# Signature Sequence Optimization in Asynchronous CDMA Systems

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***Abstract*– We characterize the *user capacity*, i.e., the maximum number of supportable users at a common SIR target level for a fixed processing gain, of a single-cell symbol *asynchronous* CDMA system. We show that the user capacity of an asynchronous system is the same as the user capacity of a synchronous system; that is there is no loss in user capacity due to asynchrony. Optimum signature sequences are those that minimize the total squared asynchronous correlation (TSAC) among the users, and depend on users' delay profile. Optimum received powers of the users are equal, and the optimum linear receiver filters in any observation window of size $M \geq 1$ symbols (i.e., $M$-shot MMSE filters) are one-shot matched filters. We present iterative and distributed signature adaptation algorithms where, at each iteration, only one user updates its signature sequence to decrease the TSAC of the set.**

## 1 Introduction

Code Division Multiple Access (CDMA) systems are interference limited. Much research has been conducted in the area of multiuser detection [1] to develop techniques to suppress/cancel interference for a fixed (arbitrarily chosen) set of user signatures. Recently, there has been an interest to understand the influence of multiuser detection schemes on the overall system capacity in single-cell CDMA systems. This recent literature can be divided into two general categories: those that assume random signature sequences [2–4], and those that solve for the optimum signature sequences [5–7].

In [2, 3], a random signature sequence analysis is applied to a large system where both the number of users, $K$, and the processing gain, $N$, go to infinity, but their ratio $\alpha = K/N$ is fixed and finite. References [2, 3] showed that for this large system with random signature sequences, the signal to interference ratio (SIR) of all users converged in probability to deterministic quantities, and calculated the *user capacity* of the system. Among all linear receiver filters, much attention is paid to the minimum mean squared error (MMSE) receiver, since it is the optimum linear filter in the sense of maximizing the SIR [8]. Reference [2] showed that the user capacity of the single-cell synchronous CDMA system with MMSE receivers is larger than that with matched filter receivers in the amount of exactly 1 user per dimension. Subsequently, [5] showed that if one can choose the signature sequences of the users carefully, then this gap between the MMSE and the matched filter receivers can be closed. The user capacity of a single-cell synchronous CDMA system using optimum signature sequences is the same for the MMSE and matched filter receivers. Further, this capacity equals the asymptotic (i.e., large system) user capacity with random signature sequences using MMSE receivers. The signature sequences that maximize the user capacity are Welch bound equality (WBE) sequences [9–11]. In an earlier work [7], WBE sequences were shown to maximize the information theoretic sum capacity of a single-cell synchronous CDMA system with equal received powers. More recently, a generalized version of this problem where users have arbitrary (unequal) received powers was solved in [6].

An iterative and distributed signature sequence adaptation algorithm that converges to a set of optimum signature sequences was given in [12–14]. The algorithm exploits the property of the optimum signature sequences that they minimize the sum of the squares of the cross correlations among the users, called total squared correlation, TSC, in [12–14]. The algorithm converges to a set of WBE sequences if $K > N$, or a set of orthogonal sequences if $K \leq N$. The algorithm, which was named the *MMSE update* algorithm, was based on the idea that the TSC–minimizing, hence optimum, signature sequences may be obtained if at each iteration one user updates its signature sequence to decrease the TSC of the whole set. In the proposed algorithm, each user updates its signature sequence to be the normalized MMSE receiver filter for that user when the signature sequences of all other users are fixed, hence the name MMSE update.

Recently, [4] generalized the analysis in [2] to a single-cell *asynchronous* CDMA system. Reference [4] showed that under matched filter reception, the user capacity of a single-cell asynchronous CDMA system is the same as the corresponding synchronous system. However, when MMSE reception is employed, there is a loss in user capacity compared to the user capacity in a synchronous system. This gap was shown to diminish as the observation window length is increased.

