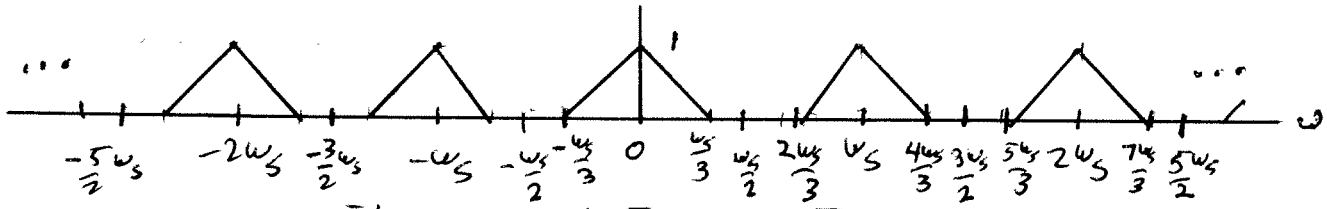
Solutions to ENEE 425 Exam 2
Spring 2010

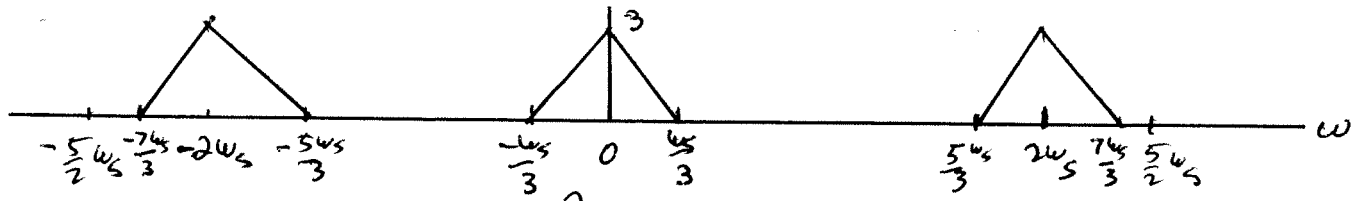
1. (a) Add a $\emptyset$ between each sample. So

$$x_e(nT/2) = \begin{cases} x(mT) & \text{for } n = 2m \\ 0 & \text{for } n \text{ odd} \end{cases}$$

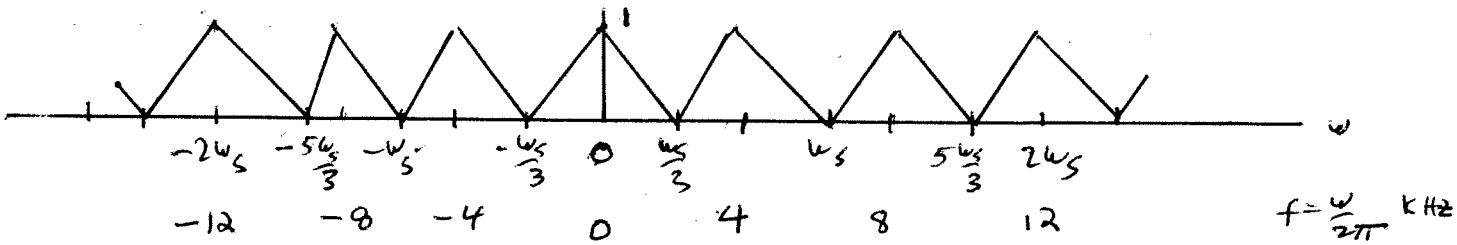
(b) $X_e(e^{j\omega T/2}) = X(e^{j\omega T})$ with Nyquist band $|\omega| < \omega_s$



(c) $Y_e(e^{j\omega T/2}) = X_e(e^{j\omega T/2})\, H(e^{j\omega T/2})$



(d) $$Y(e^{j\omega 3T/2}) = \frac{1}{3}\sum_{n=0}^{2} Y_e\left(e^{j(\omega - n\frac{2}{3}\omega_s)T/2}\right)$$

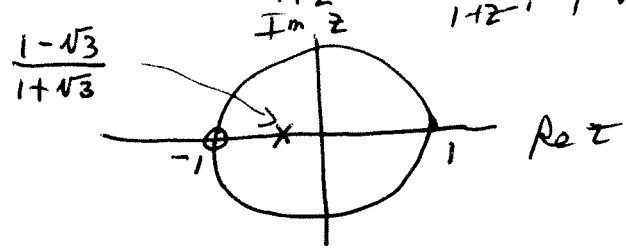


2. (a) Poles: $s_k = \Omega_c\, e^{j\left(\frac{\pi}{2} + \frac{\pi}{2N} + k\frac{\pi}{N}\right)}$ $\quad k = 0, 1, \ldots, N-1$

