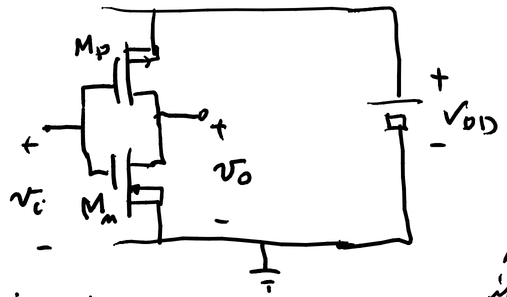
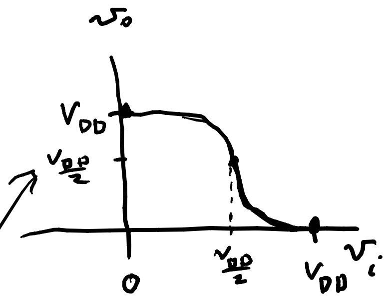
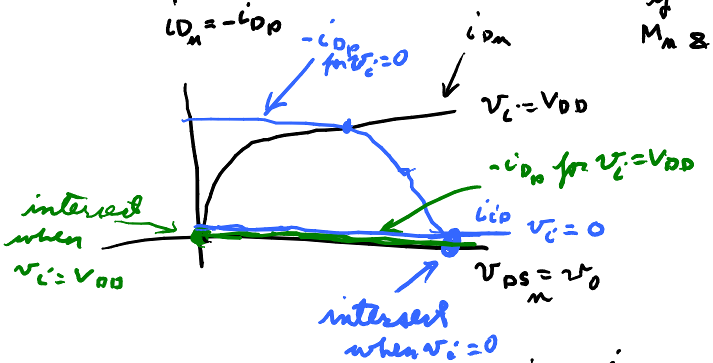
EE303
02/25/10

P. 338 Inverters (CMOS)





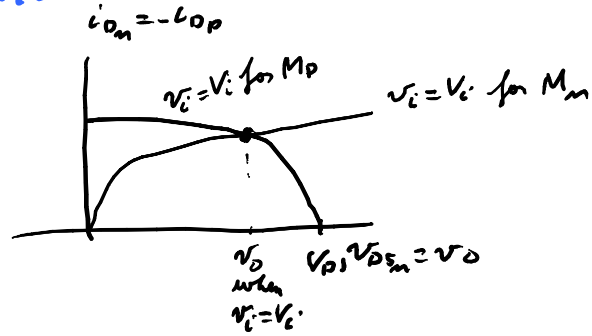
if Mn & Mp are "identical" = completely complementary



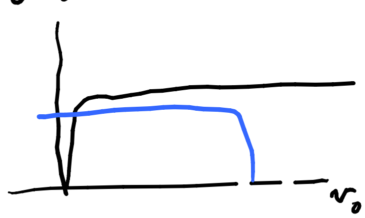
for p-channel

$v_{SD} = V_{DD} - v_o$

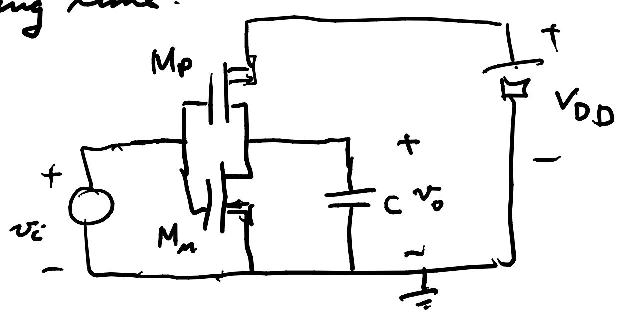
When $v_i \neq 0$ or V_{DD}
$\quad\;\; = V_i$



for no Early effect



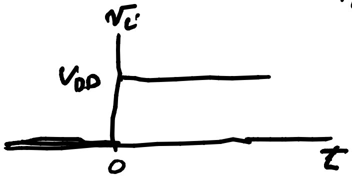
Look at switching time:



assume $v_i(t)$ is a pulse

assume @ $t=0$ $v_i(0_-)=0$ (for a long time for $t<0$)

$v_i(0_+)=V_{DD}$ $\Rightarrow$ C was charged to V_{DD}



$v_O(0_-)=V_{DD}$

$\Downarrow$

$v_O(0_+)=V_{DD}$ as assume no impulse

$i_C(t)=C\dfrac{dv}{dt}$

$-i_D(0_+)=0$ as M_P is turned off $(v_{SG}-|V_{TO}|)<0$

$\Rightarrow$ discharge of capacitor through M_n

$i_{D_n}\Big|$ initially $t>0_+$

check $v_{GS}-V_{TO}$ vs $v_{DS}=v_O$

@ $t=0_+$ $\quad v_{GS}-V_{TO} = V_{DD}-V_{TO}$, $\quad v_{DS} = V_{DD}$

$\Rightarrow$ in saturation as $v_{GS}-V_{TO}<v_{DS}$

remains in saturation until v_{DS} decreases to $V_{DD}-V_{TO}$

$\therefore$ the differential equation for the time interval for when M_n is in saturation is

