COMPRESSED SENSING:
FROM ALGORITHMS TO CIRCUITS

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Theory/Introduction

1. E. Candès, J. Romberg, T Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information", IEEE Trans. on Information Theory, 52(2) pp. 489 - 509, February 2006

Decoding/Reconstruction


Circuits and Systems Implementations


### Applications

Journals


Conferences


A few important special issues

**Special Issue on Compressive Sensing**
Gust Editors: R. Chartrand, R. Baraniuk, Y. Eldar, M. Figueiredo, J. Tanner
IEEE J. on Selected Topics in Signal Processing, **April 2010**

**Special Issue on Applications of Sparse Representation & Compressive Sensing**
Gust Editors: R. Baraniuk; E. Candes; M. Elad; Yi Ma
Proceedings of the IEEE, **June 2010**

**Special Issue on Circuits, Systems and Algorithms for Compressive Sensing**
Gust Editors: D. Allstot, R. Rovatti, G. Setti
IEEE J. on Emerging and Selected Topics in Circuits and Systems, **September 2012**
TIME FOR A DECISION....

Maybe it was too theoretical... 😞
• Foreword / On Sampling
• An Introduction to Compressed Sensing
  ▪ Sparse Signals and Basis Expansions
  ▪ Compressed Sensing (and its intuitive view in the "land of vectors")
  ▪ Incoherent acquisition and Sparse Signal Recovery by Linear Programming
  ▪ Improving Compressed Sensing with Rakeness
• Applications
  ▪ Surface ElectroMyoGraphy signal acquisition (with IIT@PoliTo)
  ▪ Embedded Security in CS
• Analog-to-Information Conversion
  ▪ Random Modulation Pre-Integration (RMPI) implementation in 180nm TI CMOS technology (hardware-algorithm co-design)
  ▪ A smart way to deal with saturation
  ▪ Measured Performances
• Digital CS Front End and Signal Feature Extraction in CS domain
• Conclusion
**Nyquist – Shannon Sampling Theorem**: to perfectly reconstruct a finite – bandwidth signal \( x(t) \) from its uniform samples \( x(jT) \), acquire them at sampling rate \( f_s > 2W \) (Nyquist rate)

\[
x(t) \quad f_s = \frac{1}{T} \quad x_j = x(jT)
\]

The theorem guarantees **(sufficient condition!!)** that \( x(t) \) can be reconstructed by the Shannon – Whittaker interpolation formula:

\[
x(t) = \sum_{j=-\infty}^{+\infty} x(jT) \text{sinc} \left( \frac{t - jT}{T} \right)
\]

**Constructive and general approach to signal reconstruction** (no a-priori information on the signal structure!!):
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Constructive and general approach to signal reconstruction (no a-priori information on the signal structure!!):

Most intuitive and used approach to sampling.
Most simple and general decoding/reconstructing solution
**A problem:** High sampling rates ⇒ high dynamic power consumption – a motivating hardware issue to try and lower the sample rate as much as possible!

... but Nyquist-Shannon theorem gives only a **sufficient condition** for general signal reconstruction..... So we trade **generality** for lower sampling rates when acquiring signals whose **structural properties** allow sampling below the Nyquist rate.

Consider this toy example: \[ x(t) = \sum_{k=1}^{8} \sin(2\pi f_k) \]

\[ \{f_k/f_s\} = \{0.05, 0.075, 0.11, 0.12, 0.2, 0.22, 0.43, 0.44\} \]

Although \( x(t) \) is a sum of eight tones, its bandwidth is large / it would require a sampling rate much higher than its actual information content.

\( x(t) \) has a very simple representation (very few terms) w.r.t. the Fourier basis in \([0, nT]\).
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Consider this toy example: \( x(t) = \sum_{k=1}^{8} \sin(2\pi f_k t) \)

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\( x(t) \) has a very simple representation (very few terms) w.r.t. the Fourier basis in \([0, nT]\).

The information content of this signal is much less than what \( W \) indicates.
How many significant terms are there in the basis representation of $x(t)$?

Let a generic basis expansion:

$$x(t) = \sum_{j=0}^{n-1} \alpha_j \psi_j(t)$$

$$\alpha_j = \langle x(t), \psi_j(t) \rangle_{[0,nT]} = \int_0^{nT} x(t) \psi_j(t) dt$$

We define sparse any signal with very few nonzero coefficients in its basis expansion w.r.t. the dimensionality of the sparsity basis $\Psi_{n \times n}$.

Our example $x(t)$ is 8-sparse. In general, a $k$-sparse signal is such that

$$x = \Psi \alpha, \; \alpha \in \mathbb{R}^n, \; k = \# \{ j \in [0, n-1] : \alpha_j \neq 0 \}$$

Note:

A signal may be sparse in some bases and dense in others. Finding a basis or dictionary such that $k < n$ coefficients represent most of the information/energy in a signal also means you have found a way to compress it after acquisition (trivial compression: discard $n - k$ negligible coefficients).
Examples: wavelet domain compression of images, biosignals; audio coding.

**ECG signals** are sparse w.r.t. a *(real)* **Gabor dictionary** – a set of scaled and shifted, Gaussian-windowed cosines.

If one could "sample" only the most significant (red) coefficients, the sampling operation would be much more efficient....
Examples: wavelet domain compression of images, biosignals; audio coding.

**ECG signals** are sparse w.r.t. a (real) **Gabor dictionary** – a set of scaled and shifted, Gaussian-windowed cosines.

If one could "sample" only the most significant (red) coefficients, the sampling operation would be much more efficient....
**Good News:** most signals are sparse when expressed in a proper basis (all biosignals, even images, radar, radiotelescope...)... **Bad News:** but how can we acquire them?

**Remark:** the sampling operation is a linear projection

\[
x(jT) = \int_0^{nT} x(t) \delta(t - jT) dt
\]

\[
= \langle x(t), \delta(t - jT) \rangle_{[0,nT]}
\]

\[
= \langle x(t), \phi_j(t) \rangle_{[0,nT]}
\]

---

How should we choose \( \phi_j(t) \) to get the information?

Projection onto \( \phi_j(t) \) to which one needs to add (projection independent) noise
- **Good News**: most signals are sparse when expressed in a proper basis (all biosignals, even images, radar, radiotelescope...)...
  - **Bad News**: but how can we acquire them?

