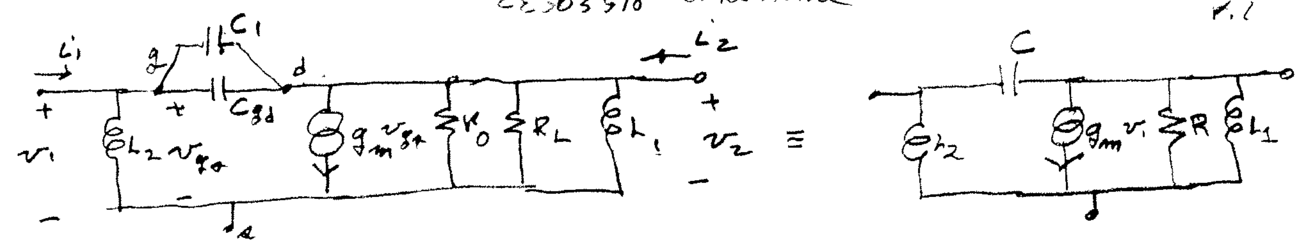
a1)



a2) $i = Yv \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} sC + \frac{1}{sL_2} & -sC \\ g_m - sC & sC + G + \frac{1}{sL_1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

a3) $\Delta Y(s) = (sC + \frac{1}{sL_2})(sC + G + \frac{1}{sL_1}) - (g_m - sC)(-sC)$

$= \cancel{s^2C^2} + sCG + \frac{C}{L_2} + \frac{C}{L_1} + \frac{G}{sL_2} + \frac{1}{s^2L_1L_2} + s g_m C - \cancel{s^2C^2}$

$= \frac{1}{s^2L_1L_2}\left[s^3(CGL_1L_2 + Cg_mL_1L_2) + s^2(CL_1 + CL_2) + s(GL_1) + 1\right]$

For oscillations; $s = j\omega_0$, as $v = Y^{-1} \cdot i = Y^{-1} \times 0 \Rightarrow \infty \cdot 0 \neq 0$ via IC's.

$\Delta Y(j\omega_0) = \frac{-1}{\omega_0^2 L_1 L_2}\left[-j\omega_0^3 CL_1L_2(G+g_m) - \omega_0^2 C(L_1+L_2) + j\omega_0 GL_1 + 1\right]$

$\mathrm{Re}\,\Delta Y(j\omega_0) = 0 \Rightarrow \omega_0^2 = \frac{1}{C(L_1+L_2)}$

$\mathrm{Im}\,\Delta Y(j\omega_0) = 0 \Rightarrow \omega_0\left[-\omega_0^2 CL_1L_2(G+g_m) + GL_1\right] = 0 \Rightarrow g_m + G = \frac{GL_1}{\omega_0^2 CL_1L_2}$

$\Rightarrow g_m = G\left[-1 + \frac{1}{\omega_0^2 CL_2}\right] = G\left[-1 + \frac{C(L_1+L_2)}{CL_2}\right] = GL_1/L_2$

$= GL_1/L_2$

a31) $\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{C(L_1+L_2)}}$

Then $s^2 + \omega_0^2$ is a factor. By long division the other factor is

$$\begin{array}{r|l} & s\cdot CL_1L_2(G+g_m) + C(L_1+L_2) \\ \hline s^2+\omega_0^2 & s^3CL_1L_2(G+g_m) + s^2C(L_1+L_2) + sGL_1 + 1 \\ & s^3CL_1L_2(G+g_m) + sCL_1L_2(G+g_m)\omega_0^2 \\ & + s^2C(L_1+L_2) + \omega_0^2C(L_1+L_2) \\ \hline & 0 \quad 0 \quad \underbrace{sL_1[G - CL_2(G+g_m)\omega_0^2]}_{=0 \text{ by } g_m = GL_1/L_2} \quad 0 \end{array}$$

Here $CL_1L_2(G+g_m) = CL_1L_2(G + G\frac{L_1}{L_2}) = CL_1G(L_1+L_2)$

$\Rightarrow sCL_1L_2(G+g_m) + C(L_1+L_2) = C(L_1+L_2)[sL_1G + 1]$

$= C(L_1+L_2)L_1G[s + 1/L_1G]$

∴ a32)

$\Delta Y(s) = \frac{C(L_1+L_2)G}{s^2L_2}\left[(s^2+\omega_0^2)(s + \frac{1}{L_1G})\right]$

a4) $\frac{v_2}{v_1}(s)\Big|_{i_2=0} \Rightarrow \quad i_2 = 0 = y_{21}v_1 + y_{22}v_2 \Rightarrow \frac{v_2}{v_1} = -\frac{y_{21}}{y_{22}}$

$$\therefore \frac{v_2}{v_1}(s) = \frac{g_m - sC}{sC + G + \frac{1}{sL_1}} = \frac{-s(s - g_m/C)}{s^2 + \frac{G}{C}s + \frac{1}{L_1C}}$$

a5) as V_{dd} is on G & D wrt S, M_n is in saturation, $V_{DS} = V_{GS} = V_{dd}$

$$I_D = \frac{KP}{2}\frac{W}{L}(V_{dd} - VTO_n)^2(1 + \lambda_n V_{dd})$$

$$\Rightarrow \frac{W}{L} = \frac{I_D}{\frac{KP}{2}(V_{dd} - VTO_n)^2(1 + \lambda_n V_{dd})} = \frac{2\times4\times10^{-3}}{10^{-4}(10-1)^2(1+0.01\times10)}$$

$$= \frac{8\times10}{81\times1.1} = 0.898$$

a6) $$g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = \frac{2I_D}{(V_{GS} - VTO_n)} = \frac{2\times4\times10^{-3}}{9} = \frac{8}{9}\,m\mho = 0.889\,m\mho$$

$$g_o = \frac{\partial i_D}{\partial v_{DS}}\Big|_Q = \frac{\lambda_n I_D}{(1 + \lambda_n V_{DS})} = \frac{0.01\times4\times10^{-3}}{1.1} = \frac{40}{1.1}\mu\mho = 36.4\,\mu\mho$$

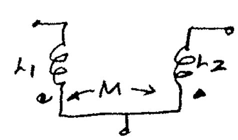
a7) $G_L = 1/R_L = 10^{-3}\mho$, as $g_m = \frac{GL_1}{L_2} \Rightarrow L_2 = \frac{GL_1}{g_m}$; $G = G_L + g_o = (1.0364)\times10^{-3}$

$$\Rightarrow L_2 = \frac{9}{8}\times(1.0364)\times3\times10^{-3} = 3.498\,mHy$$

$$\Rightarrow \omega_0 = \sqrt{\frac{1}{C(L_1+L_2)}} = \frac{1}{\sqrt{C}}\cdot\frac{1}{\sqrt{6.498\times10^{-3}}} = \frac{1}{\sqrt{C}}\cdot\frac{10^2}{\sqrt{64.98}} =$$

$$= \frac{1}{\sqrt{C}}\times\frac{10^2}{8.061} = \frac{12.4}{\sqrt{C}}$$

b) For mutually coupled coils the 2-port Y matrix of



is needed. This has $Z = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} s$

so $Y = \frac{1/s}{L_1L_2 - M^2}\begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix}$ if $L_1L_2 - M^2 \neq 0$ which is true if $k^2 = \frac{M^2}{L_1L_2} < 1$