RWN
03/26/09
v.1

Solutions EE303 Hamilton

#1. as $V_0 = R_{out} I_{SD_{Mout}}$ we need I_{SD} (which is part b)

$I_{SD_{Mout}} = I_{SD_{Min}}$ so $V_{Rin} = V_0 = 3V \Rightarrow V_{SD_{Min}} = V_{dd} - V_0 = 9-3 = 6V \Rightarrow M_{out}$ is in saturation
$= V_{SG}$

$$\Rightarrow I_{SD_{Min}} = \frac{KP}{2}\frac{W}{L}(V_{SG} - |VTO_p|)^2(1+\lambda_p V_{SG})$$

$$= \frac{10.32\times10^{-6}}{2} \times \frac{328\times10^{-6}}{8\times10^{-6}}(6-1.5)^2(1+0.015\times6)$$

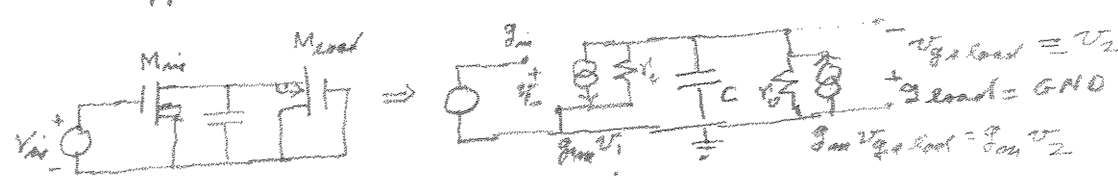
$$= 211.56\times10^{-6}(20.25)(1.09) = 4{,}669.658\times10^{-6} A$$

b) $\underline{I_{SD} = 4.670\,mA}$

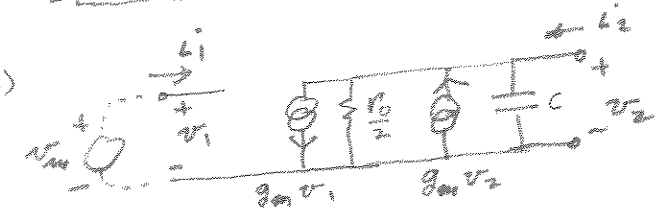
a) $R_{out} = \frac{V_0}{I_{SD}} = \frac{3}{4{,}669.658\times10^{-6}} = 0.6424\times10^3 = \underline{642\,\Omega = R_{out}}$

#2. a) By symmetry both transistors have $V_{GS} = 5$ if $V_c = 5$
and then both are in saturation
with the same current I_D giving the voltage $V_c = 5$

b) as V_{gg} & $V_{dd} \to 0$ for small signal



c)



as $M_{in} = M_{Load}$ with same bias points, $g_{m_{in}} = g_{m\,load} = g_m$
$g_{o_{in}} = g_{o\,load} = g_0 = 1/r_0$

d) $i_1 = 0$, $i_2 = g_m v_1 + [sC + 2g_0 - g_m]v_2$ by KCL

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g_m & sC + 2g_0 - g_m \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 0 & 0 \\ g_m & sC + 2g_0 - g_m \end{bmatrix}$$

Numerically for the 4007: $g_0 = \frac{\partial I_D}{\partial V_{DS}} = \frac{\lambda \cdot I_D}{1+\lambda V_{DS}}$, $g_m = 2\frac{I_D}{(V_{GS} - VTO)}$

Here $I_D = \frac{KP}{2}\frac{W}{L}(V_{GS} - VTO)^2(1+\lambda V_{DS}) = \frac{20.54\times10^{-6}}{2}\times\frac{144}{8}(5-1.3)^2(1+0.015\times5) = 184.86\times10^{-6}(13.69)\times(1.075)$