- **Remark**: the sampling operation is a linear projection

Assume that the signal to acquire is represented by a vector $\mathbf{x}$ which is sparse since it has only one component wrt the coordinate system $(a_0, a_1, a_2)$

If I know that only one component is non-null, I do not acquire all 3 components, but only the projection (i.e. 2 components only) on a plane which must be aligned to $(a_0, a_1, a_2)$ to have non-zero projections ($\theta_j \neq \pi/2$)

...and reconstruct the original signal from projections.
Compressive Sensing (E. Candès, D. Donoho 2006)

- Signal $x(t)$ defined for $0 \leq t \leq nT$
- Capturing signal information content by projections on a proper set of different waveforms
  \[
  y_j = \int_0^{nT} x(t) \phi_j(t) \, dt = \langle x(t), \phi_j(t) \rangle_{[0,nT]}
  \]

- **Compressive Sensing**: by using a proper set of sampling function $\phi_j(t)$, we can get signal information content by using only $m$ waveforms (with $m \leq n$)
Compressed Sensing (E. Candès, D. Donoho 2006) proposes a radically new approach to sampling:

- **Input Vector**: $x \in \mathbb{R}^n$ (e.g. Nyquist rate samples), $x = \Psi_{n \times n} \alpha$, where $\Psi_{n \times n}$ is the sparsity basis, $\alpha$ is $k$-sparse.

- **Sampling Operator**: Let $\Phi_{m \times n}$ be a linear operator with $m < n$ that is incoherent to $\Psi$, i.e., the coherence:

$$\mu(\Phi, \Psi) = \sqrt{n} \max_{j,l} |\langle \phi_j, \psi_l \rangle|$$

is as small as possible, i.e. the rows of $\Phi$ are properly aligned to the sparsity basis vectors to always collect information at every sample.

A $n$-dimensional, $k$-sparse signal may be acquired in a compressed form directly at the A/D interface, thus allowing a sample rate reduction.
\[ y = \Phi_{m \times n} x = \Phi_{m \times n} \Psi_{n \times n} \alpha = \Theta_{m \times n} \alpha \]

- **Sampling=Projection:** We will measure \( y = \Phi_{m \times n} x = \Phi_{m \times n} \Psi_{n \times n} \alpha = \Theta_{m \times n} \alpha \)
  - The linear operator \( \Theta_{m \times n} \) clearly performs a dimensionality reduction since it shrinks \( \mathbb{R}^n \to \mathbb{R}^m \).
1. Solving \( y = \Theta \alpha \) in \( x \) has infinitely many solutions in general because it is an **underdetermined** system of equations \((m < n!!)\).

How can one solve this????

2. Is there a way for ensuring the "best choice" for \( \Phi_{m \times n} \) in presence of a projection independent noise?

One needs to choose the projection plane to maximize the projection of each vector (i.e. "conserve" its length and energy)!!

choose \( \theta_0, \theta_1, \theta_2 \) such that

\[
\sum_j \sin^2 \theta_j = 1 \quad \text{and} \quad \cos \theta_j = \max \forall j
\]
**Second issue:** (*Restricted Isometry Property - RIP*) one requires that matrix $\Phi_{m \times n}$ is "almost an isometry", i.e. formally that, it $\exists \delta_k \in [0,1]$ such that

$$(1 - \delta_k) \| \alpha \|^2_2 \leq \| \Phi \alpha \|^2_2 \leq (1 + \delta_k) \| \alpha \|^2_2$$

i.e. the "energy of $\alpha$" is almost conserved by the projection

**First issue:** (*Sparse Signal Recovery Problem*) one can obtain the solution by solving the following optimization problem

$$\min_{\alpha \in \mathbb{R}^n} \| \alpha \|_0 = \min_{\alpha \in \mathbb{R}^n} \sum_{l=0}^{n-1} \#\{ \alpha_l | \alpha_l \neq 0 \} \text{ s.t. } y = \Theta \alpha$$

Remark: When $x$ is $k$-sparse in $\Psi$, only $k$ of $n$ columns of $\Theta$ are considered: let $S$ be the support of $\alpha$, $y = \Theta \alpha = \Theta_S \alpha_S$.

If an oracle told us the support $S$ and $k < m < n$, we would have to solve an **overdetermined system of equations** by least squares (easy and polynomial-time at worst)
Unfortunately $S$ is unknown in general, and guessing it would take $\binom{n}{k}$ attempts – given that $k$ is known.

- This is a combinatorial problem, it is NP-hard in general.
- Take $n = 1024, k = 8$.
  - Worst-case sparse support search (at $10^{-2}$ s-instance): $2.917 \times 10^{17}$ s.
  - Age of the universe: $4.339 \times 10^{17}$ s ... it is "a bit" unpractical!!

"Engineering approach":

1. Which problem can we solve?
2. Are there cases in which the problem we can solve has the same solution of the problem we must solve?

Consider the convex optimization problem:

$$\min_{\alpha \in \mathbb{R}^n} \|\alpha\|_1 = \min_{\alpha \in \mathbb{R}^n} \sum_{l=0}^{n-1} |\alpha_l| \quad s.t. \quad y = \Theta \alpha \quad \text{(basis pursuit problem)}$$

Linear programming problem (easy and polynomial-time).

Is there a set of cases for which this solves the sparse signal recovery problem?
### Good News: The minimum $1$-norm solution is also the sparsest (minimum $0$-norm) solution, i.e.,

$$\hat{\alpha}(\ell_1) = \hat{\alpha}(\ell_0) = \alpha$$

provided that:

$$m \geq C(\Theta)k \log \left( \frac{n}{k} \right) \quad \text{(in practice } m \approx 4k)$$

where $C(\Theta)$ depends on the structure of $\Theta = \Phi \Psi$.

### There is a flourishing literature on algorithms that find solutions to basis pursuit/sparse signal recovery problems.

**Basis Pursuit (BP):** Basis Pursuit with Denoising (BPDN):

$$\min_{\alpha \in \mathbb{R}^n} \sum_{l=0}^{n-1} |\alpha_l| \quad \text{subject to} \quad y = \Theta \alpha$$

1. In addition to Min-$L_1$, we have CoSaMP, Orthogonal Matching Pursuit, Iterative Hard/Soft Thresholding, Smoothed $L_0$, ... They converge, but may be slow...

