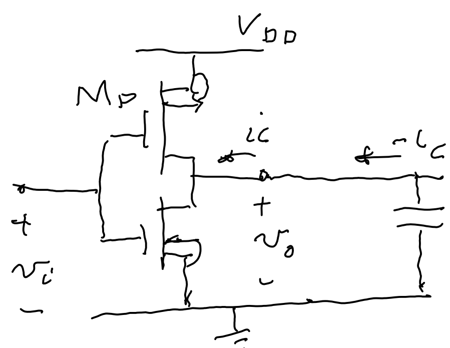
ENEE303
04/11/19

Note Title 4/11/2019

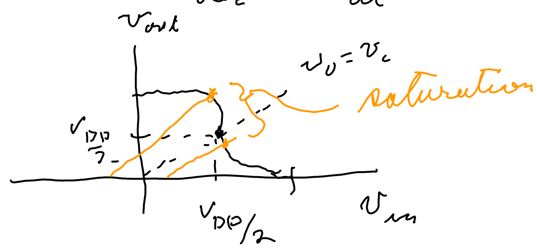
P. 1269 = OTA as sensor

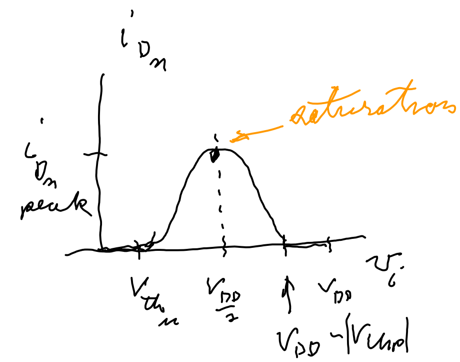
P. 1151, Fig 14.39 ⇒ charging current, P. 1150 power in

pp. 1130 – 1135 C load on inverter



$\Rightarrow i_C = C\dfrac{dv_C}{dt} = \dfrac{dq}{dt}$, $q = Cv$





$$i_{peak} = k\left(\frac{V_{DD}}{2} - V_{th}\right)^2\left(1 + \lambda \frac{V_{DD}}{2}\right)$$

power from V_{DD} $= P_{supplied} = v(t)\, i(t) = V_{DD}\, i_{D_p}(t)$

$$\text{Energy} = \int_0^{t_c} P_{supplied}\, dt = \int_0^{t_c} V_{DD} \cdot i_{D_p}(t)\, dt = V_{DD} \cdot q(t_c)$$

If we start with C not charged, $v_0 = 0$

& we pulse v_i to charge $C \Rightarrow$ Turn on M_p & off M_n $\Rightarrow$ as $q(t) = C\, v(t)$

$\Rightarrow q(t_c) = C \cdot V_{DD}$

@ $t_c \Rightarrow v_0 = V_{DD}$ (a 1 out for a 0 in)

Energy from bias source $= C V_{DD}^2$

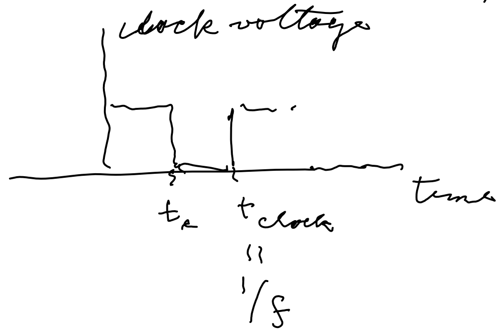
Energy into a capacitor $\Rightarrow$ $\mathcal{E}(t) = \int_0^t v_c(\tau)\, i_c(\tau)\, d\tau \Rightarrow \int_0^{t_c} v(\tau) \cdot C \frac{dv(\tau)}{d\tau}\, d\tau = \int_{v(0)}^{v(t_c)} C\, v\, dv$

$$= \frac{1}{2}\left(C\, v^2(t)\Big|_0^{t_c}\right) = \frac{1}{2} C V_{DD}^2$$

Energy into the inverter to charge C is

$$C V_{DD}^2 - \frac{1}{2} C V_{DD}^2 = \frac{1}{2} C V_{DD}^2$$

For a clock of frequency f then we do this every half cycle



$$\lim_{\Delta t \to 0} \frac{\Delta \text{ Energy}}{\Delta \text{ time}} = \frac{d\text{Energy}}{dt} \approx \frac{\left(\frac{1}{2} C V_{DD}^2\right) \times 2}{1/f}$$

transistor #

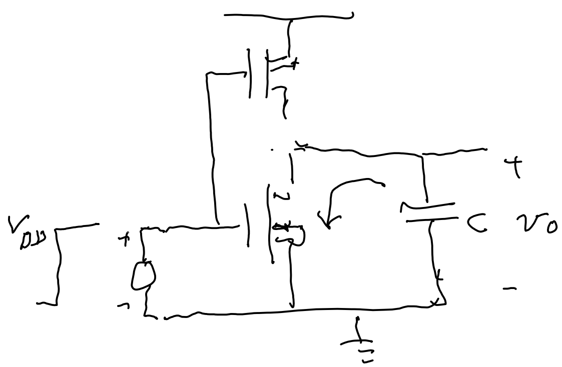
= power into the inverter $= f C V_{DD}^2$

clock f — load C — bias supply

$\underbrace{\qquad\qquad}$ determines computer speed & structure

C is from other transistors & something like $W \times L \times \frac{\varepsilon_{SiO_2}}{t_{ox}}$

discharge of a C into an NMOS



@ $t = 0_+$ NMOS $v_{GS} = V_{DD}$

$v_{DS} = V_{DD}$

$\Downarrow$

M_n is in saturation
& stays there until v_{DS} drops
to $V_{DS} - V_{th}$ (as $v_{DS} < v_{GS} - V_{th}$)
($\because V_{DD}$)

