CCITT
1989-1992

Commision d'etude | Study Group | Comision de Estudo — XVII

Contribution tardive
Delayed Contribution
Contribucion tardia

Ft. Lauderdale, 6-8 February 1991

Question: 3/XVII
WP: XVII/1

Texte disponible seulement en
Text available only in
Texto disponible solamente en

SOURCE: IBM/EUROPE

TITLE: ROTATIONALLY INVARIANT 64-STATE TWO-DIMENSIONAL TRELLIS CODE

Abstract

A nonlinear 64-state trellis code for two-dimensional (2-D) signals is presented. The code is invariant under 90° rotations. Compared to the nonlinear 8-state code specified in V.32/V.32bis, V.33, and V.17, an additional effective coding gain of 1.1 dB at an error-event probability of 10^{-6} is achieved.

Code objectives for V.FAST

An ideal modulation code for an "ultimate" modem recommendation should have the following properties:

- Significant coding gain over current nonlinear 8-state code.
- Invariance under 90° carrier-phase rotations.
- Differential precoding/postdecoding at no extra cost.
- Moderate decoding delay.
- Multi-rate capability easy to achieve.

In addition, the existence of reduced-complextity decoding algorithms would be desirable, by which appreciable coding gains can still be achieved, if full Viterbi decoding is prohibitive for initial implementations.

Nonlinear 64-state 2-D trellis code

Figure 1 depicts the encoder for the proposed a 64-state trellis code. The derivation of this code, and the construction of an encoder for it, will be described in a forthcoming paper by S.S. Pietrobon, D. Costello, and G. Ungerboeck. The code appears to offer the desired properties. Work on suboptimal decoding is still in progress. Conclusive results cannot yet be given.

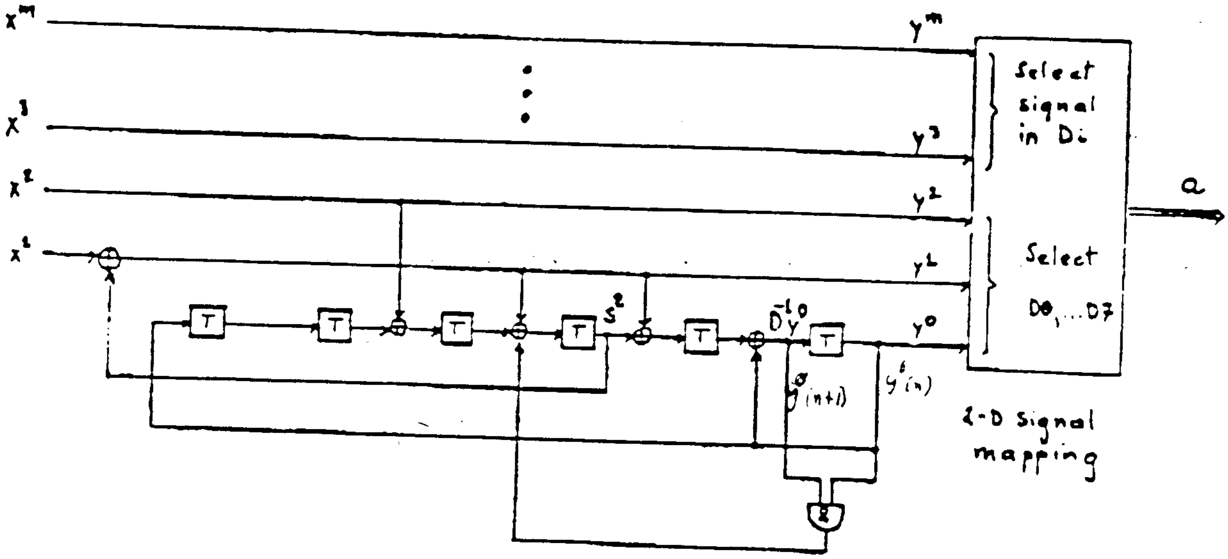


Figure 1. Nonlinear 64-state trellis encoder.

The 64-state nonlinear 2-D code exhibits the following properties:

- Effective coding gain of 1.1 dB over the CCITT 8-state code.
- Invariance under 90° carrier-phase rotations.
- Differential encoding integrated in code trellis.