$$I_D = i_{D_n} = \frac{KP}{2}\frac{W}{L}(V_{DD}-V_{TO})^2 = -C\frac{dv_O}{dt}$$

$$dv_O = -\frac{1}{C}I_D\cdot dt \Rightarrow dt = -\frac{C}{I_D}dv_O$$

integrate
$$\int_0^{t_1} dt = -\frac{C}{I_D}\int_{v_O(0)}^{v_O(t_1)} dv_O$$

$$t_1 = -\frac{C}{I_D}\left[(V_{DD}-V_{TO})-V_{DD}\right] = \frac{CV_{TO}}{I_D}$$

at t_1 $\quad M_n$ changes to the Ohmic state,

$v_{GS}-V_{TO}=V_{DD}-V_{TO}\geq v_{DS}$

then
$$i_{D_n} = \frac{KP}{2}\frac{W}{L}\left(2(V_{DD}-V_{TO})\underbrace{v_{DS}}_{v_O}-\underbrace{v_{DS}^2}_{v_O^2}\right) \text{ = ohmic region law}$$

$$= -C\frac{dv_O}{dt}$$

$$dv_O = -\frac{KP}{2C}\frac{W}{L}\left(2(V_{DD}-V_{TO})v_O - v_O^2\right)dt$$

; $b = \frac{KP}{2}\frac{W}{L}\frac{1}{C}$

$a = 2(V_{DD}-V_{TO})$

$v_O = x$

$$dt = -\frac{dx}{b(ax-x^2)}$$

$$\int_{t_1}^{t_2} dt = \frac{1}{b}\int_{x(t_1)}^{x(t_2)}\frac{dx}{x^2-ax} = \frac{1}{b}\int_{x(t_1)}^{x(t_2)}\frac{dx}{x(x-a)}$$

$$t_2 - t_1 = \frac{1}{b}\int_{v_0(t_1)}^{v_0(t_2)} dx\left[\frac{-1/a}{x} + \frac{1/a}{x-a}\right] = \frac{1}{b}\int_{v_0(t_1)}^{v_0(t_2)} -\frac{1}{a}\frac{dx}{x} + \frac{1}{b}\int_{v_0(t_1)-a}^{v_0(t_2)-a} \frac{1}{a}\frac{d(x-a)}{x-a}$$

$$\frac{1}{x(x-a)} = \frac{k_1}{x} + \frac{k_2}{x-a}$$

$$k_1 = \left(\frac{\cancel{x}}{\cancel{x}(x-a)} - \frac{k_2 x}{x-a}\right)\Bigg|_{x=0} = \frac{1}{-a}$$

$$k_2 = \left(\frac{\cancel{(x-a)}}{x\cancel{(x-a)}} - \frac{k_1(x-a)}{x}\right)\Bigg|_{x=a} = \frac{1}{a}$$

$\int \frac{dx}{x}$;

$= \ln x$

$$x = \ln e^x ; \quad \frac{dx}{dx} = \frac{d\ln y}{dy}\cdot\frac{dy}{dx} \quad ; \quad y = e^x$$

$$1 = \frac{d\ln y}{dy}\cdot e^x = \frac{d\ln y}{dy}\cdot y \Rightarrow \frac{1}{y} = \frac{d\ln y}{dy}$$

$$\Rightarrow \frac{dy}{y} = d\ln y$$

$$t_2 - t_1 = \frac{-1}{ab}\ln x\Big|_{x=v_0(t_1)}^{v_0(t_2)} + \frac{1}{ab}\ln(x-a)\Big|_{x=v_0(t_1)}^{v_0(t_2)=x}$$

$$= \frac{1}{ab}\ln\left(\frac{x-a}{x}\right)\Bigg|_{x=v_0(t_1)}^{v_0(t_2)} , \qquad t_2 > t_1$$

$$= \frac{1}{\frac{KP}{2}\frac{W}{L}\cdot\frac{2}{C}(V_{DD}-V_{TO})}\cdot\left[\ln\left(1-\frac{2(V_{DD}-V_{TO})}{v_0(t_2)}\right) - \ln\Big(\underbrace{1-\frac{2(V_{DD}-V_{TO})}{v_0(t_1)=V_{DD}-V_{TO}}}_{-1}\Big)\right]$$

as $\ln(-1)$ is complex combine terms

$$= \frac{1}{\frac{KP}{2}\frac{W}{L}\frac{2}{C}(V_{DD}-V_{TO})}\ln\left[\frac{2(V_{DD}-V_{TO})}{v_0(t_2)} - 1\right]$$

We are interested in $v_0(t_2) = 0$ but never reach (get $\ln(\infty)$)

so evaluate to get delay to $v_0(t_2) = V_{DD}/2$, as per the book.

also can get fall time, usually 0.90 to 0.1 of V_{DD}

added