2. Designing real-time algorithms which rapidly converge to the sparsest solution at very large problem sizes (large $n$) is an open challenge.
Decoding strategies divided in two family

- **minimization based procedures**
  among all the possible counterimages of the vector $y = \Theta \alpha$ we look for the “most sparse”, i.e., the one with is minimum $\|\alpha\|_0/1$.
  
  reference algorithm: Min-L1

- **iterative support guessing procedures**
  Alternate a rough, non-necessarily sparse, solution of $y = \Theta \alpha$ (pseudoinverse) with an estimate of support of $\alpha$ (for example by thresholding on the magnitudes of the components of the temporary solution) that is then exploited in a pseudoinverse-based step refining the value.
  
  reference algorithm: CoSaMP
Is anybody awake?
**CS**

\[ x = \Psi \alpha \xrightarrow{\Phi} y \xrightarrow{\min \ell_1} \hat{\alpha} \text{ s.t. } \hat{x} = \Psi \hat{\alpha} \simeq x \]

- **Note:** The most universal choice is to choose $\Phi$ at random, since it guarantees that $\Phi$ is incoherent for every $\Psi$. "Most convenient": are Gaussian RV

\[
\Phi_{j,l} \sim \mathcal{N}(0, \Sigma_{\Phi}) \text{ (i.i.d.)} \\
\Sigma_{\Phi} = \mathbb{E}(\Phi^\dagger \Phi) = I_n/n
\]

RIP can be proven
TO RECAP…

\[ x = \Psi \alpha \rightarrow \Phi \rightarrow y \rightarrow \min \ell_1 \rightarrow \hat{\alpha} \quad \text{s.t.} \quad \hat{x} = \Psi \hat{\alpha} \simeq x \]

- must be sparse
- must preserve energy (RIP)
- \( m \ll n \), i.e. less resources for acquisition
- \( \Psi \) and \( \Phi \) must be incoherent
- must be "solvable" (i.e. Min-\( L_1 \)); \( m \) must be large enough wrt \( k \)

- **Binary antipodal \( \{+1, -1\} \)** are also used as hardware-friendly solutions (more later)

\[
\Phi
\]

\[
\phi_{j,l} = \begin{cases} 
1/\sqrt{n}, & p = 1/2 \\
-1/\sqrt{n}, & 1 - p = 1/2 
\end{cases}
\]
- **Input signal**: unit-energy, \( k \)-sparse vectors \( x = \Psi \alpha \) (Non-zero entries in \( \alpha \) are i.i.d. Gaussian RVs with zero mean and variance \( 1/k \))

- \( \Psi_{n \times n} \) is the **discrete Fourier basis** (\( n = 256 \))

- **Measurement matrix**: \( \Phi \in \mathbb{R}^{m \times n} \sim \text{i.i.d Gaussian RVs with zero mean and variance } 1/n. \)

- **Reconstruction Algorithm**:
  \[ \hat{\alpha} = \arg \min \| \alpha \|_1 \text{ s.t. } y = \Phi \Psi \alpha \]
  i.e. Min-\( L_1 \)

- **Figure of Merit**:
  \[ \text{PSR} = P \left[ \frac{\| \alpha \|_2^2}{\| \hat{\alpha} - \alpha \|_2^2} > 10^6 \right] \]
  Probability of Successful Reconstruction

![Figure of Merit Graph]
- **CS of ECG signals:**
  \( m = 55 \) measurements
  Nyquist-rate samples:
  \( n = 256 \) (\( f_n = 256 \) Hz)
- **Compression rate** \((n/m)\): 4.7.
  - \( \Psi \) is a dictionary of Gabor time-frequency atoms
  - \( \Phi_{55 \times 256} \) is an antipodal (pseudo)random measurement matrix
- Intrinsic **SNR=17 dB**
- **Figure of Merit:**
  \[
  \text{ARSNR}_{\text{dB}} = E \left[ 20 \log_{10} \left( \frac{||\alpha||_2}{||\alpha - \hat{\alpha}||_2} \right) \right]
  \]
  (Average Reconstruction SNR)

This case: \( \text{ARSNR} \approx 15 \) dB.
CS of ECG signals: 
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\text{ARSNR}_{\text{dB}} = \mathbb{E} \left[ 20 \log_{10} \left( \frac{||\alpha||_2}{||\alpha - \hat{\alpha}||_2} \right) \right]
$$

(Average Reconstruction SNR)

This case: $\text{ARSNR} \approx 15$ dB.

$m = 80$; Compression rate $(n/m)$: 3.5.

Intrinsic SNR=30 dB

Reconstruction: $\text{ARSNR} \approx 28$ dB.
CS of ECG signals:

- \( m = 55 \) measurements
- Nyquist-rate samples: \( n = 256 \)

Compression rate: \( n/m = 4.7 \)

- \( \Psi \) is a dictionary of Gabor time-frequency atoms
- \( \Phi_{55 \times 256} \) is an antipodal (pseudo)random measurement matrix

Intrinsic SNR = 17 dB

Figure of Merit:

\[ \text{ARSNR}_{\text{dB}} = I \]

(Average Reconstruction SNR)

This case: \( \text{ARSNR} \approx 15 \text{ dB} \).
Standard CS is *universal* for all $k$-sparse signals in $\Psi$ *regardless of their support:* The implicit assumption is that the signal is white – this is almost always false!

Consider a wideband natural signal generated from a *nonwhite* process, whose instances are $k$-sparse (w.r.t. Fourier):

Some indices are evidently more frequent in the $k$-sparse signal representation!
Standard CS is universal for all $k$-sparse signals in $\Psi$ regardless of their support: The implicit assumption is that the signal is white – this is almost always false!

Consider a wideband natural signal generated from a nonwhite process, whose instances are $k$-sparse (w.r.t. Fourier):

Some indices are evidently more frequent in the $k$-sparse signal representation!
**Intuition:**

- *Best* random projection vectors for *white* processes (worst case, universal) → *white* (i.i.d.) Gaussian or antipodal symbols.
- *Best* random projection vectors for *localized* and stationary processes → *nonwhite* Gaussian or antipodal symbols *chosen to maximize the raked energy*.

