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Iterative Joint Optimization of CDMA Signature Sequences and Receiver Filters

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Abstract — **Optimization of the capacity of a single cell Code Division Multiple Access (CDMA) system, both from the perspective of the maximum number of users that can be served at a required quality of service level and from the information theoretic perspective, has been recently shown to be achieved by the same joint transmit and receive strategies. In this work, we propose an alternating minimization based iterative algorithm that updates the transmitters and the corresponding receivers of the users. The algorithm is suitable for online implementation in contrast with previously proposed algorithms. We show that the algorithm is provably convergent to the optimum signature sequences and the corresponding receivers.**

I. Introduction

Capacity of CDMA systems has been studied extensively [1–3]. Reference [1] defines the *user capacity* of a CDMA cell as the number of users that can be accommodated at the required quality of service level defined in terms of the signal-to-interference ratio (SIR) and computes the user capacity when random signature sequences are used for matched filter receivers and for minimum mean squared error (MMSE) receivers [4]. The *information theoretic* capacity region of a synchronous CDMA cell was derived in [2] for a given deterministic set of signature sequences. Reference [5] identified the set that maximizes the information theoretic sum capacity with equal power users, as one where all signature sequences are orthogonal to each other if possible, i.e., when the number of users is less than or equal to the processing gain, and as the Welch Bound Equality (WBE) sequences otherwise. More recently [6] identified the optimum signature sequences for arbitrary (unequal) powers. An important result that bridges the two capacity results is given by [3] where user capacity is considered for a fixed signature sequence set. It is shown that, for a single CDMA cell where each user has a required SIR level, the capacity is achieved with minimum total power by each user having equal received powers and using the signature sequence sets that are identified in [5]. Furthermore, when such sets are used, the MMSE filters reduce to scalar multiples of matched filters [3].

Following these developments that emphasize the importance of transmitter optimization, iterative algorithms that converge to the optimum signature sequence sets are proposed [7–10]. Reference [7] defines the total squared correlation (TSC), whose optimization results in the optimum signature sequence sets. Both algorithms proposed in [8–10] work on minimizing the TSC. The algorithms are distributed and their convergence can be proven only when the algorithms are run off-line in a sequential manner, i.e., one signature sequence update at a time needs to be performed. Once the optimum signature sequences are found, the corresponding receivers are set to be matched filters.

Our aim in this work is to design an algorithm that is amenable to on-line implementation. Thus, we would like to introduce the notion of receiver updates to the algorithm as well as transmitter updates until the joint optimum transmitters and receivers are reached. We also desire the algorithm to converge to the optimum set of signature sequences and the corresponding receivers when users update their transmitters in parallel since this alleviates the need to schedule user updates. We show that, working with a closely related performance measure, the *system wide* mean squared error (MSE), whose optimum point is identical to that of information theoretic sum capacity and total weighted squared correlation (TWSC), it is possible to come up with such an algorithm. Specifically, we propose an algorithm based on alternating minimization, that updates the transmitters and the receivers of the users until the optimum signature sequence set and the corresponding receivers (matched filters) are reached. We prove the convergence of the algorithm, with parallel user updates, to the optimum point.

II. System Model and Performance Metric

We consider the uplink of a single cell synchronous CDMA system with K users and processing gain N. In the presence of additive white Gaussian noise $n(t)$ with zero mean and power spectral density σ^2, the received signal in one symbol interval is

$$r(t) = \sum_{i=1}^{K} \sqrt{p_i} b_i s_i(t) + n(t) \tag{1}$$

where, for user i, p_i is the received power, b_i is the bit, and $s_i(t)$ is the signature waveform. The signature waveforms of the users, which are zero outside the symbol interval, have unit energies, and can be represented by N orthonormal basis waveforms $\{\psi_j(t)\}_{j=1}^N$ such that $s_i(t) = \sum_{j=1}^N s_{ij}\psi_j(t)$, where $s_{ij} = \langle s_i(t), \psi_j(t)\rangle$. Projecting the received signal onto these basis waveforms yields a set of sufficient statistics $\{r_j\}_{j=1}^N$ where $r_j = \langle r(t), \psi_j(t)\rangle$ [11]. By defining the *signature sequence* of user i as $\mathbf{s}_i = [s_{i1}, \cdots, s_{iN}]^\top$ and the received signal vector $\mathbf{r} = [r_1, \cdots, r_N]^\top$, we can write (1) in the equivalent vector notation

