

# Van der Pol oscillator

$\epsilon$  positive

$$\ddot{x} + \epsilon(x^2 - 1)\dot{x} + \frac{1}{LC}x = 0$$

$$x(0), \dot{x}(0)$$

$$\ddot{x} + \dot{f}(x) + \omega_0^2 x = 0$$

$$\dot{f}(x) = f'(x) \cdot \dot{x}$$

$$f' = \epsilon(x^2 - 1)$$

$$f(x) = \epsilon \frac{1}{3} x^3 - x$$

$$\ddot{x} + \dot{f}(x) + \omega_0^2 x = 0$$

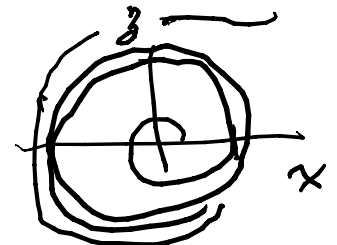
$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = \frac{d^2x}{dt^2} = -\dot{f}(x) - \omega_0^2 x = -f'(x) \cdot \dot{x} - \omega_0^2 x = -f'(x) \cdot y - \omega_0^2 x = \frac{d^2x}{dt^2}$$

$$\begin{cases} \frac{dx}{dt} = y \\ \frac{dy}{dt} = -\epsilon(x^2 - 1)y - \omega_0^2 x \end{cases}$$

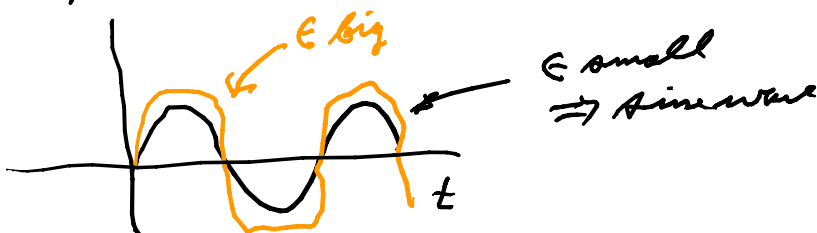
two nonlinear 1st order differential state variable eqs

$$\frac{dy/dt}{dx/dt} = \frac{dy}{dx} = -\frac{(\epsilon(x^2 - 1)y + \omega_0^2 x)}{y}$$



goes to a limit cycle

$f(\epsilon)$  varies with  $\epsilon$



structurally stable if  $\epsilon \neq 0$