Throughput Maximization in CDMA Uplinks Using Adaptive Spreading and Power Control

Sennur Ulukus and Larry J. Greenstein
AT&T Labs–Research
{ulukus, ljg}@research.att.com

Abstract — **We formulate the throughput maximization problem for CDMA uplinks as an optimization problem in terms of the spreading gains (equivalently transmission bit rates) and transmit powers of the users, and solve it using a nonlinear programming approach. In nearly all cases, the solution is to give a rate of W to the highest-path-gain user in each cell and virtually nothing to the others. This approach maximizes total throughput at the expense of fairness. We then extend the notion of one-user-at-a-time by assuming all users get served on a time-sharing basis. The result is a considerable gain in fairness, with a modest loss in total throughput. The average user throughput (per timeslot) now ranges from W near the base to $\sim 0.4W$ near the cell boundaries, and the average cell throughput drops by 33%. Also, compared to rate-control-only (no power control), we show that optional power allocation considerably improves both average cell throughput and transmit power usage at the terminals.**

I. Introduction

One of the objectives of future wireless communication systems will be to deliver high speed data services. Wideband Code Division Multiple Access (W-CDMA) [1] is a promising candidate to fulfill this ambitious objective. Second-generation CDMA systems, such as those based on the IS-95 standard, are designed primarily for voice communications. Voice is a real-time, constant-bit-rate service in which each user needs to keep a constant quality of communication connection with the base station. Since the quality of communication is commonly expressed in terms of the Signal to Interference Ratio (SIR), maintaining a certain quality of communication level for a user is equivalent to maintaining an SIR above a corresponding level (called the *target* SIR). The target SIR is fixed once the bit rate is fixed. Therefore, the power control problem in voice CDMA systems is to find the componentwise smallest system-wide power vector that provides users with their SIR targets. There is no incentive in voice communication to provide users with higher than necessary SIRs, since it only results in waste of resources in the form of higher transmission powers.

Power control for fixed bit rate services (*conventional* power control) has been very well studied. Conventional power control is mathematically a linear programming problem. Although the optimum power vector can be determined with a simple matrix inversion [2], it is not desirable to do so since it entails a centralized approach which requires the knowledge of the channel gains of all users to all base stations. Iterative and distributed power control algorithms have been proposed [2–7]. In these algorithms each user updates its own transmit power by the knowledge of only its own current and target SIRs, and the power vector converges to an optimum.

In data communications, on the other hand, the objective is not to provide users with fixed SIRs, but to maximize the system throughput (possibly subject to certain constraints). A user's throughput depends on that user's bit rate and SIR which, in turn, depends on the user's bit rate as well as the powers of all users in the system. A higher SIR for a given transmission rate translates into higher throughput, since increasing the SIR increases the probability of correct detection. Power control in conjunction with transmission rate assignment to maximize the total throughput is not very well understood and it is precisely the problem we address here. The rate assignment can also be interpreted as spreading gain assignment, since the bit rate, R_i, and the processing (spreading) gain, G_i, of a user i have the one-to-one relationship $G_i = W/R_i$, for a fixed system bandwidth of W (assuming BPSK modulation, and sinc-type chip pulses having bandwidth W and spaced by $1/W$). Processing gain can also be defined as the number of chips per symbol. In order to realize variable spreading gains, users vary their symbol durations over a fixed rate chip sequence [8]. Since a symbol cannot contain less than one chip, we have the constraint that $G_i \geq 1$, which is equivalent to $R_i/W \leq 1$. We include this constraint in our optimization problem. Another physical constraint on the processing gain is that it should be an integer number. For the sake of tractability, however, we will relax this constraint and treat the processing gain as a parameter which can take any real value larger than 1. Thus, our capacity results will be optimistic and serve as an upper bound.

Adaptive rate assignment has been considered before in the context of mixed voice-data single-cell CDMA communication systems. In [9], the problem of maximizing the sum of the rates of the users subject to fixed SIR targets, minimum synchronization bit rates, and maximum transmit powers has been considered. In [10], for fixed-rate voice users and delay-tolerant data users, two transmission schemes based on scheduling of the data users have been studied. Neither [9] nor [10] includes the $R_i/W \leq 1$ constraint.