$$\mathbf{r} = \sum_{i=1}^{K} \sqrt{p_i} b_i \mathbf{s}_i + \mathbf{n} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n} \tag{2}$$

where $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_K]$ is an $N \times K$ matrix with the users' signature sequences as its columns, $\mathbf{P} = \mathrm{diag}\{p_1, \cdots, p_K\}$ is a $K \times K$ diagonal matrix of the users' received powers, and $\mathbf{n}$ is a zero mean Gaussian random vector with $E\big[\mathbf{n}\mathbf{n}^\top\big] = \sigma^2\mathbf{I}_N$, where $\mathbf{I}_N$ denotes the $N \times N$ identity matrix.

Previous work showed that the information theoretic sum capacity of this system is given by [2]

$$C_{\text{sum}} = \frac{1}{2}\log\left[\det\left(\mathbf{I}_N + \sigma^{-2}\mathbf{S}\mathbf{P}\mathbf{S}^\top\right)\right] \tag{3}$$

When the received powers of the users are the same, $p_i = p$ for all i, C_{sum} is maximized by [5]

$$\mathbf{S}^\top\mathbf{S} = \mathbf{I}_K \tag{4}$$

if $K \leq N$, and by

$$\mathbf{S}\mathbf{S}^\top = \frac{K}{N}\mathbf{I}_N \tag{5}$$

if $K > N$. The signature sequence sets satisfying (4) contain K orthonormal signature sequences in N dimensional vector space, and the sequence sets satisfying (5) are the Welch Bound Equality (WBE) sequences [5]. For general K and N, the signature sequence sets that satisfy (4) for $K \leq N$ and (5) for $K > N$, are those that achieve the minimum *Total Squared Correlation*, TSC, defined as

$$\text{TSC} = \sum_{i=1}^{K}\sum_{j=1}^{K}(\mathbf{s}_i^\top \mathbf{s}_j)^2 \tag{6}$$

References [7–10] then devise algorithms that minimize the TSC to find the optimum signature sequence sets. Specifically, they give single signature sequence update algorithms that are guaranteed to decrease the TSC of the set at each update.

For unequal (arbitrary) powers, the signature sequences that maximize the sum capacity in (3) are identified in [6]. It was shown that, when $K \leq N$, C_{sum} is again maximized by orthonormal sequences. In the case of $K > N$, the capacity is maximized when users with relatively high received powers, termed as *oversized users* in [6], are assigned sequences orthogonal to all other users, and the remaining users are assigned generalized WBE sequences in the reduced dimensionality signal space [6]. Assuming that the users are ordered according to their powers $p_1 > \cdots > p_K$, and the first L users are oversized, the eigenvalues of $\mathbf{SPS}^\top$ with the optimum $\mathbf{S}$ are $\{p_1, \cdots, p_L, \lambda, \cdots, \lambda\}$ where $\lambda = (\sum_{l=L+1}^{K} p_l)/(N-L)$, and the multiplicity of λ is $N - L$. In other words, the characterization of the optimum signature sequences is that the eigenvalues of $\mathbf{SPS}^\top$ with the optimum signature sequences are majorized by the eigenvalues of the same matrix with any other feasible signature sequence set. The eigenvalues of $\mathbf{SPS}^\top$ corresponding to the optimum signature sequences is a *Schur-minimal* element of the space of all feasible eigenvalues corresponding to all possible signature sequence sets. We will use the fact that a Schur-minimal point maximizes all Schur-concave functions and minimizes all Schur-convex functions in the sequel [12].

Let us now turn our attention to another important quantity; the mean squared error (MSE) incurred by a user, say user i, at the output of a linear filter $\mathbf{c}_i$

$$\begin{aligned}\text{MSE}_i &= E\Big[(\mathbf{r}^\top \mathbf{c}_i - b_i)^2\Big] \\ &= \mathbf{c}_i^\top \left(\mathbf{SPS}^\top + \sigma^2 \mathbf{I}_N\right)\mathbf{c}_i - 2\sqrt{p_i}\mathbf{c}_i^\top \mathbf{s}_i + 1\end{aligned} \tag{7}$$