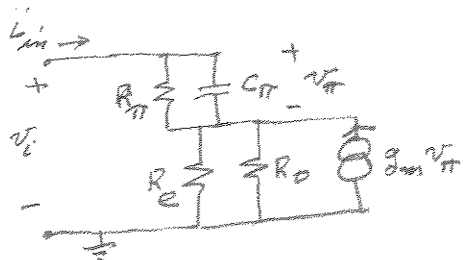
$= 2.721\,mA \Rightarrow g_0 = 0.015\times184.86\times10^{-6}\times13.69 = 37.96\times10^{-6}$ $\qquad g_m = 397.4\times10^{-6}$

$$\Rightarrow Y = \begin{bmatrix} 0 & 0 \\ 397.4 & sC\times10^6 - 321.5 \end{bmatrix}\times10^{-6}$$

(note: part 2 needs a negative R)

#3, one can use the results of the 03/24/09 lecture where its y_{11} is $Y_{in}(s)$ or redo as here

Redrawing



Let $y_\pi = g_\pi + sC_\pi \Rightarrow$ 

$G_O = 1/R_O = 1/R_E$

Here $i_{in} = y_\pi v_\pi = (v_i - v_\pi)(2G_o) - g_m v_\pi$
$= 2G_o v_i - [g_m + 2G_o] v_\pi$

$$\Rightarrow \{y_\pi + [g_m + 2G_o]\} v_\pi = 2G_o v_i \Rightarrow v_\pi = \frac{2G_o}{y_\pi + [g_m + 2G_o]} \cdot v_i$$

$$\therefore \; i_{in} = y_\pi v_\pi = y_\pi \times \frac{2G_o}{y_\pi + g_m + 2G_o} \cdot v_i \Rightarrow Y_{in}(s) = \frac{2G_o y_\pi}{y_\pi + g_m + 2G_o}$$

$$= \frac{2G_o(sC_\pi + g_\pi)}{sC_\pi + g_\pi + [g_m + 2G_o]}$$

$$= 2G_o \left[\frac{s + g_\pi / C_\pi}{s + \frac{g_\pi}{C_\pi} + \frac{[g_m + 2G_o]}{C_\pi}}\right]$$

For a BJT

$$g_m = \frac{I_C}{V_T}, \quad g_\pi = \frac{g_m}{\beta}, \quad g_o = \frac{I_C}{V_A} = G_o$$

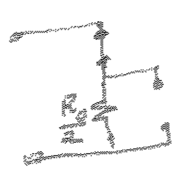
$$\Rightarrow g_m = \frac{2.6\times10^{-3}}{26\times10^{-3}} = 0.1, \quad g_\pi = \frac{0.1}{100} = 10^{-3}, \quad G_o = \frac{2.6\times10^{-3}}{100} = 2.6\times10^{-6}$$

a₁) $\therefore \; Y_{in}(s) = 5.2\times10^{-6}\left[\frac{s + 10^{-3}/C_\pi}{s + \frac{[10^{-1} + 10^{-3} + 5.2\times10^{-6}]}{C_\pi}}\right] \approx 5.2\times10^{-6}\, \frac{s + 10^{-3}/C_\pi}{s + \frac{1.01\times10^{-3}}{C_\pi}}$

a₂) zeros @ $s_0 = -g_\pi/C_\pi = -10^{-3}/C_\pi$

a₃) poles @ $s_p = -[g_m + g_\pi + 2G_o]/C_\pi \approx -1.01\times10^{-3}/C_\pi$

b) for $C_\pi = 10\,pFd = 10\times10^{-12} \Rightarrow s_0 = -10^{-3}\times10^{-1}\times10^{+12} = -10^{8}$
$p_0 = 1.01\, s_0 = -1.01\times10^{8}$

c) as $s = j\omega \rightarrow j\infty \Rightarrow Y_{in} \Rightarrow |Y_{in}(\infty)|e^{j\angle\infty} = 2G_o$
as $s \rightarrow \infty$ $C_\pi \rightarrow$ short & $v_\pi \rightarrow 0 \Rightarrow$ 

$\Rightarrow Y_{in} = \frac{1}{R_O/2} = 2G_o$