04/05/19
RWN

Solutions Midterm 303 S19

#1. $v_{GS} = V_{tr}$, $v_{DS} = v_O$; for $v_{in} = V_{tr}$, $v_{DS} = V_O = V_{tr} - V_{TO} = v_{GS} - V_{th}$

By KVL around outer loop $v_O = V_{DD} - R_L i_D$ and @ $v_{in} = V_{tr}$, M_n is in saturation (and in triode region) so

$$v_O = V_{DD} - R_L k(v_{in} - V_{TO})^2(1+\lambda v_O) = 12 - 2\times10^3 \times 4\times10^{-3}(V_{tr} - 1)^2(1+0.v_O)$$

$$V_{tr} - V_{TO} \Rightarrow V_{tr} = 13 - 8(V_{tr}-1) \Rightarrow (V_{tr}-1) = 12 - 8(V_{tr}-1)^2$$

If $x = V_{tr} - 1 \Rightarrow 8x^2 + x - 12 = 0 \Rightarrow x^2 + \frac{1}{8}x - \frac{12}{8} = 0 \Rightarrow x = -\frac{1}{16} \pm \frac{1}{2}\sqrt{(1/8)^2 + \frac{4\times12}{8}}$

$\therefore x = \frac{-1}{16}(1 \pm \sqrt{1 + 12\times\frac{4}{8}\times8^2}) = \frac{-1}{16}(1 \pm \sqrt{385}) \approx \frac{-1}{16}(1 \pm 19.62) = \frac{-1}{16}(20.62 \text{ or } -18.62)$

or $x = -1.289$ or $+1.164$ as $V_{tr} > V_{TO}$ use − sign, $V_{tr} = x + 1 = 2.164$

$\Rightarrow$ $V_{tr} = 2.16\ V$

#2 @ the gate, wrt gnd, $V_G = \frac{R_B}{R_A + R_B} V_{DD} = V_{GS} + R_S I_D$; $V_{GS} = V_{TO} + \sqrt{\frac{I_D}{k}} = 1 + \sqrt{\frac{1}{4}} = 3/2 \Rightarrow V_{DS} = 8 >$ $V_{GS} - V_{TO} = V_2$

$= 3/2 + 2 = 7/2 = \frac{1}{1 + R_A/R_B} \cdot 12 \Rightarrow R_A/R_B = \frac{1}{7/2}(12 - 7/2) \Rightarrow$ sat.

$\Rightarrow R_A = (\frac{2}{7}\times12 - 1)R_B = \frac{17}{7}R_B$

[If $k = 2mA$ is used then $v_{GS} = 1 + \sqrt{1/2} = 1.707$ & $V_G = 1.707 + 2 = 3.707$ $\Rightarrow \frac{R_A}{R_B} + 1 = 3.237$ or $R_A = 2.24 R_B$ $g_m = 2\times2\times0.707\times10^{-3} = 2.83\times10^{-3}$ $A_v = -5.86$]

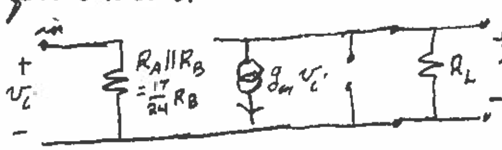
$\Rightarrow V_{DS} = 8V$, $V_{GS} = 1.5V$, $I_D = 1\ mA$

and $R_A = 2.42\ R_B$

b) We need $g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q = 2k(V_{GS} - V_{TO}) = 2\times4\times10^{-3}(\frac{3}{2} - 1) = 4\ m\text{Mho}$

and $g_o = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q = 0$ @ the input R_A & R_B are parallel

The equivalent circuit is



$v_o = R_L(-g_m v_i) \Rightarrow \frac{v_o}{v_i} = -g_m R_L = A_v$

$\Rightarrow A_v = -4\times10^{-3}\times2\times10^3 = -8 = A_v$

#3. $\frac{v_{mid}}{v_{in}} = \frac{g_{m_1} + G}{sC_1 + (g_{m_1} + G)}$ from last class & $\frac{v_{out}}{v_{mid}} = -\frac{1}{sC_2} \cdot g_{m_2}$ by KCL on OTA_2

$\therefore \frac{v_O}{v_{in}} = \frac{v_{out}}{v_{mid}} \cdot \frac{v_{mid}}{v_{in}} = \frac{-g_{m_2}(g_{m_1} + G)}{sC_2(sC_1 + (g_{m_1} + G))} = \frac{-g_{m_2}(g_{m_1} + G)}{s^2 C_1 C_2 + sC_2(g_{m_1} + G)}$

or $(s^2 C_1 C_2 + sC_2(g_{m_1} + G))v_O(t) = -g_{m_2}(g_{m_1} + G)v_i(t)$

$\Rightarrow C_1 C_2 \frac{d^2 v_O}{dt^2} + C_2(g_{m_1} + G)\frac{dv_O}{dt} = -g_{m_2}(g_{m_1} + G)v_i(t)$

#4. a) For two fins, $n = 2$, we obtain the standard NMOS

b) In saturation $\frac{\partial i_D}{\partial v_{GS}} = nk(v_{GS} - V_{th})^{n-1}(1 + \lambda v_{DS}) \Rightarrow g_m = \frac{n I_D}{(V_{GS} - V_{th})}$

$\frac{\partial i_D}{\partial v_{GS}} = k(v_{GS} - V_{th})^n \cdot \lambda \Rightarrow g_o = \frac{\lambda I_D}{(1 + \lambda V_{DS})}$

Equivalent circuit