The code satisfies the nonlinear parity-check equation

$$D^4\, y^2(D) \oplus [D^3 \oplus D^2]\, y^1(D) \oplus [D^6 \oplus D \oplus 1]\, y^0(D) = D^3 y^0(D) \cdot D^2 y^0(D) \, ,$$

(write eq. for $y^0(n)$)

where • denotes the element-wise applied *logical-and* operation. The parity-check equation remains satisfied, if as a result of a 90° rotation of the 2-D signals (see Figure 2), $y^1(D)$ and $y^0(D)$ are replaced by $y_r^1(D) = y^1(D) \oplus y^0(D)$ and $y_r^0(D) = y^0(D) \oplus 1(D) = \bar{y}^0(D)$.

$$\begin{array}{r} (y^1, y^0) \\ +\ (0, 1) \\ \hline (y^1 \oplus y^0, \bar{y}^0) \end{array}$$

(2 bit adder sum mod 4

Differential encoding is based on the rotation-invariant expression

$$\begin{aligned} x^1(D) &= s^2(D) \oplus y^1(D) \\ &= s^2(D) \oplus y^1(D) \oplus \overbrace{D^{-1}y^0(D) \oplus D^{-1}y^0(D)}^{0} \\ &= D^{-2}y^0(D) \oplus D^{-1}y^0(D) = D^{-2}y_r^0(D) \oplus D^{-1}y_r^0(D)\ . \end{aligned}$$

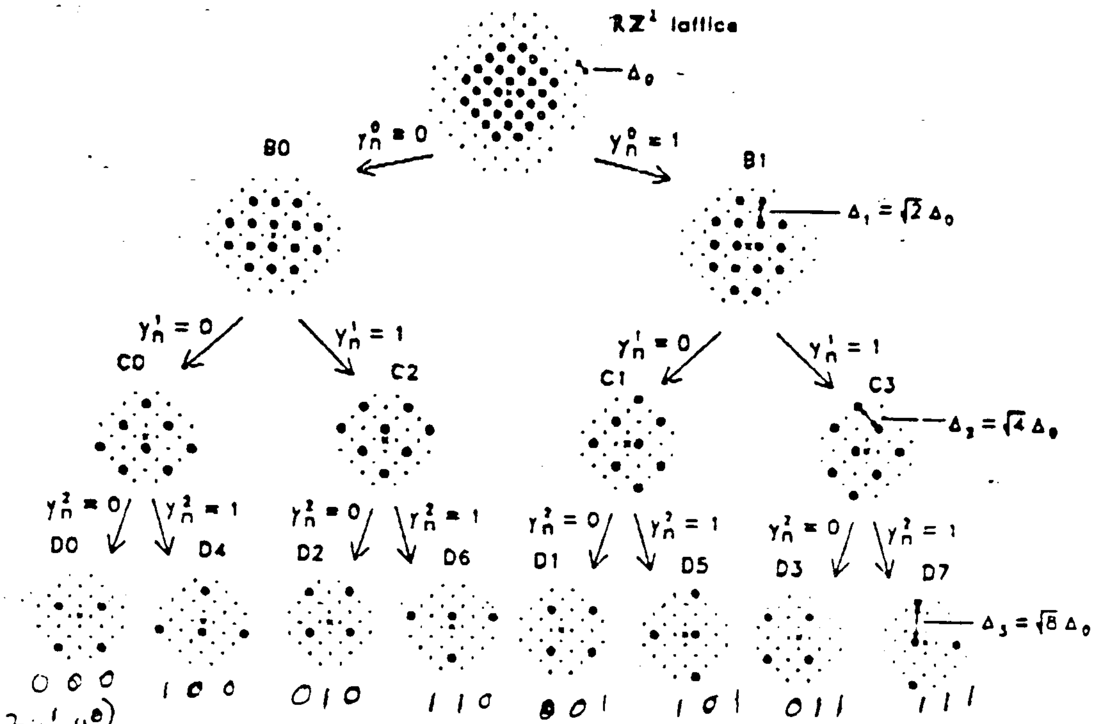
$\bar{y}^0 \qquad \bar{y}^0$

$D^{-2}y^0$ from coder diagram $D^{-1}y^0 = y^0 + D[y^1 + s^2]$ so $D^{-2}y^0 = D^{-1}y^0 + y^1 + s^2$

Signal sets for multi-rate operation

The standard *mapping-by-set-partitioning* rule (G. Ungerboeck, "Channel coding with multilevel/phase signals", *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 55-67, 1982) is applied to obtain labeled signal subsets. Figure 2 illustrates the employed partitioning of the rotated 2-D integer lattice RZ^2 into 8 subsets, D0, D1, ... D7. The subsets D1,D2,D3 are obtained by successive 90° rotations of subset D0. The same holds for D4,D5,D6,D7. Notice that the CCITT 8-state code employs a different labeling scheme.

