{-1/2}\mathbf{U}^\dagger \mathbf{H}_k^\dagger \tilde{\mathbf{Z}}_k \mathbf{H}_k \mathbf{U}\mathbf{D}^{-1/2}$ with minimum eigenvalue. Note that $\mathbf{c}_k$ is scaled to have unit norm. When this update is applied to the signature sequences of all users sequentially, the $\mathrm{TSC}_{\mathrm{eff}}$ decreases monotonically, and since it is lower bounded by Welch's bound, it is guaranteed to converge. Moreover, the signature sequence ensemble converges to the invariant set of the algorithm, which consists only of fixed points. Note, however, that $\rho(\mathbf{A})$ need not converge.

5. Optimizing Signatures Only

In the previous section, we aimed at optimizing the transmit powers as well as signature sequences of the users. The signature sequences have determined the feasibility of the power control problem, and we have chosen them to maximize the common achievable SIR. In this section, we consider the optimization of signature sequences for fixed user received powers. We will consider two criteria: minimizing the total MMSE and maximizing the sum capacity.

5.1 Minimizing the Total MMSE

The MMSE filter for the kth user is $\mathbf{c}_k = \arg\min \mathrm{MSE}_k$, where $\mathrm{MSE}_k = E[(\mathbf{r}^\dagger \mathbf{c}_k - b_k)^2]$. Therefore, the MMSE of user k, MMSE_k (i.e., MSE_k using $\mathbf{c}_k$), is

$$\mathrm{MMSE}_k = 1 - p_k \mathbf{s}_k^\dagger \mathbf{H}_k^\dagger (\mathbf{Z} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}_k \mathbf{s}_k \tag{17}$$

where

$$\mathbf{Z} = \sum_{j=1}^{K} p_j \mathbf{H}_j \mathbf{s}_j \mathbf{s}_j^\dagger \mathbf{H}_j^\dagger \tag{18}$$

It is well known that, for fixed powers and signature sequences, the MMSE filter maximizes the SIR, and MMSE_k and MSIR_k, the maximum SIR (i.e., SIR_k using $\mathbf{c}_k$), are related as

$$\text{MMSE}_k = \frac{1}{1+\text{MSIR}_k} \tag{19}$$

One can consider minimizing the total MMSE, defined as $\text{MMSE} = \sum_{k=1}^{K} \text{MMSE}_k$:

$$\begin{aligned} \text{MMSE} &= K - \text{Tr}\left\{ \left(\mathbf{Z}+\sigma^2\mathbf{I}\right)^{-1} \mathbf{Z} \right\} && (20) \\ &= K - \sum_{i=1}^{D} \frac{\lambda_i(\mathbf{Z})}{\lambda_i(\mathbf{Z})+\sigma^2} && (21) \end{aligned}$$

where $\{\lambda_i(\mathbf{Z})\}_{i=1}^{D}$ are the eigenvalues of $\mathbf{Z}$. Here $D = N + L - 1$ is the dimension of $\mathbf{Z}$. If one were free to choose the eigenvalues of $\mathbf{Z}$, then majorization theory tells us that the best is to make all of them equal [14]. This follows from the fact that the expression (21) is Schur-convex in the vector of eigenvalues.

However, the $\{\mathbf{H}_i\}_{i=1}^{K}$ matrices impose restrictions on the effective signatures, and it is not clear what combinations of eigenvalues are achievable. We therefore resort to numerical methods. In the following we present an iterative algorithm to perform the required optimization. As we have done in the case of TSC_{eff}, we first isolate the terms in MMSE that depend on a particular signature $\mathbf{s}_k$, and then look for ways of changing this signature to produce a smaller MMSE. Let us define $\mathbf{Z}_k$ as

$$\mathbf{Z}_k = \sum_{j\neq k} p_j \mathbf{H}_j \mathbf{s}_j \mathbf{s}_j^\dagger \mathbf{H}_j^\dagger = \mathbf{Z} - p_k \mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\dagger \mathbf{H}_k^\dagger \tag{22}$$

