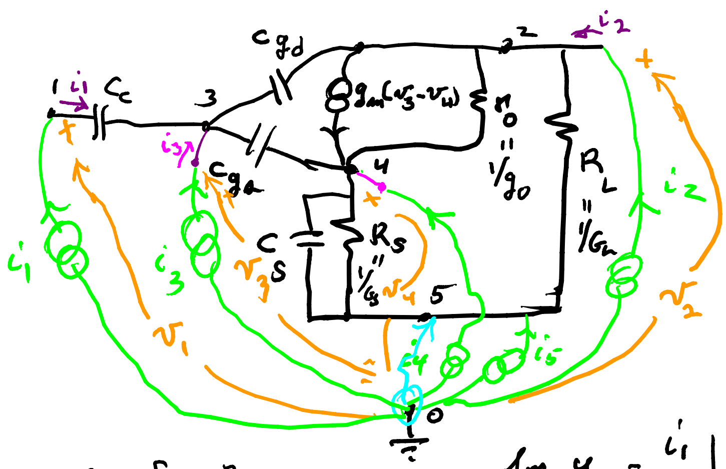
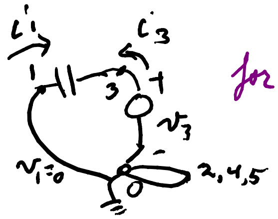
EE 303
03/04/10



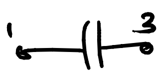


for y_{31}

$$i = Y_{ind}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

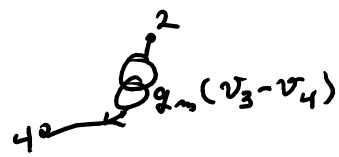
for $y_{13} = \dfrac{i_1}{v_3}\Big|_{v_1=v_2=v_4=v_5=0} = -sC$ $\quad = y_{31}$ by reciprocity

$$y_{11} = y_{33} = \frac{i_3}{v_3}\Big|_{v_1=v_2=v_4=v_5=0} = sC$$



$$\Rightarrow \begin{array}{cc} & \\ \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix} & \begin{array}{l} \leftarrow 1 \\ \leftarrow 3 \end{array} \\ \uparrow 1 \quad\quad \uparrow 3 & \end{array}$$

3.



$$i_3\big|_{g_m} = 0 \qquad i_2\big|_{g_m} = +g_m(v_3 - v_4)$$

$$i_4\big|_{g_m} = -g_m(v_3 - v_4)$$

$$\begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline 2 \rightarrow & 0 & g_m & -g_m \\ 3 \rightarrow & 0 & 0 & 0 \\ 4 \rightarrow & 0 & -g_m & g_m \end{array}$$

ind = indefinite

$$Y_{ind} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & sC_c & 0 & -sC_c & 0 & 0 \\ 2 & 0 & sC_{gd}+g_o+G_L & -sC_{gd}+g_m & -g_o-g_m & -G_L \\ 3 & -sC_c & -sC_{gd} & sC_c+sC_{gs}+sC_{gd} & -sC_{gs} & 0 \\ 4 & 0 & -g_o & -sC_{gs}-g_m & g_o+sC_{gs}+g_m+sC_s+G_s & -sC_s-G_s \\ 5 & 0 & -G_L & 0 & -sC_s-G_s & G_L+sC_s+G_s \end{array}$$

$\Downarrow$ Y_{def} (4x4)

ground node 5 →

for Y_{ind}, sum of entries in a row & in a column = 0

set $v_5 = 0 \Rightarrow$ can delete column 5 as entries multiply v_5

Now have

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = Y_{def} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$Y_{def} = Y_{ind}$ after gnd node 5 $\Rightarrow$ removed row 5 & col. 5

want these = 0 $\begin{bmatrix} i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $\begin{bmatrix} v_3 \\ v_4 \end{bmatrix} \neq 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ \hline 0 \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ \hline v_3 \\ v_4 \end{bmatrix}$$

Y_{ij} all 2x2

solve for $\begin{bmatrix} v_3 \\ v_4 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{22} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}$

$$\begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = Y_{22}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right\} = -Y_{22}^{-1} Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

into 1st 2 rows

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y_{11} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{12} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = Y_{11} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{12} \left\{ -Y_{22}^{-1} Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right\}$$

$$= \left\{ Y_{11} - Y_{12} Y_{22}^{-1} Y_{21} \right\} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Y_{2\text{-port}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{2\text{-port}} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$Y_{11} = \left[\begin{array}{c|c} sC_c & 0 \\ \hline 0 & sC_{gs} + g_o + G_L \end{array}\right] \qquad Y_{12} = \left[\begin{array}{c|c} -sC_c & 0 \\ \hline -sC_{gs} + g_m & -g_o - g_m \end{array}\right]$$