For transmission of m bits per 2-D signal, each subset contains $2^{m+1}/8 = 2^{m-2}$ signals. The signals for the D0 and D4 subsets are chosen from the D0- and D4-sublattices, respectively, by an algorithm which assigns signal indices to sublattice points in ascending order of their distance from the origin (and also observes symmetries). Thus, the cumbersome definition of signal sets of various sizes by figures as found in recommendations V.32/V.32bis, V.33, and V.17 is avoided. Instead a consistent algorithmic specification is employed. For implementation, an algorithmic definition offers the additional benefit that signal sets can be created dynamically.



$n = (y^2 y^1 y^0)$

So: D0 → D1 → D2 → D3 ; D4 → D5 → D6 → D7

D_n

Two rotation groups selected by $y^{(2)}$
rotation within group selected by $(y^1 y^0)$

Figure 2. Signal subsets D0, D1, ... D7.

Error performance and decoding complexity

Figure 3 shows error-event probabilities versus signal-to-noise ratios achieved with the proposed 64-state code and the CCITT 8-state code for transmission of m = 6 bits per 2-D signal. Approximate lower and upper bounds are given. The bounds were computed based on an evaluation of the three smallest Euclidean distances and their multiplicities, in the two codes. For unbounded subset decoding, the following values were obtained:

	d_1^2/Δ_0^2	N_1	d_2^2/Δ_0^2	N_2	d_3^2/Δ_0^2	N_3	SNR in dB for $P_{ev} = 10^{-4}$	SNR in dB for $P_{ev} = 10^{-6}$
Nonlinear 8-state code	5	16	6	72	7	320	21.95/22.20	24.81/24.83
Nonlinear 64-state code	7	80	8	300	9	*1175	21.13/21.40	23.70/23.74

In the investigated class of nonlinear 64-state codes, no code with $N_1 < 80$ was found. The asterisk (*) in the table indicates that the given value of N_3 is an average value.

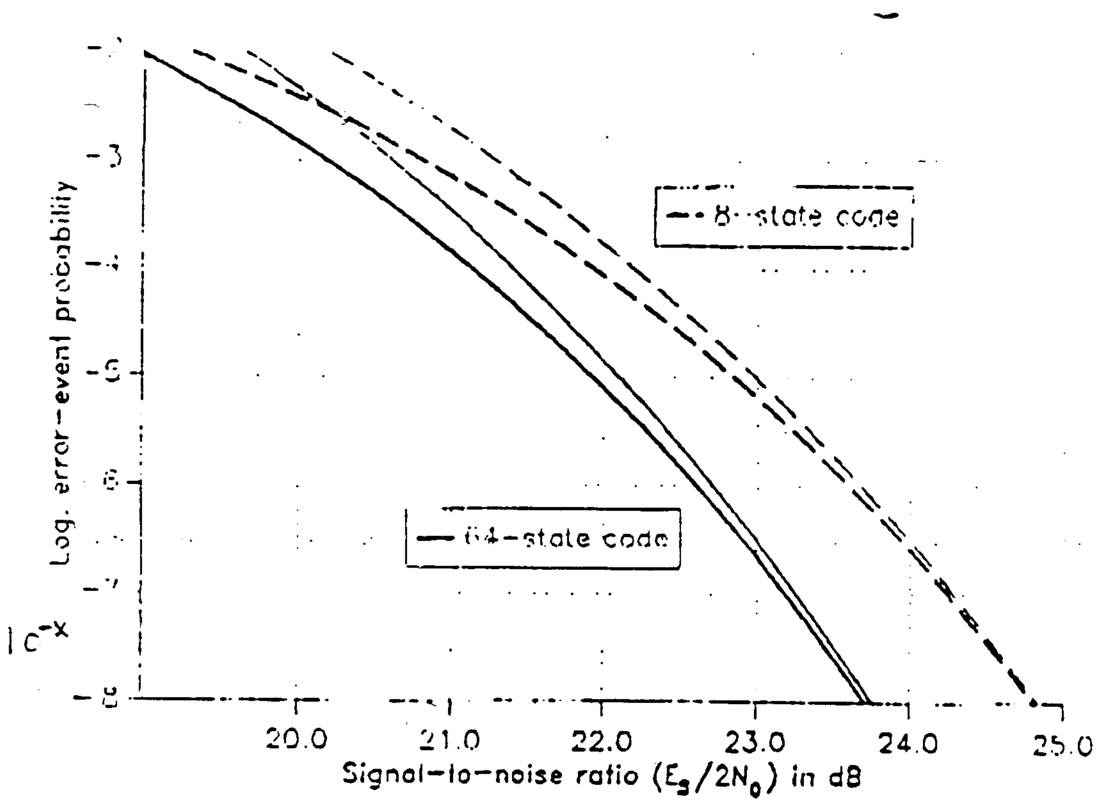


Figure 3. Comparison of error-event probabilities (6 bits per 2-D signal).

