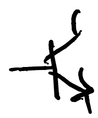
EE303
05/12/09
updated 05/13/09

Electronic circuits

nonlinear



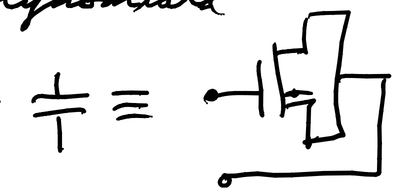
BJT

exponentials

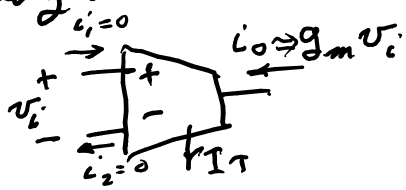
MOS

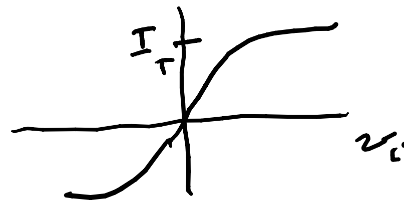
polynomials

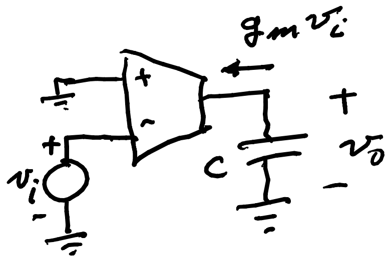
for ODE $\frac{dx}{dt} \Rightarrow$ capacitors



a handy device is the OTA

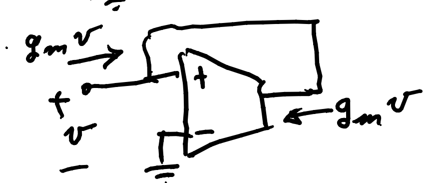




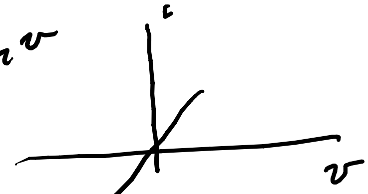


$$-g_m(-v_i) = C\frac{dv_o}{dt}$$

$$v_o = \frac{g_m}{C}\cdot\int_0^t v_i(\tau)d\tau + v_o(0)$$



$i = g_m v$

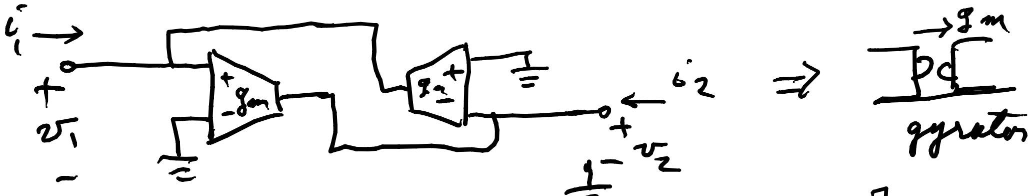


a good description is $Y(s)$; 2x2 for a 2-port

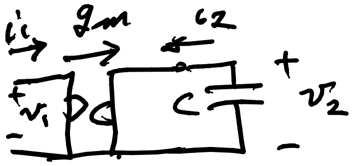
$i = Y v$

Y for OTA $\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}\begin{bmatrix} v_i \\ v_o \end{bmatrix}$; $Y\Big|_{OTA} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$

df add admittances use parallel ports

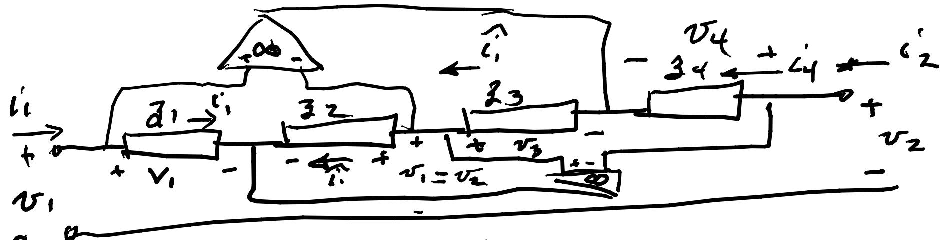


$$Y_1+Y_2=\begin{bmatrix}0 & 0\\ g_m & 0\end{bmatrix}+\begin{bmatrix}0 & -g_m\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & -g_m\\ g_m & 0\end{bmatrix}$$



$$Cs\,v_2=-i_2=-g_m v_1$$

$$Cs\left(\frac{-i_1}{g_m}\right)\Rightarrow v_i=\frac{Cs}{g_m^2}=Ls$$



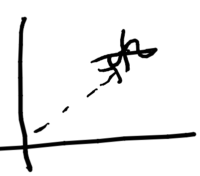
$$v_1=z_1 i_1=z_2\hat{i}_1\Rightarrow \hat{i}_1=\frac{z_1 i_1}{z_2}\qquad v_2=v_1$$

$$v_3=-z_3\hat{i}_1$$

$$=-z_3\left(\frac{z_1}{z_2}\right)i_1=v_4;\quad i_4=\frac{1}{z_4}v_4=\left(-z_3\frac{z_1}{z_2}\right)\frac{i_1}{z_4}$$

$$\left.\begin{array}{l} i_2=-\dfrac{z_3}{z_4}\dfrac{z_1}{z_2}\,i_1\ ;\quad v_2=v_1\end{array}\right\}\text{ no admittance}$$

