RWN
04/10/11

Solutions EE303 S11 Midterm Makeups

#1. a) From class notes, $-\infty < V_i < \infty$

$$I_o = \alpha I_T \tanh\left(\frac{V_i}{2V_T}\right) \Rightarrow G_m = \left.\frac{dI_o}{dV_i}\right|_{V_i=0} = \left.\frac{\alpha I_T}{2V_T}\left(1 - \tanh\left(\frac{V_i}{2V_T}\right)\right)\right|_{V_i=0}$$

$$= \frac{\alpha I_T}{2V_T} = \frac{\beta}{1+\beta}\frac{I_T}{2V_T}$$

$$\therefore G_m\Big|_{\substack{BJT\\OTA}} = \frac{\beta}{1+\beta}\frac{I_T}{2V_T} = \frac{150}{151}\cdot\frac{5\times10^{-3}}{2\times26\times10^{-3}} = 0.0955\ \mho = 95 \text{ milli Mho}$$

b) From class notes, $-V_S < V_i < V_S = \sqrt{\frac{I_T}{2KP(W/L)}}$ [which covers the bias point of $V_i = 0$]

for which

$$I_o = G_o V_i \sqrt{1 - \tfrac{1}{2}(V_i/V_S)^2}\ ,\quad G_o = \sqrt{2 I_T \tfrac{KP}{2}(\tfrac{W}{L})}$$

$$\Rightarrow G_m = \left.\frac{dI_o}{dV_i}\right|_{V_i=0} = G_o\left.\left(\sqrt{1-\tfrac{1}{2}(V_i/V_S)^2} + V_i\cdot\frac{1}{2}\frac{1}{\sqrt{1-\tfrac{1}{2}(V_i/V_S)^2}}\left(-\frac{1}{2}\times 2\frac{V_i}{V_S^2}\right)\right)\right|_{V_i=0}$$

$$= G_o$$

For the 4007 differential pair

b₁) NMOS, $G_m\Big|_{\substack{OTA\\NMOS}} = \sqrt{2\times5\times10^{-3}\times\frac{20.54}{2}\times10^{-6}\times\left(\frac{144\times10^{-6}}{8\times10^{-6}}\right)} = \sqrt{1{,}848.6\times10^{-9}}$

$$= \sqrt{1.8486\times10^{-6}} = 1.360\times10^{-3}\ \mho = 1.36 \text{ milli Mho}$$

b₂) PMOS, $G_m\Big|_{\substack{OTA\\PMOS}} = \sqrt{2\times5\times10^{-3}\times\frac{10.32\times10^{-6}}{2}\left(\frac{328\times10^{-6}}{8\times10^{-6}}\right)} = \sqrt{2{,}115.6\times10^{-9}}$

$$= \sqrt{2.1156\times10^{-6}} = 1.455\times10^{-3}\ \mho = 1.46 \text{ milli Mho}$$

#2. G_m = slope of I_o versus V_i in $-V_S \le V_i \le V_S$

$$= I_T/V_S$$

#3. a) (a) $v_c = v_{in} - v_a$, $i_o = C\frac{dv_c}{dt} = C\frac{d[v_{in} - v_a]}{dt} = G_m v_a$

$$\Rightarrow C\frac{dv_a}{dt} + G_m v_a = C\frac{dv_{in}}{dt}$$

(b) $v_c = v_{in} - v_b$, $i_o = C\frac{dv_c}{dt} = C\frac{d[v_{in} - v_b]}{dt} = -G_m v_b$

$$\Rightarrow C\frac{dv_b}{dt} - G_m v_b = C\frac{dv_{in}}{dt}$$

b) $\frac{d}{dt} \Rightarrow s$, $\mathcal{L}[v_a] = V_a$, $\mathcal{L}[v_b] = V_b$, $v_{in} = \delta(t)$, $\mathcal{L}[v_{in}] = 1$

(a) $(sC + G_m)V_a = sC\, V_{in} = sC$

$$\Rightarrow V_a(s) = \frac{sC}{sC + G_m} = \frac{s}{s + (G_m/C)} = 1 + \frac{-(G_m/C)}{s + (G_m/C)}$$

$$\Rightarrow v_a\Big|_{v_{in}=\delta}(t) = \delta(t) - \frac{G_m}{C} e^{-(G_m/C)t}\, 1(t)$$

(b) $(sC - G_m)V_b = sC$

$$\Rightarrow V_b(s) = \frac{sC}{sC - G_m} = \frac{s}{s - (G_m/C)} = 1 + \frac{+(G_m/C)}{s - (G_m/C)}$$

$$\Rightarrow v_b\Big|_{v_{in}=\delta}(t) = \delta(t) + \frac{G_m}{C} e^{+(G_m/C)t}\, 1(t)$$

c) In (a) the OTA gives a positive resistor while in (b) it gives a negative one; for (a) the response → 0 as t → ∞ while for (b) the response → ∞ as t → ∞, an unstable circuit.

#4 a) $V_{GS} = \frac{R_{b2}}{R_{b1}+R_{b2}} \cdot V_{dd} = \frac{1}{2} \cdot 6 = 3V = V_{GS}$

$I_D = \frac{KP}{2}\frac{W}{L}(V_{GS} - V_{TO_n})^2(1+\lambda V_{DS})$ if in saturation

$= \frac{20.54\times10^{-6}}{2} \cdot \frac{144}{8}(3-1.3)^2(1+0.015 V_{DS})$

$= 5.342\times10^{-4}(1+0.015 V_{DS})$

and a second equation in the two unknowns, I_D & V_{DS}

$V_{DD} = R_L I_D + V_{DS}$

$\Rightarrow \quad 6 = 0.5342(1+0.015 V_{DS}) + V_{DS}$

$\Rightarrow \quad 6 - 0.5342 = (1+0.5342\times0.015)V_{DS} \Rightarrow 5.4658 = (1.008)V_{DS}$

$\Rightarrow \quad \underline{V_{DS} = 5.42\ V} \quad \Rightarrow V_{DS} = 5.42 > 3-1.3 = 2.7 = V_{GS} - V_{TO_n} \Rightarrow$ saturation

$\Rightarrow \quad I_D = 5.342\times10^{-4}(1+0.015\times5.42) = 5.776\times10^{-4}$

$\Rightarrow \quad \underline{I_D = 0.578\ mA}$

b) $g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q = \frac{2 I_D}{(V_{GS}-V_{TO_n})} = 2\,\frac{0.578\times10^{-3}}{(1.7)} = 0.68\ m\mho$

$g_o = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q = \frac{\lambda I_D}{1+\lambda V_{DS}} = \frac{0.015\times0.578\times10^{-3}}{\underbrace{1+0.015\times5.42}_{1.0813}} = 8.02\times10^{-6} = 8.02\ \mu\mho$

$A_v = -g_m R_L = -0.68\times10^{-3}\times1\times10^{3}$
$= -0.68$