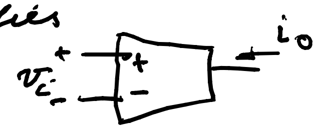
03/24/09
EE303

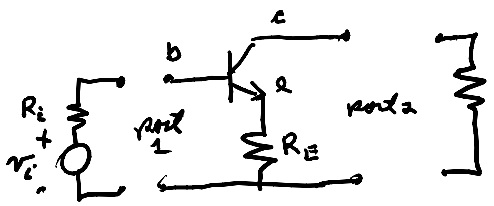
OTA = operational transconductance amplifier

p. 728

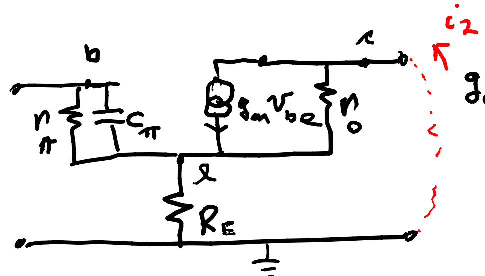


small signal $i_o = G_m v_i$

Small signal behavior $\Rightarrow$ linear behavior



$\Rightarrow$



$g_o = \frac{1}{r_o} = \frac{I_c}{V_A}$

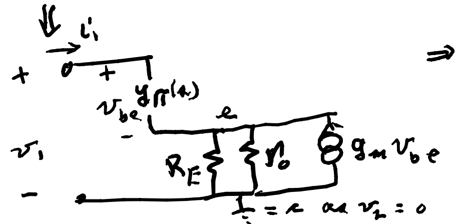
desire Y for this $\longrightarrow$ $y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$

$I = YV$

$\underline{\underline{y_\pi = g_\pi + sC_\pi}}$; $g_\pi = \frac{1}{r_\pi} = \frac{g_m}{\beta}$; $g_m = \frac{I_c}{V_T}$

$V_T = 0.026$ @ room T

$\Downarrow i_1$



let $R_E' = \frac{1}{G_E + g_o} = \frac{1}{G_E'}$

$\Rightarrow i_1 = y_\pi v_{be}$
$= G_E'(v_1 - v_{be})$
$- g_m v_{be}$

$\Rightarrow$

$(y_\pi + g_m + G_E')v_{be} = G_E' v_1$

$$i_1 = \frac{y_\pi \cdot G_E'}{y_\pi + g_m + G_E'} \cdot v_1$$

$y_{11} = y_{input}$ when $v_2 = 0$ port 2 shorted

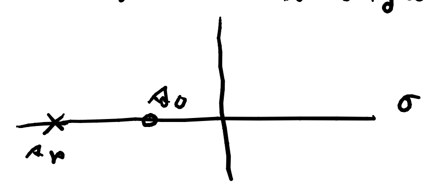
$$= G_E' \cdot \frac{y_\pi}{y_\pi + g_m + G_E'} = \frac{G_E'(sC_\pi + g_\pi)}{sC_\pi + (g_\pi + g_m + G_E')}$$

$$= \frac{G_E'(s + g_\pi/C_\pi)}{s + \frac{(g_\pi + [g_m + G_E'])}{C_\pi}} = \frac{G_E'(s - s_o)}{s - s_p}$$

$s = \sigma + j\omega$

zeros: $s_o = -g_\pi/C_\pi$

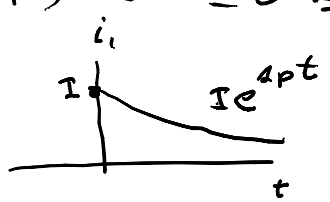
poles: $s_p = -\frac{g_\pi}{C_\pi} - \frac{g_m + G_E'}{C_\pi}$



$(s - s_p)i_1 = G_E'(s - s_o)v_1$ $\Rightarrow$ if $v_1 = 0$ we can have $i_1 \neq 0$ if $s = s_p$

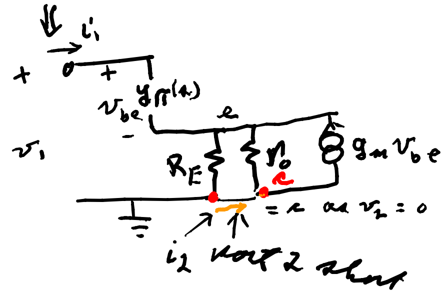
$i_1 = Ie^{s_p t}$ as look at $s = \frac{d}{dt}$; I = const.