**RAKENESS: DEFINITION**

We define *rakeness* as:

\[
\rho(\phi, x) = \mathbb{E}_{\phi, x} \left[ |\langle \phi_j, x \rangle|^2 \right]
\]

i.e., the *expectation* of the affinity of random projection vectors \( \phi_j \) to collecting the energy in \( x \).

And introduce the *maximum rakeness* optimization:

\[
\max_{\phi_j} \rho(\phi_j, x) \quad \text{s.t.} \quad \begin{cases} 
\langle \phi_j, \phi_j \rangle = e \\
\rho(\phi_j, \phi_j) \leq r^2 e
\end{cases}
\]

**Remark:** The first constraint ensures that the vectors \( \phi_j \) have constant energy, while the second ensures that they are still random enough (to guarantee incoherence)

**Why this is needed?**
We define **rakeness** as:

\[
\rho(\phi, x) = E_{\phi,j,x} \left[ |\langle \phi_j, x \rangle|^2 \right]
\]

i.e., the **expectation** of the affinity of random projection vectors \( \phi_j \) to **collecting the energy** in \( x \).

And introduce the **maximum rakeness** optimization:

\[
\begin{align*}
\max_{\phi_j} & \quad \rho(\phi_j, x) \\
\text{s.t.} & \quad \langle \phi_j, \phi_j \rangle = 1 \\
& \quad \rho(\phi_j, \phi_j) \leq r^2 e
\end{align*}
\]

- The signal is localized since the probability that the non-zero component is aligned to \( a_0 \) or \( a_1 \) is larger than it is aligned to \( a_2 \) (the first 2 happen more frequently)

- To maximize rakeness one should project on a plane aligned with \( a_0 \) and \( a_1 \), but this would spoil the capability to collect information along \( a_2 \)

**Remark:** Incoherence between input signal and sampling vectors is the equivalent of guaranteeing to collect information in every direction (generalization of duality time-frequency for sampling with \( \delta \) in the time domain)
Sensing matrix generator used on CS for signal localized in Fourier Domain

- The power spectrum of the generated sequence is fixed by the coefficient of the feedback liter.
- Slices of length $T$ of this process can be used as projection waveforms in an RMPI architecture.

**features:**
- time-invariant linear filter
- finite number of coefficient $m$ in the filter:

\[
\hat{\phi}(f) = \frac{1 + F(e^{2\pi if})^{-2}}{\int_{-1/2}^{1/2} |1 + F(e^{2\pi if})^{-2}|^{-2} df}
\]

\[
\theta_i \rightarrow U(-1, 1)
\]

\[
b_i \in [-1, 1]
\]

\[
F(z) = \sum_{j=0}^{B-1} f_j z^{-j}
\]

\[
F(z) = \sum_{j=0}^{B-1} |f_j| \leq 1
\]
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Slices of length $T$ of this process can be used as projection waveforms in an RMPI architecture.

Sensing matrix generator used on CS for signal localized in Fourier Domain features:
- time-invariant linear filter
- finite number of coefficients $m$ in the filter: $m = 80$

**Example**

$$\hat{\phi}(f) = \frac{|1 + F(e^{2\pi if})|^{-2}}{\int_{-1/2}^{1/2} |1 + F(e^{2\pi if})|^{-2} \, df}$$

$$F(z) = \sum_{j=0}^{3-1} f_j z^{-j}$$

$$|f_j| \leq 1$$
- **Input signal**: unit-energy, $k$-sparse vectors $x = \Psi \alpha$ (Non-zero entries in $\alpha$ are i.i.d. Gaussian RVs with zero mean and variance $1/k$)

- $\Psi_{n \times n}$ is the **discrete Fourier basis** ($n = 256$)

- **Measurement matrix**: $\Phi \in \mathbb{R}^{m \times n}$ ~ i.i.d Gaussian RVs with zero mean and variance $1/n$.

- **Reconstruction Algorithm**:

  $\hat{\alpha} = \text{arg min}_{\alpha} \| \alpha \|_1$ s.t. $y = \Phi \Psi \alpha$

  i.e. Min-$L_1$

- **Figure of Merit**:

  $\text{PSR} = P \left[ \frac{\| \alpha \|_2^2}{\| \hat{\alpha} - \alpha \|_2^2} > 10^6 \right]$

  Probability of Successful Reconstruction
We apply the rakeness-based design flow to the CS of ECG signals with antipodal random sequences:
We apply the rakeness-based design flow to the CS of ECG signals with antipodal random sequences:

\[ n/m \]
We apply the rakeness-based design flow to the CS of ECG signals with antipodal random sequences.
Most of real (Bio)signal are sparse and compressed sensing is an excellent way to reduce the energy necessary for signal acquisition → natural application in body area network nodes!

Estimated trough a design in 28nm FDSOI CMOS Technology

Based on on CS

may be necessary for energy harvesting systems when transmission occurs only when there is enough energy
Surface ElectroMyoGraphy (sEMG) is the gold standard for the detection and analysis of muscle activation. EMGs gives access to information about muscles activation and, therefore, to movements which are being performed.

sEMG movement recognition is performed following 3 major phases:

- signal acquisition and digitalization
- features extraction
- classification

Adopting a Wireless BAN is particularly interesting to bring sEMG as an application in the real world. Which implicitly means that a huge amount of data must be transmitted every time.

(EMG sampled at 4KHz using 12 bit/Sample) x 16 sensors = 6.5 MB/second

A technique to reduce the size of the transmitted data must be consider!!