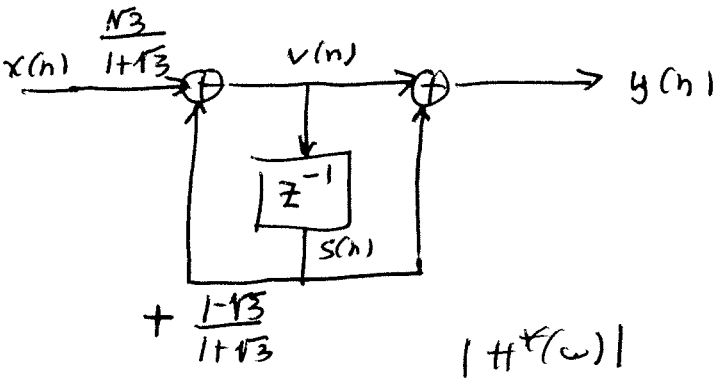
Prewarp $\Omega_c = \tan\frac{\omega_c T}{2} = \tan\frac{\omega_s}{3}\frac{T}{2} = \tan\frac{\pi}{3} = \sqrt{3}$

For $N=1$ $\quad H(s) = \dfrac{1}{\frac{s}{\sqrt{3}} + 1} = \dfrac{\sqrt{3}}{s + \sqrt{3}}$

(b) $$H(z) = H(s)\Big|_{s = \frac{1-z^{-1}}{1+z^{-1}}} = \frac{\sqrt{3}}{\frac{1-z^{-1}}{1+z^{-1}} + \sqrt{3}} = \frac{\sqrt{3}}{1+\sqrt{3}}\;\frac{1+z^{-1}}{1 - \frac{1-\sqrt{3}}{1+\sqrt{3}}z^{-1}}$$



2 (c)

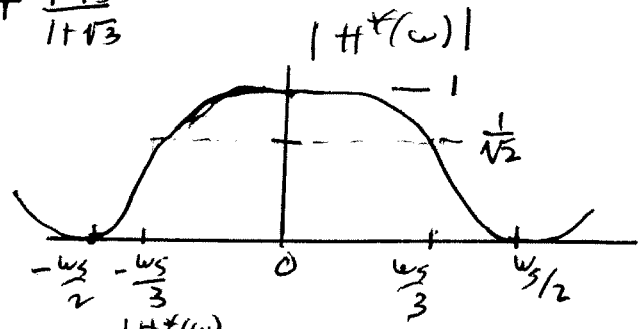


$$v(n) = \frac{\sqrt{3}}{1+\sqrt{3}}\,x(n) + \frac{1-\sqrt{3}}{1+\sqrt{3}}\,s(n)$$

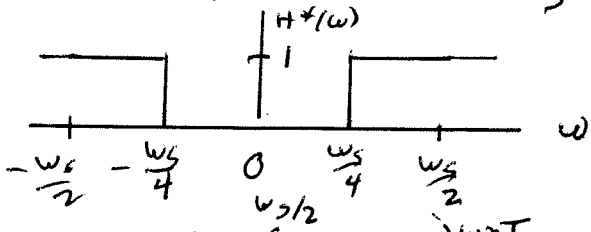
$$y(n) = v(n) + s(n)$$

$$s(n+1) = v(n)$$

(d)



3. (a)



$$h_0(n) = \frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} H^*(\omega)\, e^{j\omega nT}\, d\omega = \frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} \underbrace{H^*(\omega)\cos\omega nT}_{ev\times ev = ev}\, d\omega + j\,\frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} \underbrace{H^*(\omega)\sin\omega nT}_{ev\times odd \,\to\, odd}\, d\omega \;\to\; 0$$

$$= \frac{2}{\omega_s}\int_{\omega_s/4}^{\omega_s/2} 1\times\cos\omega nT\, d\omega = \frac{2}{\omega_s}\,\frac{\sin\omega nT}{nT}\Big|_{\omega=\omega_s/4}^{\omega_s/2}$$

$$= \frac{1}{n\pi}\left(\sin n\pi - \sin\frac{n\pi}{2}\right) = -\frac{\sin n\pi/2}{n\pi}$$

$$h_1(n) = -\frac{\sin\frac{n\pi}{2}}{n\pi}\left[0.54 + 0.46\cos\frac{2\pi}{N}n\right] p_k(n), \quad k = \frac{N-1}{2}$$

Add delay of k samples

$$H(z) = z^{-k}\sum_{n=-k}^{k} h_1(n)\, z^{-n}$$

(h) $N \geq 4\dfrac{\omega_s}{\omega_t} = 4\dfrac{\omega_s}{\omega_s/25} = 100$

4. (a) $y = 1\times H(1) = \dfrac{1}{1-0.95} = 20$

(b) $y - \dfrac{q}{2B(1)} < y_2 \leq y + \dfrac{q}{2B(1)} \Rightarrow 20 - \dfrac{1}{2\times 0.05} < y_2 \leq \dfrac{1}{2\times 0.05} + 20$

Deadband $10 < y_2 \leq 30$

(c)

n	-1	0	1	2	3	...
$y_1(n)$		31.4	30.45	29.5	29.5	...
$y_2(n)$	32	31	30	30	30	...

(d) The output stays at the value set in the deadband