Interference Avoidance for Wireless Systems

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Abstract

Motivated by the emergence of programmable radios, we seek to understand a new class of communication system where pairs of transmitters and receivers can adapt their modulation/demodulation method in the presence of interference to achieve better performance. Using signal to interference ratio as a metric and a general signal space approach, we present a class of iterative distributed algorithms for synchronous systems which results in an ensemble of optimal waveforms for multiple users connected to a common receiver (or co-located independent receivers). That is, the waveform ensemble meets the Welch Bound with equality and therefore achieves minimum average interference over the ensemble of signature waveforms. We describe fixed points for a number of scenarios.

1 Introduction

Wireless system designers have always had to contend with interference from both natural sources and other users of the medium. Thus, the classical wireless communications design cycle has consisted of measuring or predicting channel impairments, choosing a modulation method, signal pre-conditioning at the transmitter and processing at the receiver to reliably reconstruct the transmitted information. These methods have evolved from simple (like FM and pre-emphasis) to relatively complex (like CDMA and adaptive equalization). However, all share a common attribute – once the modulation method is chosen, it is difficult to change. For example, an ASK system cannot be simply modified to obtain a PSK system owing to the complexities of the transmission and reception hardware.

Software radios [1–5] change this paradigm by providing the communications engineer with a radio which can be programmed to produce almost arbitrary output waveforms and act as an almost arbitrary receiver type. Thus, it is no longer unthinkable to instruct the transmitting and receiving radios to use a more effective modulation in a given situation.

It is therefore probable that wireless systems of the near future will have elements which adapt dynamically to changing patterns of interference by adjusting modulation and processing methods in much the same way that power control [6–9], is used today, albeit on a possibly slower time-scale. Furthermore, if the release of 300MHz of unlicensed spectrum in the 5GHz range [10] is any indication, one might expect there to be an abundance of mutually interfering independent systems and no central control for efficient coordination. This provides added impetus to understand mutual interference of systems at some general level and implicit coordination in a multi-system environment.

In this paper we assume radios which can vary their output waveforms as well as their demodulation method. Our development starts with signal space to drive home the point that these techniques are applicable to a broad variety of communications scenarios and not only the usual "chip-based" CDMA system with a non-dispersive channel and a single antenna. That is, if the problem has a signal space description, then interference avoidance can be brought to bear.

In this context we consider optimal waveform selection to maximize the signal to interference ratio (SIR) for a power-constrained user in the presence of interference. Starting from the essentially classical approach of whitening [11] and showing the relation to modern methods (typified by [12]), we then consider ensembles of users and describe a class of distributed greedy algorithms which can optimize their shared use of the medium. That is, through local self-interested action, a social optimum can usually be reached. Moreover, the algorithms we will describe are simple and amenable to adaptive implementation.

2 Background

Consider the classical continuous time digital communications model, in which during an interval $[0,T]$, a signal $bS(t)$ is transmitted where $b=\pm 1$ equiprobably, p is the received power, and $S(t)$ is the a signal waveform with some energy. A receiver recovers $r(t) = bS(t) + Z(t)$, where $Z(t)$ is an independent interference stochastic waveform that may be composed of both thermal noise and interfering signals of other transmitters. For a single bit, the fundamental problem is to build a receiver which guesses b with minimum probability of error. Alternatively, when b is one bit in a stream of coded bits, we would like to produce a soft estimate of b with high signal to interference ratio (SIR). When $Z(t)$ is composed of known waveforms in addition to independent Gaussian noise, that is $Z(t) = \sum_i b_i S_i(t) + N(t)$, multiuser receivers have been designed for a variety of objectives, e.g. minimum probability of error, maximum SIR, or zero interference from other users [12]. These multiuser systems share the property that the receiver does as best it can given the set of transmitter signals $S_i(t)$.

We define the covariance of the noise process $Z(t)$ as $R_Z(t,\tau) = E[Z(t)Z(\tau)]$ and then seek a set of orthonormal functions $\Phi_i(t)$ for which $E[< \Phi_i(t), Z(t) >< \Phi_i(\tau), Z(\tau) >]$ yields uncorrelated projections. Precisely, we require

$$\int_0^T \Phi_j(t) \left(\int_0^T R_Z(t,\tau)\Phi_i(\tau)\,d\tau \right) dt = \lambda_j \delta_{ij} \qquad (1)$$

The solution to this integral equation requires

$$\lambda_i \Phi_i(t) = \int_0^T R_Z(t,\tau)\Phi_i(\tau) d\tau \qquad (2)$$

The properties of (2) are discussed in detail in [11].