Let us define the total mean squared error of the system as

$$\begin{aligned}\text{MSE} &= \sum_{i=1}^{K}\text{MSE}_i \\ &= \text{tr}\left[\mathbf{C}^\top\left(\mathbf{SPS}^\top + \sigma^2\mathbf{I}_N\right)\mathbf{C} - 2\mathbf{C}^\top\mathbf{SP}^{1/2} + \mathbf{I}_K\right]\end{aligned} \tag{8}$$

where $\mathbf{C} = [\mathbf{c}_1, \cdots, \mathbf{c}_K]$ is an $N \times K$ matrix containing the receiver filters of the users in its columns. Consider the MMSE filters for all users for fixed signature sequences. In this case $\mathbf{C}$ becomes

$$\mathbf{C} = \left(\mathbf{SPS}^\top + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{SP}^{1/2} \tag{9}$$

Substituting (9) into (8), we obtain the MMSE of the system with signature sequence set $\mathbf{S}$ as

$$\text{MMSE} = K - \text{tr}\left[\mathbf{SPS}^\top\left(\mathbf{SPS}^\top + \sigma^2\mathbf{I}_N\right)^{-1}\right] \tag{10}$$

Note that the total *weighted* square correlation (TWSC) corresponding to this system is

$$\text{TWSC} = \sum_{i=1}^{K}\sum_{j=1}^{K} p_i p_j (\mathbf{s}_i^\top \mathbf{s}_j)^2 = \text{tr}\left[(\mathbf{SPS}^\top)^2\right] \tag{11}$$

In terms of the eigenvalues of $\mathbf{SPS}^\top$, $\{\lambda_i\}$,

$$\begin{aligned}C_{\text{sum}} &= \frac{1}{2}\sum_{i=1}^{N}\log(1+\frac{\lambda_i}{\sigma^2}) \\ \text{MMSE} &= K - \sum_{i=1}^{N}\frac{\lambda_i}{\lambda_i+\sigma^2} \\ \text{TWSC} &= \sum_{i=1}^{N}\lambda_i^2\end{aligned} \tag{12}$$

It is easy to see that, C_{sum} is Schur-concave, and MMSE and TWSC are Schur-convex functions of the eigenvalues of $\mathbf{SPS}^\top$ [12]. Therefore, the signature sequence matrix yielding Schur-minimum eigenvalues, i.e., one with eigenvalues that are majorized by all other feasible eigenvalues, which maximizes C_{sum}, also minimizes the MMSE and the TWSC.

Since our aim is to obtain an on-line iterative joint receiver/transmitter update algorithm, we will concentrate on the MSE criterion given (8) which is a function of all signature sequences and receiver filters. Clearly, the minimization of (8) over the signature sequences and the receivers is equivalent to the minimization of the MMSE in (10) over the signature sequences. This, in turn, is also equivalent to the minimization of TWSC and the maximization of (3) over the signature sequences. When all users have the same received power, minimization of TSC is also equivalent to these problems.

Next, we will devise an algorithm that minimizes the total MSE in (8) over all transmitters (signature sequences) of unit energy and receivers.

III. An Alternating Minimization Algorithm and Its Convergence

Our aim is to minimize the cost function in (8) over the signature sequences $\{\mathbf{s}_i\}$ and the receivers $\{\mathbf{c}_i\}$. We assume the received powers are given, and require the resulting signature sequences to have unit energy. That is, we impose the set of constraints, $\mathbf{s}_i^\top \mathbf{s}_i = 1$, for all $i = 1, \cdots, K$. The Lagrangian for this constrained optimization problem is expressed as

$$\begin{aligned}\mathcal{L}(\{\mathbf{c}_i\},\{\mathbf{s}_i\},\{\alpha_i\}) = &\sum_{i=1}^{K}\sum_{j=1}^{K} p_j(\mathbf{c}_i^\top \mathbf{s}_j)^2 - 2\sum_{i=1}^{K}\sqrt{p_i}(\mathbf{c}_i^\top\mathbf{s}_i) \\ &+ \sigma^2\sum_{i=1}^{K}\mathbf{c}_i^\top\mathbf{c}_i + \sum_{i=1}^{K}\alpha_i(\mathbf{s}_i^\top\mathbf{s}_i - 1)\end{aligned} \tag{13}$$

where $\{\alpha_i\}$ are the Lagrange multipliers. One can devise an iterative algorithm to optimize this function based on the block coordinate descent method, also known as alternating minimization [13]. The idea is to fix the value of all but one of the vector variables in the function and optimize over that variable. One then iterates between different variables optimizing one at a time.

