ENEE 303
02/26/19

Note Title 2/26/2019

BJT $f_T \Longrightarrow$ p. 720, eq. 10.41, C_π & C_μ p. 718

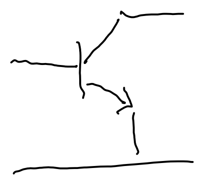
MOS, C_g

C_{gs}

C_{gd}

Differential pairs, chapter 9, MOS, p. 596, 606

BJT, p. 614, 619



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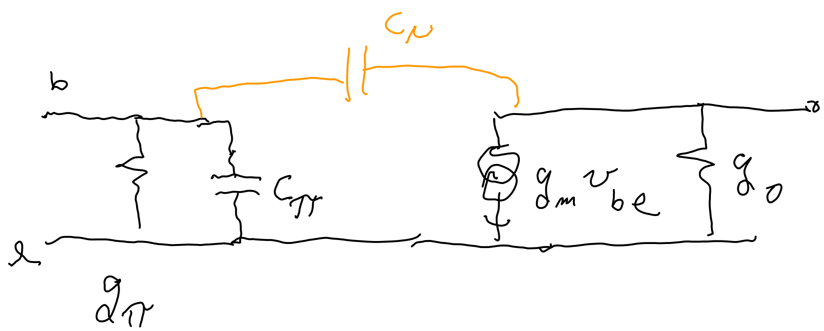


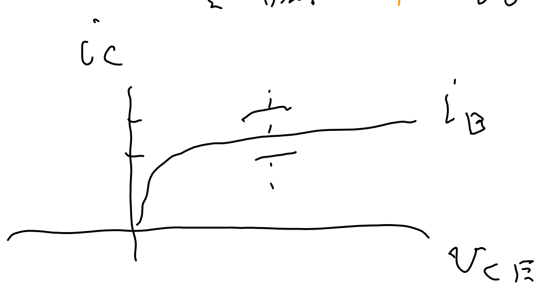
$$y_\pi = g_\pi + sC_\pi$$

$$i_c = C\frac{dv_c}{dt}$$

=

$$Y = \begin{bmatrix} y_\pi(s) + sC_\mu & -sC_\mu \\ g_m - sC_\mu & g_o + sC_\mu \end{bmatrix} ; \quad i = Yv$$

$$v = \begin{bmatrix} v_{be} \\ v_{ce} \end{bmatrix}$$



if $v_c = V_c e^{st}$

$$I_c e^{st} = i_c = \frac{d}{dt} v_c = V_c s e^{st} = s v_c$$

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}, e^{j\omega t} \quad , j = \sqrt{-1}$$

if $\sigma = 0$ $\quad e^{st} = e^{j\omega t} = \cos\omega t + j\sin\omega t$

$$\omega = 2\pi f$$

to get $\beta = \dfrac{i_c}{i_b}$,

$$\begin{bmatrix} i_b \\ i_c \end{bmatrix} = \begin{bmatrix} g_\pi + s(C_\pi + C_\mu) & -sC_\mu \\ g_m - sC_\mu & g_o + sC_\mu \end{bmatrix} \begin{bmatrix} v_{be} \\ v_{ce} \end{bmatrix} \leftarrow v_{ce} = 0 \text{ to get } \beta$$

$i_b = y_{11} v_{be}$ if $v_{ce} = 0$

$i_c = y_{21} v_{be}$ $v_{be} \neq 0$

(can cancel)

$$\Rightarrow \frac{i_c}{i_b} = \frac{y_{21}}{y_{11}} = \frac{g_m - sC_\mu}{g_\pi + s(C_\pi + C_\mu)} \quad \text{eq. (10.45)}$$

