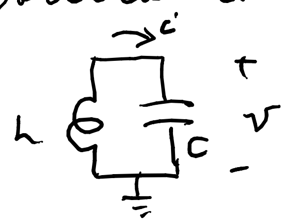
03/31/09
EE303

oscillators

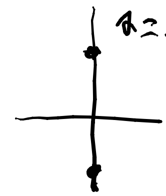


$$i = C\frac{dv}{dt}$$

$$-v = L\frac{di}{dt} \Rightarrow -v = L\frac{d}{dt}\left(C\frac{dv}{dt}\right) = LC\frac{d^2v}{dt^2}$$

$$\frac{d^2v}{dt^2} + \frac{1}{LC}v = 0 \quad \Rightarrow \left(s^2 + \frac{1}{LC}\right)v = 0$$

$s = \pm j\frac{1}{\sqrt{LC}}$



$$\ddot{x} + \omega_0^2 x = 0$$

$\rightarrow \quad \ddot{x} + \frac{d\,F(x)}{dt} + \omega_0^2 x = 0$ is a nonlinear system

van der Pol $\Rightarrow$ $\frac{dF(x)}{dt} = \epsilon(-1 + x^2)\dot{x}$

allows small signals to increase via a negative R
" large " " decrease " " positive R

$\therefore$ allows for a limit cycle

so convert to state variables for physical circuits

$$\rightarrow \quad \frac{d(\dot{x} + F(x))}{dt} + \omega_0^2 x = 0$$

$$x_2 = \dot{x} + F(x) \Rightarrow \frac{dx_2}{dt} = -\omega_0^2 x$$

set $x_1 = x \Rightarrow$ $\left.\begin{aligned} \dot{x}_1 &= x_2 - F(x_1) \\ \dot{x}_2 &= -\omega_0^2 x_1 \end{aligned}\right\}$ state variable eqs for structurally stable oscillators

replace t by $\frac{1}{\omega_0^2}\hat{t}$ gives $\omega_0^2 \rightarrow 1$

$\left.\begin{aligned} \dot{x}_1 &= x_2 - F(x_1) \\ \dot{x}_2 &= -x_1 \end{aligned}\right\}$ look at $x_2 - x_1$ plane

$$\frac{dx_2/dt}{dx_1/dt} = \frac{dx_2}{dx_1}$$

$$= \frac{-x_1}{x_2 - F(x_1)}$$