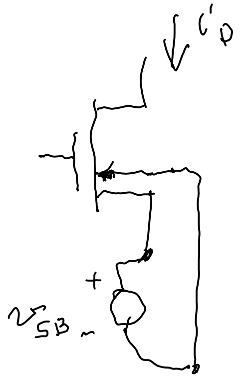
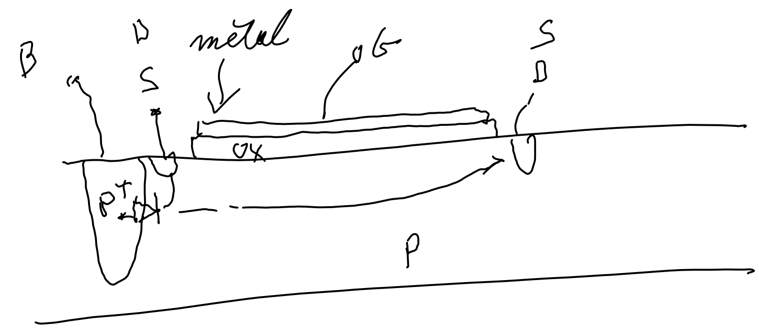
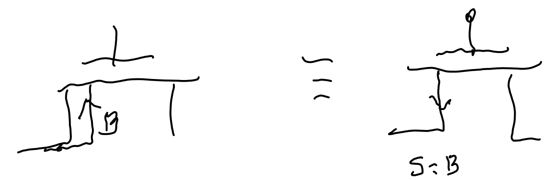
Note Title 5/9/2019







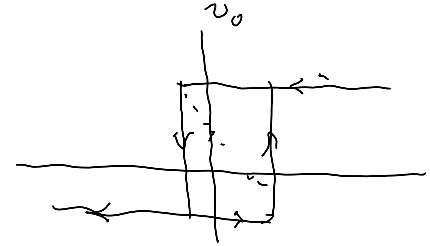
$$i_D = k(v_{GS} - V_{th})^2$$

$$V_{th} = V_{TO} + \gamma\left(\sqrt{\phi + v_{SB}} - \sqrt{\phi}\right)$$

$$g_{m_{bulk}} = \frac{\partial i_D}{\partial v_{SB}}\bigg|_{\substack{bias\\Q}} = \frac{\partial i_D}{\partial V_{th}} \cdot \frac{\partial V_{th}}{\partial v_{SB}} = 2k(v_{GS} - V_{th}) \cdot \gamma \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\phi + v_{SB}}}\bigg|_{Q} = \frac{2 I_D}{V_{ov}} \cdot \frac{\gamma}{2\sqrt{\phi + 0}}$$

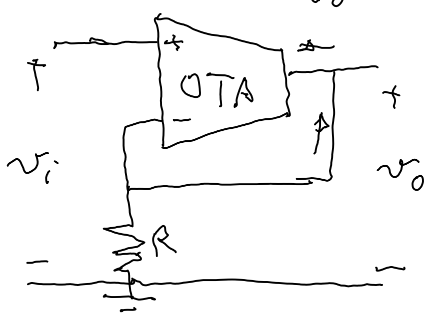
@ bias

OTA hysteresis



hysteresis schmitt trigger

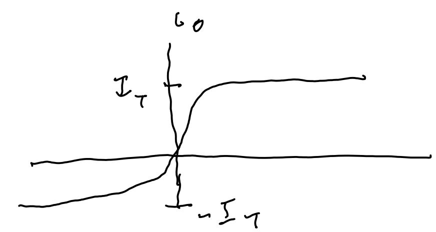
$$i_o = I_T \tanh(v_D/(2V_T)) \qquad V_T = kT/|Q| = 26\,mV \text{ @ room } T$$