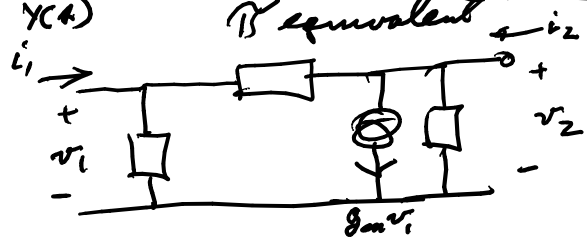
can get high order polynomials in $s=\sigma+j\omega$



Biasing; small signal behavior

$Y(s)$

π equivalent

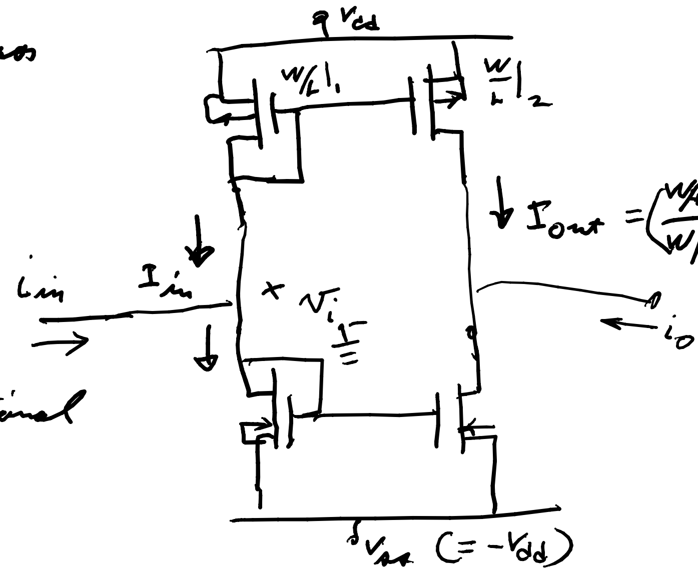


For the BJT
in the forward
active region

due to Early effect

diode at base to emitter

current mirror



$I_{out} = \left(\frac{W/L|_1}{W/L|_2}\right) I_{in} \underbrace{1(I_{in})}_{\text{unit step function}}$ for upper half

bidirectional

has current flowing (I_{in}) when $i_{in} = 0$

if $i_{in} = 0 \Rightarrow$ $I_{in} = \frac{KP_n}{2}\frac{W}{L}\Big|_n (v_i - V_{ss} - VTO_n)^2 (1 + \lambda_n [v_i - v_{ss}])$

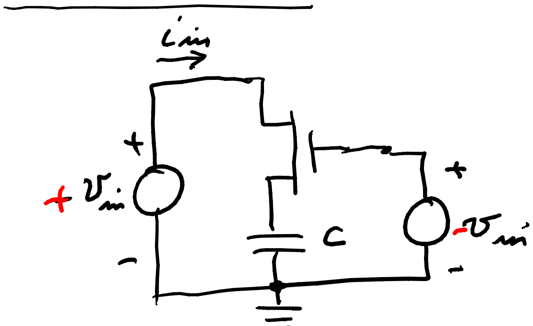
$= \frac{KP_p}{2}\frac{W}{L}\Big|_p (V_{dd} - v_i - |VTO_p|)^2 \underbrace{(1 + \lambda_p [V_{dd} - v_i])}_{\text{about equal} \approx 1 + \lambda V_{dd}}$

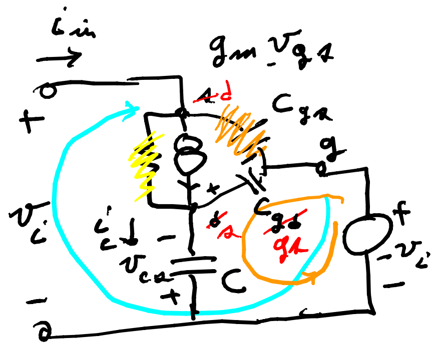
assume $\frac{KP_n}{2}\frac{W_n}{L_n} = \frac{KP_p}{2}\frac{W_p}{L_p} \Rightarrow$ $v_i - V_{ss} - VTO_n = V_{dd} - v_i - |VTO_p|$