In this paper, we investigate the user capacity of an *asynchronous* single-cell CDMA system when we can choose the signature sequences, powers and linear receiver filters of the users. We show that the optimum signature sequences minimize the total squared asynchronous correlation (TSAC) among the users, and therefore depend on users' delay profile; the optimum received powers of the users are equal; the optimum unconstrained (in observation window size) linear re-

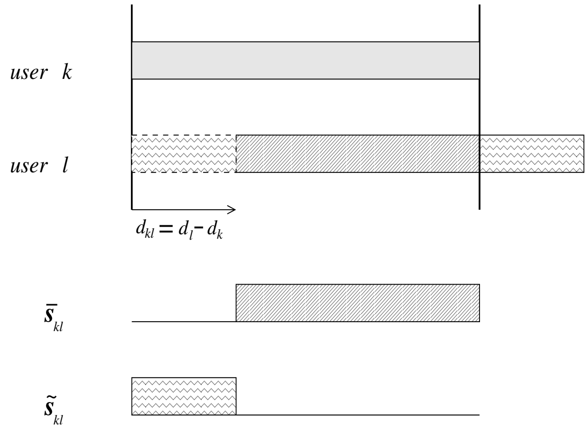


Figure 1: Asynchronous interference calculation.

ceiver filters of the users are single-user matched filters; and that the user capacity of an asynchronous system is the same as that of a synchronous system.

## 2 Problem Statement and Derivations

We consider a single-cell symbol-asynchronous (but chip-synchronous) CDMA system with $K$ users and processing gain of $N$. The received signal in the $n$th symbol interval of user $k$ is

$$\mathbf{r}_k(n) = \sqrt{p_k} b_k(n)\mathbf{s}_k + \sum_{l \neq k} \sqrt{p_l}\left(b_l(n)\bar{\mathbf{s}}_{kl} + b_l(n+1)\tilde{\mathbf{s}}_{kl}\right) \quad (1)$$

where $p_k$, $b_k(n)$ and $\mathbf{s}_k$ are the received power, $n$th transmitted symbol and signature sequence of user $k$, respectively, and $\bar{\mathbf{s}}_{kl}$ and $\tilde{\mathbf{s}}_{kl}$ are the "right" and "left" signatures of user $l$ with respect to user $k$ (Figure 1), and depend on the signature sequence of user $l$, as well as the relative delay, $d_{kl} = d_l - d_k$, between users $k$ and $l$, where $d_k$ is the delay of user $k$. The signature sequences are of unit energy, i.e., $\mathbf{s}_k^\top \mathbf{s}_k = 1$, for all $k$.

We will use one-shot matched filters as the receivers. In general, for an arbitrary set of signature sequences, single-user matched filters are suboptimum in multiuser systems, and one-shot receivers are suboptimum in asynchronous systems. However, as we will show later, when the optimum signature sequences are used, the optimum linear receivers in an observation window of arbitrary size, i.e., the $M$-shot MMSE receivers for some $M \geq 1$, turn out to be scaled matched filters, and there is no loss in capacity due to one-shot matched filtering.

The decision statistics for the $k$th user in the $n$th symbol interval is $y_k(n) = \mathbf{s}_k^\top \mathbf{r}_k(n)$, where we do assume that the matched filter of each user is perfectly aligned with the symbol interval of that user. The SIR of the $k$th user is

$$\mathrm{SIR}_k = \frac{p_k(\mathbf{s}_k^\top \mathbf{s}_k)^2}{\sum_{l \neq k} p_l \left\{(\mathbf{s}_k^\top \bar{\mathbf{s}}_{kl})^2 + (\mathbf{s}_k^\top \tilde{\mathbf{s}}_{kl})^2\right\} + \sigma^2(\mathbf{s}_k^\top \mathbf{s}_k)} \quad (2)$$

We define the $K \times K$ matrix $\mathbf{A}$ with the following entries

$$A_{kl} = \begin{cases} (\mathbf{s}_k^\top \bar{\mathbf{s}}_{kl})^2 + (\mathbf{s}_k^\top \tilde{\mathbf{s}}_{kl})^2 & k \neq l \\ 0 & k = l \end{cases} \quad (3)$$

A simple observation that will be important later is that $\mathbf{A}$ is a symmetric non-negative matrix. Since $\mathbf{s}_k^\top \mathbf{s}_k = 1$, the SIR of the $k$th user in (2) becomes

$$\mathrm{SIR}_k = \frac{p_k}{\sum_{l \neq k} A_{kl} p_l + \sigma^2} \quad (4)$$

The common SIR target $\beta$ is said to be feasible iff one can find non-negative powers $\{p_k\}_{k=1}^K$ such that $\mathrm{SIR}_k \geq \beta$, for $k = 1, \cdots, K$, i.e.,

$$p_k \geq \beta\left(\sum_{l \neq k} A_{kl} p_l + \sigma^2\right) \qquad k = 1, \cdots, K \quad (5)$$

which can be written in an equivalent matrix form as

$$\mathbf{p} \geq \beta\left(\mathbf{A}\mathbf{p} + \sigma^2 \mathbf{1}\right) \quad (6)$$

where $\mathbf{1}$ is the vector of all ones. It is well known that if the SIR target $\beta$ is feasible, then the optimum power vector, i.e., the componentwise smallest feasible power vector, is found by solving (6) with equality:

$$\mathbf{p} = \beta \mathbf{A}\mathbf{p} + \sigma^2 \beta \mathbf{1} \quad (7)$$

