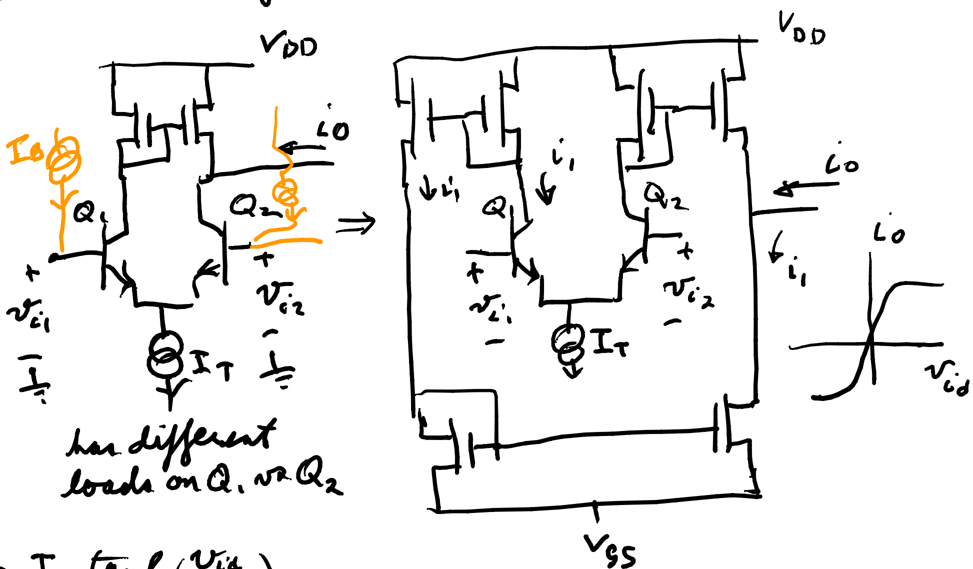
For next time: inverters p. 338 — EE303 02/18/10

Riccati eq. for, p. 343

Today, more on diff. pair, CMOS eq., p. 693

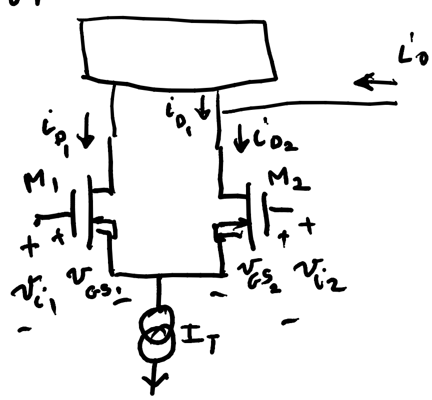


$$i_o = I_T \tanh\left(\frac{v_{id}}{2V_T}\right)$$

$$\frac{d \tanh x}{dx} = 1 - \tanh^2 x \quad \text{is nice for optimizations}$$

CMOS diff. pair

at DC



if M_1 & M_2 are in saturation

$$i_{D_1} = \frac{KP}{2}\frac{W}{L}(v_{GS_1} - VTO)^2$$

$$i_{D_2} = \frac{KP}{2}\frac{W}{L}(v_{GS_2} - VTO)^2$$

$$\beta = \frac{KP}{2}\frac{W}{L}$$

$$i_O = i_{D_2} - i_{D_1}$$

$$I_T = i_{D_1} + i_{D_2}$$

$$v_{iD} = v_{i_1} - v_{i_2} = v_{GS_1} - v_{GS_2}$$

but $v_{GS_1} = VTO \pm \sqrt{i_{D_1}/\beta} = VTO + \sqrt{i_{D_1}/\beta}$ if M_1 is on

$$v_{GS_2} = VTO + \sqrt{i_{D_2}/\beta}$$

$$v_{iD} = v_{GS_1} - v_{GS_2} = \sqrt{\frac{i_{D_1}}{\beta}} - \sqrt{\frac{i_{D_2}}{\beta}}$$

square $\beta v_{iD}^2 = i_{D_1} + i_{D_2} - 2\sqrt{i_{D_1} i_{D_2}} = I_T - 2\sqrt{i_{D_1}(I_T - i_{D_1})}$

