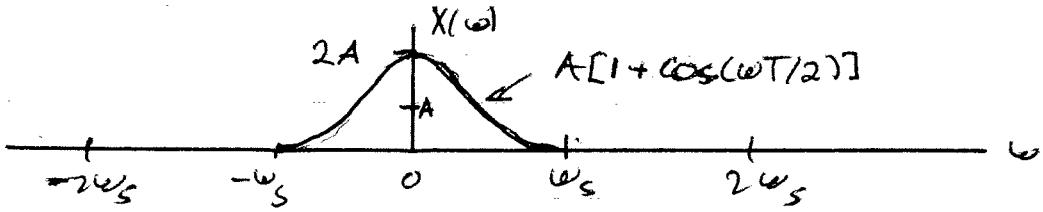
ENEE 425 Spring 2010

EXAM 1 Solutions

1.

(a)



(b) $X^*(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

For $0 < \omega < \omega_s$ $\quad X^*(\omega) = \frac{A}{T}[1+\cos(\omega T/2)] + \frac{A}{T}[1+\cos(\omega-\omega_s)T/2]$

$= 2A/T$

So $X^*(\omega) = 2A/T$ for all ω

(c)

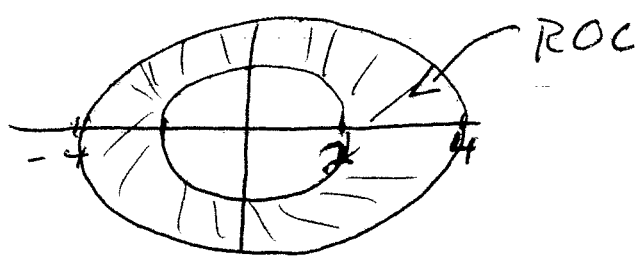
$$x(nT) = \frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} X^*(\omega)\, e^{j\omega nT}\, d\omega = \begin{cases} 2A/T & \text{for } n=0 \\ 0 & \text{otherwise} \end{cases}$$

2.

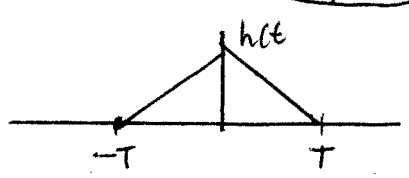
$$X(z) = \sum_{n=-\infty}^{0} 4^n z^{-n} + \sum_{n=0}^{\infty} 2^n z^{-n} - 1$$

$$= \sum_{n=0}^{\infty} (4^{-1}z)^n + \sum_{n=0}^{\infty} 2^n z^{-n} - 1$$

$$= \underbrace{\frac{1}{1-\frac{1}{4}z}}_{\text{for } |z|<4} + \underbrace{\frac{1}{1-2z^{-1}}}_{\text{for } |z|>2} - 1 \qquad \text{for } 2<|z|<4$$



3. (a)



(b) $h(t) = \frac{t+T}{T}u(t+T) - 2\frac{t}{T}u(t) + \frac{t-T}{T}u(t-T)$

$$H(s) = \frac{e^{sT} - 2 + e^{-sT}}{Ts^2} = T\left[\frac{e^{+sT/2} - e^{-sT/2}}{2sT/2}\right]^2$$

(c)
$$H(j\omega) = T\left[\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j\ \omega T/2}\right]^2 = T\left(\frac{\sin \omega T/2}{\omega T/2}\right)^2$$

4.
$$Y(z) - 1.5\left[z^{-1}Y(z) + \overset{1}{y(-1)}\right] + 0.5\left[z^{-2}Y(z) + \cancel{y(-2)}^{\,0} + z^{-1}\overset{1}{y(-1)}\right]$$
$$= \frac{1}{1-z^{-1}} - \frac{z^{-1}}{1-z^{-1}}$$

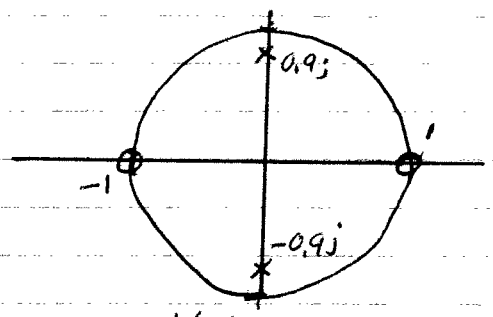
$$Y(z)\left(1 - 1.5z^{-1} + 0.5z^{-2}\right) - 1.5 + 0.5z^{-1} = 1$$

$$Y(z) = \frac{2.5 - 0.5z^{-1}}{(1-z^{-1})(1-0.5z^{-1})} = \frac{\frac{2.5-0.5}{1-0.5}}{1-z^{-1}} + \frac{\frac{2.5-0.5\times 2}{1-2}}{1-0.5z^{-1}}$$

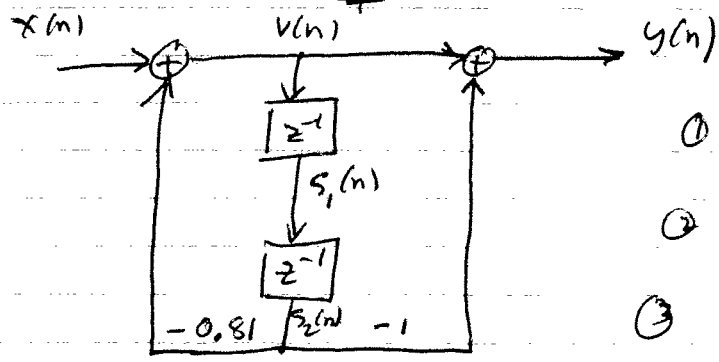
$$= \frac{4}{1-z^{-1}} + \frac{-1.5}{1-0.5z^{-1}}$$

$$y(n) = 4 - 1.5(0.5)^n \quad \text{for } n \geq 0$$

5.
(a)



(b)

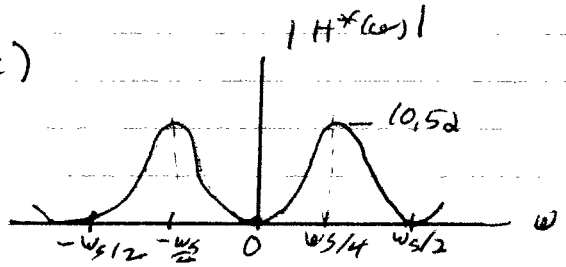


① $v(n) = x(n) - 0.81\, s_2(n)$

② $y(n) = v(n) - s_2(n)$

③ $s_2(n+1) = s_1(n)$
$s_1(n+1) = v(n)$

(c)



$H^*(0) = H^*(\omega_s/2) = 0$

$H^*\left(\frac{\omega_s}{4}\right) = H\left(e^{j\frac{\omega_s}{4}T}\right) = H\left(e^{j\frac{\pi}{2}}\right) = H(j)$

$$= \frac{j^2 - 1}{j^2 + 0.81} = \frac{-2}{-0.19} = \frac{2}{0.19} = 10.52$$

(d)