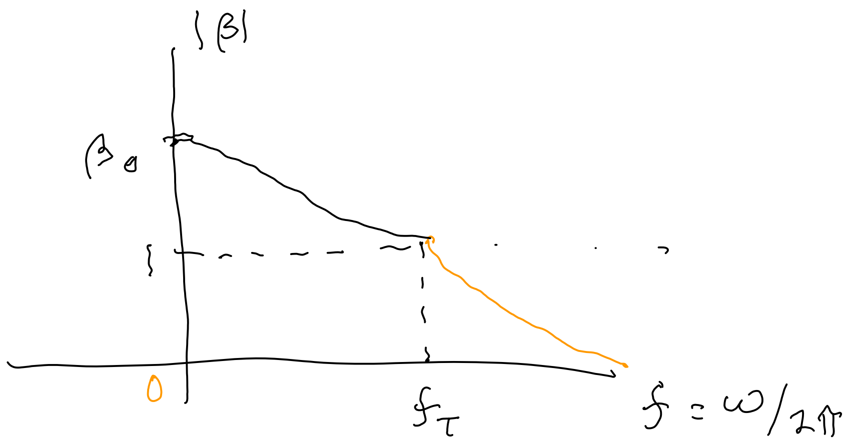
$$= \frac{g_m}{g_\pi} \cdot \frac{1 - sC_\mu/g_m}{1 + \frac{s(C_\pi + C_\mu)}{g_\pi}}$$

$\beta_o = \beta$ @ DC

$= \dfrac{g_m}{g_m/\beta_o}$

$$\approx \frac{g_m}{g_\pi} \cdot \frac{1}{1 + \frac{s(C_\pi + C_\mu)}{g_\pi}} = \frac{\beta_o}{1 + \frac{s(C_\pi + C_\mu)}{g_\pi}}$$

for f_T, this is when β drops to $|\beta(j\omega_T)| = 1$



deduce $f_T \Rightarrow \left| 1 + j\omega \dfrac{(C_\pi + C_\mu)}{g_\pi} \right| = \beta_0$

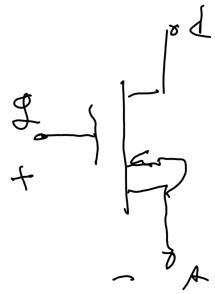
$$\omega_T \frac{(C_\pi + C_\mu)}{g_\pi} \approx \beta_0$$

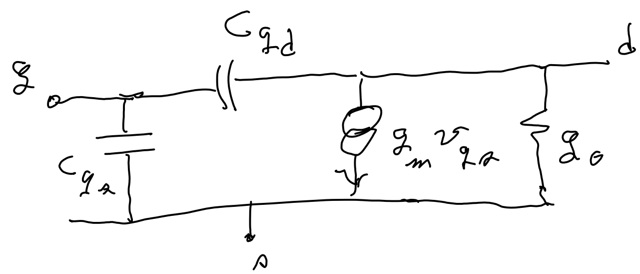
$$f_T = \frac{\omega_T}{2\pi} \approx \frac{g_\pi \times \beta_0}{C_\pi + C_\mu} = \frac{g_m}{C_\pi + C_\mu}$$

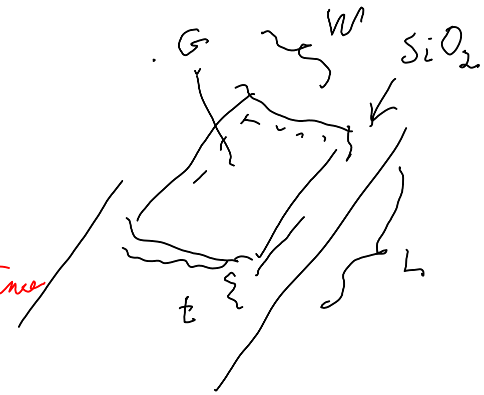
eq. (10.41)

$$\left| \frac{\beta_0}{1 + j\omega\left(\frac{C_\pi + C_\mu}{g_\pi}\right)} \right| = 1 \Rightarrow \frac{\beta_0}{\sqrt{1 + \omega^2\left(\frac{C_\pi + C_\mu}{g_\pi}\right)}} = 1 \Rightarrow 1 + \frac{\omega^2 (C_\pi + C_\mu)^2}{g_\pi^2} = \beta_0^2 \Rightarrow \omega_T \Uparrow$$