$$2\sqrt{i_{D_1}(I_T - i_{D_1})} = I_T - \beta v_{iD}^2$$

$$4(I_T i_{D_1} - i_{D_1}^2) = (I_T - \beta v_{iD}^2)^2$$

$$i_{D_1}^2 - I_T i_{D_1} + \frac{1}{4}(I_T - \beta v_{iD}^2)^2 = 0$$ an eq. for i_{D_1} vs v_{iD}

$$i_{D_1} = \frac{I_T}{2} \pm \frac{1}{2}\sqrt{\cancel{I_T^2} \underbrace{- \frac{4}{4}(I_T - \beta v_{iD}^2)^2}_{-(\cancel{I_T^2} - 2I_T\beta v_{iD}^2 + \beta^2 v_{iD}^4)}}$$

$$= \frac{I_T}{2} \pm \frac{1}{2}\sqrt{2I_T\beta v_{iD}^2 - \beta^2 v_{iD}^4} = \frac{I_T}{2} \pm \frac{1}{2}\sqrt{2I_T\beta}\cdot v_{iD}\sqrt{1 - \frac{\beta}{2I_T}v_{iD}^2}$$

need + sign for i_{D_1} as $i_{D_1}\uparrow$ when $v_{iD}\uparrow$

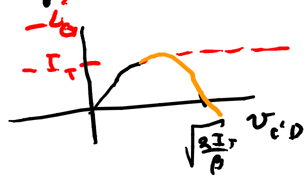
$$i_{D_2} = I_T - i_{D_1} = \frac{I_T}{2} - \frac{1}{2}\sqrt{2I_T\beta}\, v_{iD}\sqrt{1 - \frac{\beta}{2I_T}v_{iD}^2}$$

$$i_O = i_{D_2} - i_{D_1}$$

$$= \cancel{\frac{I_T}{2}} - \frac{1}{2}\sqrt{2I_T\beta}\, v_{iD}\sqrt{1 - \frac{\beta}{2I_T}v_{iD}^2} - \cancel{\frac{I_T}{2}} - \frac{1}{2}\sqrt{2I_T\beta}\, v_{iD}\sqrt{1 - \frac{\beta}{2I_T}v_{iD}^2}$$

$$= -\sqrt{2I_T\beta}\, v_{iD}\sqrt{1 - \frac{\beta}{2I_T}v_{iD}^2}$$

can't hold for large v_{iD}^2 ∴ M_1 or M_2 must go out of saturation



when $v_{iD} = 0$

$$i_{D_1} = I_{D_1} = \beta(V_{GS_1} - V_{TO})^2 = I_T/2$$

$$i_{D_2} = I_{D_2} = \beta(V_{GS_2} - V_{TO})^2 = I_T/2$$

let $V_{ov} = V_{GS} - V_{TO}$, $V_{GS_1} = V_{GS_2} = V_{GS}$

$$V_{ov}^2 = \frac{I_T}{2\beta} \Rightarrow \beta = \frac{I_T}{2V_{ov}^2}$$

$$\therefore\ i_O = -\sqrt{2I_T\cdot\frac{I_T}{2V_{ov}^2}}\cdot v_{iD}\sqrt{1 - \frac{I_T}{2V_{ov}^2}\cdot\frac{1}{2I_T}v_{iD}^2}$$

$$= -I_T\cdot\frac{v_{iD}}{V_{ov}}\sqrt{1 - \frac{v_{iD}^2}{4V_{ov}^2}} = -I_T\left(\frac{v_{iD}}{V_{ov}}\right)\sqrt{1 - \left(\frac{v_{iD}}{2V_{ov}}\right)^2}$$

$$\therefore\ \frac{i_O}{I_T} = -2\left(\frac{v_{iD}}{2V_{ov}}\right)\sqrt{1 - \left(\frac{v_{iD}}{2V_{ov}}\right)^2} \Rightarrow y = \frac{i_O}{I_T} = -2x\sqrt{1 - x^2}$$

$$x = v_{iD}/2V_{ov}$$

to get the maximum (or min)

$$\frac{dy}{dx} = 0;\quad \frac{dy}{dx} = -2\sqrt{1 - x^2} - 2x\frac{1}{2}\frac{1}{\sqrt{1 - x^2}}\cdot(-2x)$$ 1