Consider the receiver filters first. As mentioned before, minimization of the total MSE with respect to the receiver filter of user i is equivalent to minimizing MSE_i. This is a simple consequence of the fact that a user's receiver does not affect the MSE of any other user but itself. Thus, if we keep the signature sequences fixed, all receivers need to be set to the MMSE filters for all users. More specifically, for user i, setting the gradient of the Lagrangian with respect to $\mathbf{c}_i$ to zero, we have:

$$\mathbf{c}_i = \sqrt{p_i}\left(\mathbf{SPS}^\top + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{s}_i \tag{14}$$

Note that all users can update their receivers in a parallel fashion, since the receiver updates are independent of each other.

Next, consider the signature sequence updates. Once again, for user i, we need to optimize the Lagrangian by keeping all other variables fixed. The signature sequence update for user i is found as:

$$\mathbf{s}_i = \sqrt{p_i}\left(p_i\mathbf{C}\mathbf{C}^\top + \alpha_i\mathbf{I}_N\right)^{-1}\mathbf{c}_i \tag{15}$$

where α_i is the Lagrange multiplier which needs to be chosen such that the resulting signature sequence $\mathbf{s}_i$ is of unit energy. This is simply accomplished by substituting (15) into the norm constraint for $\mathbf{s}_i$ and using eigen-decomposition of $\mathbf{C}\mathbf{C}^\top = \mathbf{V}\mathbf{D}\mathbf{V}^\top$ with eigenvalue matrix $\mathbf{D} = \text{diag}\{d_1, \cdots, d_N\}$. Simple algebra reveals that we should choose α_i such that

$$\sum_{j=1}^{N} \frac{p_i x_j^2}{(p_i d_j + \alpha_i)^2} = 1 \tag{16}$$

with $\mathbf{x} = \mathbf{V}^\top\mathbf{c}_i$.

Note that the signature sequence update of user i depends on all users' receiver filters and not the signature sequences of the other users. Thus, all users can update their signature sequences in parallel as well. Note also that each signature sequence update closely resembles an MMSE-type update. More specifically, the update replaces the signature sequence of the ith user by a generalized MMSE receiver filter of a system where the signature sequences of the users are their receiver filters, and all users have equal powers equal to the power of user i, p_i. Lastly, we note that the signature sequence update is similar to one of the updates proposed in [14] for multipath channels. Iterative algorithms that aim at minimizing the total MSE have previously been presented in [14,15] without any claims on their convergence.

The convergence of the overall algorithm is established by first observing that each update decreases the total MSE function which is bounded from below, and then investigating the properties of the fixed points of the algorithm. Let us consider a complete cycle of signature sequence and receiver filter updates for all users: $\mathbf{S} \rightarrow \mathbf{C} \rightarrow \hat{\mathbf{S}}$. The receiver filters $\mathbf{C}$ are obtained from the signature sequence set $\mathbf{S}$ using (14) for all users. Then, the next set of signature sequences, $\hat{\mathbf{S}}$, are obtained from $\mathbf{C}$ using (15) for all users. The signature sequence of the ith user is obtained using (15) by inserting $\mathbf{C}$ given in terms of $\mathbf{S}$ in (9):

$$\begin{aligned}\hat{\mathbf{s}}_i &= \left[\mathbf{S}\mathbf{P}\mathbf{S}^\top\left(\mathbf{S}\mathbf{P}\mathbf{S}^\top + \sigma^2\mathbf{I}_N\right)^{-1} + \frac{\alpha_i}{p_i}\left(\mathbf{S}\mathbf{P}\mathbf{S}^\top + \sigma^2\mathbf{I}_N\right)\right]^{-1}\mathbf{s}_i \\ &= \mathbf{B}_i\mathbf{s}_i \end{aligned} \tag{17}$$

At the fixed point of the algorithm $\hat{\mathbf{s}}_i = \mathbf{s}_i$, for all i. This implies that the signature sequence of user i, $\mathbf{s}_i$, should be an eigenvector of $\mathbf{B}_i$ with eigenvalue 1. Moreover, we note that the eigenvectors of $\mathbf{B}_i$ are the eigenvectors of $\mathbf{S}\mathbf{P}\mathbf{S}^\top$. Therefore, the fixed point signature sequences satisfy,

$$\mathbf{S}\mathbf{P}\mathbf{S}^\top\mathbf{s}_i = \lambda_i\mathbf{s}_i \qquad i = 1, \cdots, K \tag{18}$$

When the signature sequences satisfy (18), the corresponding fixed point receiver filters obtained from $\mathbf{S}$ using (14) are scaled matched filters, i.e., $\mathbf{c}_i = \beta_i\mathbf{s}_i$.