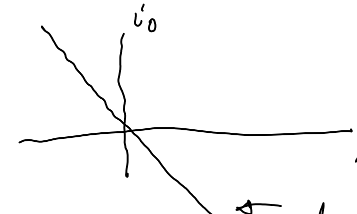
$v_D = v_o$

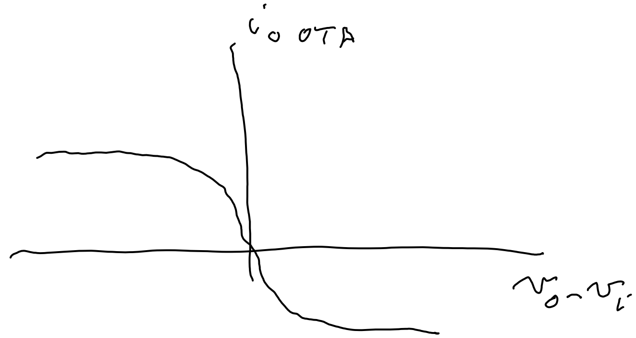
$-R i_o = v_o$

$i_o = -G v_o\ ,\ G = 1/R$

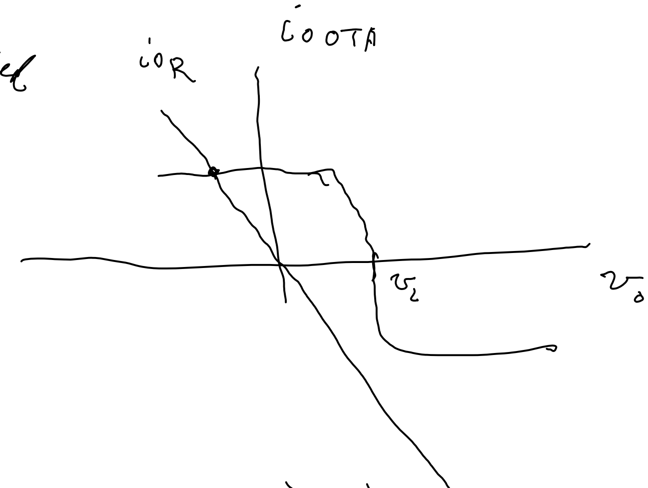


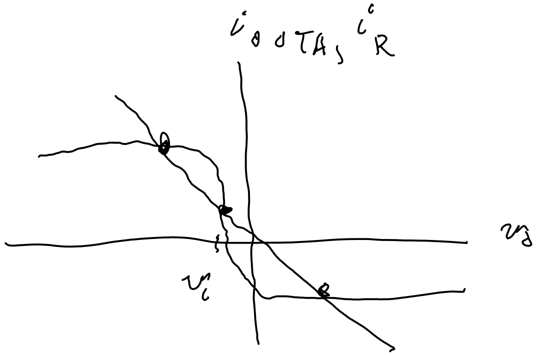
$v_D = v_i - v_o$

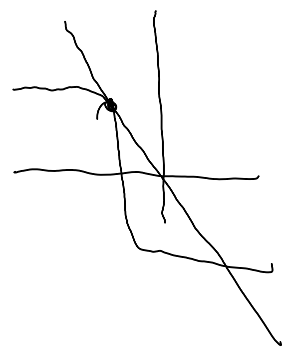




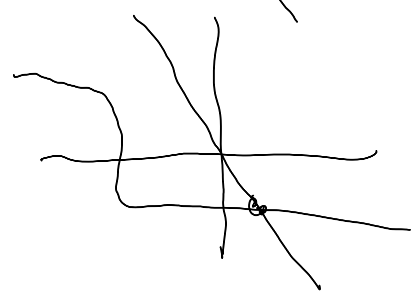
for v_i fixed

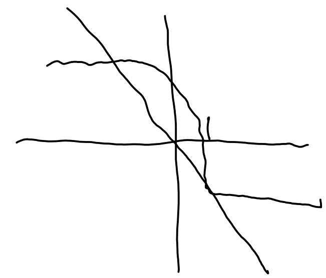




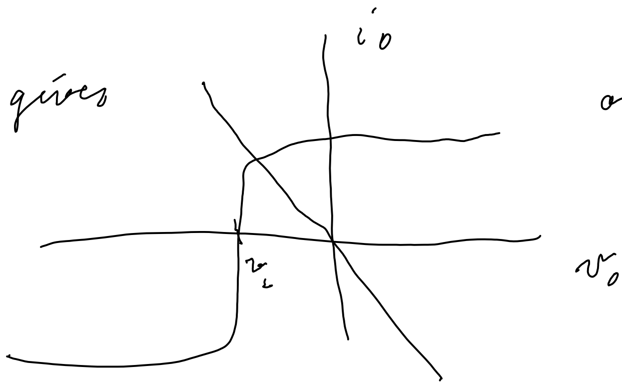


$\Rightarrow$



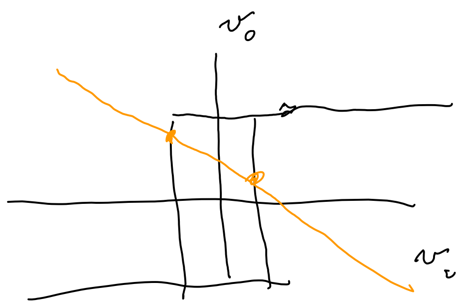


if OTA gives



only 1 intersect & no hysteresis

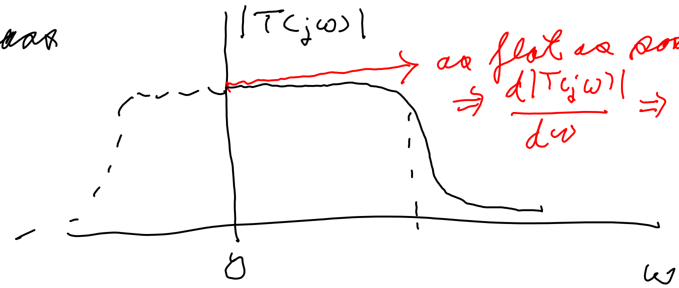
If m



oscillates

Butterworth = maximally flat magnitude

Low pass



as flat as possible $\Rightarrow \dfrac{d|T(j\omega)|}{d\omega} \Rightarrow$ Taylor series as many coefficients $= 0$ as possible

$$|T(j\omega)|^2 = T(j\omega)\,T^*(j\omega) = T(j\omega)\,T(-j\omega)$$

$$T(s) = \frac{k}{s^m + a_{m-1}s^{m-1} + \dots + \underset{\substack{\parallel \\ 1}}{a_0}} \Rightarrow T(j\omega)T(-j\omega) = |T(j\omega)|^2$$

1 by normalization

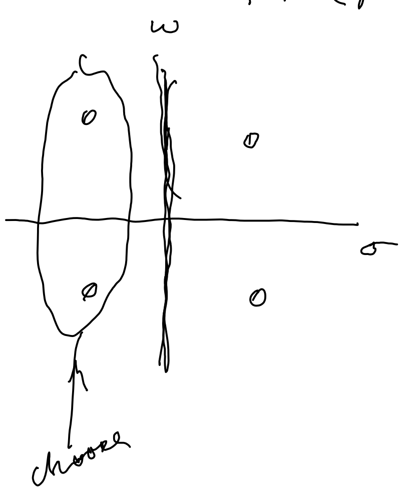
$$= \frac{k}{D(s)}$$

$$\frac{d\,|T(j\omega)|^2}{d\omega} = 2\,|T(j\omega)| \cdot \frac{d\,|T(\omega)|}{d\omega}$$

