Midterm Exam, 303 S2012

#1. $V_{CC} = R I_i + V_{BE}$, $I_i = I_o + \frac{2}{\beta} I_o = \left(\frac{\beta+2}{\beta}\right) I_o = \frac{102}{100} \times 6\times10^{-3}$
$= 6.12\times10^{-3}$

$V_{BE} \Rightarrow \quad |I_E| = I_S e^{V_{BE}/V_T}$

$$\Rightarrow V_{BE} = V_T \ln\left(\frac{|I_E|}{I_S}\right) = V_T \ln\left(\frac{I_o \times \left(\frac{\beta+1}{\beta}\right)}{I_S}\right)$$

$$= 26\times10^{-3} \ln\left(\frac{6\times10^{-3}\times\frac{101}{100}}{6\times10^{-16}}\right)$$

$$= 26\times10^{-3} \times (\ln 10^{13} \times 1.01) = 26\times10^{-3} \ln(10^{6\times2+1} \times 1.01$$

$$= 26\times10^{-3}(6\ln 100 + \ln 10 + 1.01)$$

$$= 26\times10^{-3}(27.63 + 2.30 + 0.01)$$

$$= 26\times(29.93 + 0.01)\times10^{-3}$$

$$= 0.778 + 0.0003$$

$$= 0.778$$

$$R = \frac{V_{CC} - V_{BE}}{I_i} = \frac{6 - 0.778}{6.12\times10^{-3}}$$

$$= 0.853\times10^{3}$$

$= 853$ ohm

#2. @ zero input = bias, M_p & M_n are in saturation as $|V_{GS}| = |V_{DS}| = 5 \geq |V_{GS}| - |V_{TO}| = 4$

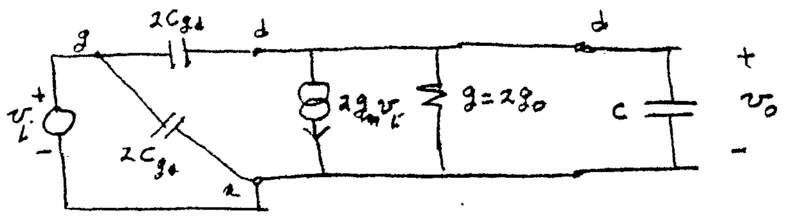
$$\Rightarrow I_{D_n} = -I_{D_p} = \frac{KP}{2}\frac{W}{L}(|V_{GS}| - |V_{TO}|)^2(1 + 0.1|V_{DS}|) = 4\times10^{-5}(5-1)^2(1+0.1\times5)/2$$

$$\frac{4}{2}\times16\times1.5\times10^{-5} = \frac{96}{2}\times10^{-5} = 0.48\ mA$$

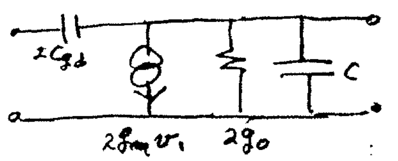
$$g_m = \frac{2I_{D_n}}{(|V_{GS}| - |V_{TO}|)} = \frac{4\times16\times1.5\times10^{-5}}{4} = 16\times1.5\times10^{-5}, \quad g_o = \lambda I_{D_n} = 0.048\,10^{-3}\ mho$$

$$= 0.24\times10^{-3}\ mho$$

Equivalent circuit (admittances double)



To find v_o/v_i consider the Y matrix for



$$Y = \begin{bmatrix} 2C_{gd}s & -2C_{gd}s \\ -2C_{gd}s + 2g_m & 2g_o + (2C_{gd} + C)s \end{bmatrix}$$