Equation (18) gives a complete description of the set of all possible fixed points. Fixed points are those for which MSE or SIR cannot be improved by linear filtering, i.e., MMSE filters are scaled matched filters. Unfortunately, the set of fixed points described by (18) includes a wide spectrum of signature sequences, ranging from the very best signature sequences that we wish our algorithm to converge to, to the absolutely worst signature sequences. For instance, the all-equal signature sequences, i.e., $\mathbf{s}_i = \mathbf{s}$, which may be considered to be the worst signature sequences as a set, satisfy (18). Note also that the best signature sequences that maximize C_{sum} and minimize MSE and TWSC, satisfy (18) too.

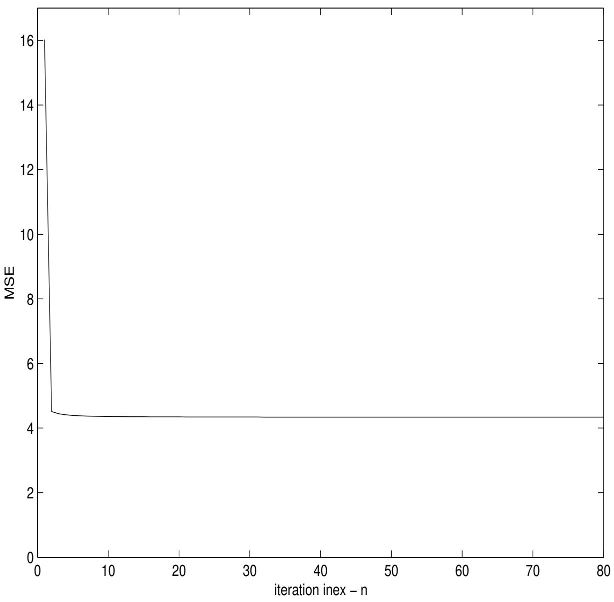

Fig. 1: Total MSE, $N = 6$, $K = 10$, equal powers, $p_i = 1$ for all i.

Next, we note that the fixed points of the algorithm proposed here are the same as the fixed points of the TSC minimization algorithm given in [9] (for equal powers). It was noted in [9] that when the algorithm was started from a set of randomly generated initial signature sequences, it *always* converged to an optimum signature sequence set. This claim is supported by the recent paper [16] which proves that suboptimum fixed points of the MMSE algorithm of [9] are unstable, in that the iterations never converge to any one of them unless they are started exactly with one of them. In fact, [16] considers the more general unequal (arbitrary) received powers for the users, hence its results apply to the algorithm proposed here without any need for further generalization. Therefore, we conclude that the algorithm proposed here converges to an optimum signature sequence set and corresponding receivers with probability one with random starting points.

IV. Numerical Results

In this section, we present numerical examples to support our analysis. We consider example CDMA systems with processing gain $N = 6$ and investigate different scenarios. In all experiments, the initial signature sequences are created randomly. All users update their receivers in parallel once followed by their update of the signature sequences in parallel once, thus a total of $2K$ updates are done between iterations n and $(n+1)$ of the algorithm.

Our first example is a system with $K = 10$ users each with equal received powers which we set to unity. Figure 1 shows the total MSE which monotonically decreases and converges to its minimum possible value. Figure 2 shows the minimum and maximum eigenvalues of $\mathbf{S}\mathbf{S}^\top$ which converge to $K/N = 1.67$, verifying that the resulting signature sequence set satisfies $\mathbf{S}\mathbf{S}^\top = K/N\mathbf{I}_N$, i.e., it is a WBE set [5].

