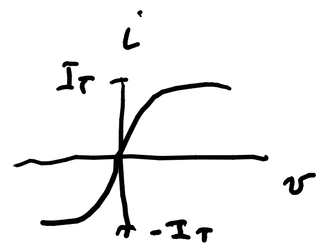
Note Title 3/14/2011

03/14/11
EE 303+

Midterm on W open book, notes
but not computers

BJT diff pair $\Rightarrow$ OTA

$$i = \alpha I_T \tanh\left(v/2V_T\right)$$



$$\left.\frac{di}{dv}\right|_{v=0} = \alpha I_T\left(\frac{1}{2V_T}\right)\left.\frac{d\tanh x}{dx}\right|_{x=0}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

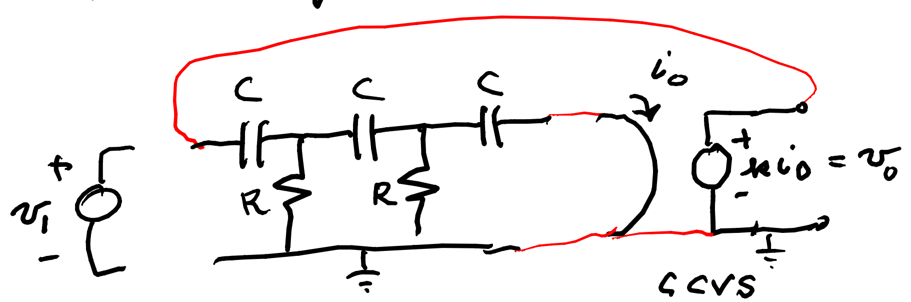
$$\frac{d\tanh x}{dx} = (e^{x}+e^{-x})\frac{1}{e^{x}+e^{-x}} - (e^{x}-e^{-x})\frac{(e^{x}-e^{-x})}{(e^{x}+e^{-x})^2}$$

$$= 1 - \tanh^2 x$$

$$\therefore \quad i = 0 + \alpha \frac{I_T}{2V_T}\left(1-\tanh^2\left(\frac{v}{2V_T}\right)\right)(v-\overset{=0 \text{ fixed at origin}}{v}) + \text{higher order terms}$$

G_m at the bias point ≥ 0

RC phase shift oscillator   p. 1345



CCVS

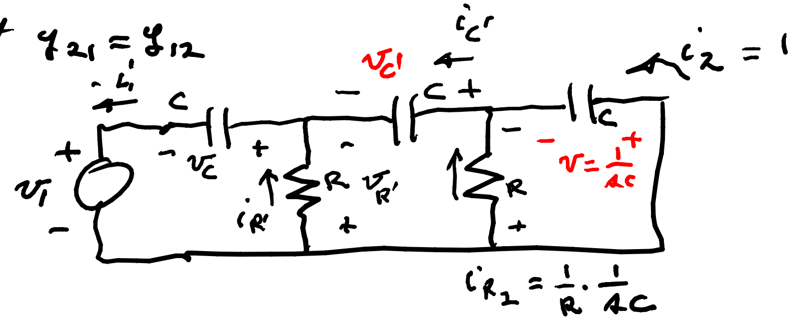
(H in Space

$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{12} & y_{11} \end{bmatrix}$ desire $i_o = -i_2$ when $v_2 = 0$

$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \overset{v_2=0}{\Longrightarrow} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix} [v_1]$

$\therefore$ want $y_{21} = y_{12}$

analyze



$$i_{c'} = i_2 + i_{R_2} = 1 + \frac{1}{RCs} \; ; \quad v_c' = \frac{1}{sC} \cdot \left(1 + \frac{1}{RCs}\right)$$

$$v_{R'} = v_{c'} + v = \frac{1}{sC} + \frac{1}{R(sC)^2} + \frac{1}{sC} = \frac{2}{sC} + \frac{1}{RC^2s^2}$$

$$i_{R'} = \frac{1}{R} v_{R'} = \frac{2}{RCs} + \frac{1}{(RCs)^2}$$

$$i_c = i_{R'} + i_{c'} = \frac{2}{RCs} + \frac{1}{(RCs)^2} + 1 + \frac{1}{RCs} = 1 + \frac{3}{RCs} + \frac{1}{(RCs)^2}$$

$$-v_1 = v_c + v_{R'} = \frac{1}{sC}\left[1 + \frac{3}{RCs} + \frac{1}{(RCs)^2}\right] + \frac{2}{sC} + \frac{1}{RC^2s^2}$$

$$= \frac{1}{R^2C^3s^3} + \frac{4}{RC^2s^2} + \frac{3}{sC} = \frac{1}{R^2C^3s^3}\left[1 + 4RCs + R^2C^2s^2\right]$$

$$y_{21} = \frac{1}{v_1} = -\left[\frac{R^2C^3s^3}{R^2C^2s^2 + 4RCs + 1}\right]$$

$$v_o = k\, i_o = k(-i_2) = k\left(\frac{-i_2}{v_1}\right) \cdot v_1 = k(-y_{21})\, v_1$$