MOS



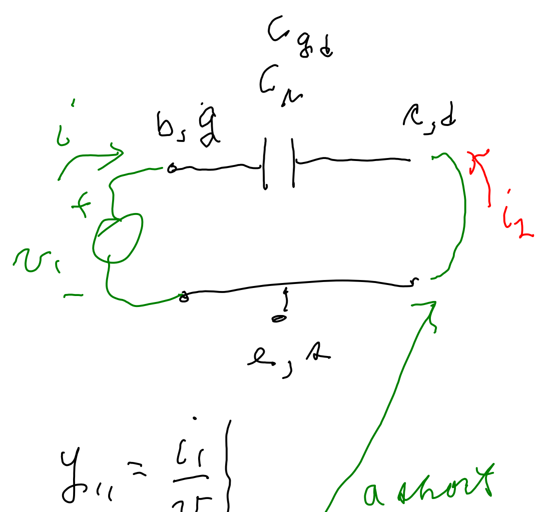




$$C_{gs}+C_{gd} = C_g = \text{gate capacitor} \simeq \frac{W \times L \times \epsilon_{SiO_2}}{\underset{\text{thickness of oxide}}{t}} = \underbrace{W \times L}_{\text{gate area}} \times \underbrace{C_{ox}}_{\text{unit area capacitance}}$$

$$C_{gs} = \frac{1}{2} C_g \quad \text{or} \quad = \frac{2}{3} C_g$$

$$C_{gd} = \underbrace{\frac{1}{2} C_g}_{\text{saturation}} \quad = \underbrace{\frac{1}{3} C_g}_{\text{triode}}$$



$\Rightarrow \quad Y = \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} sC_\mu & -sC_\mu \\ -sC_\mu & sC_\mu \end{bmatrix}$ or $\begin{bmatrix} sC_{gd} & -sC_{gd} \\ -sC_{gd} & sC_{gd} \end{bmatrix}$

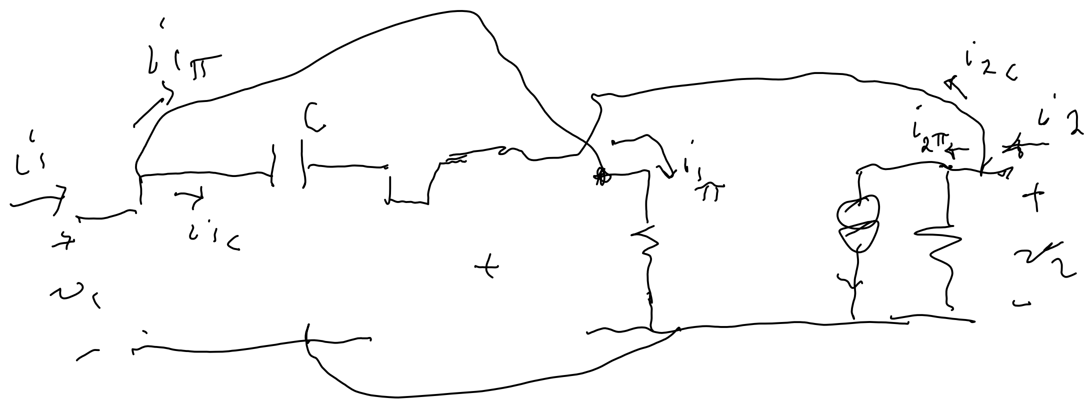
$y_{11} = \left.\dfrac{i_1}{v_1}\right|_{v_2=0}$

$i = sC \cdot v \Rightarrow i_1 = \left.\begin{matrix} sC_\mu \\ \text{or } sC_{gd} \end{matrix}\right\} \cdot v_1$

$y_{21} = \left.\dfrac{i_2}{v_1}\right|_{v_2=0} = \dfrac{-i_1}{v_1} = \begin{matrix} -sC_\mu \\ -sC_{gd} \end{matrix}$

$= y_{12}$

$y_{11} = y_{22}$ & $y_{12} = y_{21}$ by symmetry



$$\Rightarrow \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$(i_c + i_\pi) = i = (Y_C + Y_\pi) \cdot v \quad \Rightarrow \quad Y = Y_C + Y_\pi$$

$$\Downarrow$$

$$i = Yv$$

$$v = v_C = v_\pi$$

$$Y = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix} + \begin{bmatrix} y_\pi & 0 \\ g_m & g_o \end{bmatrix}$$

