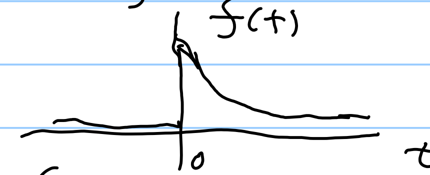
EE 307
03/12/18

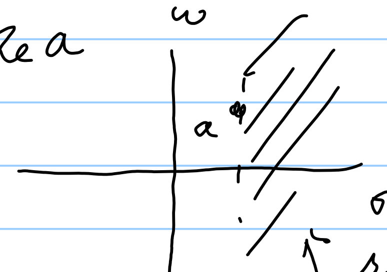
Note Title 3/12/2018

$$f(t) \Longrightarrow \mathcal{L}[f(\cdot)] = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt = F(s)$$



$$f(t) = e^{at}\, 1(t)$$

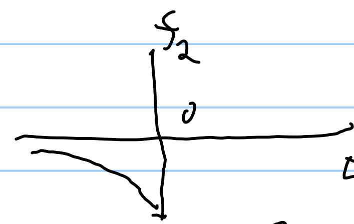
$$\mathcal{L}[f] = \frac{1}{s-a} \quad \text{for } \operatorname{Re} s > \operatorname{Re} a$$



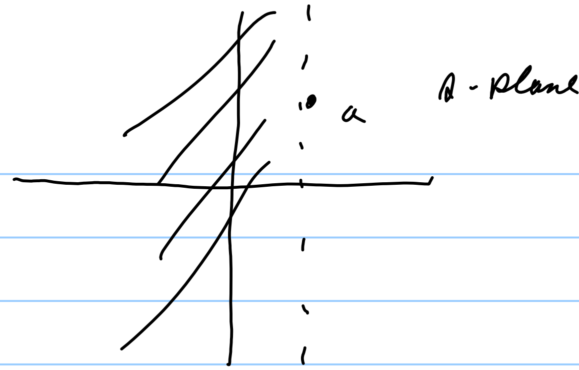
$s = \sigma + j\omega$

region of convergence of $\mathcal{L}[f]$

$$f_2(t) = -e^{at}\, 1(-t)$$



$$\mathcal{L}[f_2] = \int_{-\infty}^{\infty} [-e^{-(s-a)t}\, dt \cdot 1(-t)] = \int_{-\infty}^{0} -e^{-(s-a)t}\, dt = \left. \frac{+e^{-(s-a)t}}{(s-a)} \right|_{t=-\infty}^{t=0} =$$

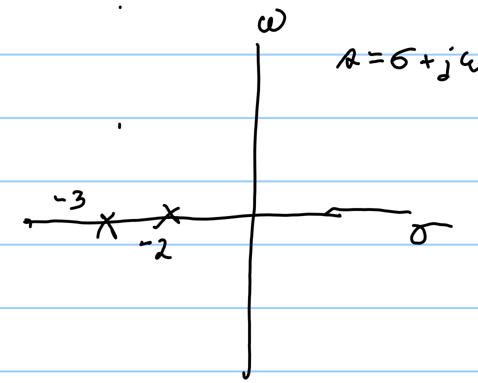


$$\Rightarrow \mathcal{L}\left[-e^{at}1(-t)\right] = \frac{1}{s-a} \qquad \mathrm{Re}\, s < \mathrm{Re}\, a$$

$$\mathcal{L}\left[e^{at}1(t)\right] = \frac{1}{s-a} \qquad \mathrm{Re}\, s > \mathrm{Re}\, a$$

To find $f(t)$ given $F(s)$ ratio

Ex: $F(s) = \dfrac{2s}{(s+2)(s+3)}$ Poles $s = -2, -3$



if $f(t)$ is causal ($=0$ for $t<0$)
then the region of convergence is $\sigma > -2$

make a partial fraction expansion

$$F(s) = \frac{k_2}{s+2} + \frac{k_3}{s+3} = \frac{2s}{(s+2)(s+3)}$$

$$k_2 \Rightarrow (s+2)F(s) = k_2 + \frac{k_3(s+2)}{s+3} = \frac{2s}{s+3} \Rightarrow \text{set } s = -2$$

$$k_2 + 0 = \frac{2(-2)}{-2+3} = -4 \Rightarrow k_2$$

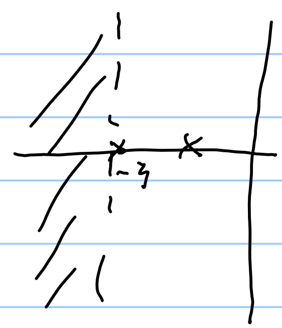
$$k_3 = \frac{2(-3)}{(-3+2)} = 6 \Rightarrow k_3$$

$$F(s) = \frac{-4}{s+2} + \frac{6}{s+3} \Rightarrow \mathcal{L}^{-1}[F(s)] = -4e^{-2t}1(t) + 6e^{-3t}1(t) \quad \text{all } t, -\infty < t < \infty$$

