EE 303
05/11/10

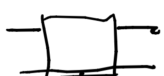
Final exam: open book, notes
part take home

topics
- current mirrors
- small signal
- biasing
- inverters
- poles-zeros
- oscillator
- BJT & CMOS
- op-amp, OTA

Review:
electronics can mimic if scale almost anything

Main devices, R, L, C, BJT, CMOS, diodes
all describe by models & use Spice
need the mathematical models

put into systems



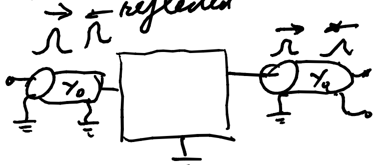
2-port

descriptions
if "linear" use Taylor series
need Q point, need to bias

small signal
$i = Y(s)v$, $Y(s)$ = admittance matrix
$Z(s) = Y^{-1}(s)$

$S(s)$ = scattering matrix
$= (Y - Y_0 1_n)(Y + Y_0 I_n)^{-1}$

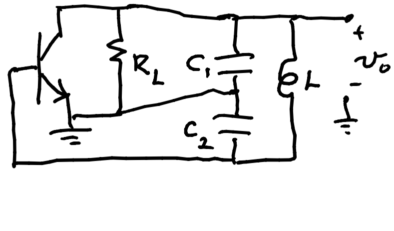
incident
reflected

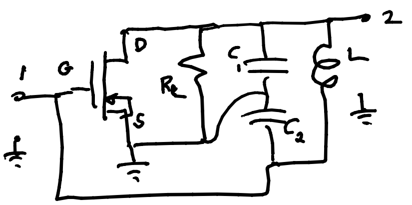


State space descriptions
use when work with nonlinearities

from cap $\rightarrow \dot{x} = a(x) + Bu$ $\quad x$ = state-vector
$y = Cx + Du$

Ex: Colpitts oscillator p. 1179





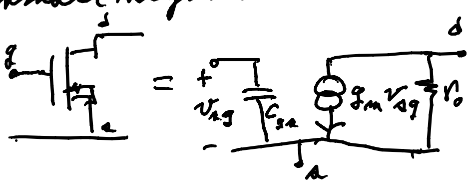
for bias to allow signal to pass

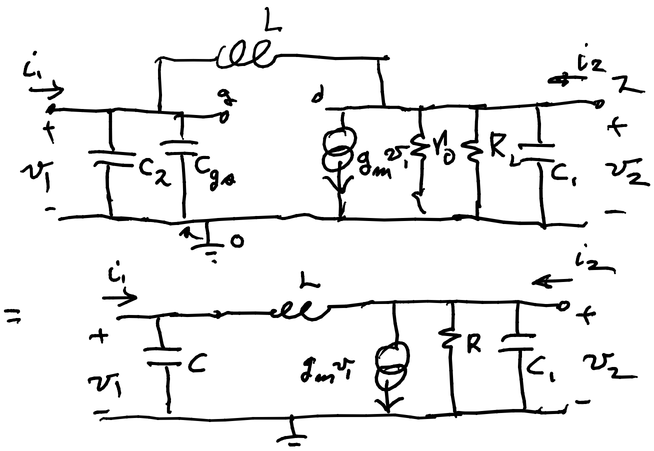
C large { bias open / signal short }

L large { bias short / signal open

an RFC = radio frequency choke

for small signal





$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} sC + \frac{1}{sL} & -\frac{1}{sL} \\ g_m - \frac{1}{sL} & sC_1 + \frac{1}{sL} + G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

; $i = Yv$

$v = Z^{-1} i$

; $G = 1/R$

$G = g_o + G_L$

$G_L = 1/R_L$

$$\det Y(s) = \left(sC + \frac{1}{sL}\right)\left(sC_1 + \frac{1}{sL} + G\right) - \left(g_m - \frac{1}{sL}\right)\left(-\frac{1}{sL}\right)$$

$$= s^2 CC_1 + \frac{C}{L} + sCG + \frac{C_1}{L} + \cancel{\frac{1}{s^2L^2}} + \frac{G}{sL} + \frac{g_m}{sL} - \cancel{\frac{1}{s^2L^2}}$$

$$= \frac{1}{sL}\left\{ s^3 CC_1L + s^2 CGL + s(C+C_1) + (G+g_m) \right\}$$

For an oscillator desire a pole of $Z(s)$ on $j\omega$ axis

$s = j\omega_0$

$$\det Y(j\omega_0) = \frac{1}{j\omega_0 L}\left\{ -j\omega_0^3 CC_1L - \omega_0^2 CGL + j\omega_0(C+C_1) + (G+g_m) \right\}$$