That is, if the common SIR target $\beta$ is feasible, the optimum power vector is given as $\mathbf{p}^* = \sigma^2\beta(\mathbf{I} - \beta\mathbf{A})^{-1}\mathbf{1}$. It is also well known that the power control problem is feasible iff [15]

$$\beta < \frac{1}{\rho_A} \quad (8)$$

where $\rho_A$ is the largest (also called the Perron-Frobenius) eigenvalue of matrix $\mathbf{A}$. We define the matrix $\mathbf{R} = \mathbf{A} + \mathbf{I}$ so that $\mathbf{R}_{kk} = (\mathbf{s}_k^\top \mathbf{s}_k)^2 = 1$ and $\mathbf{R}$ represents the squared asynchronous cross correlations of the signatures. The Perron-Frobenius eigenvalue of $\mathbf{R}$ satisfies $\rho_R = \rho_A + 1$, and the feasibility condition in (8) can also be expressed as

$$\beta < \frac{1}{\rho_R - 1} \quad (9)$$

That is, for a single-cell CDMA system, the range of common achievable SIR values are determined only by the Perron-Frobenius eigenvalue of the squared asynchronous cross correlation matrix $\mathbf{R}$ which depends only on the signature sequences of the users and their relative time delays. For a given signature sequence set $\{\mathbf{s}_k\}_{k=1}^K$ and a set of time delays $\{d_k\}_{k=1}^K$, the supremum of common achievable SIR targets equals $1/(\rho_R - 1)$. Our aim is to choose the signature sequences of the users, for any given set of time delays, such that the common achievable SIR is maximized. Therefore, we seek the signature sequence set that maximizes $1/(\rho_R - 1)$, or, equivalently, minimizes $\rho_R$, for a given set of user time delays $\{d_k\}_{k=1}^K$.

We note that it is hard to characterize the dependence of $\rho_R$ on individual signature sequences. Our approach would be to tie $\rho_R$ to another parameter of $\mathbf{R}$ which can be related to the signature sequences in a more direct way. To this end, we start our derivation with the following bounds on the Perron-Frobenius eigenvalue of $\mathbf{R}$ in terms of its row-sums [15]

$$\min_k \sum_{l=1}^{K} R_{kl} \leq \rho_R \leq \max_k \sum_{l=1}^{K} R_{kl} \tag{10}$$

We also have the following bound from a simple application of the Rayleigh quotient [16]

$$\frac{1}{K} \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \leq \rho_R \tag{11}$$

which is equivalent to $(\mathbf{1}^\top \mathbf{R} \mathbf{1})/(\mathbf{1}^\top \mathbf{1}) \leq \rho_R$. Combining (10) and (11) and the fact that the minimum row-sum lower bounds the average of the row-sums yields

$$\min_k \sum_{l=1}^{K} R_{kl} \leq \frac{1}{K} \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \leq \rho_R \leq \max_k \sum_{l=1}^{K} R_{kl} \tag{12}$$

We define the total squared asynchronous correlation (TSAC) as

$$\mathrm{TSAC} = \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \tag{13}$$

Since we want to minimize $\rho_R$, and since $\rho_R$ is lower bounded by $\mathrm{TSAC}/K$, it is reasonable to try to minimize the TSAC over the space of all possible signature sequences. It is not clear, however, that $\rho_R$ decreases as TSAC decreases. We will show, though, that the signature sequence sets that minimize TSAC are precisely those that minimize $\rho_R$.

In order to motivate the solution of the asynchronous problem, we will first revisit the synchronous problem which has been recently solved in [5].

## 3 The Synchronous Problem Revisited

In the synchronous case the $(k,l)$th component of $\mathbf{R}$ is $R_{kl} = (\mathbf{s}_k^\top \mathbf{s}_l)^2$. The following two theorems guarantee that the signature sequences that minimize the TSC (equivalent of TSAC in synchronous case) are those that minimize $\rho_R$.