Next, we consider the same system where two of the users' received powers are replaced with $p_1 = 10$ and $p_2 = 5$. The remaining eight users have unity received powers. Using the algorithm given in [6] to identify the oversized users, it is easy to see that, in

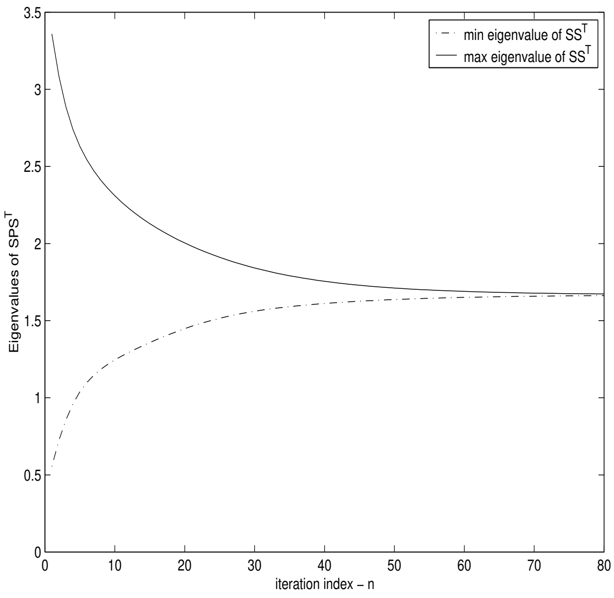

Fig. 2: Minimum/maximum eigenvalues of $\mathbf{SS}^\top$ for the system in Fig. 1.

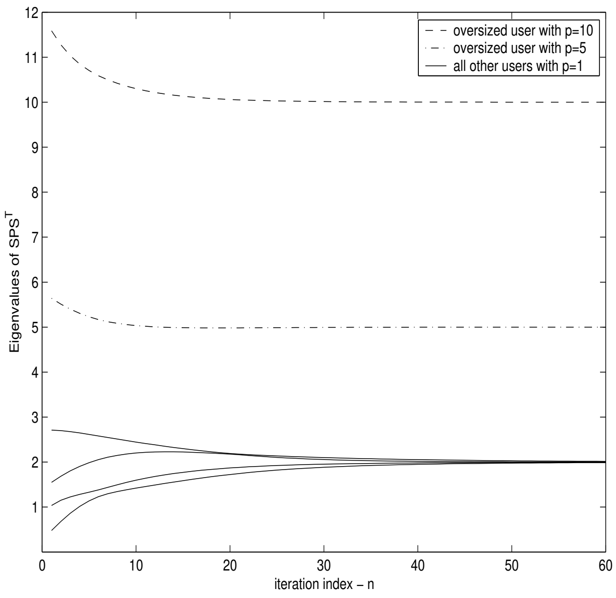

Fig. 4: Eigenvalues of $\mathbf{SPS}^\top$ for the system in Figure 3.

this system, the two users with higher powers are oversized. Thus, the optimum signature sequence set dedicates each of the oversized users their own signal dimensions, and the rest of the users are assigned generalized WBE sequences in the remaining 4 dimensions. The eigenvalues of $\mathbf{SPS}^\top$ with the optimum signature sequence set are $\lambda_1 = 10$, $\lambda_2 = 5$, and $\lambda_k = 2$ for $k = 3, \cdots, 6$. Figure 4 shows the evolution of the eigenvalues of $\mathbf{SPS}^\top$ and their convergence to the optimum values as we run the iterative transmitter receiver optimization algorithm with the corresponding MSE values plotted in Figure 3.

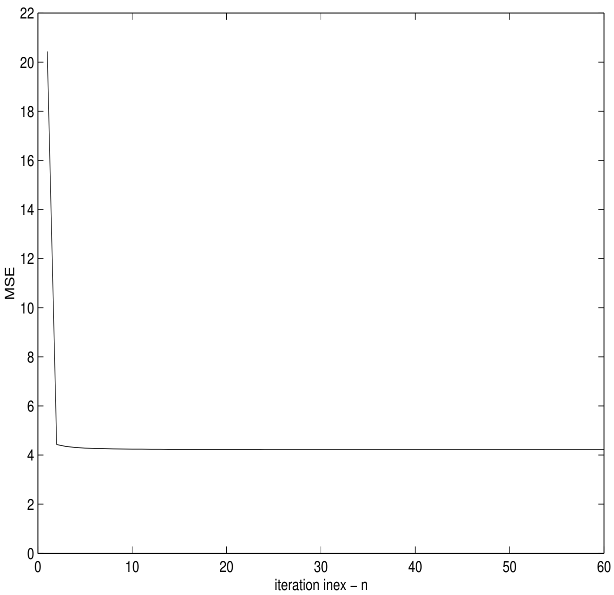

Fig. 3: Total MSE, $N = 6$, $K = 10$, two oversized users.