Since integral equations are in general difficult to solve, it is useful to derive an equivalent discrete representation of (2). This will allow us to use simple methods from linear algebra. So let us assume that $Z(t)$, and therefore the function set $\{\Phi_i(t)\}$, can be well-approximated by a *finite* set of orthonormal basis functions $\{\Psi_n(t)\}$ on the interval $[0,T]$. That is, we assume that the process $Z(t)$ has no significant energy outside some finite signal space. As an example, a process "almost" limited to bandwidth $\pm W$ has a basis function set with about $2WT$ orthonormal functions [11, 13]. Likewise, for a synchronous CDMA system with N chips per bit, the appropriate orthonormal set consists of the N time-shifted chip pulses. One could also use a space-time orthogonalization for reception/transmission antenna diversity and/or a frequency-time orthogonalization for a frequency-hopped system.

Regardless of the specifics, once we assume a convenient finite basis function set, or *signal space* for the interval, the $\Phi_i(t)$ can then be represented by the finite sum

$$\Phi_i(t) = \sum_{n=1}^{N} \phi_{in} \Psi_n(t) \qquad (3)$$

and likewise

$$Z(t) = \sum_{n=1}^{N} Z_n \Psi_n(t) \qquad (4)$$

with $\phi_{in} =< \Phi_i(t), \Psi_n(t) >$ and $Z_n =< Z(t), \Psi_n(t) >$. This allows the reduction of equation (2) to a standard matrix eigenvalue/eigenvector equation of the form

$$E\left[\mathbf{Z}\mathbf{Z}^\top\right]\phi_i = \mathbf{R}\phi_i = \lambda_i \phi_i \qquad (5)$$

where $\phi_i = [\phi_{i1} \cdots \phi_{iN}]^\top$ and $\mathbf{Z} = [Z_1 \cdots Z_N]^\top$. Each eigenvector corresponds to an eigenfunction of (2) and it is easily verified that each eigenvalue is the amount of interference signal energy carried by that eigenfunction. It is also easy to verify that since $R_Z(t,\tau)$ is an autocorrelation function, $\mathbf{R}$ is symmetric and positive semi-definite. This implies that $\mathbf{R}$ has non-negative eigenvalues and an associated full set of orthonormal eigenvectors which span $\Re^N$ [14].

The receiver observes the signal $r(t)$ as input on the interval $[0,T]$. Projecting the received signal onto the interference eigenfunctions $\Phi_1(t), \ldots, \Phi_n(t)$, we obtain the vector output

$$\bar{\mathbf{r}} = b\bar{\mathbf{s}} + \bar{\mathbf{z}} \qquad (6)$$

where $\bar{\mathbf{s}}$ and $\bar{\mathbf{z}}$ have nth components $\bar{s}_n = \langle S(t), \Phi_n(t) \rangle$, $\bar{z}_n = \langle Z(t), \Phi_n(t) \rangle$ and the $\bar{z}_n$ are mutually uncorrelated.

At this point it is instructive to consider the detection of b when $Z(t)$ is a Gaussian interference process. Because we chose the interference eigenfunctions $\{\Phi_1(t), \ldots, \Phi_N(t)\}$ to yield uncorrelated (and therefore independent) interference components $\bar{z}_n$, optimal decision rule becomes [11,15]

$$\sum_{n=1}^{N} \frac{\bar{s}_n \bar{r}_n}{\lambda_n} = \sum_{n=1}^{N} \frac{\bar{r}_n}{\sqrt{\lambda_n}} \frac{\bar{s}_n}{\sqrt{\lambda_n}} \underset{\text{say } 0}{\overset{\text{say } 1}{\gtrless}} 0 \qquad (7)$$

