Note Title

ENE:307
03/26/18

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$R, \Rightarrow \quad v = Ri \Rightarrow i = Gv, \quad G = 1/R$

$C, \Rightarrow \quad i = C\,dv/dt = \dfrac{dq}{dt} \Rightarrow I(s) = sC\,V(s)$

$L, \Rightarrow \quad v = L\,di/dt = d\psi/dt \Rightarrow V(s) = sL\,I(s)$

$$M = \begin{bmatrix} sL_{11} & sM_{12} \\ sM_{12} & sL_{22} \end{bmatrix}$$

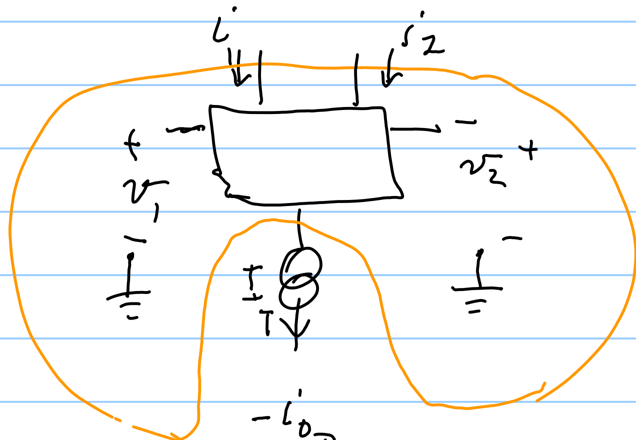
$OTA \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ G_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow$ via differential pairs

FDNR = frequency dependent negative resistor

$$I(s) = C^2 s^2 V(s) \Rightarrow s = j\omega \Rightarrow -C^2\omega^2 = G(\omega)$$

can replace R, L, C, OTA circuits by R, C, FDNR & OTA's

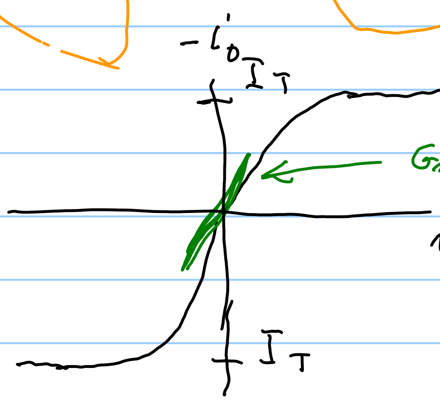
Differential pairs



I_T = tail current

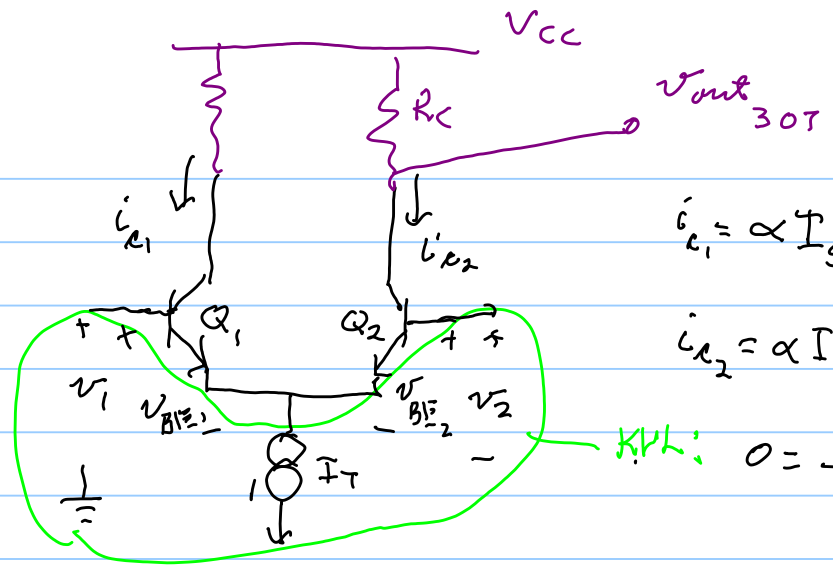
$I_T = i_1 + i_2$ by KCL + any currents into input leads ≈ 0