The last example we consider is one with $K = 6$ users and thus is not an over-saturated system in contrast with the previous two examples. The optimum signature sequence set is one where all six signature sequences are orthogonal to each other irrespective of the received powers. For this example, we have used the received powers of $p_i = i$ for $i = 1, \cdots, 6$. Figure 5 shows the MSE values as we run the iterative algorithm converging to the sum of 6 single-user MSE values as the signature sequences converge to an orthonormal set. Figure 6 shows the evolution of the eigenvalues of $\mathbf{SPS}^\top$ converging to the optimal values, i.e., the received powers of the users.

V. Conclusion

The algorithm proposed in this paper is one where users iteratively update their transmitters (signature sequences) and receivers. The receiver updates depend on the signature sequences of all users while the transmitter updates depend on the receivers of all users. Thus, users do not need to be scheduled for transmitter updates as is required for the convergence proof of the TSC minimization based MMSE or eigen update algorithms [7–10]. Furthermore, the algorithm can be implemented online and receivers can be constructed in an adaptive or blind adaptive fashion at each iteration of the algorithm [4, 17]. The algorithm is shown to converge to the joint optimum transmitters and receivers with probability one with random initial points. We have presented numerical results that support the analysis under different system scenarios.

It is worth noting that, the MMSE algorithm of [9] can be viewed as a sequential (one user at a time) transmitter–receiver update algorithm where the receivers are instantaneously set to matched filters. To that end, a related problem is the convergence of the parallel version of the MMSE update of [9].

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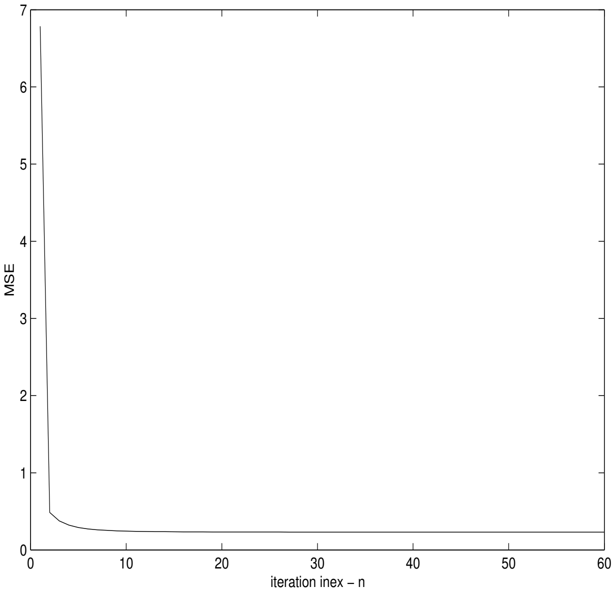

Fig. 5: Total MSE, $N = 6$, $K = 6$, $p_i = i$ for all i.

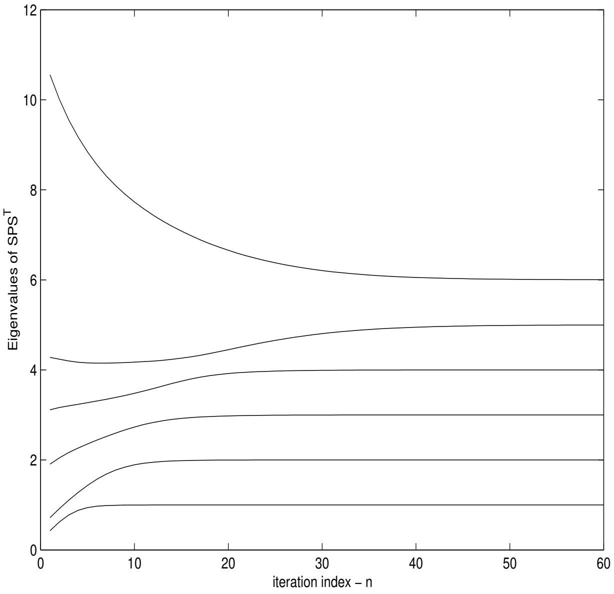

Fig. 6: Eigenvalues of $\mathbf{SPS}^\top$ for the system in Figure 5.

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