The implied receiver is called a whitening filter since it can be viewed as an initial rescaling of the input to make interference components ($\{\bar{z}_n\}$), already uncorrelated, have equal energy ($\{\bar{z}_n/\sqrt{\lambda_n}\}$) – just as would be the case for a white noise process without rescaling. A matched filter on the rescaled signal vector components $\bar{s}_n/\sqrt{\lambda_n}$ is then performed to complete the detection process and the signal to interference/noise ratio is

$$\mathrm{SIR}_W = \sum_{n=1}^{N} \frac{\bar{s}_n^2}{\lambda_n} \qquad (8)$$

We now note that in a CDMA system where $Z(t)$ consists of the other users' known signature waveforms and additive white Gaussian noise, the vector $\bar{\mathbf{c}}$ with components $\bar{c}_n = \bar{s}_n/\lambda_n$ is a scaled version of the well known minimum mean squared error (MMSE) linear filter [16] and the decision rule (7) is the MMSE multiuser detector. We see that the filter output (and decision statistic) is

$$X = \bar{\mathbf{c}}^\top \bar{\mathbf{r}} = \sum_n \bar{c}_n \bar{r}_n = \left(\sum_{n=1}^{N} \frac{\bar{s}_n^2}{\lambda_n} \right) b + \sum_{n=1}^{N} \frac{\bar{s}_n \bar{z}_n}{\lambda_n} \qquad (9)$$

and that the output SIR is

$$\mathrm{SIR}_X = \sum_{n=1}^{N} \frac{\bar{s}_n^2}{\lambda_n} \qquad (10)$$

and identical to equation (8). The MMSE filter maximizes the output SIR [16] over all linear filters. Thus, the classical whitening approach and the newer MMSE approach lead to the same result.

However, equation (8) and equation (10) also demonstrate that it is possible to obtain a higher output SIR by altering the components $\bar{s}_n$ of the desired signal $S(t)$. That is, when $S(t)$ is subject to an energy constraint $\sum_n \bar{s}_n^2 = P$, we can maximize SIR_X by choosing $\bar{s}_n = P$ for any $\lambda_n = \lambda^* = \min_k \lambda_k$. In this case, we have $S(t) = \sqrt{P}\Phi_n(t)$. Equivalently, we could distribute the signal energy in some arbitrary way over all such $\Phi_n(t)$. Regardless, this result has a simple intuitively pleasing physical interpretation:

To obtain maximum SIR, place all the signal energy where there is least interference

We call the process of altering the signal waveform of a user *interference avoidance* and for a single user with a given interference process, the method is straightforward. We now examine the implications of this simple Karhunen-Loeve inspired rule for an ensemble of users.

3 Interference Avoidance for Multiple Users

We now consider a multiuser system in which the received signal $r(t)$ explicitly includes M users and white Gaussian noise. Given the existence of a finite set of N orthonormal basis functions $\Psi_i(t)$ for the signal space, we can express the received signal as the vector

$$\mathbf{r} = \sum_{i=1}^{M} b_i \mathbf{s}_i + \mathbf{n} \tag{11}$$

where $\mathbf{n}$ is the projection of the additive white Gaussian noise onto the basis. The classical communications scenario presumes that each user signature $S_i(t)$ is fixed. Assuming software radio transceivers we now allow the use of tailored signature waveforms $S_i(t)$. Without loss of generality, we assume each $S_i(t)$ has unit energy. The relationship between signature selection and multiuser system capacity has been studied in several papers [17–20]. In [17], it is shown that for a set of users' rates $R_1, \ldots, R_M$ belonging to the information theoretic achievable rate region $\mathcal{C}$, the *sum* capacity is

$$C_{\mathbf{s}} = \max_{(R_1,\cdots,R_M)\in\mathcal{C}} \sum_{i=1}^{M} R_i = \frac{1}{2}\log\left[\det\left(\sigma^2\mathbf{I}_N + \mathbf{S}\mathbf{S}^\top\right)\right] \tag{12}$$

In (12), $\mathbf{I}_N$ is the $N\times N$ identity matrix, $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_M]$ is the $N \times M$ matrix with columns $\mathbf{s}_i$, and σ^2 is the Gaussian white noise variance.

3.1 Sum Capacity and Total Square Correlation

3.1.1 Sum Capacity

It was shown in [18] that the sum capacity for equal received powers is maximized if the signature sequences are chosen such that if $M \le N$,

$$\mathbf{S}^\top\mathbf{S} = \mathbf{I}_M \tag{13}$$

and if $M \ge N$,

$$\mathbf{S}\mathbf{S}^\top = (M/N)\mathbf{I}_N \tag{14}$$

In [19], the *user* capacity of a CDMA system is defined in terms of the maximum number of admissible users. Given the signal space dimensionality N and a common SIR target β, M users are said to be admissible if there exist signature sequences $\mathbf{s}_i$ with powers $p_i = |\mathbf{s}_i|^2$ such that each user has an SIR at least as large as β. The user capacity was found for two kinds of linear receiver structures in [19]: matched filters and MMSE filters [16, 21]. It was shown in [19] that the user capacity with MMSE receivers is maximized if the signature sequence set is chosen to satisfy (13) or (14) and that the MMSE filter *is the matched filter* in these cases. Thus, the user capacity of a system with matched filter receivers is the same as that using MMSE filters [19].