$2v_i = V_{dd} + V_{ss} + VTO_n - |VTO_p|$

if $V_{dd} = -V_{ss}$, $VTO_n = +|VTO_p|$ $\Rightarrow$ $2v_i = 0 \Rightarrow v_i = 0$

$I_{in} = \left(\frac{KP}{2}\frac{W}{L}\right)(V_{dd} - |VTO_p|)^2 (1 + \lambda_p V_{dd})$





$i_{in} = g_m v_{gs}$; $v_{gs} = -v_i - v_i = -2v_i$

$= -2g_m v_i \Rightarrow Y_{in} = -2g_m$

modified

still $i_{in} = g_m v_{gs}$

here $v_i + v_{cs} + v_{cgs} = 0$; $i_c = g_m v_{gs} + C_{gs} \cdot s \cdot v_{gs}$

$$v_i + \frac{-1}{sC}\left[g_m v_{gs} + C_{gs} \cdot s v_{gs}\right] + v_{c_{gs}} = 0, \qquad v_{c_{gs}} \to -v_{gs}$$

$$v_i = \left(+1 + \frac{g_m}{sC} + \frac{C_{gs}}{C}\right) v_{gs} \Rightarrow v_{gs} = \frac{sC}{sC + (g_m) + sC_{gs}} \cdot v_i$$

$$i_{in} = g_m v_{gs} = \frac{g_m C s}{s(C_{gs}+C) + g_m} \cdot v_i$$

$$\Rightarrow y_{in}(s) = \frac{g_m C s}{s(C_{gs}+C) + g_m} = \frac{g_m C}{C_{gs}+C} + \frac{-\left(\frac{g_m}{C_{gs}+C}\right)^2 \cdot C}{s + \frac{g_m}{C_{gs}+C}}$$

by partial fraction expansion

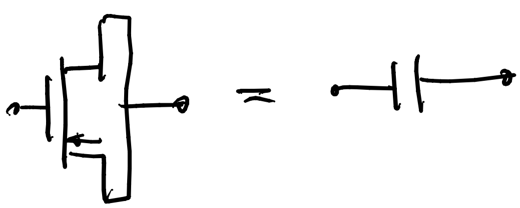
pole at $s_p = -\frac{g_m}{C+C_{gs}}$, zero at $s_z = 0$

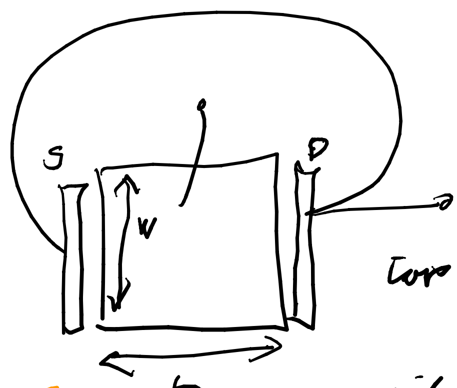
$$= \frac{\frac{g_m C}{C_{gs}+C}\, s}{s + \frac{g_m}{C_{gs}+C}}.$$

Note: actually $g_m \to -g_m$ if this is an NMOS device as it probably should be, then the pole is in the RHP ⇒ unstable

CMOS

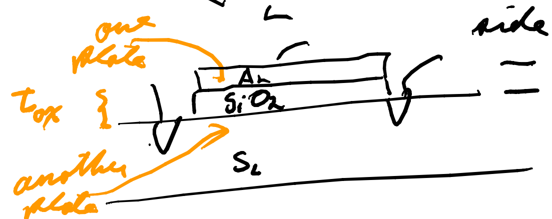
Capacitor





Thickness of oxide

$t_{ox} \approx 100\,\mathring{A}$; $\mathring{A} = 10^{-10}$ m



$$C = \frac{\epsilon A}{t_{ox}} = \frac{\epsilon_{SiO_2} \cdot W \times L}{t_{ox}}$$

free space, ϵ_0

$$\epsilon_{SiO_2} \approx \epsilon_0 \times 4 \approx 8 \times 10^{-12} \times 4$$

$C \approx 1-10$ fFd for a $1\mu \times 1\mu$

$$C \approx \frac{32 \times 10^{-12}}{100 \times 10^{-10}} \times W \times L = 32 \times 10^{-4} \times WL$$

$$= 32 \times 4 \times 10^{-4} \times 10^{-12} \Leftarrow \text{if } W = L = 2\mu$$

$$= 128 \times 10^{-16} = 1.28 \times 10^{-2} \times 10^{-12} = 0.0128\ \text{pFd}$$

if $L = L = 100\mu$

$$\Rightarrow C \approx \frac{32}{100} \times 10^{-2} \times 100^2 \times 10^{-12} = 32\ \text{pFd}$$