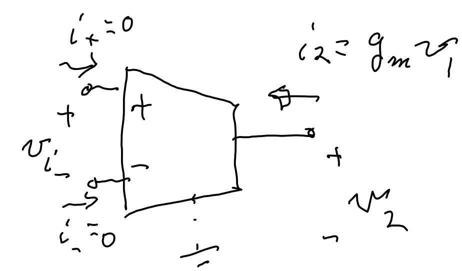
$$Y = \begin{bmatrix} y_\pi + sC_\mu & -sC_\mu \\ g_m - sC_\mu & g_o + sC_\mu \end{bmatrix}$$ for BJT or $C_\mu \to C_{gd}$ for MOS

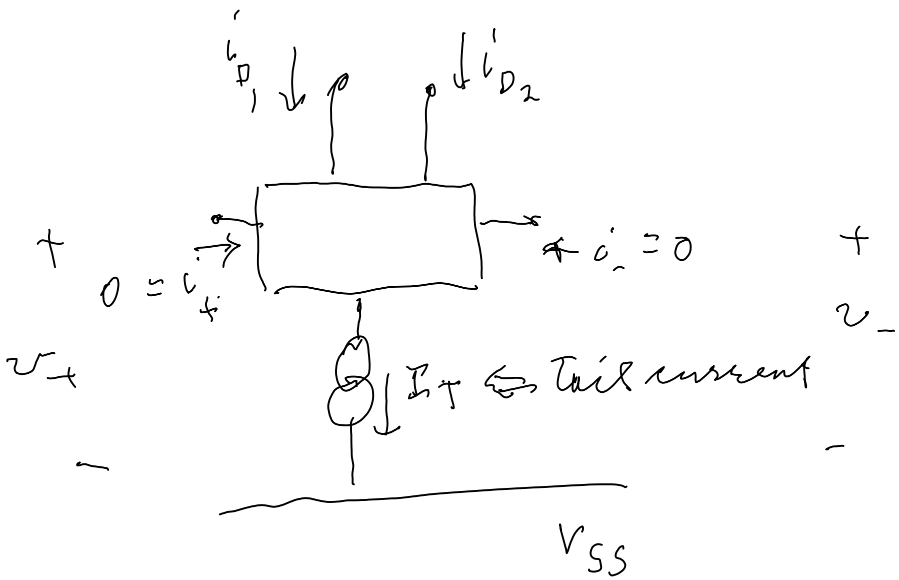
$y_\pi \to C_{gs} \cdot s$

differential pairs $\Rightarrow$ OTA = operational transconductance amplifiers

(g_m = mutual conductance = transconductance)

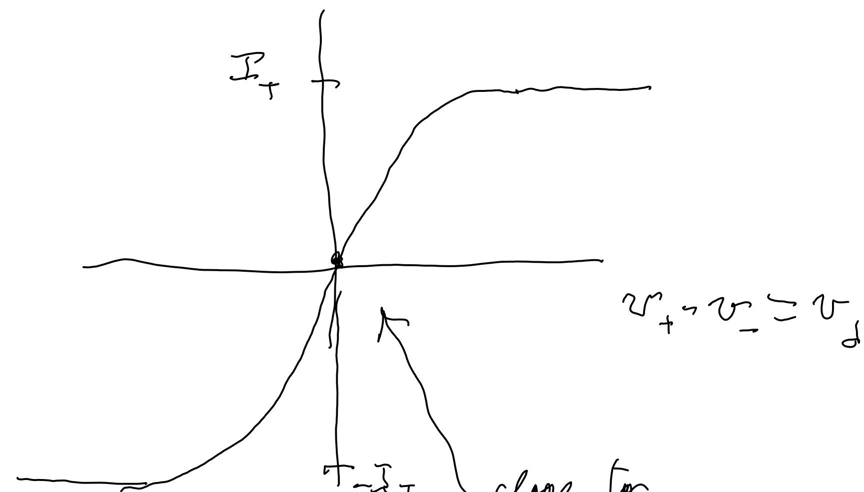
OTA $\Rightarrow$ $Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$ $\Rightarrow$