Adaptive transmission techniques have also been investigated in the narrow-band wireless communications context. A topic closely related to adaptive spreading in CDMA is adaptive modulation in TDMA. Qualitatively, the equivalent of decreasing the processing gain in adaptive spreading is increasing the number of signal constellation points in adaptive modulation. Both of these actions increase the bit rate and take place when users experience favorable channel conditions, such as little multiaccess interference or a strong signal. Earlier work [11] considered adaptation of the modulation scheme and transmit power to the time varying nature of the channel for a single-user case. In [12], joint power control and adaptive modulation is considered for a multi-user channel.

In this work, we first find the optimum uplink rate (or, equivalently, the optimum spreading gain) of all users given the system power vector. We have found that, for a given user, the optimum rate is linearly proportional to the SIR of the user before despreading as long as this proportionality yields a rate that is less than the system bandwidth, i.e., spreading is greater than 1. The condition that the spreading gain must exceed 1 introduces a *threshold* effect. If the SIR of a user before despreading is too large, i.e., this direct proportionality between the SIR and the rate yields a rate that is larger than the bandwidth, or equivalently, a spreading gain less than 1, the user must invoke the highest allowable rate ($R_i = W$), or equivalently, lowest allowable spreading gain ($G_i = 1$).

When the optimum spreading value is inserted into the throughput expression of a user, it reduces to being a function of only the user's SIR before despreading which, in turn, is a function of the system power vector. Thus, the original throughput maximization problem over the rates and the powers of the users reduces to being a maximization problem over only the powers. Unfortunately, the throughput after optimum spreading is a nonlinear function of the unspread SIR. In order to simplify the problem, we apply a linear approximation to it, and solve the simplified problem using a feasible-directions-based nonlinear programming algorithm.

II. Problem Definition and Analysis

We consider a general multi-cell CDMA system with fixed base station assignment of users. We assume that the signature sequences of the users are created randomly, so that the SIR of a user at the uplink receiver can be written as

$$\mathrm{SIR}_i = \frac{W}{R_i} \frac{p_i h_{ii}}{\sum_{j \neq i} p_j h_{ij} + \sigma^2} \tag{1}$$

where p_i and R_i are the transmit power and bit rate of user i, respectively, h_{ij} is the path gain of a user j to the base station of user i, W is the bandwidth and $\sigma^2 = \eta_0 W$ is the background noise power in this bandwidth. As noted earlier, the processing gain (spreading factor) of user i is $G_i = W/R_i$, and therefore, the SIR of user i can also be written as

$$\mathrm{SIR}_i = G_i \gamma_i \tag{2}$$

where γ_i, the SIR of user i *before despreading* at the receiver, is given as

$$\gamma_i = \frac{p_i h_{ii}}{\sum_{j \neq i} p_j h_{ij} + \sigma^2} \tag{3}$$

and depends only on the power vector. Let L denote the block length. We define the throughput of user i, normalized by the system bandwidth W, as

$$T_i = \frac{R_i}{W} \left(1 - f(\mathrm{SIR}_i)\right)^L \tag{4}$$

where $f(\mathrm{SIR}_i)$ is the BER as a function of the SIR of the user. The second term represents the probability of error-free block (or frame) reception, and applies to an uncoded system with block error detection. The function $f(\cdot)$ depends on the modulation scheme being used. The following are some examples for $f(\cdot)$:

$$\text{(a) } f(x) = Q(\sqrt{x}), \qquad \text{(b) } f(x) = \frac{1}{2} e^{-x}, \tag{5}$$

These BER functions correspond to (a) the AWGN channel using BPSK, and (b) the AWGN channel using differential BPSK or the AWGN channel using FSK. We will not assume a specific functional form for $f(x)$ in formulating the problem, but we will use the examples given in (5) for numerical results.