$(s - s_p) I e^{s_p t} = 0 \Rightarrow \frac{d\, Ie^{s_p t}}{dt} - s_p I e^{s_p t} = s_p(1-1)Ie^{s_p t} \equiv 0$



$$i_1 = y_{11} v_1 \Big|_{v_2=0} = \frac{G_E'(s-s_0)}{(s-s_p)} \cdot v_1 \Rightarrow (s-s_p)\, i_1 = G_E'(s-s_0)\, v_1$$

$$y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$$



$$i_2 = g_m v_{be} + g_o(-[v_1 - v_{be}])$$
$$= -g_o v_1 + (g_m + g_o) v_{be}$$

but $\Rightarrow$

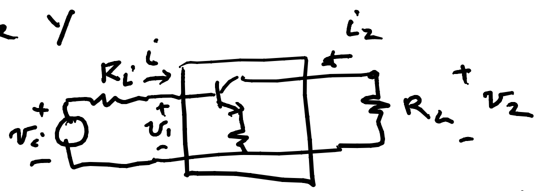
$$(y_\pi + g_m + G_E') v_{be} = G_E' v_1$$

$$\therefore \quad i_2 = \left[-g_o + \frac{[g_m + g_o]\, G_E'}{y_\pi + g_m + G_E'}\right] v_1$$

$$y_{21} = -g_o + \frac{[g_m + g_o]\, G_E'/C_\pi}{s + \left[\frac{g_\pi + g_m + G_E'}{C_\pi}\right]}$$ has the same pole as y_{11}

$$= -g_o \left[\frac{s + \frac{g_\pi + g_m + G_E'}{C_\pi} + \left[\frac{g_m}{g_o} + 1\right] G_E'/C_\pi}{s + \left[\frac{g_\pi + g_m + G_E'}{C_\pi}\right]}\right]$$

To use Y



find v_2/v_i

1) $$i_1 = y_{11} v_1 + y_{12} v_2 = y_{11}[v_i - R_i i_1] + y_{12} v_2$$

2) $$-G_L v_2 = i_2 = y_{21} v_1 + y_{22} v_2 = y_{21}[v_i - R_i i_1] + y_{22} v_2$$

1) $\rightarrow i_1 \Rightarrow \quad (1 + y_{11} R_i)\, i_1 = y_{11} v_i + y_{12} v_2$

$$i_1 = \frac{y_{11} v_i + y_{12} v_2}{(1 + y_{11} R_i)}$$

$$-G_L v_2 - y_{22} v_2 = y_{21} v_i - \frac{R_i}{1 + y_{11} R_i}\, y_{21}[y_{11} v_i + y_{12} v_2]$$

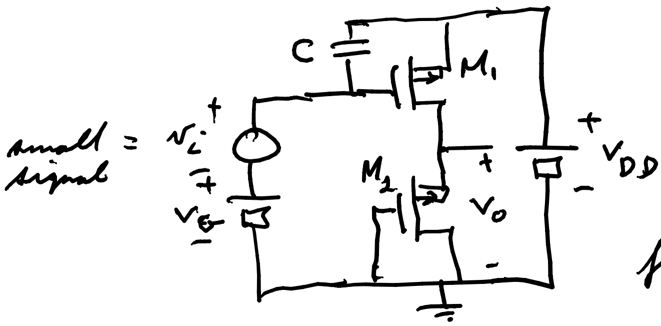
$$-\left(G_L + y_{22} - \frac{R_i y_{12} y_{21}}{1 + y_{11} R_i}\right) v_2 = \left(y_{21} - \frac{R_i y_{21} y_{11}}{1 + y_{11} R_i}\right) v_i$$