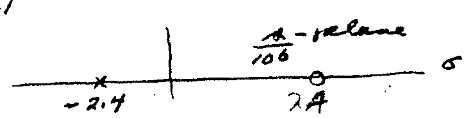
and set $i_2 = 0 \Rightarrow -y_{21}v_1 = y_{22}v_2 \Rightarrow$

$$\frac{v_2}{v_1} = -\frac{y_{21}}{y_{22}} = \frac{-(2g_m - 2C_{gd}s)}{2g_o + (2C_{gd}+C)s} = -\frac{g_m}{g_o}\left(\frac{1 - (C_{gd}/g_m)s}{1 + \left(\frac{2C_{gd}+C}{2g_o}\right)s}\right)$$

$$\approx -\frac{0.24\,10^{-3}}{0.048\,10^{-3}}\left(\frac{1 - (10^{-11}/0.24\times10^{-3})s}{1 + \left(\frac{40\times10^{-12}}{0.096\times10^{-3}}\right)s}\right) = -5\times\left(\frac{1 - 4.167\times10^{-8}s}{1 + 41.67\times10^{-8}s}\right)$$

$$= +0.5\left(\frac{s - 24\times10^{6}}{s + 2.4\times10^{6}}\right)$$

Zero @ $s = 24\times10^{6}$
pole @ $s = 2.4\times10^{6}$



#3. $v_o = -\frac{R_f}{R_i} v_i - \frac{R_f}{R_1} v_1 = -\frac{R_f}{R_i} v_i - \frac{R_f}{R_1} \cdot T(s) v_o \Rightarrow (1 + K \cdot T(s)) v_o = -K v_i$

$$\Rightarrow \frac{v_o}{v_i} = \frac{-(R_f/R_i)}{1 + (R_f/R_1) \cdot T(s)} \quad \text{where } T(s) = V_1/V_0 = \frac{1}{C_1C_2LRs^3 + C_2Ls^2 + (C_1+C_2)Rs + 1}$$

$$= \frac{-(K)\left[C_1C_2LRs^3 + C_2Ls^2 + (C_1+C_2)Rs + 1\right]}{C_1C_2LRs^3 + C_2Ls^2 + (C_1+C_2)Rs + (1+K)}$$

For oscillations, $s = j\omega_0$ & denominator $= 0$

$$\Rightarrow -j\omega^3 C_1C_2LR + j\omega(C_1+C_2)R - C_2L\omega^2 + (1+K) = 0$$

Imaginary $= 0 \Rightarrow \boxed{\omega_0^2 = \frac{C_1+C_2}{C_1C_2L}}$

Real $= 0 \Rightarrow (1 + R_f/R_1) = +C_2L\omega_0^2 = 1 + \frac{C_2}{C_1} \Rightarrow \underline{\frac{R_f}{R_1} = \frac{C_2}{C_1} = K}$

Poles are zeros of denominator, two from $s^2 + \omega_0^2$

$$\Rightarrow \underline{s_{1,2} = \pm j\omega_0 = \pm j\sqrt{\frac{C_1+C_2}{C_1C_2L}}}$$

The other is from $s + \sigma_0$ found by long division

$$\begin{array}{r|l} & C_1C_2LRs + C_2L \\ \hline s^2 + \omega_0^2 & C_1C_2LRs^3 + C_2Ls^2 + (C_1+C_2)Rs + C_2L\omega_0^2 \\ & C_1C_2LRs^3 \quad C_2L \quad + C_1C_2LR\omega_0^2 s + C_2L\omega_0^2 \\ \hline & 0 \qquad\qquad 0 \end{array}$$

$\Rightarrow C_1C_2LRs + C_2L = C_1C_2LR\left(s + \frac{1}{C_1R}\right)$

$$\Rightarrow \underline{\sigma_0 = \frac{1}{C_1R}}$$

$$\Rightarrow C_1C_2LRs^3 + C_2Ls^2 + (C_1+C_2)Rs + (1 + R_f/R_1) = C_1C_2LR(s^2 + \omega_0^2)(s + \sigma_0)$$

$$\Rightarrow \text{poles: } s_{1,2} = \pm j\sqrt{\frac{C_1+C_2}{C_1C_2L}}, \quad s_0 = -\sigma_0 = -\frac{1}{C_1R}$$

Note $C_{series} = C_s = \frac{C_1C_2}{C_1+C_2} \Rightarrow \underline{\omega_0^2 = \frac{1}{C_s \cdot L}} \Rightarrow \omega_0$ from L & C_s is reasonable as L & C_s are in parallel