Using the matrix inversion lemma and some straightforward algebra

$$\begin{aligned} \text{MMSE} &= (K-D) + \sigma^2 \text{Tr}\left\{ \left(\mathbf{Z}_k+\sigma^2\mathbf{I}\right)^{-1} \right\} \\ &\quad -\sigma^2 \frac{p_k \mathbf{s}_k^\dagger \mathbf{H}_k \left(\mathbf{Z}_k+\sigma^2\mathbf{I}\right)^{-2} \mathbf{H}_k \mathbf{s}_k}{1+p_k \mathbf{s}_k^\dagger \mathbf{H}_k^\dagger \left(\mathbf{Z}_k+\sigma^2\mathbf{I}\right)^{-1} \mathbf{H}_k \mathbf{s}_k} && (23) \end{aligned}$$

It can be shown that updating $\mathbf{s}_k$ to $\mathbf{c}_k$ given below (maximally) decreases MMSE

$$\mathbf{c}_k = \frac{1}{(\mathbf{z}_0^\dagger \mathbf{D}^{-1} \mathbf{z}_0)^{1/2}} \mathbf{U}\mathbf{D}^{-1/2}\mathbf{z}_0 \tag{24}$$

where $\mathbf{U}$ and $\mathbf{D}$ are the matrices with the eigenvectors and eigenvalues of $\mathbf{I} + p_k \mathbf{H}_k^\dagger (\mathbf{Z}_k + \sigma^2\mathbf{I})^{-1}\mathbf{H}_k$ and $\mathbf{z}_0$ is an eigenvector of $\mathbf{D}^{-1/2}\mathbf{U}^\dagger\mathbf{H}_k^\dagger(\mathbf{Z}_k+\sigma^2\mathbf{I})^{-2}\mathbf{H}_k\mathbf{U}\mathbf{D}^{-1/2}$ with the maximum eigenvalue. The algorithm proceeds by repeating the above calculation for one user at a time, in a round-robin fashion; and MMSE decreases monotonically.

5.2 Maximizing the Sum Capacity

The ISI-free multipath channel model adopted here is equivalent to a synchronous vector multiple access channel where user k transmits with the vector $\sqrt{p_k}\mathbf{H}_k\mathbf{s}_k$. The information theoretic sum capacity of this channel is [12]

$$C_{\text{sum}} = \frac{1}{2}\log\det\left(\mathbf{I}+\sigma^{-2}\mathbf{Z}\right) \tag{25}$$

Again, we will follow the method of separating the effect of any one user on our objective function. Using the matrix inversion lemma along with $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$,

$$\begin{aligned} C_{\text{sum}} &= \frac{1}{2}\log\det\left(\mathbf{I}+\sigma^{-2}\mathbf{Z}_k\right) \\ &\quad + \frac{1}{2}\log\left(1+p_k\mathbf{s}_k^\dagger\mathbf{H}_k^\dagger\left(\mathbf{Z}_k+\sigma^2\mathbf{I}\right)^{-1}\mathbf{H}_k\mathbf{s}_k\right) && (26) \\ &= \bar{C}_{\text{sum}} + \frac{1}{2}\log(1+\text{MSIR}_k) && (27) \end{aligned}$$

where $\bar{C}_{\text{sum}}$ denotes the sum capacity of the users when the kth user is not present. The result in (27) says that the contribution of the kth user to the sum capacity is $1/2\log(1+\text{MSIR}_k)$. It is interesting to note the similarity of this to the capacity of a single user AWGN channel: $C = 1/2\log(1+\text{SNR})$. Using (19), one can write (27) as

$$C_{\text{sum}} = \bar{C}_{\text{sum}} - \frac{1}{2}\log(\text{MMSE}_k) \tag{28}$$

As in the previous formulations, we look for an update of $\mathbf{s}_k$ to $\mathbf{c}_k$ that increases the C_{sum}. From (27) and (28), we see that such an update must increase MSIR_k or equivalently, decrease MMSE_k. Let $\mathbf{U}$ and $\mathbf{D}$ denote the matrices with the (orthonormal) eigenvectors and eigenvalues of $p_k\mathbf{H}_k^\dagger(\mathbf{Z}_k + \sigma^2\mathbf{I})^{-1}\mathbf{H}_k$. Then $\mathbf{c}_k = \mathbf{u}$ where $\mathbf{u}$ (a column of $\mathbf{U}$) denotes the eigenvector that corresponds to the largest eigenvalue yields the largest increase in C_{sum}. When this update is repeated by users sequentially, C_{sum} increases monotonically.