3.1.2 Total Square Correlation

We define total square correlation (TSC) as

$$\text{TSC} = \text{Trace}[(\mathbf{S}\mathbf{S}^\top)^2] = \sum_{i=1}^{M}\sum_{j=1}^{M}(\mathbf{s}_i^\top\mathbf{s}_j)^2 \tag{15}$$

and it is related to sum capacity. First we define the eigenvalues of $\sigma^2\mathbf{I}_N + \mathbf{S}\mathbf{S}^\top$ as λ_i, $i = 1, \cdots N$ and rewrite sum capacity as

$$C_{\mathbf{s}} = -N\log\sigma + \frac{1}{2}\sum_{i=1}^{N}\log\lambda_i \tag{16}$$

essentially as was done in both [18] and [20]. Now note that if $\{\lambda_i\}$ are the eigenvalues of $\sigma^2\mathbf{I}_N + \mathbf{S}\mathbf{S}^\top$ then

$$\text{Trace}[(\sigma^2\mathbf{I}_N + \mathbf{S}\mathbf{S}^\top)^2] = \sum_{i=1}^{N}\lambda_i^2 \tag{17}$$

since the eigenvalues of $(\sigma^2\mathbf{I}_N + \mathbf{S}\mathbf{S}^\top)^2$ are $\{\lambda_i^2\}$ [14].

For brevity's sake, we follow [19, 20] and note that the function described in equation (16) is Schur concave while that of equation (17) is Schur convex. Since any constraints on the eigenvalues must be identical, and in fact form a convex set [20], we can conclude that any set $\{\lambda_i\}$ which maximizes equation (16) must also minimize equation (17) and *vice versa*. Therefore, minimization of TSC is *completely equivalent* to maximization of $C_{\mathbf{s}}$ assuming a convex constraint on the $\{\lambda_i\}$. For those unfamiliar with majorization and Schur convexity, an alternate development based on Lagrange methods is provided in [22].

3.2 Iterative Methods of TSC Reduction

There are a number of methods which might be used to determine codeword sets which minimize TSC [18–20]. Here we explore simple iterative methods which can be applied by each transmitter/receiver pair asynchronously and independently.

For a single user k, we observe that $\mathbf{S}\mathbf{S}^\top = \mathbf{R}_k + \mathbf{s}_k\mathbf{s}_k^\top$ where $\mathbf{R}_k = \sum_{i\neq k}\mathbf{s}_i\mathbf{s}_i^\top$, the correlation matrix of the interference faced by user k, is analogous to the matrix $\mathbf{R}$ introduced in Section 2. When user k replaces its signature vector $\mathbf{s}_k$ with a vector $\mathbf{x}$, the resulting difference in TSC is

$$\Delta = \text{Trace}[(\mathbf{R}_k + \mathbf{s}_k\mathbf{s}_k^\top)^2] - \text{Trace}[(\mathbf{R}_k + \mathbf{x}\mathbf{x}^\top)^2] \tag{18}$$

After some linear algebraic manipulations we find that $\Delta \ge 0$ iff

$$2\mathbf{s}_k^\top\mathbf{R}_k\mathbf{s}_k + |\mathbf{s}_k|^2 \ge 2\mathbf{x}^\top\mathbf{R}_k\mathbf{x} + |\mathbf{x}|^2 \tag{19}$$

which reduces to

$$\mathbf{s}_k^\top\mathbf{R}_k\mathbf{s}_k \ge \mathbf{x}^\top\mathbf{R}_k\mathbf{x} \tag{20}$$

if $|\mathbf{x}| = |\mathbf{s}_k|$ as we will hereafter assume. When the interference faced by user k includes additive white Gaussian noise with power spectral density σ^2, we may replace $\mathbf{R}_k$ by $\mathbf{Z}_k = \mathbf{R}_k + \sigma^2\mathbf{I}$ if desired, as the terms depending on σ^2 cancel each other in (20) since $|\mathbf{x}| = |\mathbf{s}_k|$.