Compressed Sensing can reach this goal!!
Compression Ratio = $\frac{n}{m}$

Average SNR = $\mathbb{E}\left[\frac{\|x\|_{l2}}{\|x - \bar{x}\|_{l2}}\right]_{dB}$

The rakeness-based approach presents a Average SNR increase up to 7 dB.

first 2048 samples of the original signal (top), the associated reconstructed signals adopting both rakeness-based CS (middle) and standard CS (bottom) with $m = 60$ which correspond to a CR= 8.5
This task consisted in recognizing three well separated movements: 1. hand in neutral and relaxed position, 2. finger flexion (hand close as in a power grip), and 3. finger extension (hand opening). To test the quality of the recognition the Average Recognition Rate was computed as:

\[
ARR = \frac{\text{number}_{\text{correct\_recognition}}}{\text{number}_{\text{total\_samples}}}
\]

Results confirm the quality of the CS approach for the task of hand motion recognition. Indeed, depending on the specific application under study, very high levels of CR could be accepted. CS is particularly interesting when coupled with rakeness as it improves the performance of the hand motion recognition system of about 7% with all CR from 5 to 60. Furthermore, the rakeness-based CS seems to maintain the performance at their top level with a CR up to 5.
Remote ECG Monitoring

Alice
Heart Patient
Transmitter

Steve
A Relative
Low-quality, non-diagnostic ECG

Marvin
Eavesdropper
No recoverable information/noise

Bob
Medical Doctor
High-quality ECG
A receiver who wants exact signal recovery from the measurements $y$ must know the exact encoding matrix $\Phi$, or is otherwise subject to measurement noise as:

$$y = \Phi x + \nu$$

Additive perturbation model:

$$y = (\Phi x + \Delta \Phi) x$$
A **controlled** (pseudorandom) **perturbation** $\Delta \Phi$ can be **exploited** to devise two decoder classes:

- **First-class**: completely knows the true encoding matrix $\Phi^{(1)} = \Phi^{(2)} + \Delta \Phi$.

- **Second-class**: knows $\Phi^{(2)}$ but doesn't know $\Delta \Phi$!
A simple and effective choice to differentiate decoder classes is to pseudorandomly invert the sign of \( c \) symbols in an \( m \cdot n \) i.i.d. sequence.

First-class decoders are unaffected; second class decoders can only decode noisy approximations of the input. The noise power is controlled by \( \gamma = \frac{c}{m \cdot n} \).
Pseudorandom Antipodal Sequence Generator

+ 

Pseudorandom Pattern of Sign Flips

\[ S_{\Phi(2)} \rightarrow \text{PRNG}_{\Phi(2)} \]

\[ m \cdot n \text{ symbols} \]

\[ \Phi(2) \]

\[ + \quad - \quad - \quad \cdots \quad + \quad - \]

\[ - \quad + \quad - \quad \cdots \quad + \quad + \]

\[ \cdots \]

\[ - \quad - \quad + \quad \cdots \quad + \quad - \]

\[ \Phi(1) \]

\[ S_{\Delta \Phi} \rightarrow \text{PRNG}_{\Delta \Phi} \]

\[ c \text{ sign flips} \]

\[ \Delta \Phi \]
Example: CS of ECG signals.
- First-class user: medical equipe
- Second-class user: relatives

Let $n = 256$ Nyquist rate samples. For varying $m$ and $\gamma$ we measure the average reconstruction SNR.
- $\Psi_{256 \times 1440}$ is a dictionary of Gabor time-frequency atoms.
- $\Phi_{m \times 256}$ is the devised measurement matrix generator.
- Intrinsic SNR=30 dB

We choose $m = 80$, $\gamma = 0.03$ such that $\frac{m}{n} \approx 3$ and $\text{ARSNR}^{(1)} \approx 25$ dB, $\text{ARSNR}^{(2)} \approx 6$ dB.

$$\text{ARSNR}_{dB} = 20 \log_{10} \left( \frac{||\alpha||_2}{||\alpha - \hat{\alpha}||_2} \right)$$
A CASE STUDY: SECURE CS OF ECGs II

First class receiver
Original signal

Second class receiver
**Two user classes** with different information access levels

- **UNIQUE ENCODING**
  - The full image frame is block-processed; each block is encoded and transmitted.
  - When a block contains high-level information, a class B user is only able to reconstruct the transmitted block with low SNR.
  - $c \geq 40\%$ ensures total corruption of the information contained in the image.

- **TWO-CLASS DECODING**
  - $c = 2\%$
  - $c = 40\%$
- It can be shown that the CS measurements are such that

\[ y_j \xrightarrow{d} N(0, W) \]

where \( W \) is the input signal power.

- All \( W \)-power signals have *the same* measurement PDF

\[ f_y(y) \sim N(0,1) \]

so for large \( n \) the system is asymptotically secret in the Shannon sense (except for \( W = 0 \)).

\[ f_y(y) \equiv f_{y|x}(y|x) \]

- A malicious eavesdropper cannot infer any information from the statistical analysis of \( y \), which is *practically identical to Gaussian noise for* \( n \to \infty \).
How do we translate these algorithms into analog circuitry for signal acquisition?

In general, sampling is the projection of $x(t)$ over a set of waveforms:

$$x(jT) = \int_0^{nT} x(t) \delta(t - jT) dt$$

$$= \langle x(t), \delta(t - jT) \rangle_{[0,nT]}$$

$$y_j = \int_0^{nT} x(t) \phi_j(t) dt$$

$$= \langle x(t), \phi_j(t) \rangle_{[0,nT]}$$
Main architecture: Random Modulation Pre Integration

- RMPI

Acquisition sequences: either Gaussian random sequences or binary antipodal random sequences
Main architecture: Random Modulation Pre Integration

- **RMPI**

Acquisition sequences: either Gaussian random sequences or binary antipodal random sequences.
RMPI: Gaussian vs Binary sequences

Implementation overview

- Switched capacitor based architecture – *low voltage, fully differential*
- Additional I/O pins for testing purpose
- Detailed schematic (single-ended version):

Binary antipodal RMPI, 16 channels (external $\phi$ for maximum flexibility)

Innovative part

Buffer + 16 channels (distributed) multiplexer
Switched capacitor fully differential integrator with embedded +1/-1 multiplication and buffer/multiplexer: main challenges in the design

- Minimization of **leakage currents** (long integration time)
- Ensuring that **no pieces of signal are lost** between two time frames (two feedback capacitors)
- Coping with **saturation of OP-AMPS**

+1/-1 multiplication simply by swapping differential inputs

Single sampling capacitor to remove input common mode

Double feedback capacitors

IN DETAILS... – I
Event sequence:
- Time frame #1: $C_F^{(I)}$ is used to accumulate $y^{(#1)}$
- Time frame #2: $C_F^{(I)}$ is used by the buffer, then resetted; $C_F^{(II)}$ is used to accumulate $y^{(#2)}$
- Time frame #3: $C_F^{(II)}$ is used by the buffer, then resetted; $C_F^{(I)}$ is used again to accumulate $y^{(#3)}$