$$Y_{21} = \left[\begin{array}{c|c} sC_c & -sC_{gd} \\ \hline 0 & g_o \end{array}\right] \qquad Y_{22} = \left[\begin{array}{c|c} sC_c + sC_{gs} + sC_{gd} & -sC_{gs} \\ \hline -sC_{gs} - g_m & g_o + sC_{gs} + g_m + sC_s + G_s \end{array}\right]$$

need $Y_{22}^{-1} = \dfrac{1}{\Delta} \left[\begin{array}{c|c} g_o + sC_{gs} + g_m + sC_s + G_s & +sC_{gs} \\ \hline sC_{gs} + g_m & s(C_c + C_{gs} + C_{gd}) \end{array}\right]$

Δ = determinant of $Y_{22} = s(C_c + C_{gs} + C_{gd})(s[C_s + C_{gs}] + [g_o + G_s + g_m]) - (-sC_{gs})(-sC_{gs} - g_m)$

Δ = degree 2 polynomial in s

$$Y_{2\,port} = \begin{bmatrix} sC_c & 0 \\ 0 & g_o+G_L+sC_{gd}\end{bmatrix} - \frac{1}{\Delta}\begin{bmatrix} -sC_c & -sC_{gd} \\ 0 & g_d\end{bmatrix}\begin{bmatrix} g_o+sC_{gd}+g_m+sC_g+G_S & sC_{gd} \\ sC_{gs}+g_m & s(C_c+C_{gs}+C_{gd})\end{bmatrix}$$

$$\times\begin{bmatrix} -sC_c & 0 \\ -sC_{gs}+g_m & -g_o-g_m\end{bmatrix}$$

to get voltage gain $v_o/v_{in} = \left.\dfrac{v_2}{v_1}\right|_{i_2=0}$ if no load

$$Y_{2\text{-}port} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22}\end{bmatrix} \qquad z \longrightarrow = \text{open circuit voltage gain}$$

$$i_2 = 0 \Rightarrow y_{21}v_1 + y_{22}v_2 = 0$$

$$v_2 = -\frac{y_{21}}{y_{22}}\cdot v_1 \Rightarrow A_v = \left.\frac{v_2}{v_1}\right|_{\text{no load}} = -\frac{y_{21}}{y_{22}}$$

$$0 = \begin{bmatrix} 0 & g_o+G_L+sC_{gd}\end{bmatrix}\begin{bmatrix} v_1 \\ v_2\end{bmatrix} - \frac{1}{\Delta}\begin{bmatrix} 0 & g_d\end{bmatrix}\begin{bmatrix} 0 & 0 \\ sC_{gs} & s(C_c+C_{gs}+C_{gd})\end{bmatrix}\underbrace{\begin{bmatrix} -sC_c & 0 \\ -sC_{gs}+g_m & -g_o-g_m\end{bmatrix}}_{\begin{bmatrix} 0 & 0 \\ sC_{gs}(-sC_c)+(sC_{cc}+C_{gs}+C_{gd})\times(-sC_{gd}+g_m) & \uparrow \end{bmatrix}}\begin{bmatrix} v_1 \\ v_2\end{bmatrix}$$

$$y_{21} = 0 - \frac{g_d\left(-s^2C_c + s(C_c+C_{gd}+C_{gs})(-sC_{gd}+g_m)\right)}{\Delta}$$

$$y_{22} = \left\{(g_o+G_L+sC_{gd})\Delta + (-g_d)(-g_o-g_m)\left(s[C_c+C_{gd}+C_{gs}]\right)\right\}/\Delta$$

$(-g_o-g_m)$
$(s(C_c+C_{gs}+C_{gd}))$

$$A_v = \frac{\text{degree three in numerator (in } s)}{\text{degree three in denominator (in } s)}$$

in summary: can carry out these steps

1) draw small signal equivalent circuit [which is "linear"]
2) move ground off of circuit
3) bring out leads from all nodes
4) use variables, i & v, from ground to nodes & write indefinite admittance, $Y_{ind}(s)$, by inspection [it also has sums of row & of column entries to 0]
5) move the ground to a circuit node

(This removes a column [as its $v=0$] and a row [as its current is a combination of others by KCL])

• This gives a "definite" admittance, Y_{def}

6) If some nodes do not have access externally eliminate their rows & columns by setting their currents to zero. By numbering them last $Y_{ports} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

7) If there are two ports the y_{ij} are 1x1 matrices and

$$A_v = -\frac{y_{21}}{y_{22}} = \frac{v_{out}}{v_{in}} = \text{open circuit voltage gain when no load}$$

$$A_i = \frac{-y_{21}}{y_{11}} = \frac{-i_2}{i_1} = \text{short circuit current gain}$$