04/25/19
RWN

Midterm Supplement S19 Solutions

#1. $I_D = 1\times10^{-3} = \frac{KP}{2}\cdot\frac{W}{L}(V_{GS}-V_{TO})^2(1+\lambda V_{DS})$; $V_{GS} = \frac{1}{2}V_{DD} - R_S I_D = 6-2 = 4$; $V_{DS} = V_{DD} - (R_S+R_L)I_D = 12-4 = 8$

$= \frac{2\times10^{-5}}{2}\cdot\frac{W}{L}(4-1)^2(1+0.01\times8)$

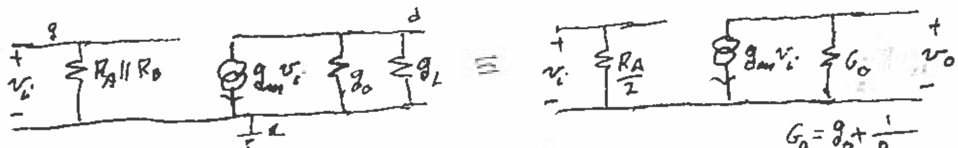
a) $\Rightarrow \frac{W}{L} = \frac{10^{-3}\times10^{5}}{3^2\times(1.08)} = \frac{100}{9.72} = 10.3 = W/L$, $V_{GS} = 4V$, $V_{DS} = 8V$

as $V_{DS} = 8 > 4-1 = 3 = V_{GS} - V_{th} \Rightarrow$ saturation

b) $g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = k\cdot2(V_{GS}-V_{TO})(1+\lambda V_{DS}) = \frac{2 I_D}{(V_{GS}-V_{TO})} = \frac{2\times10^{-3}}{3} = \frac{2}{3}m\mho = 0.667\, m\mho = g_m$

$g_o = \frac{\partial i_D}{\partial v_{DS}}\Big|_Q = \frac{KP}{2}\frac{W}{L}(V_{GS}-V_{TO})^2\cdot\lambda = \frac{I_D\cdot\lambda}{1+\lambda V_{DS}} = \frac{1\times10^{-3}\times1\times10^{-2}}{1.08} = \frac{10^{-5}}{1.08} = 9.26\times10^{-6} = 9.26\mu\mho = g_o$

c)



$G_o = g_o + \frac{1}{R_L}$

$A_v = v_o/v_i = -R_o g_m \approx -2\times10^3\times\frac{2}{3}10^{-3} = -\frac{4}{3} = -1.33 = A_v$

$= 9.26\mu + \frac{1}{2}m = 509.26\mu\mho$

$\approx 1/R_L = 0.5 m\mho$

#2. $v_{mid} = -g_{m_1}\frac{1}{sC_1}v_i$, $v_o = -g_{m_2}\frac{1}{sC_2}(v_o - v_{mid}) \Rightarrow (1+g_{m_2}\frac{1}{sC_2})v_o = g_{m_2}v_{mid}$

$= g_{m_2}\cdot(-g_{m_1}\frac{1}{sC_1})v_i$

a) $\Rightarrow \frac{v_o}{v_i} = \frac{-g_{m_1}g_{m_2}}{s^2C_1C_2(1+\frac{g_{m_2}}{sC_2})} = \frac{-g_{m_1}g_{m_2}}{s^2C_1C_2 + sC_1g_{m_2}}$

2 zeros @ ∞

poles @ $s=0$, $s = -\frac{g_{m_2}}{C_2}$

b) as $s \Rightarrow d/dt$ & $(s^2C_1C_2 + sC_1g_{m_2})v_o = -g_{m_1}g_{m_2}v_i$

$\Rightarrow C_1C_2\frac{d^2v_o}{dt^2} + C_1 g_{m_2}\frac{dv_o}{dt} = -g_{m_1}g_{m_2}v_i$

#3. a) For $n=2$, $i_D = \begin{cases} 0 & \text{off}, v_{GS}-V_{th} \le 0 \\ k(v_{GS}-V_{th})^2(1+\lambda v_{DS}) & \text{saturation}, 0 \le v_{GS}-V_{th} \le v_{DS} \\ k[2(v_{GS}-V_{th})^1 v_{DS} - v_{DS}^2](1+\lambda v_{DS}) & \text{triode}, 0 \le v_{DS} \le v_{GS}-V_{th} \end{cases}$ law of an NMOS

b) as in triode $i_D = k[2(v_{GS}-V_{th})^{n/2}v_{DS}^{n/2} - v_{DS}^n](1+\lambda v_{DS})$

$\frac{\partial i_D}{\partial v_{GS}}\Big|_Q = k[2\cdot\frac{n}{2}(v_{GS}-V_{th})^{(\frac{n}{2}-1)}\cdot v_{DS}^{n/2}](1+\lambda v_{DS})\Big|_Q = k\cdot n(V_{GS}-V_{th})^{\frac{(n-2)}{2}}\cdot V_{DS}^{n/2}(1+\lambda V_{DS}) = g_m$

$\frac{\partial i_D}{\partial v_{DS}}\Big|_Q = \Big\{k[2\cdot\frac{n}{2}(v_{GS}-V_{th})^{n/2}v_{DS}^{(\frac{n}{2}-1)} - n v_{DS}^{n-1}](1+\lambda v_{DS}) + \lambda\cdot k[2(v_{GS}-V_{th})^{n/2}v_{DS}^{n/2} - v_{DS}^n]\Big\}\Big|_Q$

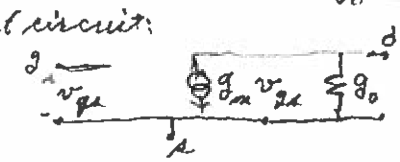
$= k\Big\{[n(V_{GS}-V_{th})^{n/2}V_{DS}^{\frac{n-2}{2}} - nV_{DS}^{n-1}](1+\lambda V_{DS}) + \lambda[2(V_{GS}-V_{th})^{n/2}V_{DS}^{n/2} - V_{DS}^n]\Big\}$

Using Mathcad (see attached file)

$g_m = 4.004\times10^{-11}\mho$

$g_o = 3.966\times10^{-10}\mho$

equivalent circuit:



file: g:/coursesS19/303/midsupProb3_sol.mcd

$n := 4 \qquad k := 1\cdot 10^{-9} \qquad VGS := 1.1 \quad VDS := 0.1 \qquad Vth := 0.1 \qquad \lambda := 0.01$

$$gm := k\cdot n\cdot (VGS - Vth)^{\left[\frac{(n-2)}{2}\right]}\cdot VDS^{\left(\frac{n}{2}\right)}\cdot (1 + \lambda\cdot VDS) \qquad gm = 4.004 \times 10^{-11}$$

$$go := k\cdot\left[n\cdot\left[(VGS - Vth)^{\frac{n}{2}}\cdot VDS^{\frac{-n}{2}} - 1\right]\right]VDS^{n-1}\cdot (1 + \lambda\cdot VDS) + \lambda\cdot k\cdot\left[2\cdot (VGS - Vth)^{\frac{n}{2}}\cdot VDS^{\frac{n}{2}} - VDS^{n}\right]$$

$$go = 3.966 \times 10^{-10} \qquad ro := \frac{1}{go} \qquad ro = 2.521 \times 10^{9}$$