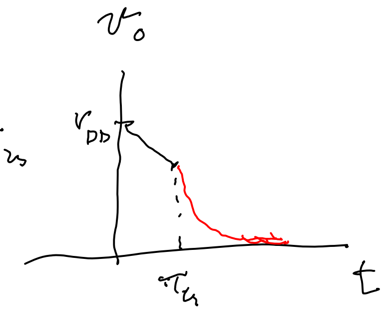
@ $t = 0_+$ until T_{tr} to $V_o = V_{DD} - V_{th}$

$I_{D_n} = i_{D_n} = k(V_{DD} - V_{TO})^2 \Rightarrow i_{D_n} = -i_C = -C \dfrac{dv_o}{dt} = I_{D_n}$

$k = \dfrac{KP}{2} \cdot \dfrac{W}{L}$

$= \dfrac{\mu C_{ox}}{2} \cdot \dfrac{W}{L}$

$v_o(t) = \displaystyle\int_{0_+}^{t} -\frac{1}{C} I_{D_n} d\tau = -\frac{1}{C} I_{D_n} \cdot t + (V_{DD})$ for $0_+ < t \leq T_{tr}$



@ $t = T_{tr}$ transitions from saturation to triode state for M_n

then for $t \geq T_{tr}$, $i_{D_n} = k\left(2(v_{GS} - V_{th})^2 v_{DS} - v_{DS}^2\right)$

to find v_0, $T_{th} < t < \infty$

$$k\left(2(V_{DD}-V_{th})^2 v_0 - v_0^2\right) = i_{D_n} = -i_C = -C\frac{dv_0}{dt} \implies \text{a Riccati equation}$$

$$-\frac{k}{C}\,dt = \frac{dv_0}{2(V_{DD}-V_{th})^2 v_0 - v_0^2} \implies \int_{T_{th}}^{t} -\frac{k}{C}\,dt = \int_{v_0(T_{th})}^{v_0(t)} \underbrace{\frac{1}{2(V_{DD}-V_{th})^2 v_0 - v_0^2}}_{\frac{1}{\left(2(V_{DD}-V_{th})^2 - v_0\right) v_0}} \cdot dv_0$$

$$\frac{-1}{v_0\left(v_0 - 2(V_{DD}-V_{th})^2\right)} \qquad \text{let } a = 2(V_{DD}-V_{th})^2$$

$$= \frac{-1}{v_0(v_0-a)} = \frac{k_1}{v_0} + \frac{k_2}{v_0-a} \; ; \quad k_1 = \left.\frac{-\cancel{v_0}}{\cancel{v_0}(v_0-a)} - \frac{k_2 v_0}{v_0-a} = -\frac{1}{v_0-a} - \frac{k_2 v_0}{v_0-a}\right|_{\text{set } v_0 = 0}$$

$$k_1 = \frac{-1}{-a} = 1/a$$

$$\text{for } k_2: \quad k_2 = \left.-\frac{(v_0-a)}{v_0(v_0-a)} - \frac{k_1(v_0-a)}{v_0}\right|_{\text{set } v_0 = a} = \left.-\frac{1}{v_0}\right|_{v_0=a} = \frac{-1}{a}$$

$$-\frac{k}{c}\int_{T_{ts}}^{t} d\tau = -\frac{k}{c}t + \frac{k}{c}T_{ts} = \int_{v_0(T_a}^{v_0(t)} \frac{-1}{v_0(v_0-a)}\,dv_0 = \int_{v_0(T_{ts}}^{v_0(t)} \left(\frac{1/a}{v_0} + \frac{-1/a}{v_0-a}\right) dv_0$$

$$-\int_{v_0(T_{ts})-a}^{v_0(t)-a} \frac{+1/a}{(v_0-a)}\,d(v_0-a)$$

$$=$$

$$-\frac{\ln}{a}\Big[(v_0-a)\ t\Big] - \frac{\ln}{a}\left(v_0(T_{ts}-a\right]$$

$$\Uparrow$$

$$= \frac{1}{a}\Big\{\ln(v_0(t)) - \ln(v_0(T_{ts}))$$
$$-\ln(v_0(t)-a) + \ln(v_0(T_{ts})-a)\Big\}$$

gives

$$\frac{1}{a}\ln\left[F(v_0(t))\right] = -\frac{k}{c}t + \frac{k}{c}T_{ts}$$

$$\Longrightarrow F(v_0) = e^{\left(-\frac{k}{c}t + \frac{k}{c}T_{ts}\right)a}, \quad T_{ts} < t < \infty$$

where $F(v_0) = \dfrac{v_0(t)/v_0(T_{ts})}{(v_0(t)-a)/(v_0(T_{ts})-a)}$ $\Longrightarrow$ can solve for $v_0(t)$ now, $T_{ts} \le t < \infty$