Given the channel gains of the users (h_{ij} for all i and j), we are interested in the rate and power distributions of the users such that the total throughput is maximized. Therefore, the optimization problem can be expressed as

$$\begin{array}{rll} \max\limits_{\{p_i, R_i\}} & \sum\limits_{i=1}^{N} T_i & \\ \text{s.t.} & R_i/W \leq 1 & i = 1, \cdots, N \\ & P_{\min} \leq p_i \leq P_{\max} & i = 1, \cdots, N \end{array} \tag{6}$$

where $P_{\min}$ and $P_{\max}$ are the minimum and maximum allowable transmit power levels. The condition $R_i \leq W$ merely states that a bit cannot contain less than one chip; in other words, the spreading gain must be at least 1.

First we will investigate the properties of the throughput of user i, T_i, as a function of powers and rates. Using (1), (3) and (4), the throughput of user i can be expressed as a function of R_i and γ_i as

$$T_i = \frac{R_i}{W} \left(1 - f\left(\frac{W\gamma_i}{R_i}\right)\right)^L \tag{7}$$

The throughput, T_i, as a function of γ_i for several fixed values of R_i/W, and T_i as a function of R_i/W for several fixed values of γ_i are plotted in Figures 1 and 2, for $f(x)$ in (5)(b) and $L = 100$. Note that T_i is an increasing function of γ_i for a fixed value of R_i/W, since $f(x)$ is a decreasing function of x, see Figure 1. However, T_i as a function of R_i/W is more complicated and deserves a little more elaboration. First we equate the derivative of T_i with respect to R_i for a fixed γ_i to zero to find its local extrema.

$$\begin{aligned} \frac{\partial T_i}{\partial R_i} &= \frac{1}{W} \left(1 - f\left(\frac{W\gamma_i}{R_i}\right)\right)^{L-1} \\ &\quad \times \left[1 - f\left(\frac{W\gamma_i}{R_i}\right) + Lf'\left(\frac{W\gamma_i}{R_i}\right)\frac{W\gamma_i}{R_i}\right] = 0 \end{aligned} \tag{8}$$

Since the first term is strictly positive, the local extrema of T_i for a given γ_i are found by equating the second term to zero. Defining $a_i = \frac{W\gamma_i}{R_i}$, this can be written as

$$1 - f(a_i) + Lf'(a_i)a_i = 0 \tag{9}$$

Note that the a_i's which satisfy the fixed point equation in (9) depend only on L. Let us denote the fixed points of (9) with $\bar{a}$. Then, the extrema of T_i occur at

$$\frac{R_i}{W} = \frac{1}{\bar{a}} \gamma_i \tag{10}$$

The value of $\bar{a}$, and how many $\bar{a}$'s there are, depend on the functional form of $f(x)$. For the two example BER functions in (5) with $L = 100$, the fixed point equation is satisfied at two $\bar{a}$ values. T_i as a function of R_i/W has a local minimum at the smaller value of $\bar{a}$ and a local maximum at the larger value of $\bar{a}$. The smaller value of $\bar{a}$ is 6×10^{-4} and 10^{-2}, and the larger value of $\bar{a}$ is 8.08 and 5.64, for BER functions in (a) and (b) of (5), respectively. Note also that, as $R_i/W \to \infty$, $T_i \to \infty$, in almost a linear fashion with R_i/W.

In order to solve for the optimum spreading gain, we maximize T_i with respect to R_i subject to the constraint that $R_i/W \leq 1$. Note that although T_i is increasing as a function

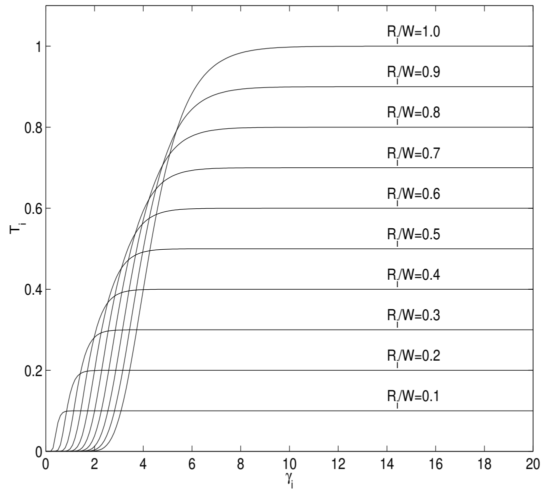

Fig. 1: Throughput versus γ_i for several fixed values of R_i/W.