but when connect output to input $v_o = v_1$

$$v_o \simeq v_1 = k\left[\frac{R^2C^3s^3}{1 + 4RCs + R^2C^2s^2}\right] v_1$$

look at $\left(1 - k\,\dfrac{R^2C^3s^3}{1 + 4RCs + R^2C^2s^2}\right) v_1 = 0$

$\Rightarrow \quad -k R^2C^3s^3 + R^2C^2s^2 + 4RCs + 1 = 0 \qquad j = \sqrt{-1}$

set $s = j\omega$ for sinusoidal steady state

$\Rightarrow$ oscillations

$$-k R^2C^3(-j\omega^3) - R^2C^2\omega^2 + 4jRC\omega + 1 = 0$$

$\Rightarrow$ set real & imaginary parts $= 0$

Re: $\quad 1 - R^2C^2\omega^2 = 0$

Im: $\quad k R^2C^3\omega^3 + 4RC\omega = 0 \Rightarrow RC + kR^2C^3\omega^2 = 0$

oscillates at $\omega_0^2 = 1/R^2C^2 \Rightarrow \omega_0 = 2\pi f_0 = \frac{1}{RC}$

if we set k: $\quad k = \frac{-RC}{R^2C^3\omega_0^2} = -\frac{R^2C^2}{R^2C^3} = -\frac{1}{C}$

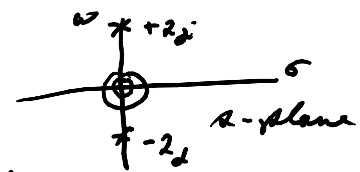
$\therefore$ can make an oscillator with this circuit

Impulse response: $\quad$ Input $= \delta(t) =$ unit impulse

$= \frac{d1(t)}{dt}, \quad 1(t) =$ unit step $= \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$

$\mathcal{L}[\delta] = 1 \qquad \frac{\text{output}}{\text{input}}(s) = T(s)$

Ex: $T(s) = \dfrac{s^2}{s^2+4}$



$\mathcal{L}^{-1}[T(s)]$ = inverse laplace transform

use partial fraction

$$T(s) = \frac{k}{s+j2} + \frac{k^*}{s-j2} + 1 \qquad * = j \to -j$$

$$k = k_r + j k_i$$

$$= \frac{s^2}{s^2+4} = \frac{ks - j2k}{s^2+4} + \frac{k^* s + j k^* 2}{s^2+4} + \frac{s^2+4}{s^2+4}$$

$$= \frac{s^2 + (k+k^*)s + j(2k^* - 2k) + 4}{s^2+4}$$

$$(k+k^*)s = 0, \quad j(2k^* - 2k) + 4 = 0$$

$$\Downarrow$$

$$\operatorname{Re} k = k_r = 0 \quad ; \quad 2j(k^* - k) = -4$$

$$2j(2(-k_i)) = -4 \implies k_i = \frac{4}{4j} = -j$$

$$T(s) = \frac{s^2}{s^2+4} = \frac{-j}{s+j2} + \frac{+j}{s-j2} + 1 = \frac{-js - 2 + js - 2 + s^2 + 4}{s^2+4}$$

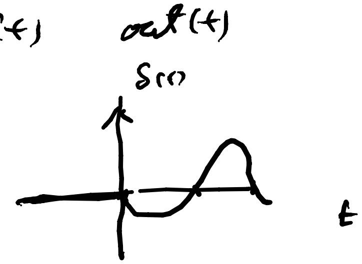
$$\mathcal{L}[e^{-at} 1(t)] = \frac{1}{s+a} \text{ for } \operatorname{Re} s > -a$$

$$\mathcal{L}^{-1}[T(s)] = -je^{-j2t}\,1(t) + je^{-(-2j)t}\,1(t) + \delta(t)$$

$$= j\left[e^{j2t} - e^{-j2t}\right] 1(t) + \delta(t)$$

$$= -2\,\frac{e^{j2t} - e^{-j2t}}{2j}\,1(t) + \delta(t)$$

$$= -2\sin(2t)\,1(t) + \delta(t)$$



in $= v_{in}$, out $= v_{out}$

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = T(s) = \frac{s^2}{s^2+4} \implies \text{let } s = \frac{d}{dt}$$

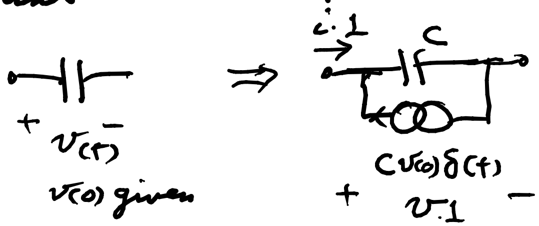
$$= \frac{v_{out}(t)}{v_{in}(t)}$$

$$(s^2+4)\,v_{out}(t) = s^2 v_{in}(t)$$

if $v_{in}(t) = \delta(t)$

this gives $v_{out} = -2\sin(2t)\,1(t) + \delta(t)$

Initial conditions on a capacitor



$t > 0$

$$C\frac{dv}{dt} = i(t) \xRightarrow{\times 1(t)} C\frac{dv}{dt}1(t) = i(t)1(t) \qquad t > 0$$

$$C\frac{d(v(t)1(t))}{dt} = C\frac{dv}{dt}1 + C\,v(t)\frac{d1(t)}{dt} = C\frac{dv}{dt}1 + C\,v(0)\delta(t)$$

$$i(t)1(t) = C\frac{dv1(t)}{dt} - C\,v(0)\delta(t)$$