Note that equation (20) defines a class of replacement algorithms whereby a given user can reduce (or at least not increase) the total squared correlation assuming other users' codewords remain fixed during the replacement. Such an algorithm may be used by each user sequentially until all users have updated their codewords. At that point the cycle may begin anew. Cycles (iterations) would then be repeated until there were no further change in the TSC by individual codeword updates. Consideration of this process begs at least two questions. First, what is an example of such an algorithm? Second, do such algorithms eventually minimize TSC?

In answer to the first question we present two algorithms. We call the first algorithm the *MMSE algorithm* since we replace $\mathbf{s}_k$ by the normalized MMSE receiver filter $\mathbf{c}_k = \left(\mathbf{s}_k^\top \mathbf{Z}_k^{-2} \mathbf{s}_k\right)^{-1/2} \mathbf{Z}_k^{-1} \mathbf{s}_k$. The MMSE algorithm was introduced previously in [9, 23]. For the details of the algorithm and its convergence properties see [9, 23, 24]. We call the second algorithm which we introduce here the *eigen-algorithm* since we replace $\mathbf{s}_k$ with $\mathbf{x} = \phi_k^*$ where ϕ_k^* is a minimum eigenvalue eigenvector of $\mathbf{R}_k$. Using equation (20) we see that both algorithms guarantee $\Delta \geq 0$; for the MMSE algorithm for a proof that $\Delta \geq 0$ see [9,24], and for the eigen-algorithm $\Delta \geq 0$ follows from the Rayleigh quotient [14] since both the right and left hand sides of the condition are under-bounded by $(\phi_k^*)^\top \mathbf{R}_k \phi_k^*$. Note also from (10) that one step of the eigen-algorithm maximizes the SIR of user k by allowing nonzero signal energy only along those basis functions with absolute minimum λ_n.

Since both algorithms decrease the TSC monotonically and since TSC is bounded below by the Welch bound [25] both must converge. At fixed points of both algorithms, each $\mathbf{s}_k$ is an eigenvector of $\mathbf{Z}_k$. For neither algorithm is the resulting codeword set unique. For example, any rotation of the codeword set will have the same cross-correlation properties. When $M \leq N$, the signatures converge to an orthonormal set. When $M \geq N$, the algorithms may converge to a Welch Bound Equality (WBE) signature set $\mathbf{S}$ satisfying equation (14).

Alternatively, the algorithms may converge to a local minimum for TSC. In [9, 23, 24] mild conditions are derived under which the MMSE algorithm converges. For the eigen-algorithm a modification of the procedure guarantees convergence to a global optimum [22]. In numerical experiments, both algorithms have *always* converged to the optimal signature set when starting from randomly chosen initial waveforms.

The intuition behind all interference avoidance algorithms which obey equation (20) is embodied by the simple requirement $\mathbf{x}^\top \mathbf{R}_k \mathbf{x} \leq \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k$, i.e., the replacement vector $\mathbf{x}$ attempts to reduce the interference from the ensemble of other user vectors and noise. From the standpoint of implementation, in the MMSE algorithm, user k must identify $\mathbf{Z}_k^{-1} \mathbf{s}_k$. In the eigen-algorithm, user k seeks a minimum eigenvalue eigenvector ϕ_k^* of $\mathbf{Z}_k$. These points, taken together suggest that the class of algorithms governed by equation (20) could be implemented by blind techniques at the receiver along with a feedback channel to the transmitter. Specifically, in the MMSE algorithm, the receiver for user k could be a blind adaptive MMSE filter [26] based on the observable $\mathbf{Z}_k$. Likewise, for the eigen-algorithm, $\mathbf{x}$ can be found by minimizing $\mathbf{x}^\top \mathbf{Z}_k \mathbf{x}$, which can also be implemented using blind techniques. Thus, interference avoidance algorithms are based on a measurable quantity – the interference/noise signal correlation $\mathbf{Z}_k$.

In the MMSE algorithm, a codeword replacement by user k requires first that the receiver filter for user k converges. Further, the MMSE filter coefficients $\mathbf{c}_k$ must be communicated to the transmitter via a feedback channel. Consequently, at each iterative step, the speed of the algorithm is limited since (1) the convergence to the MMSE filter may require several hundred bits and (2) several hundred bits may be needed for the feedback transmission of the new signature. These same conclusions will also hold for the eigen-algorithm. Therefore, these signature adaptation algorithms operate on a slower time scale than the algorithms for multiuser interference suppression. Thus, if the channel is not stable for a sufficient number of bit intervals, it is not clear whether interference avoidance will confer an advantage. However, for channels which are stable over a sufficient number of bit intervals, signature adaptation may offer potentially large capacity increases.