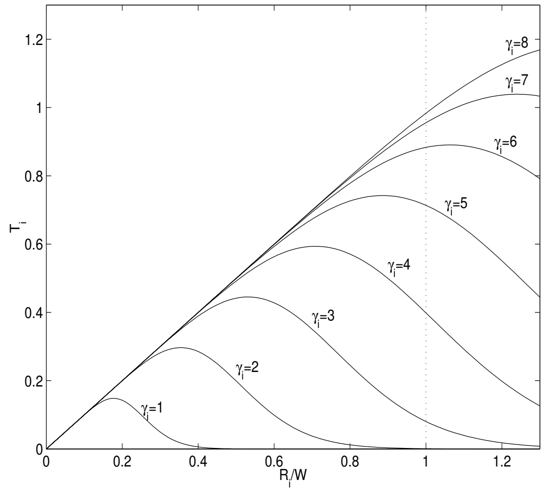

Fig. 2: Throughput versus R_i/W for several fixed values of γ_i.

of R_i/W after the local minimum at $R_i/W = \gamma_i/\bar{a}$, for the smaller value of $\bar{a}$, we do not need to consider that part of the throughput curve, since, for those values of R_i/W to be less than 1, and therefore be candidates for the optimum R_i/W, γ_i should be extremely small, in which case T_i is essentially zero. Let us denote the fixed points of (9) where T_i has a local maximum as a^*. Thus, for the BER functions given in (5)(a) and (b), we have $a^* = 8.08$ and $a^* = 5.64$, respectively. For a given power vector, hence γ_i, the optimum R_i/W, which defines the optimum spreading, is given as

$$\frac{R_i}{W} = \begin{cases} \frac{1}{a^*}\gamma_i & \text{if} \quad \gamma_i \leq a^* \\ 1 & \text{if} \quad \gamma_i > a^* \end{cases} \tag{11}$$

After optimum spreading, the throughput, T_i, reduces to being a function of γ_i only:

$$T_i = \begin{cases} \frac{(1-f(a^*))^L}{a^*}\gamma_i & \text{if} \quad \gamma_i \leq a^* \\ (1-f(\gamma_i))^L & \text{if} \quad \gamma_i > a^* \end{cases} \tag{12}$$

The throughput as a function of γ_i after optimum spreading is shown by the solid curve in Figure 3 for the BER in (5)(b). Note that this curve is also the envelope of the curves shown

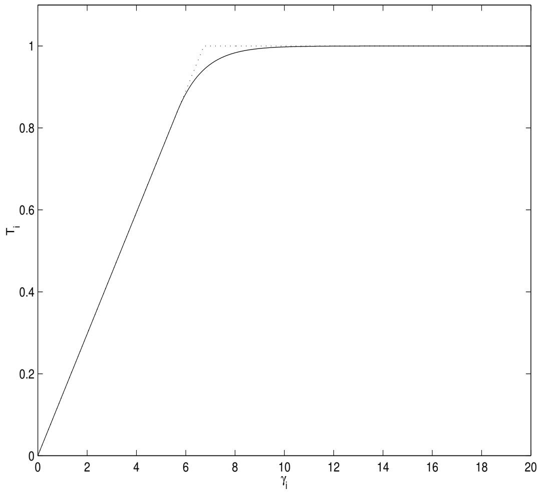

Fig. 3: Throughput vs γ_i after optimum spreading, for the BER function in (5)(b) and a linear approximation.

in Figure 1. We use a linear approximation to the throughput after optimum spreading in order to simplify the problem (see the dotted curve in Figure 3). The linear approximation curve reaches $T_i = 1$ at $\gamma^* = 6.74$ for the BER in (5)(b) and at $\gamma^* = 10.11$ for the BER in (5)(a), and can be expressed as

$$T_i = \begin{cases} \frac{(1-f(a^*))^L}{a^*}\gamma_i & \text{if} \quad \gamma_i \leq \gamma^* \\ 1 & \text{if} \quad \gamma_i > \gamma^* \end{cases} \tag{13}$$