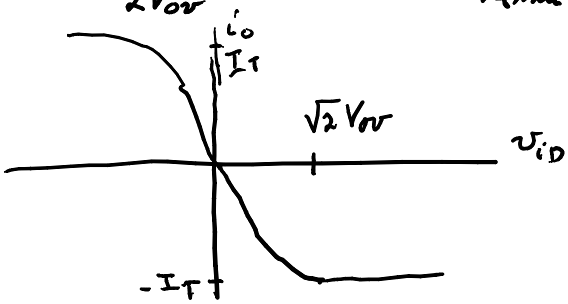
$$0 = -2(1 - x^2) + 2x^2 \qquad (\text{as } x^2 \neq 1)$$

$$= -2 + 4x^2 \Rightarrow x^2 = 1/2$$

$$y = \frac{i_o}{I_T}\Big|_{\text{at min}} = -2\frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{2}} = -\sqrt{2}\cdot\frac{1}{\sqrt{2}} = -1$$

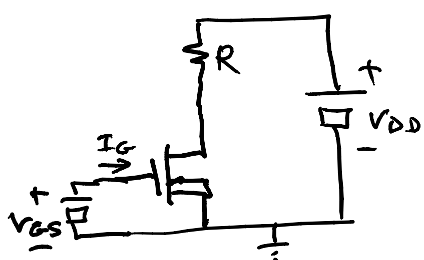
$\therefore$ at the minimum of i_o vs v_{iD}, $i_o = -I_T$

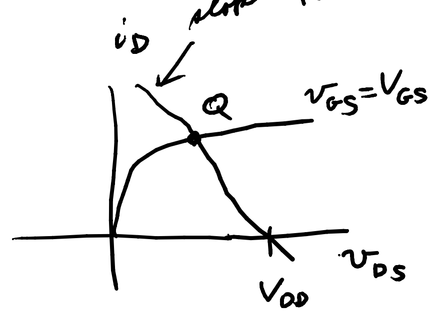
$$\frac{v_{iD}}{2V_{ov}} = x = \frac{1}{\sqrt{2}} \Rightarrow v_{iD}\Big|_{\text{at min}} = \sqrt{2}\cdot V_{ov}$$

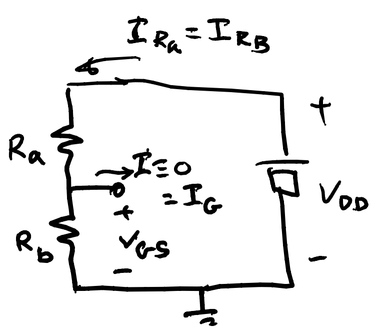


$$\therefore \quad i_o = \begin{cases} -I_T & v_{iD} \geq \sqrt{2}V_{ov} \\ -2I_T\left(\frac{v_{iD}}{2V_{ov}}\right)\sqrt{1-\left(\frac{v_{iD}}{2V_{ov}}\right)^2} & -\sqrt{2}V_{ov} \leq v_{iD} \leq \sqrt{2}V_{ov} \\ +I_T & v_{iD} \leq -\sqrt{2}V_{ov} \end{cases}$$

Biasing for "small signal" behavior



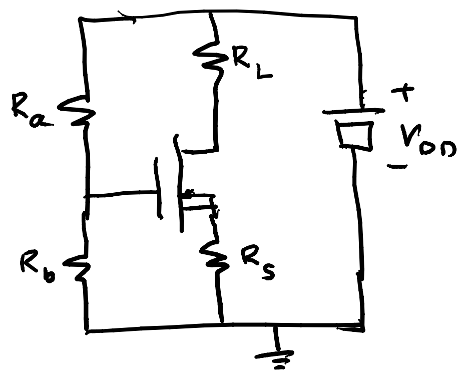




$$V_{GS} = I_{RB}\cdot R_b$$

$$I_{RB} = \frac{V_{DD}}{R_a+R_b}$$

$$V_{GS} = \frac{R_b}{R_a+R_b}V_{DD} \Big\} \text{ voltage divider}$$



R_L
R_a
+
V_{DD}
−
R_b
R_S