**Theorem 1 ( [9], [10], [11])**

$$TSC = \sum_{k=1}^{K} \sum_{l=1}^{K} (\mathbf{s}_k^\top \mathbf{s}_l)^2 \geq \frac{K^2}{N} \tag{14}$$

**Theorem 2 ( [11])** *(The Uniformly-Good Property) If the sequences $\{\mathbf{s}_k\}_{k=1}^K$ are such that the equality holds in (14) then*

$$\sum_{l=1}^{K} (\mathbf{s}_k^\top \mathbf{s}_l)^2 = \frac{K}{N} \qquad k = 1, \cdots, K \tag{15}$$

Theorem 1, combined with (12) and $R_{kl} = (\mathbf{s}_k^\top \mathbf{s}_l)^2$ for the synchronous case, says that

$$\rho_R \geq \frac{K}{N} \tag{16}$$

Since our aim is to minimize $\rho_R$, the best we can do is to choose signature sequences so as to achieve (16) with equality. Theorem 2 says that when the signature sequences are chosen such that the TSC is minimized, i.e., the bound on the TSC is satisfied with equality, then all of the row-sums are equal, and they are all equal to $K/N$. Since the row-sums sandwich $\rho_R$, (16) is also satisfied with equality, and the lowest possible $\rho_R$ is obtained: $\rho_R = K/N$. Therefore, using (9), in the synchronous case, the bound on the common achievable SIR target is

$$\beta < \frac{1}{K/N - 1} \tag{17}$$

which is equivalent to the user capacity expression

$$\frac{K}{N} < 1 + \frac{1}{\beta} \tag{18}$$

given in [5] where this problem was originally solved.

## 4 The Asynchronous Problem

In this section, we derive asynchronous versions of Theorems 1 and 2. The following two theorems guarantee that the signature sequences that minimize the TSAC are those that minimize $\rho_R$.

**Theorem 3** *For any delay profile, the total squared asynchronous correlation satisfies*

$$TSAC = \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \geq \frac{K^2}{N} \tag{19}$$

**Theorem 4** *(The Asynchronous Uniformly-Good Property) If the sequences $\{\mathbf{s}_k\}_{k=1}^K$ are such that the equality holds in (19) then*

$$\sum_{l=1}^{K} R_{kl} = \frac{K}{N} \qquad k = 1, \cdots, K \tag{20}$$

We will continue our derivation similar to the synchronous case. For this asynchronous CDMA system, Theorem 3 combined with (12) says that

$$\rho_R \geq \frac{K}{N} \tag{21}$$

Similar to the synchronous case, our aim is to minimize $\rho_R$, and we cannot do better than to choose signature sequences that achieve (21) with equality. Theorem 4 says that when the signature sequences are chosen such that the TSAC is minimized, i.e., the bound on the TSAC is satisfied with equality, then all of the row-sums are equal, and they are all equal to $K/N$. Since the row-sums sandwich $\rho_R$, (21) is also satisfied with equality, and the lowest possible $\rho_R$ is obtained: $\rho_R = K/N$. Therefore, using (9), the bound on the common achievable SIR target, in this asynchronous case, is

$$\beta < \frac{1}{K/N - 1} \tag{22}$$

which is the same as (17) found in the synchronous case.

Next, we show that the optimum received powers of the users are equal. First, we set the received power of each user $k$ to be

$$p_k = p = \frac{\sigma^2}{1+1/\beta - K/N} \qquad k=1,\cdots,K \tag{23}$$

From (4), the SIR of the $k$th user is

$$\mathrm{SIR}_k = \frac{p}{p\left(\sum_{l\neq k} A_{kl}\right)+\sigma^2} = \frac{p}{p\left(\sum_l R_{kl}-1\right)+\sigma^2} \tag{24}$$

If the signature sequences satisfy (19) with equality in Theorem 3, then using (20) in Theorem 4, we can write (24) as

$$\mathrm{SIR}_k = \frac{p}{p\,(K/N-1)+\sigma^2} \tag{25}$$

It is now straightforward to show that when (23) is inserted into (25), we have $\mathrm{SIR}_k = \beta$. Note that $p>0$ as long as $K$, $N$ and $\beta$ satisfy the user capacity inequality (22).