$$\Rightarrow -\omega_0^3 CC_1L + \omega_0(C+C_1) = 0; \quad \omega_0 \neq 0 \quad \omega_0^2 = \frac{C+C_1}{CC_1L}$$

$$\omega_0 = \frac{1}{\sqrt{L \cdot \frac{CC_1}{C+C_1}}}$$

$$\Rightarrow -\omega_0^2 GCL + (G+g_m) = 0$$

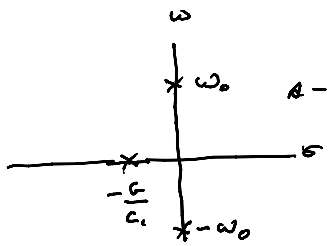
$$g_m = -G + G(CL\omega_0^2) = G\left[-1 + \cancel{CL} \times \frac{C+C_1}{\cancel{C}C_1\cancel{L}}\right] = G\left[\frac{-C_1+C+C_1}{C_1}\right]$$

$$\Rightarrow \; g_m = G\frac{C}{C_1} > 0 \text{ if pos. element values}$$

$$\det Y(s) = \frac{k_0}{s}(s^2+\omega_0^2)(s+\sigma_0) =$$

$$= \frac{CC_1L}{sL}\left\{ s^3 + \frac{s^2 CGL}{CC_1L} + \frac{s(C+C_1)}{CC_1L} + \frac{(G+g_m)}{CC_1L}\right\}$$

shows $\omega_0^2\sigma_0 = \dfrac{G+g_m}{CC_1L} \Rightarrow \sigma_0 = \dfrac{G+g_m}{CC_1L}\cdot\dfrac{1}{\omega_0^2} > 0$



$$= \frac{G+g_m}{CC_1L}\cdot L\cdot\frac{CC_1}{C+C_1}$$

$$= \frac{G+g_m}{C+C_1} = \frac{G+G\,C/C_1}{\left(\frac{C}{C_1}+1\right)C_1}$$

$$= \frac{G}{C_1}\cdot\frac{(1+C/C_1)}{(1+C/C_1)} = \frac{G}{C_1}$$

For $\left.\dfrac{v_2}{v_1}(s)\right|_{i_2=0}$: 2nd row of Y: $\underset{0}{\overset{\shortparallel}{i_2}} = y_{21}v_1 + y_{22}v_2 \Rightarrow \dfrac{v_2}{v_1} = -\dfrac{y_{21}}{y_{22}}$

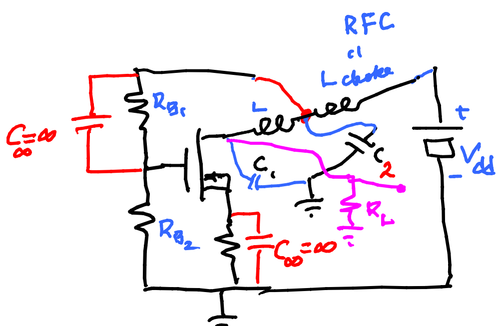
$$= -\frac{y_{21}}{y_{22}} = \frac{-\left(g_m - \frac{1}{sL}\right)}{Cs+\frac{1}{sL}+G} = \frac{-(g_mLs-1)}{CLs^2+GLs+1} = \frac{-g_mL\left(s-\frac{1}{g_mL}\right)}{CL\left(s^2+\frac{G}{C}s+\frac{1}{LC}\right)}$$

denominator polynomial

$$s^2+\frac{G}{C}s+\frac{1}{LC} = s^2+\frac{\omega_m}{Q}s+\omega_m^2 \Rightarrow \frac{\omega_m}{Q} = \frac{G}{C} \;\&\; Q = \frac{\omega_m C}{G}$$

$\dfrac{v_2}{v_1}$ has a zero in RH s-plane @ $s = 1/g_mL$

Desire to bias otherwise it won't work



choose R_{B_1} & R_{B_2} very large

if properly bias can put the NMOS in saturation

$$i_D = \frac{KP}{2}\frac{W}{L}(v_{GS}-V_{TO_n})^2(1+\lambda_n v_{DS})$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q = \left.\frac{2\,i_D}{(v_{GS}-V_{TO_n})}\right|_Q = \frac{2I_D}{V_{GS}-V_{TO_n}}$$