Since the throughput after optimum spreading (12), and the linear approximation to it (13) are functions of γ_i only, which, in turn, are functions of the power vector only, the optimization problem in (6) can be written only in terms of the powers:

$$\begin{array}{lll} \max\limits_{\{p_i\}} & \sum\limits_{i=1}^{N} \gamma_i & \\ \text{s.t.} & \gamma_i \leq \gamma^* & i = 1, \cdots, N \\ & P_{\min} \leq p_i \leq P_{\max} & i = 1, \cdots, N \end{array} \tag{14}$$

Although the objective function of this optimization problem is convex in each component of the power vector, it is not convex in the power vector as a whole. The constraint functions are linear and the constraint set is convex in powers. We use a feasible-directions-based method to solve this problem.

It is worth mentioning that we have the first set of constraints ($\gamma_i \leq \gamma^*$) in (14) essentially because of the $R_i/W \leq 1$ constraints we imposed on the bit rates; without these constraints the optimization problem in (14) has a rather simple and intuitively pleasing solution: the components of the optimum power vector are either $P_{\min}$ or $P_{\max}$. If we restrict ourselves to a single cell system, the optimum power vector can actually be found by checking only N candidate vectors which have k $P_{\max}$'s followed by $N-k$ $P_{\min}$'s for some $1 \leq k \leq N$, given that the users are ordered according to their channel gains to the base station, from the strongest user to the weakest, i.e., the ith component of the power vector, p_i, denotes the transmit power of the ith strongest user.

III. Simulation Results and Discussion

We consider a 3×3 multicell system with square cell areas of size 1000 m $\times$ 1000 m. The positions of N users are generated according to a uniform distribution. The channel gains of the users have three components: a distance-based component, shadow fading and Rayleigh fading:

$$h_{ij} = K(d_{ij}/100)^{-\alpha} s_{ij} r_{ij} \quad (15)$$

where d_{ij} is the distance of the jth user from the base station of the ith user (in meters), $K = 10^{-8}$, $\alpha = 2$ if $d_{ij} \leq 100$ meters and $\alpha = 4$ otherwise; and s_{ij} and r_{ij} denote the log-normal (shadow) fading and Rayleigh (multipath) fading factors, respectively. The mean and standard deviation of $10 \log s_{ij}$ are 0 and 8 dB, respectively, and the average of r_{ij} is 1. We choose $P_{\min} = 0$ and $P_{\max}/\sigma^2 = 10^{12.4}$. This particular $P_{\max}/\sigma^2$ value is chosen in order to give a single user an SIR (equivalently SNR) of approximately 10 dB, without spreading, at the edge of a cell, when there is no fading. In the experiments, we use the BER function given in (5)(b).

Through extensive simulations for this system, we have seen that in each cell only one user operates at $\gamma_i = \gamma^*$ and the rest of the users have negligible γ_i's. Users having $\gamma_i = \gamma^*$ have throughputs (T_i's) of 1 and the rest of the users have negligible T_i's. Thus, the total throughput is roughly equal to the number of base stations. Stating the same result in terms of the spreading gains of the users, in each cell, only one user operates at $R_i/W = 1$, i.e., with no spreading, and all other users operate at varying amounts of spreading to suppress the interference, but have negligible total throughput. We have also observed that the user that transmits at the highest possible rate achieving a throughput of 1 in each cell is the one that has the highest channel gain to the base station. An example simulation result for a system with $N = 100$ users is shown in Figures 4, 5 and 6, where the positions of the base stations and the users are indicated by $\triangle$ and $\times$, respectively, and the users who have $\gamma_i = \gamma^*$, and therefore, $R_i/W = 1$ and $T_i = 1$, are circled. The channel gains are calculated using distance, distance and shadowing, and distance, shadowing and Rayleigh fading, in Figures 4, 5 and 6, respectively.