6. Simulation Results

Figures 1 to 2 show simulation results for the TSC-minimizing algorithm of Section 4, for two channel realizations, A and B. The system has $K = 8$ users, processing gain $N = 5$, and $L = 2$ paths. Figure 1 shows (for Channel A) $\text{TSC}_{\text{eff}}/K-1$, Welch's bound, and the Perron-Frobenius eigenvalue $\rho(\mathbf{A})$, which determines the achievable SIRs. Note that Welch's bound on TSC_{eff} is asymptotically achieved, which means that the normalized effective signatures, $\{\tilde{\mathbf{s}}_i\}_{i=1}^{K}$, are asymptotically WBE sequences in $N+L-1$ dimensional space. Figure 2 shows the performance of the algorithm for Channel B. In this case, the asymptotic effective TSC is away from Welch's bound.

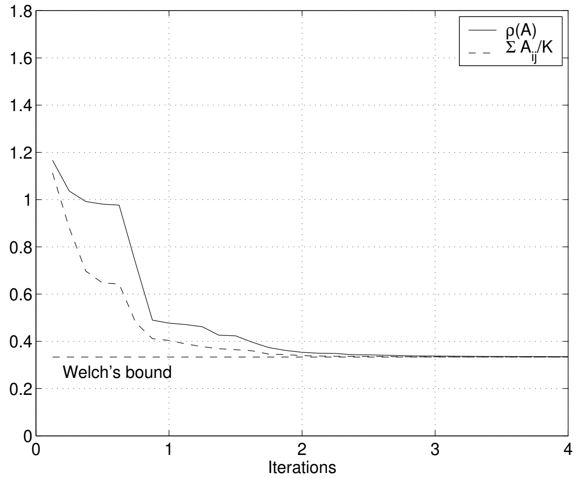

Figure 1. TSC-minimizing algorithm for Channel A ($K = 8$, $N = 5$, $L = 2$).

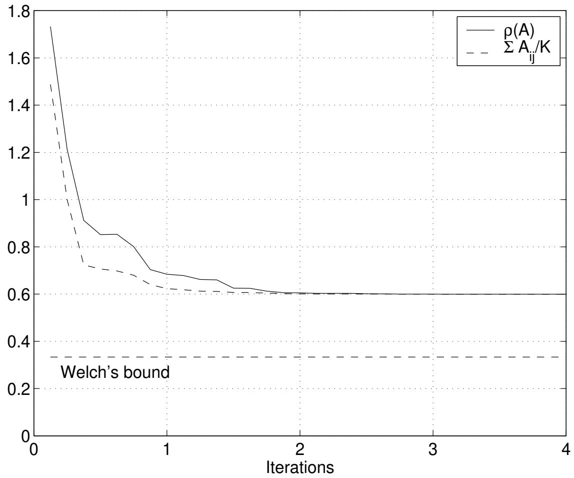

Figure 2. TSC-minimizing algorithm for Channel B ($K = 8$, $N = 5$, $L = 2$).

7. Conclusions

We proposed three algorithms to iteratively update the signature sequences of K users, with the goal of minimizing some measure of the interference produced by a known multiple-access multipath channel. The first algorithm minimizes the total square correlation of the set of effective signatures at the output of the channel, with MF receivers. The effective TSC is related to the achievable SIRs in a power-controlled scenario. The TSC decreases at each step and is guaranteed to converge. However, Welch's bound is not always achieved. An interesting open problem is to find conditions on the channel such that the effective signature sequences can be made to be WBE sequences in $N+L-1$ dimensional space. The second algorithm minimizes the total mean square error, when MMSE receivers are used, without power control, and the third algorithm maximizes the information theoretic sum capacity for fixed received powers.

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