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Solutions 303A Final Spring 2009

#1. a) For M_2, $V_{SG} - |VTO_p| = V1 - \frac{1}{2} = \frac{V_2}{2} - \frac{1}{2} = \frac{1}{2}(V_2 - 1)$; $V_{SD} = V_2$

$\therefore$ M_2 is in saturation if $V_2 \geq \frac{1}{2}(V_2 - 1) \Rightarrow V_2 \geq -1$. As $V_{dd} > 0$ then $V_2 > 0 \Rightarrow V_2 \geq -1 \Rightarrow M_2$ is in saturation

b) $I_1 = \frac{KP_p}{2}\frac{W}{L}(V1 - |VTO_p|)^2(1 + \lambda_p \cdot V1)$

$I_2 = \frac{KP_p}{2}\frac{W}{L}(V_1 - |VTO_p|)^2(1 + \lambda_p V2) = \frac{KP_p}{2}\frac{W}{L}(V_1 - |VTO_p|)^2(1 + 2\lambda_p V_1)$

$\Rightarrow \frac{I_2}{I_1} = \frac{(1 + 2\lambda_p V_1)}{(1 + \lambda_p V_1)} = \frac{1 + 0.04V1}{1 + 0.02V1}$

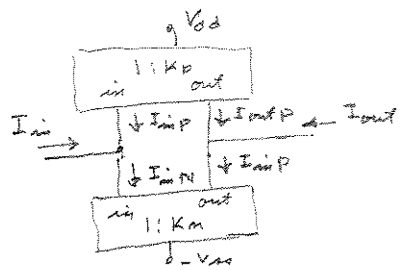
c) If $V2 = 4 \Rightarrow V1 = 2 \Rightarrow I_1 = \frac{10^{-5}}{2} \cdot 2\left(2 - \frac{1}{2}\right)^2(1 + 0.02 \times 2) = 10^{-5}\underbrace{\left(\frac{9}{4}\right)(1.04)}_{2.34}$ 0.26

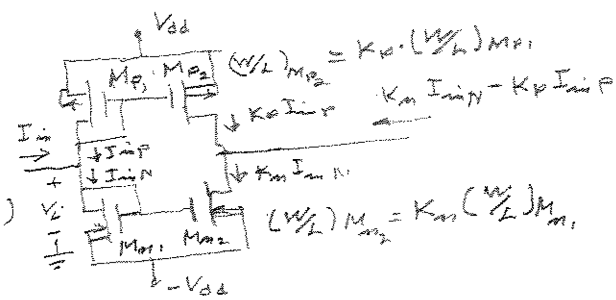
$\Rightarrow I_1 = 23.4\,\mu A$

$I_2 = \frac{(1 + 0.02\times4)}{1 + 0.02\times2} \cdot I_1 = 10^{-5}\left(\frac{9}{4}\right)\cdot(1.08) = 10^{-5}(9 \times 0.270) =$

$= 24.3\,\mu A$

#2.

a) 

b) 

c) When $I_{in} = 0 \Rightarrow I_{inp} = \frac{KP_p}{2}\left(\frac{W}{L}\right)(V_{dd} - V_i - |VTO_p|)^2(1 + \lambda_p[V_{dd} - V_i])$

$= I_{inN} = \frac{KP_n}{2}\left(\frac{W}{L}\right)(V_i - (-V_{dd}) - VTO_n)^2(1 + \lambda_n[V_i - (-V_{dd})])$

for $V_i = 0 \Rightarrow I_{inp} = I_{inN} = 10^{-5}\left(5 - \frac{1}{2}\right)^2(1 + 0.02\times5) = 10^{-5}\left(\frac{81}{4}\right)(1.1) = 22.275 \times 10$

$= 222.75\,\mu A$

d) When $K_n \neq K_p$ the output current has an offset so that $I_{out} = K_n I_{inN} - K_p I_{inp} = K_n(I_{inN} - I_{inp}) + (K_n I_{inp} - K_p I_{inp}$