The result of the power and rate optimization problem seems to be that in order to maximize the total throughput, the system must operate in a TDMA mode with a reuse factor of 1. In order to maximize the total throughput, the user in each cell that has the most favorable channel condition (and, only that user) transmits to the base station at the maximum allowable rate. If there is no fading, this means that, at each instant the user that is closest to the base station transmits at the full rate (see Figure 4). This scheme, although optimum, may have serious problems in terms of "fairness". Shadow and Rayleigh fading introduce some amount of fairness to the system in the sense that, because of the fading conditions, users that are further away from the base stations may become the user with the best channel gain and may get to transmit at the full rate (see lower left cell in Figures 5 and 6, for instance).

In this optimum strategy, users who never have the highest channel gain to their base stations may never get to transmit any of their data. One may design scheduling strategies to overcome this difficulty. One possible strategy is to serve all users having traffic, only sequentially (one at a time) in each cell. In order to understand how well such a strategy might work, we have done the following experiment: In each frame, we allow each cell to have only one user. Each user's position and associated fadings are created randomly. For each realization of the positions and fadings of users (9 users in this case), we solve the throughput maximization problem and find

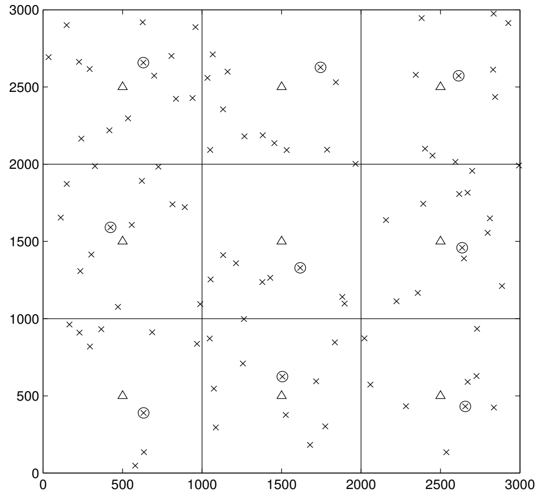

Fig. 4: Distance based channel gains.

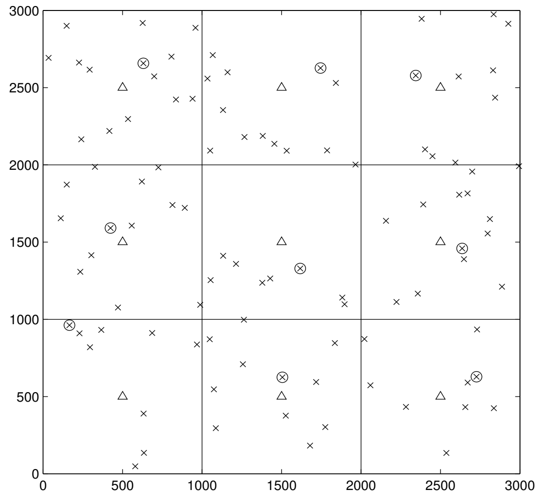

Fig. 5: Distance and shadowing based channel gains.

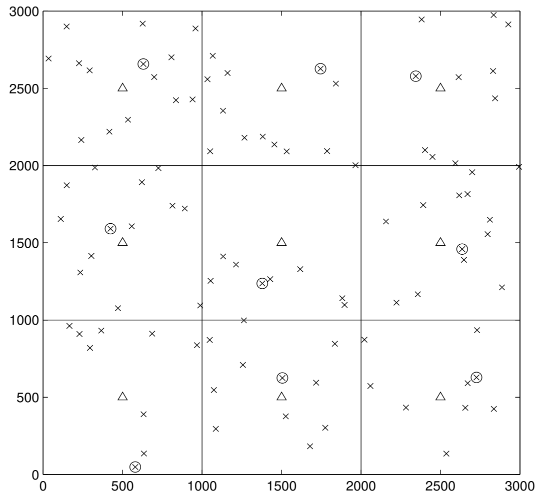

Fig. 6: Distance, shadowing and Rayleigh fading based channel gains.