4 Fixed Point Properties of the Eigen-Algorithm

For the procedures given here convergence to the optimal is not assured. For example, Δ from equation (20) can be zero over a full cycle of an interference avoidance algorithm even though the minimum eigenvalues λ_k^* might not all be equal as required for $\mathbf{S}\mathbf{S}^\top = (M/N)\mathbf{I}$. We now examine the nature of such suboptimal fixed points in more detail. Convergence properties are described in [9,23,24] for the MMSE algorithm and in [22] for the eigen-algorithm. In this paper, our focus will be on the eigen-algorithm.

Equal energy codewords in white noise: There are the optimal fixed points with $\lambda_k^* = M/N - 1$ for $k = 1, \ldots, M$, and the suboptimal fixed points with vectors $\{\mathbf{s}_k\}$ in mutually orthogonal subspaces corresponding to each different λ_k^* – which cannot differ from each other by more than 1. If all the λ_k^* are identical, then the ensemble correlation $\mathbf{S}\mathbf{S}^\top$ is an identity matrix. That is, the ensemble is essentially whitened.

Furthermore, we have identified a procedure for use with the eigen-algorithm which is able to escape suboptimal fixed points through greedy and *aggressive* user action [22].

Mixtures of fixed and agile users: It is likely that users with both fixed and agile waveform radios may need to occupy the same signal space. For this case we find the intuitively pleasing result that the agile users perform a sort of *aggregate waterfilling* over the portion of the signal space with least fixed user energy to achieve an optimum SIR. Put another way, the fixed users appear as colored noise to the agile users which causes an appropriate Shannon-esque distribution of agile user signal energy over the signal space. It is readily understood that this feature is of benefit, *on aver-*

age, to the fixed users as well since it implies that the agile users avoid the fixed users where possible.

The following two theorems summarize our results:

Theorem 1 *If there are $L < N$ agile users, then the optimal codeword set will occupy at most the L least noisy signal dimensions.*

Theorem 2 *Suppose* $\mathbf{S}\mathbf{S}^\top = \mathbf{A}\mathbf{A}^\top + \mathbf{F}\mathbf{F}^\top$ *with* $\mathbf{F}$ *fixed,* $\mathbf{A}$ *agile, and* $\mathbf{F}\mathbf{F}^\top$ *having eigenvalues* σ_i^2, $\sigma_i^2 \geq \sigma_{i+1}^2$. *We have*

$$Trace[(\mathbf{S}\mathbf{S}^\top)^2] \geq (N-h^*)\left(\frac{M-\sum_{i=1}^{h^*}\sigma_i^2}{N-h^*}\right)^2 + \sum_{i=1}^{h^*}\sigma_i^4 \tag{21}$$

where $h^* = \arg\min_h (M - \sum_{i=1}^{h}\sigma_i^2)/(N-h)$. *This bound is met with equality when the eigenvalues of* $\mathbf{S}\mathbf{S}^\top$ *are* $\{\sigma_1^2, \ldots, \sigma_{h^*}^2, c^*, \ldots, c^*\}$ *where* $c^* = (M - \sum_{i=1}^{h^*}\sigma_i^2)/(N - h^*)$ *has multiplicity* $N-h^*$

Theorem 2 is a classical waterfilling result. Thus, greedy interference avoidance algorithms seek a minimum mutual interference set of agile vectors by "water filling" the energy levels provided by the fixed users, and avoiding completely energetic interference above a certain threshold.

Also note that as a byproduct, we have also shown that the algorithm seeks a minimum mutual interference set of vectors in a background of colored noise.

Unequal power users in white noise: When each user has arbitrary but fixed received power $|\mathbf{s}_k|^2 = p_k$ we obtain a similar waterfilling result and one identical to that derived in [20] for sum capacity. Note that we explicitly add white Gaussian background noise so that capacity is well-defined although this is not a necessary feature for interference avoidance to be effective. We assume at least as many users M as signal dimensions N since if $M < N$, the users will confine themselves to the M dimensions with lowest noise power.