$i_0 = i_2 - i_1$



G_m = slope @ $v_d = 0$

$v_1 - v_2 = v_d \Rightarrow$ difference input voltage



$i_{c_1} = \alpha I_S e^{v_{BE_1}/V_T}$

$i_{c_2} = \alpha I_S e^{v_{BE_2}/V_T}$

KVL: $0 = -v_1 + v_{BE_1} - v_{BE_2} + v_2 = v_{BE_1} - v_{BE_2}$
$= v_1 - v_2$
$= v_d$

$$\frac{i_o}{I_T} = \frac{i_{c_2} - i_{c_1}}{i_{c_2} + i_{c_1}} = \frac{\alpha I_S\left(e^{v_{BE_2}/V_T} - e^{v_{BE_1}/V_T}\right)}{\alpha I_S\left(e^{v_{BE_2}/V_T} + e^{v_{BE_1}/V_T}\right)} = \frac{\left(1 - e^{(v_{BE_1} - v_{BE_2})/V_T}\right)}{\left(1 + e^{(v_{BE_1} - v_{BE_2})/V_T}\right)}$$

$$= \frac{e^{\frac{(v_{BE_1} - v_{BE_2})}{2V_T}}\left(e^{-(v_{BE_1} - v_{BE_2})/2V_T} - e^{+(v_{BE_1} - v_{BE_2})/2T}\right)}{e^{\frac{v_{BE_1} - v_{BE_2}}{2V_T}}\left(\quad " \quad + \quad " \quad\right)}$$

$$= -\tanh\left(\frac{v_{BE_1} - v_{BE_2}}{2V_T}\right) \Rightarrow \quad i_o \simeq \alpha I_T \tanh\left(\frac{v_d}{2V_T}\right)$$

; $i_{c_1} + i_{c_2} = \alpha I_T$
$= -(i_{e_1} + i_{e_2})$

$$\tanh x = (e^{x} - e^{-x})/(e^{x} + e^{-x}) \Rightarrow \frac{d \tanh x}{dx} = \frac{e^{x}+e^{-x}}{(e^{x}+e^{-x})} - \frac{(e^{x}-e^{-x})(e^{x}-e^{-x})}{(e^{x}+e^{-x})^2}$$

$$= 1 - \tanh^2(x)$$

$$\left.\frac{d i_0'}{d v_d}\right|_{v_d=0} = -I_T\left(1 - \tanh^2\left(\frac{v_d}{2V_T}\right)\right)\cdot \left.\frac{1}{2V_T}\right|_{v_d=0} \simeq -\frac{I_T}{2V_T}$$

interested in lab in $R_c \cdot i_2$

But $i_2 = I_T - i_1'$, $i_0' = -I_T \tanh\left(\frac{v_d}{2V_T}\right) \simeq i_2' - i_1'$ if $\alpha \approx 1$

$$\Rightarrow i_1' = i_2 - i_0' = i_2' - I_T \tanh\left(\frac{v_d}{2V_T}\right)$$

$$i_2' = I_T - i_1' = I_T - \left(i_2' - I_T \tanh\left(\frac{v_d}{2V_T}\right)\right) \Longrightarrow 2 i_2' = I_T\left(1 - \underbrace{\tanh\left(\frac{v_d}{2V_T}\right)}_{x}\right)$$

$$i_2' = \frac{I_T}{2}\left(\frac{e^{x}+e^{-x} - e^{x} - (-e^{-x})}{e^{x}+e^{-x}}\right) = \frac{I_T}{2}\cdot\frac{2e^{-x}}{e^{x}+e^{-x}} = \frac{I_T}{1+e^{\frac{2v_d}{2V_T}}} = \frac{I_T}{1+e^{v_d/V_T}}$$

$$v_{out} = V_{cc} - R_c i_2' \quad \Rightarrow \quad v_o(t) = -R_c \left.\frac{d i_2'}{d v_{sig}}\right|_{origin} \cdot v_d$$