$$\frac{d\,{}^1\!/f(\omega)}{d\omega} = -\frac{1}{f^2(\omega)} \cdot \frac{d f(\omega)}{d\omega} \Rightarrow \text{work with } \frac{1}{|T(j\omega)|^2} = \frac{D(j\omega)D(-j\omega)}{k^2} = \underbrace{1 + \dots}_{\text{set all these to be zero}} + (j\omega)^m(-j\omega)^m$$

$$T(s) = \frac{k}{D(s)} \qquad D(s) = s^m + a_{m-1} s^{m-1} + \dots a_1 s + 1$$

$$1 + (j\omega)^m (-j\omega)^m = |D(j\omega)|^2$$



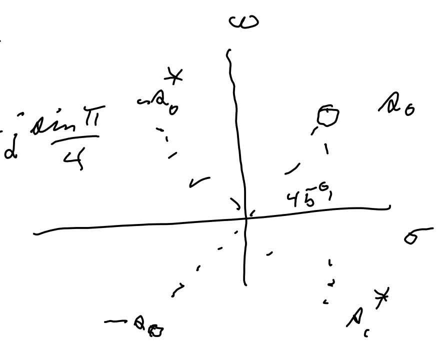
$\omega = s/j$ gives an analytic extension for $D(s)D(-s)$

$$1 + (s^m)\left(-s\right)^m = 1 + (-1)^m s^{2m}$$ factor this by finding zeros

$$(-1)^m s^{2m} = -1 \qquad \text{for } m = 2 \Rightarrow s^{2m} = -1 = e^{j\pi + 2k\pi}$$

$$s_k = e^{j\left(\frac{\pi + 2k\pi}{2m}\right)} \Rightarrow k = 0 \Rightarrow s_0 = e^{j\pi/4}$$

$$= \cos\frac{\pi}{4} + j\sin\frac{\pi}{4}$$



$$-s_0^* = -\cos\pi/4 + j\sin\pi/2$$

$$-s_0 = -\cos\pi/4 - j\sin\pi/4$$

$$D(s) = (s - (-s_0^*))(s - (-s_0)) = s^2 + (s_0^* + s_0)s + s_0 s_0^* = s^2 + \frac{2}{\sqrt{2}} s + 1$$

$$T(s) = \frac{k}{s^2 + \sqrt{2}\, s + 1} \quad \Leftarrow \text{ degree 2 Butterworth low pass}$$

for high pass $0 \leftrightarrow \infty$ $\quad s \to 1/s = p$, $\quad T(p) = \dfrac{p^2}{1 + \sqrt{2}\, p + p^2} \longrightarrow$ high pass Butterworth