by KCL: $i_{D_1} + i_{D_2} = I_T$

$i_{D_2} - i_{D_1}$

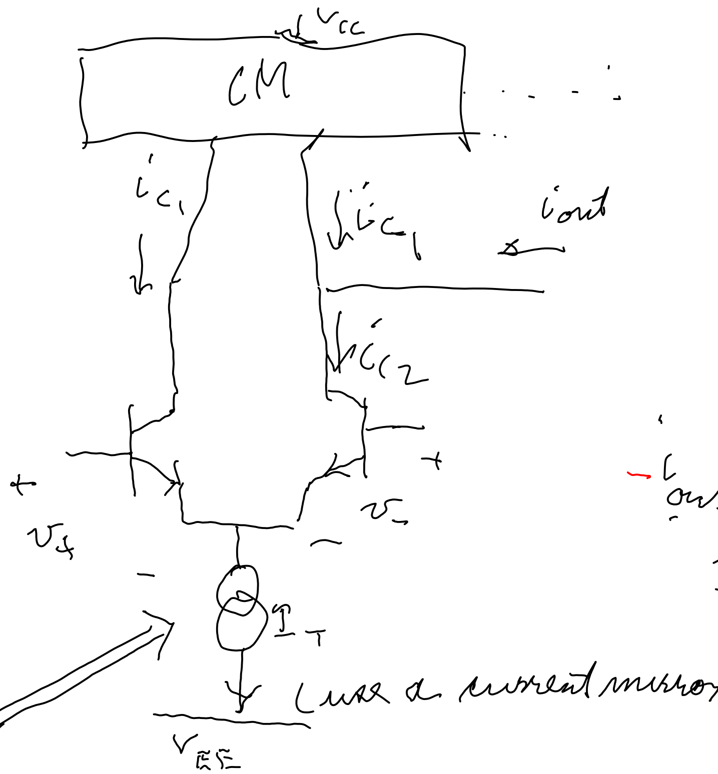


close to

$I_T \tanh(v_i / V_T)$

if BJT, $V_T = \frac{kT}{q}$

BJT
diff. pair



desire $\Rightarrow$ $i_B = 0$ so get i_B from biasing

desire a good I_T get from current mirror

$$i_{C_1} = \alpha I_S e^{v_+/V_T} \quad ; \quad i_{C_2} = \alpha I_S e^{v_-/V_T}$$

$$-i_{out} \simeq i_{C_1} - i_{C_2} = \alpha I_S \left(e^{v_+/V_T} - e^{v_-/V_T}\right)$$

$$I_T = i_{C_1} + i_{C_2} = \alpha I_S \left(e^{v_+/V_T} + e^{v_-/V_T}\right)$$

$$\text{ratio} = \frac{-i_{out}}{I_T} = \frac{e^{v_+/V_T} - e^{v_-/V_T}}{e^{v_+/V_T} + e^{v_-/V_T}} = \frac{e^{(v_+ - v_-)/V_T} - 1}{e^{(v_+ - v_-)/V_T} + 1} = \frac{e^{(v_+ - v_-)/2V_T} - e^{-(v_+ - v_-)/2V_T}}{e^{(v_+ - v_-)/2V_T} + e^{-(v_+ - v_-)/2V_T}}$$

$$-i_{out} = I_T \tanh\left(\frac{v_+ - v_-}{2V_T}\right) = \underbrace{I_T \tanh\left(\frac{v_d}{2V_T}\right) = -i_{out}}_{\text{key result}}$$

linearized @ $v_d = 0 \Rightarrow \left.\dfrac{d(-i_{out})}{dv_d}\right|_{v_d=0} = \left. I_T\left(1-\tanh^2\left(\dfrac{v_d}{2V_T}\right)\right) \times \dfrac{1}{2V_T}\right|_{v_d=0} = \dfrac{I_T}{2V_T} = g_m$ of OTA

as $\tanh(0) = 0$

$\therefore \; g_m = \dfrac{I_T}{2V_T}$ is set by the tail current via a current mirror