We define the quantity $\beta_k^* = (\lambda_k^* + \sigma^2)/p_k$ as the inverse of the SINR achieved by the kth user. We find, similar to fixed and agile users, that there exists a "water level" c^*

$$c^* = \frac{E - \sum_{i=1}^{h^*} p_i}{N-h^*} \tag{22}$$

where

$$h^* = \arg\min_h = \frac{E - \sum_{i=1}^{h} p_i}{N-h} \tag{23}$$

so that

$$\beta_k^* = \begin{cases} \sigma^2/p_k & k \leq h^* \\ (\sigma^2 + c^* - p_k)/p_k & h^* < k \leq M \end{cases} \tag{24}$$

Overall, these results have a facetious but memorable interpretation

Might makes right

That is, those users with greater received power p_k obtain better performance. In fact, the excessively energetic h^* users command private channels. Such users are called *oversized* in [20], a term coined by S. Verdú. When no $p_k > E/N$, we have $h^* = 0$ and

$$\beta_k^* = \beta^* = \frac{\sigma^2}{p_k} + \frac{E}{p_k N} - 1 \tag{25}$$

for all k. This implies an absolute minimum TSC which in turn implies an absolute maximum sum capacity. This result is in agreement with that provided in [20].

5 Summary and Conclusion

Starting from a general signal space foundation, we have derived a class of *interference avoidance* algorithms whereby individual users asynchronously adjust their transmitted waveforms and corresponding matched filter receivers to minimize interference from other sources, including other users. This method presupposes that transmitters and receivers are *waveform agile* as would be the case assuming software radios [1–5] are inexpensive and ubiquitous.

The interference avoidance procedure, based on SIR as opposed to individual capacity maximization, minimizes total square correlation (TSC). Minimization of TSC is equivalent to maximization of sum capacity and this minimization/maximization is achieved through a satisfyingly information theoretic "waterfilling" of the signal space by the ensemble of users. Note that this waterfilling is essentially an "emergent" property of the ensemble since individual users seek not to maximize their capacity (via waterfilling), but rather, only seek to maximize their own SIR. Results for combinations of fixed and agile users (or agile users in colored background noise) as well as users with unequal received powers in a white background were provided.

We note that despite the assumption of perfect channels used in this paper, wireless channels are notoriously dispersive, and when signal energy for each bit is not contained within a single received bit interval, the problem formulation changes. Nonetheless, recent extensions show that the basic interference avoidance methods proposed herein may be directly applicable in a suitable multicarrier modulation context [27].

Similar considerations may apply for the asynchronous case where users may have non-coincident bit intervals and differing bit rates, but the problem as yet is unsolved. We are hopeful that the methodology developed for asynchronous multi-user detection [12] which is similar in spirit to the "interval-lengthening" approach taken for dispersive channels [27] will provide a useful starting point.

As noted, we have found the speed and stability with which codeword ensembles converge surprising. Especially intriguing is the seemingly certain convergence to an optimal set when starting from a randomly chosen starting set and the ease with which suboptimal fixed points are escaped through perturbation. In fact, a modification of greedy interference avoidance called "class warfare" can be used to escape the finite number of local minima and results in provable convergence to minimum TSC [22]. But frustratingly, a general proof which reflects the uncannily uniform convergence seen in practice *without escape methods* remains elusive.

The issue of codeword representation and fidelity in a real software/universal radio has not yet been considered. Specifically, instead of uniform-amplitude codeword "chips", interference avoidance presumes real-valued "chips" or more generally, real-valued coefficients for a set of orthonormal signal basis functions used by the transmitter and receiver. To be effective, these values must be communicated to the transmitter, and the amount of information which may be fed back to the transmitter will determine quantization methods for codewords. It is not currently known how quantization affects the performance of interference avoidance, although empirical investigation seems to indicate that in the vicinity of 4 bits per chip using Gaussian quantization is sufficient [28].

Finally, it must be emphasized that only a single receiver (or co-located multiple receivers) were assumed for this study. An understanding of the multiple receiver problem is paramount in determining the utility of interference avoidance in real systems.

Nevertheless, owing to the simplicity of the concept and the ever increasing sophistication of radio hardware, we expect interference avoidance to be an interesting new addition to the study of wireless systems. Furthermore, with any luck, it may also evolve into a practical method for wireless system design. If so, we expect it to be especially useful in unlicensed bands such as the U-NII, where users can mutually interfere with officially sanctioned impunity [10].

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