$= K_n I_{in} + \underbrace{[K_n I_{inp} - K_p I_{inp}]}_{\text{output offset}}$

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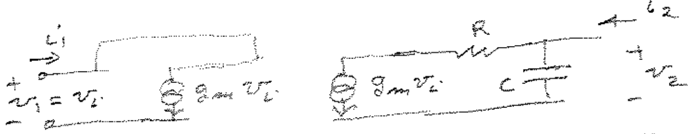
#3. as $\left.\frac{d \tanh x}{dx}\right|_{x=0} = 1$ and $g_m = \left.\frac{d I_{OTA}}{d V_i}\right|_{V_i = 0}$

a)

$$\Rightarrow g_m = \alpha I_T \cdot \frac{1}{2V_T}$$

b)



$$i_1 = g_m v_1 ,\quad i_2 = g_m v_1 + sC v_2 \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} g_m & 0 \\ g_m & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\Rightarrow Y(s) = \begin{bmatrix} g_m & 0 \\ g_m & sC \end{bmatrix}$$

c) as $i_2 \equiv 0$, $-g_m v_1 = sC v_2 \Rightarrow v_2 = -\frac{g_m}{sC} v_i$

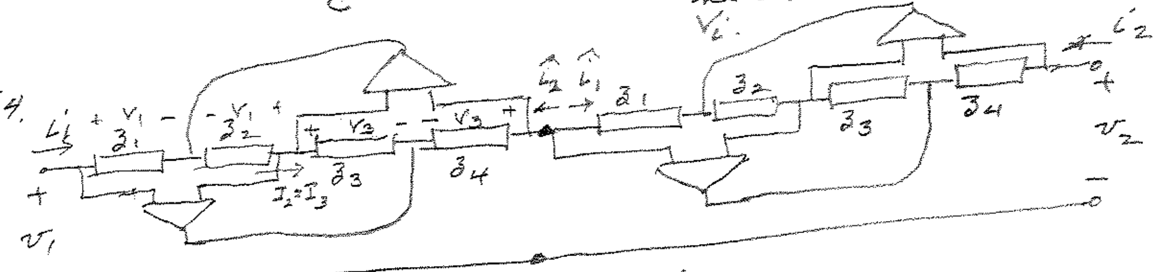
$$\therefore v_2(t) = -\frac{g_m}{C} \int_{-\infty}^{\infty} V_i 1(t-\tau)\, d\tau = -\frac{g_m}{C} V_i \left(\int_0^t 1 \cdot d\tau\right) 1(t)$$

$$= -\frac{g_m}{C} V_i \cdot t\, 1(t)$$

Note: The output increases even though the OTA does not saturate for small V_i.

#4.

a)



By virtual ties of op-amps inputs, $v_2 = v_1$

b) By virtual ties for currents $i_1 = V_1/z_1 \Rightarrow I_2 = -V_1/z_2 = -\frac{z_1}{z_2} i_1 = I_3 = \frac{V_3}{z_3} \Rightarrow V_3 = -\frac{z_1 z_3}{z_2} i_1 \Rightarrow \hat{i}_2 = \frac{V_3}{z_4}$

$$\Rightarrow \hat{i}_2 = -\frac{z_1 z_3}{z_2 z_4} i_1 \Rightarrow i_2 = \Big(-\frac{z_1 z_3}{z_2 z_4}\Big) \underbrace{\hat{i}_1}_{-\hat{i}_2} \Rightarrow i_2 = -\Big(-\frac{z_1 z_3}{z_2 z_4}\Big)^2 i_1$$

$$\therefore \quad v_2 = v_1$$
$$i_2 = -\left(\frac{z_1 z_3}{z_2 z_4}\right)^2 i_1$$

c) $z_1 = \frac{1}{sC_1}$, $z_3 = \frac{1}{sC_3}$, $z_2 = R_2$, $z_4 = R_4 \Rightarrow i_2 = -\left(\frac{1}{s^2} \cdot \frac{1}{R_1 C_1 R_2 C_2}\right)^2 i_1$

$$\Rightarrow \frac{i_2}{i_1} = \frac{1}{(R_1 C_1 R_2 C_2)^2} \circ \frac{-1}{s^4}$$

gives i_2 as 4th integral of i_1 with a negative coefficient

$\therefore$ May be useful where frequency dependent current mirrors are of interest.