**Results:**
No need for external sample/hold,
No lag between CS conversions,
Time for A/D conversion and reset capacitor charge
Additional feature: saturation checking

- each integrator has two comparators for continuous checking for both positive and negative saturation

\[ y_k \approx \frac{C_S}{C_F} \sum_{j=0}^{n-1} \phi_j^k x(jT) \]

**Central Limit Theorem**

**theoretically**: measurements are Gaussian and not bounded;

**practically** the peak value is very large compared with common values
The ADC suffers from **SATURATION**!

\[ V_{ADC}^{sat-} \leq y_k \leq V_{ADC}^{sat+} \]

Saturated measurements can be **detected** and **removed** by the reconstruction algorithm…
The ADC suffers from SATURATION!

\[ V_{\text{ADC}}^{\text{sat}^-} \leq y_k \leq V_{\text{ADC}}^{\text{sat}^+} \]

Also the integrator (OP-AMP) may suffer from saturation!
(saturation may occur at ANY integration step)

\[ V_{\text{INT}}^{\text{sat}^-} \leq y_k \leq V_{\text{INT}}^{\text{sat}^+} \]

The OPAMP reaches saturation. The measurement is invalid, and the final value could be in the ADC range!

It is IMPOSSIBLE to detect that a measurement is corrupted only by observing the converted value.
our proposal: RMPI Smart Saturation Checking, RMPI-SSC

If an OP-AMP saturation is detected, integration is stopped, and the time in which saturation has occurred is used into the reconstruction algorithm.

Ternary \((0,+1,-1)\) Random Modulation pre-Integration

This approach is perfectly compatible with the CS guidelines on \(\Phi\) (Incoherence and RI).
A/D: standard split capacitor SAR (main caps array + secondary caps array), 11 bit

- Main arrays
- Secondary arrays
- Low power, low gain static pre-amp
- Dynamic comparator

*Increases comparator accuracy, decouples main capacitive array from comparator*
The BioCOMP prototype embeds all analog circuits for Compressed Sensing based acquisition of Biomedical Signals:

- 16 RMPI channels, switched-capacitors binary antipodal modulation, saturation checking capabilities
- Single, shared buffer
- Successive Approximation Register (SAR) based ADC

Technology:
- TI CMOS 180 nm 9T5V 1.8V
- Total size (incl. pads) 2.6 x 4.0 mm²
- Circuit size 2.3 x 3.7 mm²
The testing board:

- BioCOMP chip (PLCC84)
- External DAC (TLV5630)
- Test signal and references voltage generation
- Spartan 3E FPGA, embedding necessary digital logic
- Some digital control logic...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>$320 \times 770 \ \mu m^2$</td>
</tr>
<tr>
<td>Power consumption $(100kS/s)$</td>
<td>$10.08 \ \mu W$</td>
</tr>
<tr>
<td>SFDR $(100kS/s, f_{in} = 100 \ Hz)$</td>
<td>$64.2 \ dB$</td>
</tr>
<tr>
<td>Effective number of bits</td>
<td>$8.99 \ bit$</td>
</tr>
<tr>
<td>INL ($best \ fit$)</td>
<td>$&lt; 3.4 \ LSB$</td>
</tr>
<tr>
<td>Max. conversion speed</td>
<td>$200 \ kS/s$</td>
</tr>
<tr>
<td>Figure of merit $(100kS/s)$</td>
<td>$198 fJ/c.-l.$</td>
</tr>
</tbody>
</table>
Artificial sparse signal #1: signal sparse on the canonical basis (*unit pulse* step)

- 2-sparse signal \((k=2)\) (sum of 2 pulses at random position)
- Time-window length: 20 steps \((n=20)\)
- Limited number of used channels \((m=8)\)

**Example:**

\[
x(t) = \sum_{i=1}^{n} a_i g \left( \frac{t - i}{T} \right)
\]

**Measurements:**

\[
y_j = \sum_{i=1}^{n} a_i \phi_{j,i} = \begin{cases} 
1.8 \text{ V} & \phi_{j,2} = 1, \phi_{j,7} = 1 \\
1.08 \text{ V} & \phi_{j,2} = 1, \phi_{j,7} = -1 \\
-1.08 \text{ V} & \phi_{j,2} = -1, \phi_{j,7} = 1 \\
-1.8 \text{ V} & \phi_{j,2} = -1, \phi_{j,7} = -1 
\end{cases}
\]

Only 4 possible values since \(K=2\)

**REMINDER:**

the RMPI computes

\[
y_j = \int_{0}^{nT} x(t) \phi_j(t) dt 
\approx \sum_{i=1}^{n} x(iT) \phi_{j,i}
\]
... when using the sensing vectors

\[ \phi_1: \ +1, -1, -1, -1, +1, -1, +1, -1, -1, -1, +1, -1, -1, -1, +1, -1, -1, +1 \]
\[ \phi_2: \ -1, +1, +1, +1, -1, +1, +1, -1, -1, -1, +1, +1, -1, -1, -1, +1, +1, -1 \]
\[ \phi_3: \ +1, -1, +1, +1, -1, +1, +1, -1, -1, +1, +1, +1, -1, -1, -1, +1, +1, -1 \]
\[ \phi_4: \ +1, +1, -1, +1, +1, +1, -1, -1, -1, -1, +1, -1, +1, +1, +1, +1, -1, +1 \]
\[ \phi_5: \ -1, -1, -1, -1, +1, +1, +1, -1, +1, -1, -1, +1, +1, -1, -1, +1, -1, -1 \]
\[ \phi_6: \ -1, -1, +1, -1, -1, -1, -1, +1, +1, -1, -1, +1, +1, -1, -1, +1, -1, -1 \]
\[ \phi_7: \ +1, +1, -1, -1, +1, +1, +1, -1, -1, +1, +1, +1, +1, +1, +1, +1, -1, -1, +1 \]
\[ \phi_8: \ +1, -1, -1, -1, +1, -1, +1, -1, -1, -1, +1, -1, -1, -1, +1, -1, -1, -1 \]

\[ \phi_j, \phi_j, 2, \phi_j, 7: \]

\[ y_j \text{ (external output buffer enabled)} \]