As stated earlier, in general, using the matched filter as we have done in this section for the asynchronous case and in the previous section in the treatment of the synchronous case is suboptimum. However, an important observation in the original solution for the optimum signature sequences in the synchronous case in [5] was that once the optimum signature sequences were chosen, the MMSE receiver filters reduce to scaled matched filters. As stated in the following theorem, a very similar statement is true in the asynchronous case.

**Theorem 5** *If the sequences $\{\mathbf{s}_k\}_{k=1}^K$ are such that the equality holds in (19) then the M-shot MMSE filters for all users reduce to corresponding scaled matched filters, for $M\geq 1$.*

## 5 TSAC Reduction: Iterative Algorithms

Following the closed-form expressions for the signature sequence sets maximizing the information theoretic sum capacity [6, 7] and user capacity [5], references [12–14] introduced the iterative adaptation of signature sequences for synchronous CDMA systems. Since the optimum signature sequences minimize the TSC in the synchronous case, the algorithm presented in [12–14] was designed to decrease the TSC at each iteration. Since we have shown in the previous section that the optimum signature sequences minimize the TSAC in the asynchronous case, we will design algorithms which decrease the TSAC at each iteration. To this end, we will first separate the terms that depend on the signature sequence of the $k$th user in the TSAC. Using the TSAC definition (13) and the definition of $\mathbf{R}$ as $\mathbf{R} = \mathbf{A}+\mathbf{I}$, along with the definition of $\mathbf{A}$ in (3), we have

$$\mathrm{TSAC} = (\mathbf{s}_k^\top \mathbf{s}_k)^2 + 2\mathbf{s}_k^\top \left(\sum_{l\neq k} \bar{\mathbf{s}}_{kl}\bar{\mathbf{s}}_{kl}^\top + \tilde{\mathbf{s}}_{kl}\tilde{\mathbf{s}}_{kl}^\top\right)\mathbf{s}_k + \gamma_k \tag{26}$$

where we used the fact that $\mathbf{A}$ is symmetric in deriving (26) from the definition of TSAC in (13). Also in (26),

$$\gamma_k = \sum_{i\neq k}\sum_{j\neq k} R_{kl} \tag{27}$$

denotes the squared asynchronous correlation terms that do not depend on $\mathbf{s}_k$. Let us define $\mathbf{B}_k$ as

$$\mathbf{B}_k = \sum_{j\neq k}\left(\bar{\mathbf{s}}_j\bar{\mathbf{s}}_j^\top + \tilde{\mathbf{s}}_j\tilde{\mathbf{s}}_j^\top\right) \tag{28}$$

Therefore the TSAC in (26) can be written as

$$\mathrm{TSAC} = (\mathbf{s}_k^\top \mathbf{s}_k)^2 + 2\mathbf{s}_k^\top \mathbf{B}_k\mathbf{s}_k + \gamma_k \tag{29}$$

In order to minimize the TSAC, we are looking for updates of the signature sequence of the $k$th user from $\mathbf{s}_k$ to some $\mathbf{c}_k$ that is guaranteed to decrease (not to increase) the TSAC. Let us denote the TSAC after the $\mathbf{s}_k \to \mathbf{c}_k$ update as $\overline{\mathrm{TSAC}}$. Then,

$$\overline{\mathrm{TSAC}} = (\mathbf{c}_k^\top \mathbf{c}_k)^2 + 2\mathbf{c}_k^\top \mathbf{B}_k\mathbf{c}_k + \gamma_k \tag{30}$$

Restricting the new signature sequence of the $k$th user to be of unit energy as well, i.e., $\mathbf{c}_k^\top\mathbf{c}_k = 1$, we note that $\overline{\mathrm{TSAC}} \leq \mathrm{TSAC}$ iff

$$\mathbf{c}_k^\top \mathbf{B}_k\mathbf{c}_k \leq \mathbf{s}_k^\top \mathbf{B}_k\mathbf{s}_k \tag{31}$$

Although there are many possible $\mathbf{s}_k \to \mathbf{c}_k$ updates that would guarantee that (31) holds, we will propose two of them here. The two similar updates used in the synchronous CDMA context were given in [12–14] and in [17]. We call the first update the *asynchronous MMSE update* which is given as

$$\mathbf{c}_k = \frac{\left(\mathbf{B}_k + a^2\mathbf{I}_N\right)^{-1}\mathbf{s}_k}{\left[\mathbf{s}_k^\top\left(\mathbf{B}_k + a^2\mathbf{I}_N\right)^{-2}\mathbf{s}_k\right]^{1/2}} \tag{32}$$

and we call the second update the *asynchronous eigen update* which is defined as the normalized version of the eigenvector of $\mathbf{B}_k$ corresponding to its smallest eigenvalue. The proof that the asynchronous eigen update decreases the TSAC follows from the Rayleigh quotient applied to the matrix $\mathbf{B}_k$ [16]. The proof that the asynchronous MMSE update decreases the TSAC can be carried out in a very similar fashion to the proof that the MMSE update decreases the TSC in [12–14].

