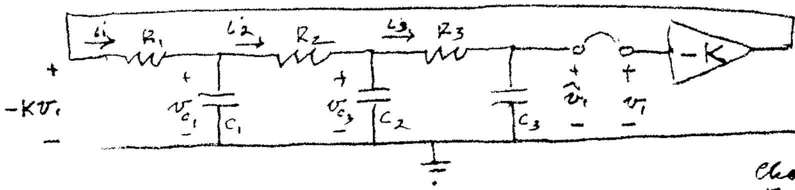
analysis of RC phase shift oscillator

02/21/12
RWN



Desire $\hat{v}_1 = v_1$ for natural response & for $v = Ve^{j\omega_0 t} + V^* e^{-j\omega_0 t}$

Choose input to $-K$ of $R_{in} \Rightarrow \infty$ so all of R_3 current is through C_3 [else add R_{in}]

1a,b,c) $i_1 = \frac{1}{R_1}(-Kv_1 - v_{c_1}) = sC_1 v_{c_1} + i_2 \Rightarrow -Kv_1 = (1+sR_1C_1)v_{c_1} + R_1 i_2$

2a,b,c) $i_2 = \frac{1}{R_2}(v_{c_1} - v_{c_2}) = sC_2 v_{c_2} + i_3 \Rightarrow v_{c_1} = (1+sR_2C_2)v_{c_2} + R_2 i_3$

3a,b,c) $i_3 = \frac{1}{R_3}(v_{c_2} - \hat{v}_1) = sC_3 \hat{v}_1 \Rightarrow v_{c_2} = (1+sR_3C_3)\hat{v}_1$

3b) & 3c) → 2c)

4a,b,c) $v_{c_1} = (1+sR_2C_2)v_{c_2} + sR_2C_3\hat{v}_1 = (1+sR_2C_2)(1+sR_3C_3)\hat{v}_1 + sR_2C_3\hat{v}_1$

4c) & 3c) → 2a)

5a) $i_2 = \frac{1}{R_2}\left([(1+sR_2C_2)(1+sR_3C_3)+sR_2C_3] - [1+sR_3C_3]\right)\hat{v}_1$

5b) $= \frac{1}{R_2}\left(sR_2C_2(1+sR_3C_3) + sR_2C_3\right)\hat{v}_1$

5b) & 4c) ⇒ 1c)

6a) $-Kv_1 = (1+sR_1C_1)\{(1+sR_2C_2)(1+sR_3C_3)+sR_2C_3\}\hat{v}_1 + \frac{R_1}{R_2}\left(sR_2C_2(1+sR_3C_3)+sR_2C_3\right)\hat{v}_1$

setting $v_1 = \hat{v}_1 \neq 0 \Rightarrow$

6b) $0 = (1+sR_1C_1)(1+sR_2C_2)(1+sR_3C_3) + (sR_2C_3)(1+sR_1C_1) + sR_1C_2(1+sR_3C_3) + sR_1C_3 + K$

or

6c) $0 = (1+[R_1C_1+R_2C_2]s + R_1C_1R_2C_2 s^2)(1+sR_3C_3) + R_2C_3 s + R_1C_1R_2C_3 s^2$
$+ R_1C_2 s + R_1C_2R_3C_3 s^2 + R_1C_3 s + K$

6d) $= 1 + [(R_1C_1+R_2C_2)+R_3C_3]s + [R_1C_1R_2C_2 + (R_1C_1+R_2C_2)R_3C_3]s^2 + (R_1C_1R_2C_2R_3C_3)s^3$
$+ (R_2C_3 + R_1C_2 + R_1C_3)s + (R_1C_1R_2C_3 + R_1C_2R_3C_3)s^2 + K$

6e) $= (R_1C_1R_2C_2R_3C_3)s^3 + \{R_1C_1+R_2C_2+R_3C_3+R_2C_3+R_1C_2+R_1C_3\}s$
$+ \{R_1C_1R_2C_2+R_1C_1R_3C_3+R_2C_2R_3C_3+R_1C_1R_2C_3+R_1C_2R_3C_3\}s^2 + (1+K)$

set $s = j\omega_0$ in 6e) & equate real and imaginary parts both $= 0$

7a) $0 = j\omega_0\{-R_1C_1R_2C_2R_3C_3\omega_0^2 + [R_1C_1+R_2C_2+R_3C_3+R_2C_3+R_1C_2+R_1C_3\}$

7b) $0 = -\omega_0^2\{R_1C_1R_2C_2 + R_1C_1R_3C_3 + R_2C_2R_3C_3 + R_1C_1R_2C_3 + R_1C_2R_3C_3\} + (1+K)$

set $R_1 = R_2 = R_3 = R$, $C_1 = C_2 = C_3 = C$, $\omega_0 \neq 0$

8a) $0 = -R^3C^3\omega_0^2 + 6RC \Rightarrow \omega_0^2 = 6/R^2C^2$

8b) $0 = -\omega_0^2 \cdot 5R^2C^2 + (K+1) \Rightarrow K = -1 + 5R^2C^2\omega_0^2 = -1 + 30 = 29$

If $R = 10K\Omega$, $C = 18\,nFd \Rightarrow \omega_0 = \frac{\sqrt{6}}{10^4 \times 18 \times 10^{-9}}$, $f_0 = \frac{1}{2\pi} \times \frac{\sqrt{6}}{18} \times 10^5\,Hz = 2.17\,KHz$

Note if $C_1 = 0$ then 7a) requires a negative R or C