$$\downarrow \qquad \downarrow$$

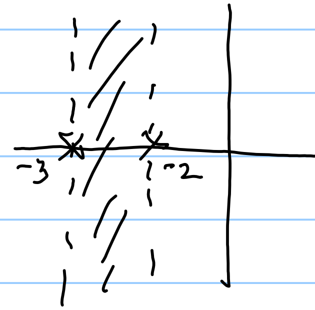
$$\sigma > -2 \qquad \sigma > -3$$

3 possibilities for $\mathcal{L}^{-1}[F(s)]$ depending on region of convergence

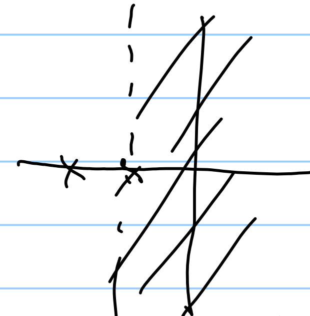


anticausal

$-e^{-2t}1(-t) - e^{-3t}1(-t)$



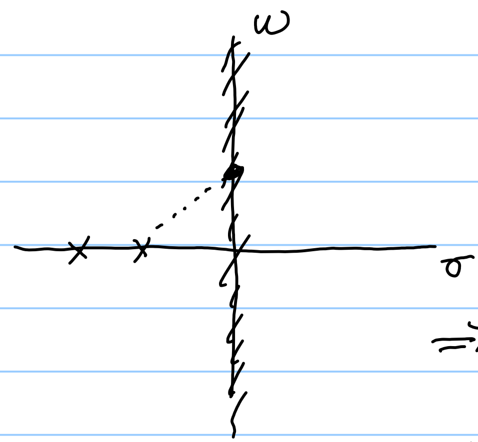
$-e^{-2t}1(-t) + e^{-3t}1(t)$



causal

$e^{-2t}1(t) + e^{-3t}1(t)$

If $\sigma = 0$, $s = j\omega$ & we are on the $j\omega$ axis



$$\mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt$$

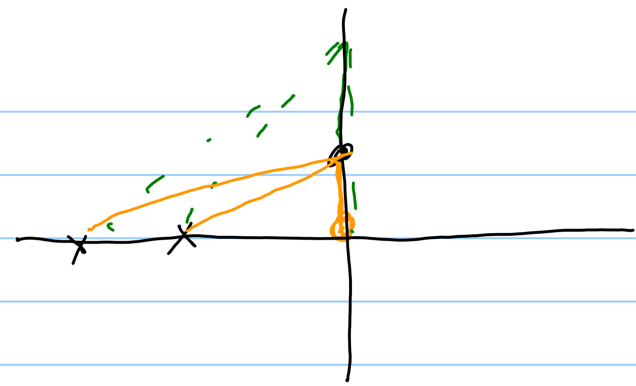
$$= \mathcal{L}[f(t)] \Big|_{s = j\omega} \quad (\sigma = 0)$$

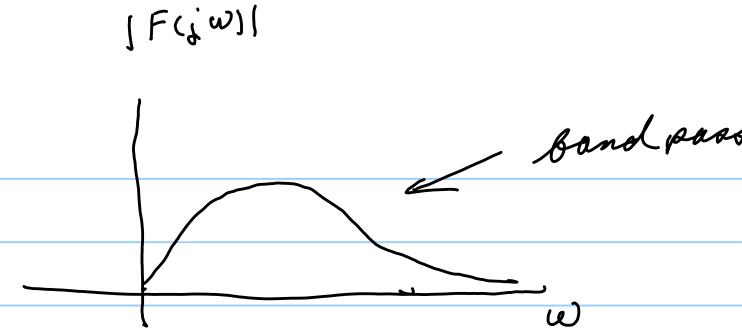
$\Rightarrow$ needs region of convergence to include $\sigma = 0$

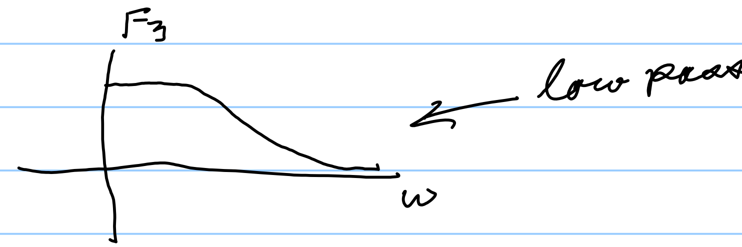
$$F(s) \Big|_{s = j\omega} = |F(j\omega)| \, e^{j \angle F(j\omega)}$$

to get $|F(j\omega)|$

if $F(s) = \dfrac{2s}{(s+2)(s+3)}$ $\Rightarrow$ $\dfrac{|2s|}{|s+2||s+3|} \Big|_{s = j\omega} = \dfrac{|2\omega|}{|j\omega+2||j\omega+3|}$

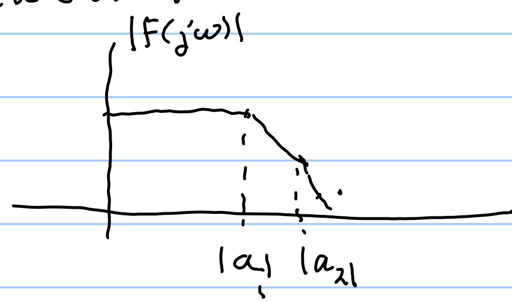




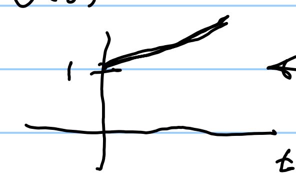


For $F_3(s) = \dfrac{2}{(s+2)(s+3)}$

For a Bode plot assume constant to 3db point & then break



If have a term $\frac{1}{s-a}$, $\operatorname{Re} a > 0 \Rightarrow f(t) = e^{at} 1(t)$



unstable