For full Viterbi decoding, the 64-state code requires eight times more trellis-decoding operations than needed for the CCITT 8-state code, with identical effort for subset decoding (as is obvious).

The relative position of a similar linear 64-state 2-D code in terms of effective coding gain and decoding complexity is illustrated in Figure 4 (the figure was presented in Delayed Document D-92: "Advances in modem technology since recommendation V.32"; originally it was given by G.D. Forney in "Coset codes - Part I: Introduction and geometrical classification", *IEEE Trans. Inf. Theory*, vol. IT-34, pp. 1123-1151, 1988). The figure shows that 2-D codes are superior to 4-D and 8-D codes, if decoding complexities of more than about four times the complexity of the CCITT 8-state code are allowed. Other advantages of 2-D codes over higher dimensional codes are: simple subset decoding, shorter decoding delay, the same decoding operations in every modulation interval, tentative decisions with zero-delay available.

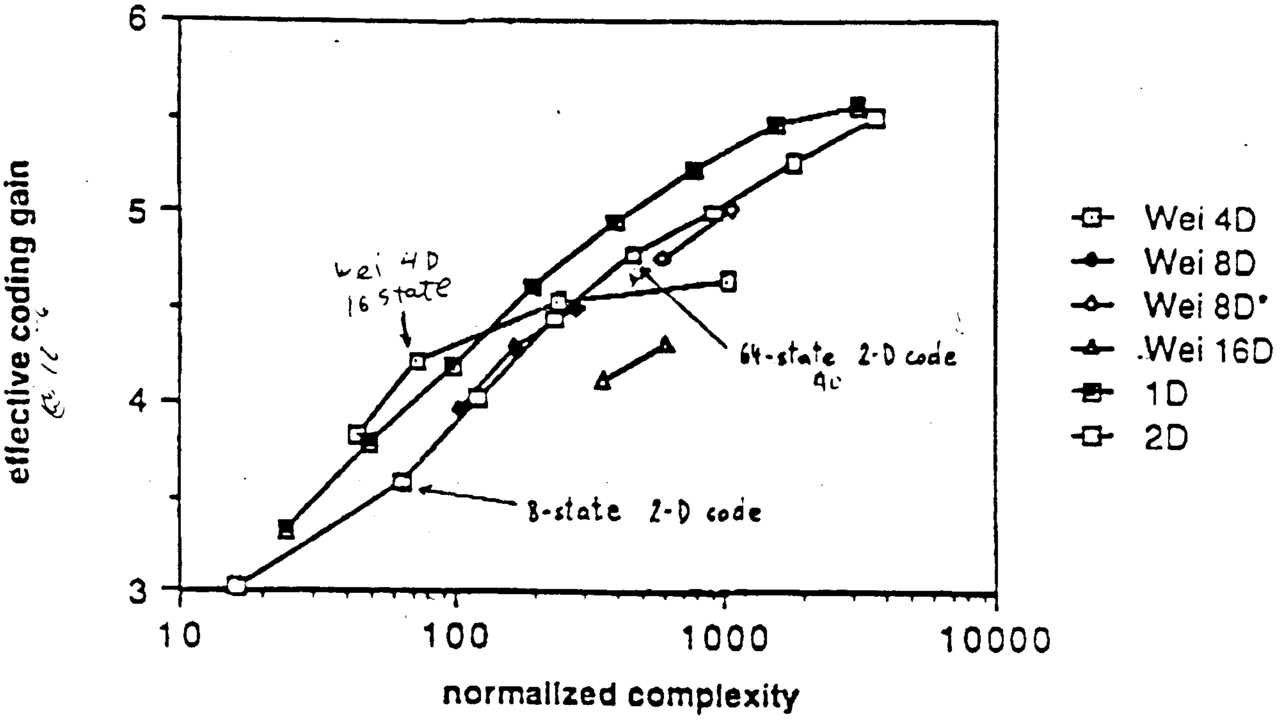


Figure 4. Performance vs. complexity of 1-D to 16-D trellis codes. (from Delayed Contribution D-92).

The 64-state code can be combined with trellis shaping/precoding, as introduced by G.D. Forney and V. Eyuboglu in recent papers, to achieve an additional coding gain of 0.8-1.0 dB (see references given in Delayed Document D-92). In this case, signal labelling within subsets D0 and D4 should be based on continued set partitioning until each signal in these subsets receives a unique label. Again, an algorithmic definition should be preferred over explicite specification by tables,