## 6 Numerical Results

We provide a simple experimental result where we implement both the asynchronous MMSE update and the asynchronous eigen update algorithms and plot TSAC and the supremum of the common achievable SIR target, $1/(\rho_R - 1)$, as a function of the iteration index in Figure 2. In this example system, $K=20$, $N=10$, and the initial signature sequence set and the delays of the users are generated randomly. In Figure 2, one iteration is equivalent to $K$ intermediate iterations where at each intermediate iteration one user updates its signature sequence. As we see from Figure 2, TSAC decreases to the bound $K^2/N = 40$ and $\rho_R$ decreases to $K/N = 2$, and the supremum of the common achievable SIRs increases to $1/(K/N-1) = 1$. Figure 2 verifies, for this instance, that the signature sequences achieving the lower bound on TSAC exist and the two algorithms we proposed here converge to those signature sequences.

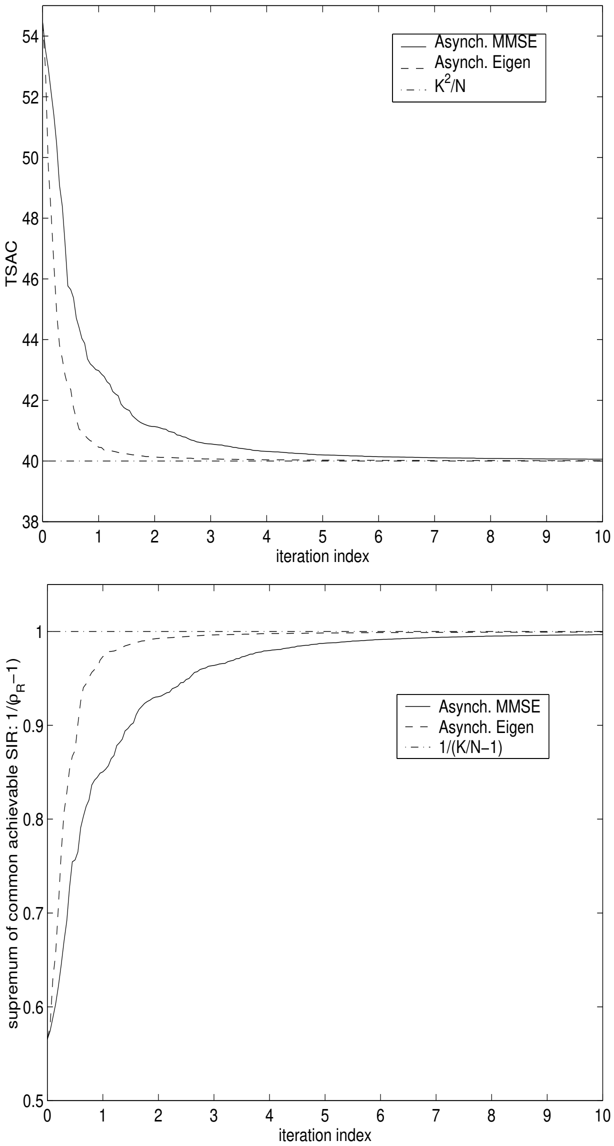


Figure 2: TSAC and $1/(\rho_R - 1)$ versus iteration index.

# 7 Conclusions

We have identified the user capacity of a symbol-asynchronous single-cell CDMA system, and showed that it is equal to that of a synchronous system. We have shown that the optimum signature sequences are those that minimize the TSAC, and that they depend only on the users' delay profile; the optimum received powers are all equal; and optimum linear receivers of any length are one-shot single-user matched filters. We have proposed two iterative algorithms which are based on the objective of decreasing the TSAC at each iteration. The existence of these optimum signature sequences for all delay profiles is an important open problem. Experimentally, when we ran the proposed algorithms starting with randomly generated initial signature sequences, we have always arrived at signature sequences having TSAC $= K^2/N$, i.e., the optimum signature sequences.

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