1.8 V
1.08 V
-1.08 V
-1.8 V
Performance summary
(5 different time windows averaging)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sparsity level</td>
<td>( n = 20, k = 2 )</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>( f_s = 2.78 \text{ kHz} )</td>
</tr>
<tr>
<td>Time window length</td>
<td>( T_W = 7.2 \text{ ms} )</td>
</tr>
<tr>
<td>Power consumption</td>
<td>251 ( \mu \text{W} )</td>
</tr>
<tr>
<td>Measurement SNR</td>
<td>39.6 ( \text{dB} )</td>
</tr>
<tr>
<td>Reconstruction SNR</td>
<td>37.7 ( \text{dB} )</td>
</tr>
</tbody>
</table>

Each time windows has only \( K=2 \) pulses at different position

Average reconstruction SNR: 37.7 dB
Artificial sparse signal #2: signal sparse on the canonical basis (*unit pulse step*)

- 3-sparse signal ($k=3$) (sum of 3 sinusoidal tones)
- Time-window length: 64 steps (n=64)
- All channels used (m=16)

$$x(t) = \sum_{i=1}^{n/2-1} a_i \sin(2\pi i t) + \sum_{i=n/2n} a_i \cos(2\pi(n - i)t)$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sparsity level</td>
<td>$n = 64, k = 3$</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>$f_s = 2.78 \text{ kHz}$</td>
</tr>
<tr>
<td>Time window length</td>
<td>$T_W = 23 \text{ ms}$</td>
</tr>
<tr>
<td>Power consumption</td>
<td>$495 \mu W$</td>
</tr>
<tr>
<td>Measurement SNR</td>
<td>$40.9 \text{ dB}$</td>
</tr>
<tr>
<td>Reconstruction SNR</td>
<td>$30.0 \text{ dB}$</td>
</tr>
</tbody>
</table>

**Average reconstruction SNR: 30.0 dB**
By using artificial sparse signal #2 working speed range of the circuit has been investigated

- **Upper bound**: maximum switched capacitor frequency: 120 KHz (8.3 usec)
- **Lower bound**: minimum switched capacitor frequency: 0.625 Hz (1.6 sec)

### Basic System Testing – V

- Average measurements SNR is 41 dB
- Average reconstruction SNR is 30 dB
- Max time windows length (-3 dB) is approx. 1.6 sec

Maximum time window length: **1.6 sec**

*(enough for all Biomedical signals acquisition)*
The sinusoidal test signal (artificial sparse signal #2) has been scaled by a factor $s$ to test saturation checking capabilities.

$$x(t) = s \left( \sum_{i=1}^{n/2-1} a_i \sin(2\pi i t) + \sum_{i=n/2}^{n} a_i \cos(2\pi (N - i) t) \right)$$

**Better performance**

with respect to *standard case* (no saturation events)

- **Low $s$:** low amplitude, quantization noise is high.
- **Low measurement SNR, low reconstruction SNR**

- **High $s$:** some saturation events are detected.

By using saturation time information, we correctly reconstruct the signal.

Not using saturation time information, reconstruction is not possible.

We can reconstruct the input signal with **more than 60% saturated measurements**.

**Standard case:** $s=1$

**High $s$:** some saturation events are detected.

**Low $s$:** low amplitude, quantization noise is high.

Low measurement SNR, low reconstruction SNR.

SNR [dB]
Test on a real ECG signal (available on PhysioNet database)

```
- Signal length: 5 s
- Approx. heart beat: 1 beat/s
- Sampling frequency: 256 Hz
- Time windows length: 0.5 s (n = 128)
- Channel used: m=16, compression ratio is 8
- Rakeness
  (more than 32 measurements required for standard CS approach)
- Sparisty basis: Wavelet
- Three different amplitudes
  (for testing saturation behavior)
```
Test on a **real EMG signal** (available on PhysioNet database)

- **Signal length**: 128 ms
- **Sampling frequency**: 20 kHz
- **Time windows length**: 12.8 ms (n= 256)
- **Channel used**: m=24, compression ratio is ≈ 10 (two circuits are used)
- **Rakeness**
  - (more than 64 measurements required for standard CS approach)
- **Three different amplitudes for testing saturation behavior**
**ANALOG VS DIGITAL CS ARCHITECTURES**

**Analog Implementation Scheme:**
- RMPI Architecture
- Binary antipodal RMPI, 16 channels (external $\phi$ for maximum flexibility)

**Digital Implementation Scheme:**
- CS sampling used for compression
- Buffer + 16 channel (distributed) multiplexer

$j = 0, 1, ... m - 1$
As reported in [1] a Digital Implementation Scheme of CS is more efficient in terms of energy consumption.

Main aim of this contribution is 1) to present an enhanced C-ACQ block based on a "puncturing" algorithm that yields the optimal sampling positions for a fixed number of samples and 2) to use it to estimate a single waveform parameter in the CS-domain.

• The innovative part is in C-MAC (enhanced compressed multiply and accumulate) which computes the quantity

\[ y_j = \sum_{k=0}^{n-1} \phi_{jk} x_k \quad \phi_{jk} = \{-2, -1, 1, 2\} \cup \{0\} \]

• Multiplication by 2 correspond to SHL and by 0 correspond to avoid sampling

• Let \( B_{AFE}, B_x, B_{ACC} \) and \( B_y \) the (resolution) bits of the analog front-end the ADC, the accumulator and the C-MAC output

• In general \( m \) parallel stages are necessary (i.e. \( j = 1, 2, \ldots, m \)), but for a single parameter estimation we may assume \( j = 1 \)
Assume that the signal to acquire can be written as

\[ x(t) = \alpha z(t) + \eta(t) \]

with \( z(t) \) is known and represented by the samples \( z = (z_0, \ldots, z_{n-1})^\top(t) \) is a realization of a zero mean disturbance processes producing the samples \( \eta = (\eta_0, \ldots, \eta_{n-1})^\top \)

Starting from the variance of \( x \), we want to compute an estimate \( \hat{\alpha} \) of \( \alpha \)

Set \( \hat{\alpha} = \frac{y}{\phi^\top z} \) and since \( \mathbf{E}[\hat{\alpha}] = \frac{\alpha \phi^\top z}{\phi^\top z} + \frac{\phi^\top \mathbf{E}[\eta]}{\phi^\top z} = \alpha \) we must minimize the estimator variance

\[
\min_{\phi} \sigma_{\hat{\alpha}}^2 = \min_{\phi} \mathbf{E}[(\hat{\alpha} - \alpha)^2] = \min_{\phi} \sigma_{\eta}^2 \frac{\phi^\top \phi}{(\phi^\top z)^2},
\]