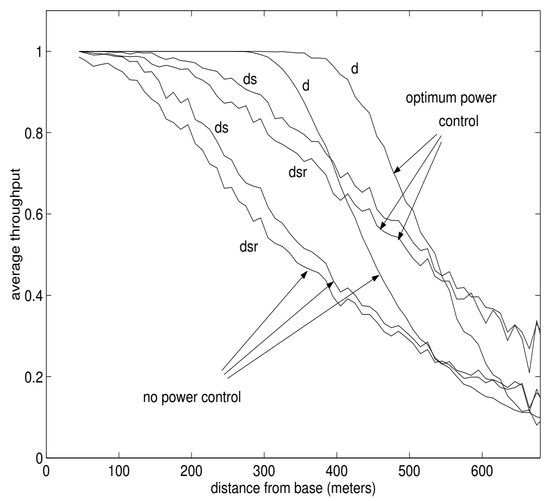

Fig. 7: Average throughput versus user distance.

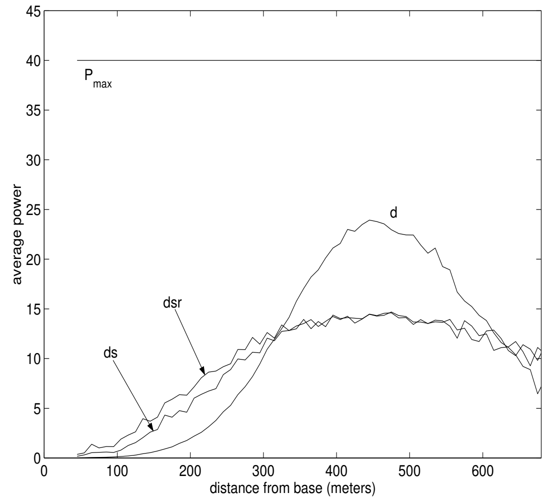

Fig. 8: Average transmit power versus user distance.

the optimum transmit powers, rates and throughputs of individual users. We do this for thousands of simulated frames and plot average throughput per user versus the distance of the user from its serving base station in Figure 7 (see curves marked as "optimum power control"). In this and the following figure, a 'd', 'ds' and 'dsr' next to a curve means that, in the experiments to generate this curve, the channel gains were calculated using distance, distance and shadowing, and distance, shadowing and Rayleigh fading, respectively.

We observe from Figure 7 (curves marked as "optimum power control") that, when there is no fading, if the distance of the user to its base station is less than approximately 350 meters, then it achieves a throughput of 1; but, when it is not that close to its base station, even if it is the only user in the cell, it cannot achieve a throughput of 1. We also observe that the effect of fading is to decrease the dependence of the throughput curve on the distance of the users from their serving base stations (i.e., more spread-out curves). The average throughput over all simulations (representing the average cell throughput over time) is 0.81, 0.71 and 0.67, for the 'd','ds' and 'dsr' cases, respectively.

In order to see the effectiveness and necessity of power control we simulated an algorithm which assigns optimum spreading gains to the users but does not consider power control. When there is no power control, all users transmit with $p_i = P_{\max}$. Since the users' powers are known, we can calculate their SIRs before despreading (γ_i's) at the receivers, and use the curve in Figure 3 to map γ_i to T_i, the optimum throughput of user i. The average throughput curves are shown in Figure 7 marked as "no power control". We observe that power control increases the total throughput of system significantly compared to no power control. The average cell throughput over time without power control is 0.66, 0.50 and 0.46, for the 'd','ds' and 'dsr' cases, respectively.

The advantages of optimum power control become further evident if we look at the average transmit power versus distance from the serving base station for optimum power control and no power control cases, Figure 8. In this figure, the horizontal line shows the average transmit power when there is no power control (this curve is of course the same for 'd', 'ds' or 'dsr' cases). The remaining three curves show the average transmit power for the cases 'd', 'ds' and 'dsr', when optimum power control is implemented. Thus, optimum power control yields not only higher total throughput but also significant power savings.

IV. Acknowledgments

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