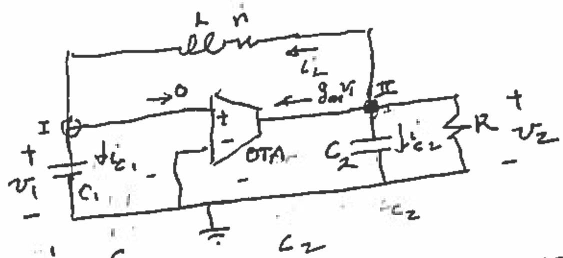
Improved Colpitts Using OTA

04/12/14
RWN



Make OTA as differential pair with current mirror output & tail current I_T; $i_{out} = I_T \tanh\left(\frac{v_{in}}{2V_T}\right) \approx g_m v_{in}$

KCL@II: $0 = i_L + g_m v_1 + i_{C_2} + i_R = \frac{1}{sL+r}(v_2 - v_1) + g_m v_1 + sC_2 v_2 + G v_2$

1) $\Rightarrow \left(\frac{1}{sL+r} - g_m\right) v_1 = \left(\frac{1}{sL+r} + sC_2 + G\right) v_2$

KCL@I: $0 = -i_L + i_{C_1} = -\frac{1}{sL+r}(v_2 - v_1) + sC_1 v_1$

2) $\Rightarrow \left(\frac{1}{sL+r} + sC_1\right) v_1 = \frac{1}{sL+r} v_2$

1) ÷ by 2) (when $v_1 \neq 0$, $v_2 \neq 0$)

$$\frac{1 - g_m(sL+r)}{1 + sC_1(sL+r)} = \frac{1 + (sC_2 + G)(sL+r)}{1}$$

$$\Rightarrow (1 - g_m(sL+r)) = [1 + (sC_2+G)(sL+r)][1 + sC_1(sL+r)]$$

$$\Rightarrow 1 - 1 = [g_m + (sC_2+G) + sC_1 + (sC_2+G)(sL+r)sC_1](sL+r)$$

$$\Rightarrow 0 = (g_m + sC_2 + G + sC_1 + s^3 C_2 L C_1 + s^2 G L C_1 + s^2 C_2 r C_1 + sGrC_1](sL+r)$$

$\Rightarrow$ as $(sL+r) \neq 0$

3) $P(s) = C_1 C_2 L s^3 + [GLC_1 + C_1 C_2 r] s^2 + [C_1 + C_2 + GrC_1] s + (g_m + G)$

Let $s = j\omega_0 \Rightarrow$

4) $\{-j\omega_0^3 C_1 C_2 L + j\omega_0 [C_1 + C_2 + GrC_1]\} + \{-\omega_0^2 [GLC_1 + C_1 C_2 r] + (g_m + G)\}$

as $\omega_0 \neq 0$

5) $Im = 0 = -\omega_0^2 C_1 C_2 L + [C_1 + C_2 + GrC_1]$

6) $Re = 0 = -\omega_0^2 [GLC_1 + C_1 C_2 r] + (g_m + G)$

5*) $\Rightarrow \omega_0^2 = \frac{C_1 + C_2}{C_1 C_2 L} + \frac{Gr}{C_2 L}$

6*) $g_m + G = \omega_0^2 [GLC_1 + C_1 C_2 r] = \left[\frac{C_1+C_2}{C_1 C_2 L} + \frac{Gr}{C_2 L}\right][GLC_1 + C_1 C_2 r]$

$$= \frac{(C_1 + C_2)G}{C_2} + (C_1 + C_2) r + \frac{C_1}{C_2} G^2 r + G r^2 \frac{C_1}{L}$$

If $Q_L > 10$, then $r \to 0$

7a, b) $\omega_0^2 \approx \frac{1}{L \frac{C_1 C_2}{C_1 + C_2}}$; $g_m \approx \frac{C_1}{C_2} G$

(note if $G = 0 \Rightarrow$ checks as an LC resonator)

Note $Q_L = \frac{\omega_0 L}{r}$ if > 10 can ignore r as $|j\omega_0 L + r| \approx |\omega_0 L|$

for BJT OTA $g_m = \frac{I_T}{2V_T} \Rightarrow I_T = \frac{C_1}{C_2} \cdot G \cdot 2V_T$

Example: $L = 100\ \mu H$, $f_0 = 1\ MHz$, $C_1 = C_2 = \frac{2}{\omega_0^2 L} = \frac{2}{4\pi^2 \times 10^{12} \times 10^{-4}} = 0.0507 \times 10^{-8} = 507\ pfd$

If $I_T = 10\ mA$, $G = 0.192 = 1/R$, $R = 5.2\ \Omega$. $\Rightarrow$ use MOS OTA

for MOS OTA $g_m = \sqrt{2 I_T \cdot \beta}$, $\beta = \frac{KP}{2}\frac{W}{L}$. $\Rightarrow g_m = G \Rightarrow I_T = \frac{G^2}{2\beta}$; if $\beta = 5 \times 10^{-4} \Rightarrow I_T = 10^3 G^2$

if $R = 1/G = 1\ k\Omega$, $I_T = 1\ ma$

an OTA for Colpitts

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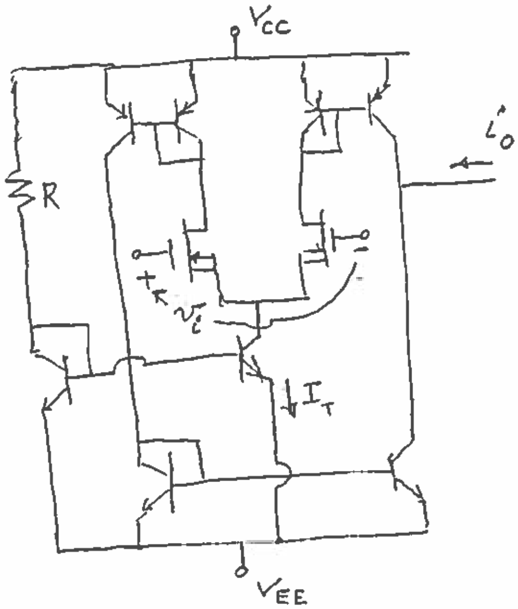

$$i_o = \begin{cases} -I_T\,; & v_i \le -\sqrt{\dfrac{I_T}{\frac{KP}{2}W/L}} \\ \sqrt{2I_T\frac{KP}{2}\frac{W}{L}}\cdot v_i\cdot\sqrt{1-\dfrac{\frac{KP}{2}\frac{W}{L}v_i^2}{2I_T}}\,; & 0 \le |v_i| \le \sqrt{\dfrac{I_T}{\frac{KP}{2}\frac{W}{L}}} \\ I_T\,; & \sqrt{\dfrac{I_T}{\frac{KP}{2}\frac{W}{L}}} \le v_i \end{cases}$$

$$\Rightarrow g_m = \sqrt{2I_T\cdot\frac{KP}{2}\frac{W}{L}} \quad @\ bias\ (i_o = I_o = 0)$$