Clearly \( \phi \) cannot be chosen randomly since it may let to \( \phi^\top z \rightarrow 0 \) and also \( \phi = z \) (which would minimize \( \hat{\alpha} \)) is not possible, since \( \phi \in \{-2, -1, 0, 1, 2\}^n \) and only \( s \) elements must be non zero to have \( s \) samples
A possible solution of the discrete optimization problem is given by

1: \( J \leftarrow \text{permute } \{0, \ldots, n-1\} \text{ s.t. } |z_{J(j)}| \geq |z_{J(j+1)}| \)
   for \( j = 0, \ldots, n-2 \)
2: Initialize \( a \leftarrow (0, \ldots, 0)^T \) and \( \bar{\omega} \leftarrow \infty \)
3: for \( j = 0, \ldots, s-1 \) do
4: \( A \leftarrow 0 \)
5: for \( A = 2, \ldots, 1 \) do
6: \( \omega \leftarrow \frac{\phi^T \phi + A^2}{(\phi^T z + A|z_{J(j)}|)^2} \)
7: if \( \omega < \bar{\omega} \) then
8: \( \bar{\omega} \leftarrow \omega \) and \( \bar{A} \leftarrow \text{sign}(z_{J(j)})A \)
9: end if
10: end for
11: \( \phi_{J(j)} \leftarrow \bar{A} \)
12: end for
13: \( \phi \leftarrow \min \{|\phi_j| \text{ s.t. } \phi_j \neq 0, \ j = 0, \ldots, n-1\} \)

which simply consider that when a large-modulus component of \( z \) is present the denominator tend to be larger than the numerator \( \Rightarrow \) one orders \( z \) in non-decreasing order and associate it with the coefficient of \( \phi \) minimizing \( \omega \) (i.e. \( \sigma^2_\hat{\alpha} \)). Note that \( n \) and \( s \) are assumed known.
Assume $z(t)$ and RC-type charge-discharge waveform (in the normalized $t \in [0,1]$ interval) and assume that $n = 16$ samples of which only $s = 4,8,12$ are used (i.e. the sample density $\delta = 0.25,0.5,0.75$)

$$z(t) = -1 + 2 \begin{cases} 
\frac{1 - e^{-\frac{t}{\tau}}}{1 - e^{-\frac{1}{2\tau}}} & \text{if } t \leq \frac{1}{2} \\
1 - \frac{1 - e^{-\frac{(t-1/2)}{\tau}}}{1 - e^{-\frac{1}{2\tau}}} & \text{if } t > \frac{1}{2}
\end{cases}$$

---

**CASE STUDY**

Optimal sampling (in terms of noise rejection)

1. Concentrate when $z$ is largest
2. The use of 4 instead of 2 values for $\phi$ becomes useful when $\delta$ increases and small values of $z$ are sampled
Goal: estimate $\alpha$ with a resolution of $B_\alpha$ minimizing the energy necessary for the operation. Of course $B_{AFE} < B_\alpha$ and $B_\alpha \leq B_x$ (assume $B_\alpha = B_x$ at the maximum acquisition frequency $f_s$)

The estimator will reduce the noise on $\alpha$ with respect to $\eta$, i.e.

$$\frac{\sigma^2_\alpha}{\sigma^2_\eta} \leq 2^{2(B_{AFE} - B_\alpha)}$$

so that $B_\alpha - B_{AFE}$ is the **processing gain in the estimation**

Six different sensor nodes as combination of:

1. Full sample ($n$ samples) or sub-samples ($s < n$ samples) (_FS or _SS)
2. On-board or off-board estimation (ON_ or OFF_)
3. Immediate transmission or storage in memory (_TX or _NV)

OFF_ means immediate transmission of all samples, i.e. $m = s$ and $B_y = B_\alpha$

Total Power = $P_{AFE} + P_{ADC} + P_{TX}$
ON_ rely on local estimation, i.e. $m = 1$ and $B_y = B_\alpha$

Total Power = $P_{AFE} + P_{ADC} + P_{C-MAC} + P_{TX}$

Total Power = $P_{AFE} + P_{ADC} + P_{C-MAC} + P_{NV}$

(no $P_{TX}$ is present since the data is stored in memory till enough energy is harvested)

Determined using the Algorithm to find the couple $n, s$ (i.e. $f_s$ and $\delta$) which gives a gain $\Delta B$ with minimum power consumption

$f_s (n)$ has practically the same behavior in all cases
Larger $\delta$ must be used to obtain larger $\Delta B$ i.e. (as seen before) use all the coefficient values that the architecture allow (i.e. 5 possible values for $\phi$ are better than 3).

Maximum reduction is obtained with ON_SS. The impact of the difference between TX and NV is not so important.
Compressed Sensing (CS) is a set of theoretical tools and algorithms which has found many applications when sparsity can be exploited, i.e., when we want to acquire signals with a structure.

This technique has been successfully applied to:
- Single-pixel Camera (CS Imager)
- Distributed/wireless sensor networks
- Magnetic Resonance Imaging (MRI, fMRI)
- CS of ECG/EEG signals
- Analog-to-Information Conversion (RMPI, Random Sampling,....)

It is a field in which CASS community can strongly contribute since it requires algorithms/circuits co-design, something on which our community is particularly strong.
Rakeness-based Projection Waveforms

- I.i.d. random matrices $\Phi$ may not be the best choice in general.
- When signals being sampled are localized in energy (e.g. their power spectrum is wideband but non-white, as it is almost always the case), one can use sequence maximizing rakeness to optimize performances


Information Concealing and Compressed Sensing

- Since $\Phi$ is sensitive to perturbation noise, we might change some coefficients (by sign flipping) and embed a mild form of cryptography directly in the A2I conversion.


Analog A2I Implementation

First implementation of an Analog A2I including saturation checking and exploiting rakens to achieve a truly hardware-software co-design


Digital CS Front-end for BAN and WSN

Design of a simple Digital CS-front end exploiting on-board feature extraction in BAN and WSN

Thank you for your attention.
Questions?