$$\frac{v_2}{v_1} = \text{Voltage gain} = -\frac{(y_{21} + y_{21} y_{11} R_i - y_{21} y_{11} R_i)}{G_L + G_L R_i y_{11} + y_{22} + y_{11} y_{22} R_i - y_{12} y_{21} R_i} = \frac{-y_{21}}{G_L(1 + R_i y_{11}) + y_{22} + R_i \Delta Y}$$

where ΔY = determinant of Y

another circuit with MOS

assume bias so upper M_1 is in saturation; M_2 is in saturation



assume $M_1 = M_2$

& Early effect small

find v_o at bias, $v_i = 0$

@ bias $\frac{d}{dt} \cong 0$ so ignore C for bias

$$I_{SD_1} = I_{SD_2} \Rightarrow \frac{KP}{2}\frac{W}{L}(V_{SG} - |VTO|)^2 (1+\lambda(V_{DD}-V_o))$$

$$\cancel{\frac{KP}{2}\frac{W}{L}}(V_{DD} - V_G - |VTO|)^2 (1+\lambda(V_{DD}-V_o)) = \cancel{\frac{KP}{2}\frac{W}{L}}(V_o - |VTO|)^2 (1+\lambda V_o)$$

$$\Downarrow$$

$$V_{DD} - V_G - |VTO| = V_o - |VTO|$$

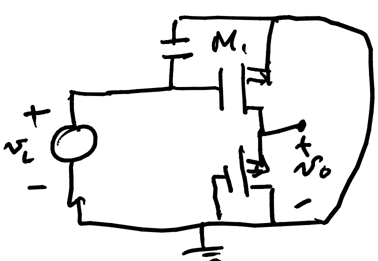
$$\Longrightarrow V_o = V_{DD} - V_G$$

true in the case of Early effect if $V_o = V_{DD} - V_o$ $\Rightarrow V_o = V_{DD}/2 \Rightarrow V_G = V_o = \frac{V_{DD}}{2}$

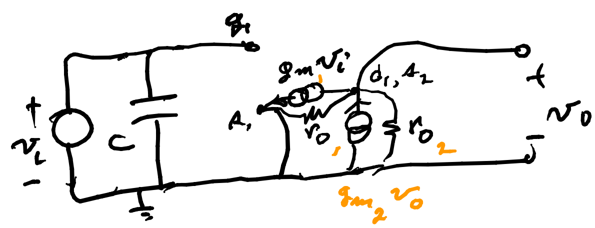
assume $V_G = \frac{V_{DD}}{2}$ & find g_m

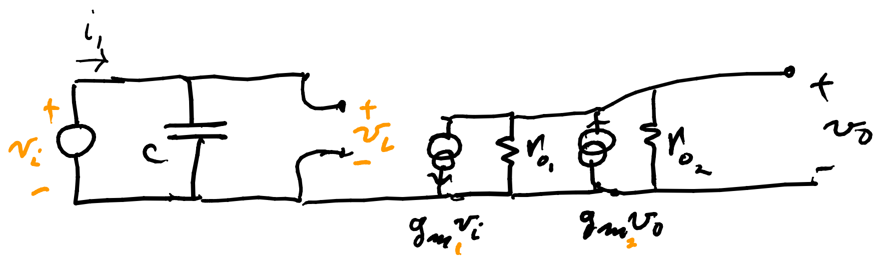
$$g_m = \frac{\partial I_S}{\partial v_{SG}} = \frac{KP}{2}\frac{W}{L} \cdot 2(V_{SG} - |VTO|) = 2\frac{I_S}{(V_{SG} - |VTO|)}$$

find small signal equivalent



$\Rightarrow$





$g_{m_1} = g_{m_2} = g_m$

$r_{o_1} = r_{o_2} = \frac{1}{g_o}$

2-port Y as seen by v_i & v_o: $y_{11} = sC$, $y_{12} = 0$, $y_{21} = g_m$, $y_{22} = 2g_o - g_{m_2} = 2g_o - g_m$

by inspection

using $y_{ik} = \left.\frac{i_i}{v_k}\right|_{v_j = 0}$ $\quad Y(s) = \begin{bmatrix} sC & 0 \\ g_m & 2g_o - g_m \end{bmatrix}$

$i, k \in \{1, 2\}$